NEWS ABOUT NEWS:

INFORMATION ARRIVAL AND IRREVERSIBLE INVESTMENT

by

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ABSTRACT

We analyze how uncertainty about *when* information about future returns to a project may be revealed affects investment. While "good news" about future returns boosts investment, "good news about news" (that is, news that information may arrive sooner) is shown to depress investment. We show that early revelation increases the value of an irreversible investment project to a risk-neutral investor. We relate our results on preference for early revelation to results in non-expected utility theory. Our framework allows us to study irreversible investment projects whose value has a time-variable volatility. We also consider how heterogeneity of revelation information across firms may induce a better-informed firm to share its information with competitors.

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1. Introduction

When decisions have an irreversible component, uncertainty about future outcomes plays a key role in the decision to commit to a course of action. Since it is costly to reverse a decision, waiting to commit until some of the uncertainty is resolved may yield benefits that more than outweigh the forgone short-run returns. The possible arrival of significant new information about outcomes thus can make the option of waiting to commit quite valuable.

If "news" about future possible outcomes is valuable when decisions are costly to reverse, then information about *when* such news might arrive ("news about news") should be valuable as well. The likelihood of receiving new information should affect the timing of irreversible decisions, as should changes in that likelihood, even if these changes convey no new information about what outcomes may be. Trading on asset exchanges, for example, often slows in anticipation of release of new economic data; political decisions are often delayed if it is believed that relevant new information will soon become available.

The most complete discussion of the implications of uncertainty about future returns when decisions are irreversible is in the theory of irreversible investment, where it is shown that the option value of waiting to invest may lead firms not to invest in projects which have a positive expected present discounted value. This literature demonstrates that uncertainty about future returns to a project may in itself depress investment, and that positive information about future returns will increase investment.[1]

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[1] Cukierman (1980) and Bernanke (1983) considered models in which the arrival of information makes future returns less uncertain, providing a channel for valuing the option to wait and gather more information. In McDonald and Siegel (1986), Pindyck (1988), and Bertola and Caballero (1994) among others, information arrives each period and updates the conditional distribution of future returns. An excellent treatment of much of this literature can be found in a recent book by Dixit and Pindyck (1994).

While the theory of irreversible investment yields a general framework for studying uncertainty about the value of an installed project, uncertainty about when information about outcomes may be revealed has not been treated explicitly. In this paper we present a model that separates the effect on investment of uncertainty about the value of an installed project into the effects of uncertainty about eventual returns ("outcome uncertainty") and the effects of uncertainty about when outcome uncertainty itself may be resolved ("revelation uncertainty").

It seems intuitive that a higher probability of knowing outcomes sooner, which could be characterized as "good news" about revelation, would decrease *current* investment as a firm waits to learn about outcomes. We show how our results on uncertainty about the arrival of new information are related to the time-varying volatility in the value of a new project. (In contrast, most of the literature has studied the impact of uncertainty on investment when the variance of value, or some underlying fundamental stays constant over time.) We further show that the higher (lower) variance of value at any period may delay (speed up) commitment. In fact, a common form of behavior under uncertainty -- wait a pre-specified length of time, then act if information has still not arrived -- is *inconsistent* with constant volatility of returns over time.

Since irreversibility implies a preference for early resolution of uncertainty in an expected utility framework, one may ask how our results relate to the Kreps-Porteus (1978) results on preference for early resolution of uncertainty, where a nonexpected utility framework is key. We show that formally irreversibility implies a convex "aggregator" function in an expected utility framework, making our results fully consistent with those in the nonexpected utility literature.

Though "good revelation news" can decrease investment it will, in general, increase the firm's value. The positive effect on firm valuation has interesting implications when many firms compete for a project under uncertainty. Suppose firms are asymmetrically informed in their knowledge about when further information may be revealed, so that the worse informed firm would commit immediately. (This difference could reflect differential ability to process signals and hence glean information.) Though a firm better informed about outcomes often has no incentive to share information on outcomes with a competitor, we show that a firm better informed about when news may arrive will share this information costlessly with the less well-informed firm.

Some examples may illustrate the relevance of our approach. Generally speaking, it is quite common for a decision to be postponed, at some cost, in the hope of getting more information. In such cases, a decision-maker will often plan to make a decision anyway by a certain date even if no new information has arrived. Though such a scenario seems to describe many decisions, we show that the standard model of irreversible investment with constant volatility over time cannot represent temporary postponement of this sort.

On a macroeconomic level, a leading class of examples would be in the news contained in government policy choices, since the process by which government decisions are made often gives rise to uncertainty about when information will be revealed. This process could be, for example, multilateral negotiations, or political events which affect the outcome of a reform process. Two recent examples may help illustrate how the nature of political decision making may engender revelation uncertainty, and, in turn, how such uncertainty may affect investment decisions: Mexico during the period of the NAFTA negotiations; and the experience of several Eastern European countries during the period of large-scale privatization and market reform. It will be evident that these examples involve a variance of returns that changes over time.

During the period of negotiation between 1991 and 1993, ratification of the North American Free Trade Agreement was seen as highly favorable to the profitability of doing business in

Mexico. During these two years, however, there was a good deal of uncertainty as to when negotiations and discussions in Congress would end and the outcome would be revealed. Sometimes, smooth negotiations indicated an early resolution of uncertainty, while at other times the process slowed down and it appeared that the outcome would not be known for a long while. How might changes in the speed of the process have affected investment in Mexico? (This is a separate issue from how investment is affected by a change in the probability of NAFTA ratification.)

After the collapse of communism, several countries in Eastern Europe undertook a large-scale effort to privatize state-owned enterprises, coupled with extensive market reform. Poland at the beginning of the 1990's is a good example. During this period Poland experienced a collapse in output, as well as a collapse in both domestic and foreign direct investment (FDI). One possible explanation for the collapse in FDI is that uncertainty about future returns to projects in Poland (outcome uncertainty) was so magnified that investors held off from any commitment until the outcome of the reforms was clearer. It is puzzling however that FDI in Poland did not pick up when the prospects for future returns began to look better. The explanation may be that whenever large-scale reforms begin to succeed, so that expected future returns increase, the rate at which information about future returns flows in also becomes faster. "Good news" about outcomes is accompanied by "good news" about revelation. If the effect of waiting for more information dominates the effect of newly acquired positive information, investment decreases.

The plan of the paper is as follows. In the next section we present a simple example to illustrate the connection between our work and a standard model of irreversible investment, and introduce time-varying volatility into this simple model in section three. In section four we present a model of the effects of uncertain time arrival of a single relevant piece of information.

Section five relates our results to those in the nonexpected utility literature. In section six we show how heterogeneity of information across firms affects our results. The final section contains conclusions.

2. A Simple Example

We begin with the simple two-period model of irreversible investment presented in chapter 2 of Dixit and Pindyck (1994). A risk-neutral firm can invest in a factory that produces one widget per year forever, with zero operating cost. The factory can be built instantaneously, at cost I and the investment is irreversible. The price of a widget is currently P_0, but will either rise to $1.5P_0$ (with probability ½) or fall to $0.5P_0$ (with probability ½) tomorrow, remaining at those levels forever after. (Note that the expected value of the future price as of today is P_0.) The firm has the option of delaying commitment to the project. Suppose the firm discounts future cash flows by a factor β. If the firm invests in the factory today, the expected net present value of the investment, which we call the value of *committing* to the investment, is

$$V_{\text{com}} = \frac{P_0}{1-\beta} - I \tag{1}$$

As in Dixit and Pindyck, assume that this net present value is positive (*i.e.*, $P_0/(1\text{-}\beta) > I$).

Nonetheless, it may not be optimal to commit to the investment today. The alternative is for the firm to remain uncommitted, deciding whether or not to build the factory after widget price uncertainty has been resolved. The option of waiting is valuable if in some states of nature, the firm would prefer not to have invested. In this example, this refers to the state in which the price falls to $0.5P_0$ in the second period. We assume that $0.5P_0/(1\text{-}\beta) < I$, so that it is not optimal to invest in the second period if the low price obtains. (If this inequality did not hold, the firm would never regret having committed to the investment.) Hence, if the firm waits till the second period, thus forgoing first-period revenues, it will invest only under the high price realization, so that the expected net present value of the investment as of the first period, which we call the value of remaining *uncommitted*, is

$$V_{\text{unc}} = \frac{1}{2}\beta\left(\frac{1.5P_0}{1-\beta} - I\right). \qquad \textbf{(2)}$$

The decision of whether it is optimal to commit to investment in the first period or to remain uncommitted depends on the relative values of the three parameters in the model: P_0, I, and β. Specifically, the firm will find it optimal to wait if $V_{\text{unc}} \geq V_{\text{com}}$, which by (1) and (2) becomes

$$\frac{P_0}{I} \leq \frac{(2-\beta)(2-2\beta)}{4-3\beta}. \qquad \textbf{(3)}$$

Equivalently, the firm will find it optimal to wait if the value of the option to wait, namely V_{unc} - V_{com} is positive.

Now suppose that with probability 1-π the price remains at P_0 tomorrow, whereas the price rises or falls by half its value, each with probability $\pi/2$. How will the possibility of no change in the price affect the decision of whether to commit to investment today? (The decision tree is represented by the part of Figure 1 describing only periods 0 and 1.) The net present value of the project if the firm commits to investment today is independent of π. The value of being uncommitted, however, is affected and becomes

$$V_{\text{unc}} = \frac{\pi}{2}\beta\left(\frac{1.5P_0}{1-\beta} - I\right) + (1-\pi)\beta\left(\frac{P_0}{1-\beta} - I\right). \tag{4}$$

Suppose that (3) holds with inequality, so that if the price were certain to change, it would be optimal for the firm to remain uncommitted. To derive a critical value of π, call it π^*, for which the firm is indifferent between committing and not committing, one equates (1) and (4) to obtain (after some manipulation):

$$\pi^* = \frac{1-\beta}{2\beta}\,\frac{\dfrac{P_0}{1-\beta} - I}{-\left(\dfrac{P_0/2}{1-\beta} - I\right)}. \tag{5}$$

The assumption that under the bad outcome, it is optimal not to invest (*i.e.*, $0.5P_0/(1\text{-}\beta) < I$), ensures that the denominator is positive, so that $\pi^* > 0$. When (3) holds with inequality, π^* is strictly less than one. For $\pi < \pi^*$, the firm will find it optimal to commit to investment today. Since the decision of whether or not to commit to investment today can be characterized, for given values of the other parameters, by the critical value of π^* relative to the actual π, equation (5) also illustrates Bernanke's (1983) "bad news principle". For a given value of committing to the project today (the numerator of the second fraction in (5)), it is the (absolute) value of the bad outcome that determines whether or not to wait. The same result may be obtained by calculating the value of the option to wait, as in the next paragraph.

One can also calculate the first-period value of the investment opportunity to the firm as a function of π, which is the project's net present value under the optimal strategy. If $\pi \leq \pi^*$, the value of the project is V_{com} , given by (1), while if $\pi > \pi^*$, the value of the project is V_{unc} , given by (4). Since (4) is greater than (1) for $\pi > \pi^*$ and since (4) is itself increasing in π, the value of the firm is weakly increasing in π. The value of the option to wait is the maximum of zero and V_{unc} - V_{com} , so the option will have positive value when $\pi > \pi^*$ and will be increasing in π. Hence an increase in π will increase the value of the firm but will decrease (if it crosses the critical threshold π^*) current investment. This is not surprising. An increase in π is a mean-preserving spread of the distribution of future returns. Specifically, the variance of second-period returns is $\pi \dfrac{P_0^2 \beta^2}{4(1-\beta)^2}$. Higher uncertainty about future returns increases the value of the firm as it decreases investment.

3. Time-Varying Volatility

Having studied a two-period model in which high variance in returns may create an incentive to wait before committing to an irreversible project, we now consider a three-period model in which information arrives in the second and third periods, $t = 1$ and $t = 2$. This setup will allow us to study irreversible investment when the volatility of returns may vary over time. In contrast, in the standard framework, as represented by the discrete-time models in Dixit and Pindyck (1994) or the continuous-time models with geometric Brownian motion (as in McDonald and Siegel [1986]), the instantaneous variance is constant.

As above, a risk-neutral firm can invest in a factory that produces one widget per year forever, with zero operating cost, where the investment cost, I, is sunk. The initial price of a widget, as before, is P_0, and may rise to $(1 + u)P_0$ in the second period with probability $\pi/2$ or fall to $(1 - d)P_0$ with probability $\pi/2$, while with probability $1-\pi$, it stays the same. In the third period, the price may rise to $(1 + u)P_1$ with probability $\rho/2$ or fall to $(1 - d)P_1$ with probability

$\rho/2$, while with probability 1-ρ, it remains equal to P_1. The probabilities π and ρ may be unequal, implying time-varying volatility of returns. To see this, note that the variance of the value of the installed project (where $u = d$) is $\sigma^2 = \pi(u\beta P_0/(1\text{-}\beta))^2$ at t =0 and is $\sigma^2 = \rho(u\beta P_1/(1\text{-}\beta))^2$ at t =1. The decision tree is represented in Figure 1, where the middle node in each period represents the event of no change in price.

For given values of the four parameters, P_0/I, u, d, and β, we can calculate combinations of π and ρ such that the firm is indifferent between committing to investment in the first period or remaining uncommitted. More generally, we could think of an indifference "surface" in these six parameters. A higher value of either of the probabilities π or ρ, holding the remaining five parameters constant implies that the firm prefers to remain uncommitted. One way to solve the problem is by backward induction, which we employ to produce such a surface in π and ρ, the solid line in Figure 2. (The values for the remaining parameters are $P_0/I = 0.15$, $u = d = ½$, and β = .91 .) In this figure, combinations of π and ρ below the solid curve imply that the firm commits to the project at $t = 0$. Note that these could involve situations in which the variance is high in the first period but low in the second, or *vice versa*. In the case that π and ρ lie above the curve *and* there is no change in the price between the first and the second period, the firm will commit at t = 1 for $\rho < \rho^* = 0.19$, represented by the dashed line in Figure 2.

Let us first consider the standard model of constant volatility of returns over time, which corresponds to points along the diagonal, where $\pi = \rho$. A standard result from the literature is that higher uncertainty (*i.e.*, a higher σ) implies a higher value to remaining uncommitted. Hence, using the parameter values of the figure, a firm which would be indifferent at $\pi = \rho = 0.55$, would prefer to remain uncommitted at $t = 0$ for any value above 0.55. Note that if there is no price change between the two periods (that is, $P_0 = P_1$), constant volatility of returns implies that the firm chooses either to commit at $t = 0$ or wait until $t = 2$, after which time no more information will be revealed. This characteristic can be shown to be more general: *with constant volatility of returns a firm which finds it optimal not to commit in the beginning will remain uncommitted as long as the price is unchanged but may change in the future*.

In contrast, consider allowing the volatility of returns to vary over time, corresponding to points off the diagonal, where $\pi \neq \rho$. As the solid-line indifference surface in the figure makes clear, higher variance of returns in the first period (corresponding to $\pi > 0.55$) can be offset by lower variance in the second period (corresponding to $\rho < 0.55$). Committing at $t = 1$ (corresponding to combinations of π and ρ in the northwest part of the figure) is possible even if there has been no change in price.

4. Uncertainty about the Time of Arrival of Information

In the above model, information arrived in each period, but the volatility of returns was allowed to vary over time, in contrast to the standard model with constant volatility. In many situations involving irreversible investment, there is a *single* important piece of information, which may be revealed at an unknown point in time. Prior to the revelation of this piece of information, uncertainty about returns is large; subsequent to its revelation this uncertainty is significantly reduced (or perhaps eliminated).

Toward this end, we consider a multiperiod model of optimal timing of irreversible investment in a single risky asset. The asset yields a known return r each period until a known time $\bar{T}$, after which it yields a net return of either R^h with probability p or R^l with probability 1-p. Assume $R^h > 0 > R^l$ and $pR^h + (1-p)R^l > 0$. A risk-neutral firm discounts returns by a factor β per period. The return to no investment is normalized to zero.[2] Conceptually, the return to the risky asset will be affected by the realization of some future event which is known to occur at time $\bar{T}$,[3] where the realization may be known with certainty at some time T before $\bar{T}$.[4] (One may call $\bar{T}$ the "outcome date" and T the "revelation date.") Uncertainty about the value of an installed project stems from two sources: uncertainty about the eventual returns to the installed project (outcome

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[2] This modeling is equivalent to existence of a second asset whose return is riskless and investment is reversible, rather than irreversible. In this case the return to the risky asset is defined as the excess over the return to the safe asset.

[3] An example of such an event would be an election, where the new party takes office on a given day, but where the election's outcome may be known beforehand; or, a possible policy change scheduled to take effect on a given date, where it is known well beforehand whether or not it will take place.

[4] In this paper we assume the outcome date $\bar{T}$ is known and concentrate on uncertainty about the timing of revelation. We hold $\bar{T}$ independent of T so that changes in the distribution of T can be seen as pure changes in revelation uncertainty, not affecting the stream of returns. Alternatively, one can concentrate on uncertainty about the outcome date, with no possibility of early revelation, as in Drazen and Helpman (1990) and Calvo and Drazen (1994).

uncertainty) and uncertainty about when information about outcomes may be revealed (revelation uncertainty). The return structure of the currently risky asset is time-invariant, so that the probability that information will be revealed in each period will depend on time, but the nature of the information that will be revealed does not change. We further assume that bad outcomes matter, in the sense of ruling out parameter values such that investment is undertaken immediately even though it is known that the bad outcome will occur.[5]

The revelation date T ($\leq \bar{T}$) (the first date at which the outcome will be known with certainty) is stochastic, with a subjective probability distribution represented by the cumulative distribution function H(T). The distribution H(T) implies probabilities of revelation in each period, conditional on uncertainty not having been previously resolved. Looking at the problem from period 0, the probability π_t of uncertainty being resolved in period t, conditional on no previous resolution, is

$$\pi_t = \frac{h(t)}{1 - H(t-1)} \quad . \tag{6}$$

The timing of decisions and events if uncertainty has not been resolved and the firm has not committed to investment is as follows. At the beginning of period t, the firm decides whether to commit to investment in the risky asset, or remain uncommitted. If it commits (irreversibly) the firm earns *r* in every period from *t* to $\bar{T}$-1 and R^i at $\bar{T}$. Uncertainty is then resolved with probability π_t. If the firm is uncommitted and the news is good ($R^i = R^h$ with probability *p*), the

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[5] Formally, we assume that for t= 0, $\sum_{s=0}^{s=\bar{T}-t-1} \beta^s r + \beta^{\bar{T}-t} R^l < 0$. Note that the condition being satisfied at t = 0 implies that it will be satisfied for all t > 0, since $\beta < 1$. Intuitively, the condition is that the discounted flow of returns until $\bar{T}$ cannot be so high as to offset a certain bad outcome.

firm undertakes the project at *t*+1. However, if the news is bad, the firm decides never to undertake the project. Because returns are stationary and the distribution H(T) is known , the firm's decision may be described as choosing a date T^* to commit conditional on uncertainty not having been previously resolved. Equivalently, the firm chooses a maximum number of periods to wait before committing to investment.

One way to find the optimal solution is first to calculate the expected value as of t of waiting j periods to commit to investment in excess of the expected value of committing at t. Call this excess value $V_{t+j}(t)$. The optimal length of time to wait before committing to investment is then found by simply choosing the maximum value $V_{t+j*}(t)$ in the set $\{V_{t+j}(t)\}$ and waiting j^* periods to invest. We formalize this as

PROPOSITION 1: The optimal waiting time is given by

$$T^* = \underset{0 \le j \le \overline{T}}{argmax} \quad \{V_j(0)\} \tag{7}$$

(The proof of all propositions is in the Appendix.) An implication of Proposition 1 is

COROLLARY 1: A necessary and sufficient condition for it to be optimal to postpone investment at time 0 is that some $V_j(0)$ (j>0) be positive.

We now derive some basic results on the optimal length of time to wait to invest as a function of the π_i. A first result is that a higher likelihood of knowing early makes waiting more attractive.

PROPOSITION 2 : An increase in the probability of revelation in any future period I will increase the value of waiting to invest at least I periods, that is, $\partial V_j(0)/\partial \pi_i > 0$, for all j $\ge$ I.

Note that in contrast to the effect of good outcome news, "good" news about revelation (in the sense of uncertainty being resolved sooner) may depress current investment and can never increase it. We formalize this result as

PROPOSITION 3: Suppose that the optimal action is to invest immediately. An increase in revelation probabilities can lead to a postponement of investment; an increase in the probability of the good outcome cannot.

More generally, one can characterize the optimal length of postponement in terms of revelation probabilities as follows: *Postpone investment as long as the probability of revelation in some future period is "sufficiently" high. A necessary condition for the optimality of investing at time T^* is that all future revelation probabilities are "sufficiently" low.*

In the previous model, in which information arrived each period, constant volatility of returns implied that a firm would either commit immediately or wait until there was no uncertainty, if price was unchanged. There is an intuitive analogue to this in this model. If the revelation probability, π_i, is the same in each period, then the firm either commits immediately or waits until all information has been revealed.[6]

The results we have presented up to now have stressed that earlier revelation of uncertainty makes initial investment less likely by raising the value of the option to wait. However, a more interesting question is whether early revelation depresses investment. Is the accumulation of capital higher or lower, in the sense of the number of projects being undertaken *ex post*, when information is more likely to be revealed earlier? Since our model is one of irreversible investment in a single project of exogenous size, the best measure to address this issue is the probability that the project will be undertaken over the firm's horizon.

Let us denote the probability that a project will be undertaken as $P(I)$. When $T^* = 0$, then $P(I) = 1$. However, when the optimal decision is to postpone investment, that is, when $T^* > 0$, the probability of investment is less than unity. The probability that the project will not be undertaken

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[6] Formally, one can show that the critical value of π such that the firm is indifferent is monotonically non-increasing over time.

is equal to the probability that there is revelation before the decision date (call this probability $\Lambda(T^*)$) multiplied by the probability that the revelation is bad $(1 - p)$. Thus $P(I) = 1 - (1 - p)\Lambda(T^*)$. One can show that the probability of investment depends (strictly) negatively on π_k for $k < T^*$ and (weakly) negatively on π_k for $k \geq T^*$. Thus, early revelation that leads the firm to postpone undertaking a project reduces the probability of investment because postponement allows the firm the possibility of learning that the project will be loss-making.

It is instructive to establish the relation between changes in revelation uncertainty and the variability of the value of an installed project. In a discrete time framework it is convenient to characterize the variability of installed value V_t in terms of the one-period-ahead variance $\sigma_t^2 = E_{t-1}(V_t - E_{t-1}V_t)^2$. In our model, this variance is $\beta^{2(\overline{T}-t)}\pi_t \mathrm{var}(R)$, if there has not been revelation before t, and zero otherwise, where $\mathrm{var}(R) = p(R^h)^2 + (1 - p)(R^l)^2 - [pR^h + (1 - p)R^l]^2$ is the variance of returns. For simplicity of exposition, we set $\beta = 1$ for the rest of the discussion. The variance of the value is affected by revelation uncertainty (through π_t) and by outcome uncertainty (through $\mathrm{var}(R)$). It is clear that this variance is variable over time as long as the revelation probabilities are themselves time-varying. Good revelation news through an increase in the π_t increases the (one-period ahead) variance of installed value at time t while leaving all other variances the same. On the other hand, good outcome news through a decrease in $\mathrm{var}(R)$ decreases the variance of installed value in *all* time periods by the same proportion. The optimal length of postponement of investment can be characterized in terms of variances: *Postpone investment as long as the variance of installed value in some future period is "sufficiently" high. A necessary condition for the optimality of investing at time T^* is that all subsequent variances are "sufficiently" low*.

So far we have considered the effect of revelation uncertainty on the decision to undertake or postpone investment, a key result being that "good" revelation news can lead to a fall both in current investment and in expected investment over the long horizon. We end this section with an observation on the effect of revelation news on the value of the option to postpone commitment. While "good" revelation news may reduce investment, it will always increase the value of the project.[7]

5. The Preference for Early Revelation

Since the value of the project increases with π_t, the firm has a preference for early resolution of uncertainty. How can this be reconciled with the result that expected utility maximizers are indifferent to the timing of resolution of uncertainty? (Our firm here is risk-neutral.) Irreversibility must be key to the result.

Consider first the case where investment is fully reversible, so that the decision at time 0 as to whether or not to commit to investment would not constrain the state at time 1. Let us denote by V_t the value associated with being uncommitted at the beginning of period t, and by $f(A_0)$ the current flow return from either committing or not committing ($A_0 = \bar{c}, \bar{n}$), where committing today implies a price today and an expected price tomorrow of P_0. We could then write the firm's period zero maximization problem as

$$V_0 = \max_{A_0} \{f(A_0) + \beta E_0 V_1\} \quad \textbf{(8)}$$

[7] This follows from proposition 2, on observing that the option value is $\max_{0 \le j \le \bar{T}} V_j(0)$.

where E_0 is the expectation as of time 0. Since the second term is independent of A_0 , (8) may be written

$$V_0 = \max_{A_0} f(A_0) + \beta E_0 V_1 . \tag{8'}$$

In the case where investment is irreversible, so that $A_1 = \bar{c}$ if $A_0 = \bar{c}$, the maximization problem if the firm is uncommitted at time 0 may be written

$$V_0 = \max_{A_0} \{f(A_0) + \beta (E_0 W_1)\} \quad . \tag{9}$$

Next period's expected utility, $E_0 W_1$, equals $E_0 V_1$ if the firm chooses to remain uncommitted at time 0 or $E_0(P_1)/(1\text{-}\beta)$ if the firm commits to investment at time 0. Equation (9) may then be written

$$\begin{aligned} V_0 &= \max \{P_0 - I + \beta \frac{E_0(P_1)}{1 - \beta}, 0 + \beta E_0 V_1\} \\ &= P_0 - I + \beta \max \{\frac{E_0(P_1)}{1 - \beta}, -\frac{P_0 - I}{\beta} + E_0 V_1\} \end{aligned} \tag{9'}$$

From (8) and (9), we see that there is a linear relation between current and future expected utility whether investment decisions are reversible or not, as must be the case for von Neumann-Morgenstern preferences. However, comparing (8') and (9'), we see that while future utility is linear in $E_0 V_1$ and hence in underlying period 0 randomness when investment is reversible, it is convex in $E_0 V_1$ and hence randomness as seen from period zero in the irreversible case. Hence the preference for early resolution of uncertainty can be related to an inherent convexity in utility aggregation, as in nonexpected utility preferences (see Kreps and Porteus [1978] or Epstein and Zin [1991]), but here it arises under expected utility maximization due to irreversibility.

6. Many Firms

So far we have implicitly assumed that a single firm has sole access to the project. In many cases, however, an investment or project may be available to more than one potential investor. For an individual firm this means that there is some probability that by waiting, the opportunity to invest in a future period will be lost, implying an incentive to commit earlier. In this section we enrich the framework in order to study the interaction between the possibility of early revelation and of investment being pre-empted. We show that the possibility of being pre-empted implies not only that the firm may commit earlier, but also that a firm with a superior ability to process information and hence benefit from early revelation will find it optimal to share its information *costlessly* with a firm with an inferior information processing ability.

To make these ideas more precise, suppose that at the beginning of each period t, there is an exogenous probability $1-\theta_t$ that the investment opportunity will disappear if the firm remains uncommitted. (If the firm has committed earlier, the investment is "locked in" and cannot disappear.) If the investment opportunity is still available (this occurring with probability θ_t), there is a probability π_t that uncertainty will be resolved. One can then show that the "disappearance" probabilities θ_t have a similar effect to revelation probabilities π_t, as follows.

PROPOSITION 4: A decrease in any of the survival probabilities θ_i will decrease the value of waiting to invest at least I periods and will decrease (or leave unchanged) the optimal waiting time.

If the firm takes the θ_t as parametric and exogenous to its decisions, then it is irrelevant for its decisions whether the disappearance of the investment possibility comes from an act of nature or from a competitor grabbing the project; in either case the result in Proposition 4 will hold. In this sense, under the assumption that θ_t is taken as exogenous, the simple framework presented above can capture the interaction of many firms competing for the same project.

In the case where the firm takes account of the influence its own actions may have on θ_t, the analysis is more complicated. Each firm will take account of other firms' strategies in deciding when to commit. In a two-player game, for example, one can derive optimal strategies and the critical levels of the π_i consistent with commitment not taking place immediately[8], but that is not our interest here. Rather, we want to point out an interesting implication of heterogeneity in information, namely that *a firm with a better ability to process information may find it optimal to share some of its information costlessly with a less well-informed firm*.

Suppose that two firms are asymmetric in their ability to *process* information in the following sense: some events or pieces of information that would reveal the ultimate outcome to the first firm will not reveal it to the second. Formally, the first firm (which has better ability to process information, or firm B for short) perceives higher revelation probabilities $\{\pi_i\}$ than the second (which has worse information processing ability, or firm W for short). We argue that if there is a possibility that firm B may be pre-empted by firm W, it will want to share its ability to process information with its competitor to induce the competitor not to commit to investment. (When both firms move simultaneously, they split the returns from the project.)

To make this more specific, suppose that firm B perceives a high enough chance of early revelation that it is optimal for it to wait (say, until period j). Its competitor, firm W, perceives such a low chance of early revelation, that it would invest immediately. In other words, firm W's

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[8] Assume that if both firms move simultaneously the returns will be split, but if one firm moves first and pre-empts, it gets the entire project. Then the critical value of the $\{\pi_i\}$ must be larger than in the case of a sole potential investor in order to make it optimal to wait.

optimal behavior given low revelation probabilities implies that firm B faces $\theta_1 = 0$. Firm B would therefore find it optimal to choose to commit immediately as well, and they would split the expected value of the project at time 0. If firm B can induce firm W to wait until j, it can do no worse than split the project at j. Since the expected value of waiting till j exceeds that of committing at 0, it will be optimal for firm B to try to induce firm W to wait. It could do this by sharing its knowledge on how to process information (that is, how to learn about early revelation), thus raising the $\{\pi_i\}$ that firm W perceives. Hence, costlessly sharing its ability to process information may be welfare improving for a firm.

7. Conclusions

In this paper we have presented a basic framework for investigating the effect of the rate at which new information is expected to arrive on irreversible investment decisions; our model separates the effect on investment of total uncertainty about the value of an installed project into the effects of uncertainty about eventual returns (outcome uncertainty) and revelation uncertainty. Some of our conclusions are straightforward, others less so. As was argued in the Introduction, if an event conveys good news both about the possibility of early revelation and about outcomes, the net effect on investment will depend on which effect dominates. This appears quite relevant in understanding investment dynamics during a multi-stage reform program, in which good progress at one stage suggests not only better ultimate outcomes, but also that residual uncertainty will be resolved faster. Many economies currently undergoing long and difficult transitions, with investment remaining low in spite of what appear to be large profit opportunities. A crucial step in understanding such transitions is a framework to analyze how investment is affected by when it is known whether the transition will be successful.

More generally, an implication of this paper is that investors benefit from earlier resolution of uncertainty. As in the case of economic transitions, it is unavoidable that the political process creates uncertainty about when important information will arrive. Nonetheless, government policy should attempt to do nothing which needlessly increases this uncertainty, or increases the information differential between firms.

APPENDIX

PROOF OF PROPOSITION 1: Strategies as of time 0 are: commit to investment immediately, wait one period to commit to investment, wait two periods to commit, etc. The optimal strategy is the one yielding the highest expected value. The expected return to each of these strategies corresponds to the associated value $V_j(0)$, so that the optimal strategy is to wait $T^* = argmax_j V_j(0)$ periods. □

Before proving Proposition 2, we need to specify $V_j(t)$ in terms of the parameters of the model. Let us denote by $A_{t+i}(t)$ the expected gain (as seen from t) of waiting from period $t+i-1$ to period $t+i$. This gain is discounted to period $t+i-1$. Then, $V_{t+j}(t)$ will be the present discounted value of the $A_{t+i}(t)$ from $i = 1$ to j, with each term also being multiplied by the probability of reaching that date with no resolution of uncertainty. With no resolution of uncertainty at t, we have

$$V_{t+j}(t) = A_{t+1}(t) + \beta(1-\pi_{t+1})A_{t+2}(t) + \ldots + \beta^{j-1}(1-\pi_{t+1})\ldots(1-\pi_{t+j-1})A_{t+j}(t) \qquad \text{(A.1)}$$

where $V_t(t) = 0$. The above equation implies a simple relation between the $V_{t+j}(t)$ in different time periods of the form

$$V_{t+j}(t) = V_{t+j-m}(t) + \beta^{j-m}\left[\prod_{s=t+1}^{t+j-m}(1-\pi_s)\right] V_{t+j}(t+j-m).$$

This says that the value of waiting until period $t+j$ may be thought of as the value of waiting until period $t+j-m$ plus the value of waiting another m periods. Thus, any $V_{T^*+k}(0)$ may be thought of as the value of waiting until T^* plus the value of waiting another k periods. If $V_{T^*}(0)$ is maximum, the value of waiting any longer once T^* has been reached must be negative.

Now we provide an expression for $A_i(t)$. Define $Q^U(t)$ as the expected return (as of $t-1$) from time t to $\bar{T}$ if uncertainty is resolved at t and the firm has not committed itself to investment before t. Define $Q^C(t)$ as the expected return (also as of $t-1$) from time t to $\bar{T}$ if uncertainty is resolved at t and the firm already has committed itself to investment. Both $Q^U(t)$ and $Q^C(t)$ are discounted to time t and may be written as

$$Q^U(t) = p\left[\sum_{s=t}^{\bar{T}-1} \beta^{(s-t)} r + \beta^{(\bar{T}-t)} R^h\right] \tag{A.2}$$

$$Q^C(t) = \sum_{s=t}^{\bar{T}-1} \beta^{(s-t)} r + \beta^{(\bar{T}-t)} \left[pR^h + (1-p)R^l\right] \tag{A.3}$$

where we have used the fact that if the firm has not committed prior to t, it will invest only if the realization is R^h and obtain a present value as of $\bar{T}$ of R^h, while if it has committed, the expected present discounted value of returns at $\bar{T}$ is $[pR^h + (1-p)R^l]$.

The net expected gain from waiting to invest until period $t+i$ rather than investing in period $t+i-1$, that is, $A_{t+i}(t)$, will be the excess of $Q^U(t+i)$ over $Q^C(t+i)$ multiplied by both the discount rate, β, and the probability of uncertainty being resolved in $t+i$ (i.e. π_{t+i}), net of the return r. We have, then,

$$A_{t+i}(t) = \beta\pi_{t+i}\left(Q^U(t+i) - Q^C(t+i)\right) - r, \tag{A.4}$$

which allows us to calculate each of the $V_j(t)$ in terms of underlying parameters. The excess value of remaining uncommitted, $Q^U(t+1) - Q^C(t+1)$, will be positive under the reasonable assumption that the bad outcome occurring with certainty leads to no investment.

PROOF OF PROPOSITION 2: Using equations (A.1), (A.4) and differentiating with respect to π_i we obtain:

$$\begin{aligned}
\frac{\partial V_j(0)}{\partial \pi_i} \quad = \quad & \beta^i(1-\pi_1)\ldots(1-\pi_{i-1})Q^U(i) \\
& -\beta^{i+1}(1-\pi_1)\ldots(1-\pi_{i-1})\pi_{i+1}Q^U(i+1) - \ldots \\
& -\beta^j(1-\pi_1)\ldots(1-\pi_{i-1})(1-\pi_{i+1})\ldots(1-\pi_{j-1})\pi_j Q^U(j) \\
& -\beta^j(1-\pi_1)\ldots(1-\pi_{i-1})(1-\pi_{i+1})\ldots(1-\pi_j)Q^C(j).
\end{aligned}$$

When $i = j$ this reduces to $\partial V_j(0)/\partial \pi_j = \beta^j(1-\pi_1)\ldots(1-\pi_{j-1})[Q^U(j)-Q^C(j)] > 0$. When $i < j$ we simplify the expression for $\partial V_j(0)/\partial \pi_i$ by making repeated use of the relationships:

$$\begin{aligned}\beta Q^U(k) &= Q^U(k-1) - pr\\ \beta Q^C(k) &= Q^C(k-1) - r,\end{aligned}$$

for $k = i+1, \ldots, j$. This leads to

$$\frac{\partial V_j(0)}{\partial \pi_i} = \beta^i(1-\pi_1)\ldots(1-\pi_{i-1})(1-\pi_{i+1})\ldots(1-\pi_j)[Q^U(i) - Q^C(i)] + r\Omega,$$

where Ω is a positive constant and $Q^U(i) > Q^C(i)$ (from (A.2) and (A.3)). □

PROOF OF PROPOSITION 3: The first part follows from Proposition 2 since an increase in π_i raises the excess value of waiting. To show the second part we differentiate equation (A.1) with respect to p and obtain $\partial V_j(0)/\partial p = \partial A_1(0)/\partial p + \beta(1-\pi_1)\partial A_2(0)/\partial p + \ldots$. Note that $\partial A_j(0)/\partial p = \beta\pi_j[Q^U(j) - Q^C(j)]/(p-1) < 0$. This implies that $\partial V_j(0)/\partial p < 0$. □

When there are many firms considering the project, as in section 6, the possibility of the opportunity disappearing, with probability $1 - \theta_{t+i}$, reduces the expected gain of waiting from $t+i-1$ to $t+i$, $A_{t+i}(t)$. Then,

$$A_{t+i}(t) = \theta_{t+i}\beta\pi_{t+i}\left(Q^U(t+i) - Q^C(t+i)\right) - (1-\theta_{t+i})\beta Q^C(t+i) - r.$$

The $V_{t+j}(t)$ are, then, formed as a discounted sum of the $A_{t+i}(t)$, where the terms are also discounted by the probability that the opportunity is still available at each date:

$$V_{t+j}(t) = A_{t+1}(t) + \beta\theta_{t+1}(1-\pi_{t+1})A_{t+2}(t) + \ldots + \beta^{j-1}\theta_{t+1}\ldots\theta_{t+j-1}(1-\pi_{t+1})\ldots(1-\pi_{t+j-1})A_{t+j}(t).$$

PROOF OF PROPOSITION 4: The gain from waiting to invest may be written as

$$V_{t+j}(t) = \theta_{t+1}\beta\pi_{t+1}Q^U(t+1) + \theta_{t+1}\theta_{t+2}\beta^2(1-\pi_{t+1})\pi_{t+2}Q^U(t+2) + \ldots$$

$$+\beta[\theta_{t+1}(1-\pi_{t+1})-1]Q^C(t+1)+\beta^2[\theta_{t+1}\theta_{t+2}(1-\pi_{t+1})(1-\pi_{t+2})-1]Q^C(t+2)+\ldots$$

$$-r-\beta r-\ldots.$$

Note that $Q^U(i)>0, Q^C(i)>0, 0<\theta_i<1$, and $0<\pi_i<1$ for all i. Differentiating this expression with respect to any of the θ_i (where $i>t$) immediately implies that $\partial V_{t+j}(t)/\partial\theta_i>0$ and that $|\partial V_{t+k}(t)/\partial\theta_i|<|\partial V_{t+h}(t)/\partial\theta_i|$ for $k<h$. These two results imply that a decrease in θ_i will cause the $V_{t+j}(t)$ to fall and that the value of j for which V_{t+j} is maximized will remain the same or fall. □

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