

# Discrete Clock Auctions: An Experimental Study

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## *Abstract*

We analyze the implications of different pricing rules in discrete clock auctions. The two most common pricing rules are highest-rejected bid (HRB) and lowest-accepted bid (LAB). Under HRB, the winners pay the *lowest* price that clears the market; under LAB, the winners pay the *highest* price that clears the market. In theory, both the HRB and LAB auctions maximize revenues and are fully efficient in our setting. Our experimental results indicate that the LAB auction achieves higher revenues. This revenue result may explain the frequent use of LAB pricing. On the other hand, HRB is successful in eliciting true values of the bidders both theoretically and experimentally.

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# 1 Introduction

A common method to auction diamonds, radio spectrum, electricity, gas, and other products is the discrete clock auction. The auctioneer names a price and each bidder responds with her desired quantity. If there is excess demand, the auctioneer then names a higher price. The process continues until there is no excess demand. This format is used for auctioning one or more products, where each product is either a single lot or many identical lots.

The clock auction is best thought of as a dynamic version of a sealed-bid uniform-price auction. In the uniform-price auction, the auctioneer collects a demand curve for each product from each bidder, forms the aggregate demand curve, and crosses it with the supply curve to determine the market clearing price and the quantity won by each bidder. The clock auction does the same thing, but gathers the demand curves from each bidder in a sequence of discrete rounds, and bidders receive information about excess demand at the end of each round. Indeed, the sealed-bid uniform-price auction is just a single-round clock auction. In both clock auctions and uniform-price auctions, two pricing rules are commonly used: highest-rejected bid (HRB) and lowest-accepted bid (LAB).

The LAB rule is used in treasury auctions in the US and elsewhere. These are sealed-bid uniform-price auctions. In contrast, clock auctions for diamonds, electricity and gas products in Europe have used the HRB rule.

Discrete rounds are used in practice to simplify communication, make the process robust to communication failures, and mitigate tacit collusion (Ausubel and Cramton 2004). An implication of discrete rounds is that the pricing rule matters. Another issue is whether the bidder specifies exit bids—the prices at which the bidder reduces the quantity demanded. In the limit as the size of the bid increment goes to zero, the distinction between pricing rules is irrelevant and exit bids are unnecessary. However, in practical auctions where the number of rounds often ranges from 4 to 10, discreteness matters. For example, in BHP Billiton’s diamond auctions, to limit participation costs the auctions are scheduled to complete in a half-day, which entails about 4 to 10 comfortably paced rounds (Cramton et al. 2010). Similarly, medium-term electricity and gas auctions are scheduled to complete in one day to avoid overnight positions. These auctions

typically last 6 to 12 rounds (Ausubel and Cramton 2004). In these settings, best practice is to have the bidder specify exit prices at which quantity reductions are desired. Exit bids make the discrete round auction much closer to a continuous auction and yet all the benefits of a discrete clock auction are retained. Exit bids are used in virtually all the major clock auctions in practice: diamonds, electricity, gas, and emission allowances (Ausubel and Cramton 2004, 2010; Cramton et al. 2010).

In this paper, we examine bidding behavior in HRB and LAB versions of discrete clock auctions. In each version, after each round the bidders learn the aggregate demand. To prevent bid-sniping, an activity rule requires that a bidder's quantity demanded cannot increase at higher prices. Bidders can only maintain or reduce quantity as the price rises. With many products, the simplest implementation is to require this monotonicity constraint for each product, as is done in diamond auctions. If the bidder reduces quantity in a round, the bidder names a price for each quantity reduction. The price of each reduction must be greater than the prior price and less than or equal to the current price. Each exit price is interpreted as the price at which the bidder is indifferent between the higher quantity and the lower quantity. If there is no excess demand at the current price, the supply is awarded to the highest bidders. The two versions differ in the pricing rule. Each winner pays the highest-rejected bid in HRB and the lowest accepted bid in LAB for the quantity won.

At first glance, it would seem that selecting the highest market clearing price (LAB) would result in greater revenue than selecting the lowest market clearing price (HRB). The argument is incomplete, since the pricing rule influences behavior. LAB introduces an incentive for shading one's bid below value. In simple cases (assuming one-lot per product, symmetry, independence, and risk neutrality), the bid shading under LAB exactly offsets the revenue gain from selecting the higher clearing price. Revenue equivalence obtains, and the two pricing rules result in the same expected revenue—at least in theory when bidders seek to maximize profits.

The purpose of this paper is to examine the bidding behavior and outcomes, especially efficiency and revenue, under the two pricing rules experimentally. Our main hypothesis is that subjects will overbid under lowest-accepted bid, consistent with the first-price sealed-bid auction; whereas, under highest-rejected bid, bidders will bid truthfully. Thus, revenues under

LAB will be higher than revenues under HRB. However, the efficiency comparison may go either way depending on how heterogeneous the bidding behavior in LAB is. Although the formats apply to the general case of auctioning many lots of multiple products, for simplicity we restrict attention to the case of auctioning a single item. This simple case is important in practice. For example, diamond auctions often sell large stones as individual lots, each with its own price clock. Many lots are sold simultaneously, but each lot is cleared separately and monotonicity of demand is enforced lot-by-lot.

It is a robust and puzzling finding of the experimental literature that in second-price sealed-bid auctions subjects bid more than their value although they bid truthfully in its dynamic counterpart, the English auction (Cooper and Fang 2008; Coppinger et al. 1980; Kagel et al. 1987, Kagel and Levin 1993). The HRB auction is somewhere between the English and second-price auctions and these two are the limit cases of HRB. HRB converges to the English auction (continuous clock) when the number of rounds approaches infinity. When there is only one round, HRB is the same as the second-price auction. The exit bid decision in HRB requires similar strategic thinking as in a second-price sealed-bid auction. On the other hand, whether to stay in the auction is the same binary decision as in the English auction. Harstad (2000) finds that experience with the English auction leads to less overbidding in second-price auctions. Moreover, he argues that the binary choice aspect of English auctions explains an important part of this learning. The HRB format allows subjects to make that kind of binary choice for early price increments and when they come to the price interval where they want to exit, they have this experience. In this respect, studying discrete clock auctions helps us understand what makes subjects overbid in second-price auctions and bid truthfully in English auctions.

The LAB pricing rule is analogous to a first-price sealed-bid auction. In particular, if there is only one round and one good, the LAB and first-price auctions are the same. Another robust finding of the experimental literature is that bidders in a first-price auction overbid compared to the risk-neutral Nash equilibrium prediction (see Cox, et al. 1982, 1988, as the seminal papers;

Kagel and Levin 2008 for a detailed survey).<sup>2</sup> Therefore, we expect to see overbidding in the LAB auction.

Ausubel et al. (2009) and Kagel and Levin (2001) are the most related papers to our work. Ausubel et al. (2009) compares a particular discrete clock with the sealed-bid auction in common value settings. The study considers reverse auctions where the bidders have liquidity needs. It tests whether dynamic or sealed bid formats lead to better price discovery (of the commonly valued goods) and identifies the format that allows bidders to satisfy the liquidity needs of the bidders. Kagel and Levin (2001) uses a continuous clock auction in a multi-unit setting; the aim of the study is to understand demand reduction in sealed-bid uniform-price and continuous clock auctions experimentally. Our study aims to understand the implications of different pricing rules in discrete clock auctions in private value settings.<sup>3</sup> No other paper to date makes this comparison.

Since we find experimentally that subjects do not deviate from the straightforward bidding strategy (bidding true value) in HRB, it is confirmed that dynamic formats make it easier for the bidders to recognize equilibrium strategies. Having a discrete clock makes HRB more practical than the English auction, and yet it still is successful in eliciting the true values in our experiment.

Our results indicate that revenues under LAB are significantly higher than under HRB. Thus, in settings where revenue is the predominant objective, the seller may favor LAB, but in settings where simplicity and gathering the true value of the bidders are of greater concern, then

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<sup>2</sup> Risk aversion offers one explanation, but this has proven inadequate (see Kagel 1995). Several papers explain the overbidding phenomena with behavioral motives (for example, Goeree et al. 2002, Crawford and Iriberry 2007; Delgado et al. 2008; Engelbrecht-Wiggans and Katok 2007; Filiz-Ozbay and Ozbay 2007; Lange and Ratan 2009).

<sup>3</sup> Although many auctioned goods have some common value characteristics, it is certainly the case that in electricity, gas, radio spectrum, and diamond auctions, each firm has private production and operation costs and may face different demands. We believe it is useful to begin with the private values case as it is also argued to be important in the literature. For example, Goeree, Offerman, and Sloof (2009) and Kagel and Levin (2001) study the issues related with spectrum auctions (such as demand reduction or preemptive bidding) in private value settings.

the seller may favor HRB. In terms of efficiency, we do not find any difference between the two formats experimentally.

In Section 2, we begin with a presentation of the theory. Equilibrium bidding strategies for the two versions of the discrete clock auction are characterized in Cramton and Sujarittanonta (2010). Here we summarize the results and provide the equilibrium bidding strategies for our experimental setting. The experimental design and the results are discussed in Sections 3 and 4. Section 5 concludes.

## 2 Theory

There is one indivisible good for sale to  $N > 1$  risk neutral bidders. Bidder  $i$ 's private value for the good is  $v_i$  where each  $v_i$  is independently drawn from a commonly known distribution  $F$  on  $[\underline{v}, \bar{v}]$  with corresponding positive density function  $f$ . Assume that the hazard rate function associated with  $F$  is increasing. Bidder  $i$ 's payoff if she wins the good at a price  $p$  is  $v_i - p$ . Otherwise, it is equal to 0. The seller values the good at 0.

Before the auction starts, the seller announces a vector of bid levels,  $P = (P_0, P_1, \dots, P_{T-1})$  where  $P_t$  is the price at round  $t$  and  $T$  is the number of bid levels. The clock price increases every round so that  $\underline{v} = P_0 < P_1 < \dots < P_{T-1} = \bar{v}$ . The auction begins in round one at a price  $P_1$ .

In each round  $t$ , each bidder chooses either to bid at the current clock price or to exit. The bidders who bid at the current clock price are called active bidders. At the beginning of each round, the bidders know the number of active bidders. When a bidder exits, she submits an exit bid. Once a bidder exits she cannot bid again. If more than one bidder stays in, the auction proceeds to the next round. If all but one bidder exit, then the bidder who stayed in wins the good. If all bidders exit, then the bidder who submitted the highest exit bid wins the good. The payments depend on the auction format.

### 2.1 *Highest-rejected-bid (HRB)*

In the HRB format, the final price is determined by the highest-rejected bid. If a bidder exits in round  $t$ , the bidder submits an exit bid—a price between  $P_{t-1}$  and  $P_t$  at which she wants to

exit. If more than one bidder remains, the auction continues to another round; if all but one bidder exits, the remaining bidder wins and pays the highest exit bid; if all bidders exit, the bidder with the highest exit bid wins and pays the second-highest exit bid.

**Proposition 1.** *In the HRB auction, truthful bidding (bidding up to one's value) is a weakly-dominant strategy. The HRB auction is efficient and maximizes seller revenue.*

The dominant strategy result is standard and holds regardless of the number of bidders, the number of goods, or how values are drawn. All that is required is that each bidder demands only a single good. Thus, highest-rejected bid is the Vickrey price, thereby inducing truthful bidding. In our setting, efficiency is an immediate implication of truthful bidding. With the additional assumption of an increasing hazard rate, the optimal seller revenue can be achieved with the optimal reserve price. Nevertheless, among auctions with the same reserve price, the HRB auction yields the highest revenue due to the revenue equivalence theorem.

The HRB auction has extremely desirable properties in our setting. It is both efficient and maximizes seller revenues. Moreover, the bidding strategy is simple—just bid up to your true value—and is best regardless of what the other bidders are doing. Another important property of the HRB auction is that a bidder cannot lose at an affordable price as long as she bids her value. Hence, neither the winner nor the loser ever regrets having bid as they bid. The winner could not do better by exiting earlier; the loser could not do better by staying in longer.

## 2.2 *Lowest-accepted-bid (LAB)*

In the LAB format, the final price is determined by the lowest-accepted bid. If a bidder exits in round  $t$ , the bidder submits an exit bid—a price between  $P_{t-1}$  and  $P_t$  at which she wants to exit. If more than one bidder remains, the auction continues to another round; if all but one bidder exits, the remaining bidder wins and pays the current price; if all bidders exit, the bidder with the highest exit bid wins and pays her exit bid.

**Proposition 2.** *In the symmetric equilibrium of the LAB auction, a bidder, who exits at  $P_t$ , bids  $b(v_i, P_{t-1}) = E[Y | P_{t-1} < Y_k < v_i]$  where  $Y_k$  is the highest of  $k - 1$  independently drawn*

values and  $k$  is the number of bidders present at  $P_{t-1}$ . The LAB auction is efficient and maximizes seller revenue.

Although it is discontinuous, the equilibrium bid function in the LAB auction is monotone in value. Therefore, the LAB auction is efficient in our symmetric setting. Similar to the HRB auction, the highest seller revenue in the LAB auction can be achieved with an optimal reserve price and among auctions with the same reserve price, the LAB auction yields maximum revenue. Nonetheless, one might favor the HRB auction because of its simple bidding strategy without bid shading. In the LAB auction, bidders must do a difficult equilibrium calculation to determine the optimal level of bid shading.

### ***2.3 Theoretical predictions for the parameters used in the experiment***

In our experiment, two bidders compete to buy a single good. Bidder  $i$ 's private value for the good is  $v_i$  where each  $v_i$  is independently drawn from the uniform distribution on  $[50,100]$  and  $\mathbf{P}=(50,60,70,80,90,100)$ .

By Proposition 1, in HRB the bidder submits her value as an exit bid.

By Proposition 2, in LAB the equilibrium strategy is given in Table 1. Any equilibrium exit should occur either in round 1, 3 or 5. Here is some intuition. Given the prior distribution, the equilibrium bid of a first price sealed bid auction is  $25+\frac{1}{2}v_i$ . A bidder with a value higher than 70 would like to submit an exit bid higher than 60. Hence, any bidder with value less than 70 exits in round 1 and submits the exit bid  $25+\frac{1}{2}v_i$ . Once, the price level increases in round 2, each active bidder updates her prior such that the values of the remaining bidders are 70 or more. Given the updated distribution (uniform between 70 and 100), the equilibrium bid is  $35+\frac{1}{2}v_i$ . But this requires the lowest bid to be 70. Hence, no bidder drops out in the second round. Since no one drops out in round 2, nothing more is learned. In round 3, any bidder with a value less than 90 exits and submits the corresponding exit bid. Once, the price level increases to 90 in round 4, each active bidder updates that the lowest possible value should be 90. Since bidders with values higher than 90 would like to submit bids that are higher than 80, they will wait for



round 5. Given the updated distribution (uniform between 90 and 100) the equilibrium bid is  $45 + \frac{1}{2}v_i$ . The active bidders wait for round 5 and submit the corresponding exit bids.

-Table 1-

Ex ante symmetry—the fact that the bidders’ values are drawn from the same distribution—is critical for Proposition 2. This allows for a symmetric equilibrium. Since the exit bid functions are the same and strictly increasing, the bidder with the highest value wins and the outcome is efficient. Revenue maximization then follows from the revenue equivalence theorem. The assignment is the same as in the HRB auction, and both auctions give the bidder with a value of 50 a payoff of 0.

Ex post revenues differ between HRB and LAB. For each realization of values, the equilibrium revenue in HRB is equal to the second-highest value. However, the revenue in LAB, may not even be equal to the exit bid of the high value bidder. For a given price level, if one bidder stays in while the other one drops out, the winner pays the current price level rather than her intended equilibrium exit bid.

### **3 Experimental method**

The experiments were run at the Experimental Economics Lab at the University of Maryland. There were 184 participants, all undergraduate students at the University of Maryland. The experiment involved two treatments: HRB and LAB. We conducted six sessions for each of the auction formats, HRB1 to HRB6 and LAB1 to LAB6. For each treatment, the numbers of participants in session one to six were 16, 14, 16, 16, 16 and 14, respectively. The random draws were balanced in the sense that we used the same sequence of random number “seed” values for each auction format, so the random value draws for HRB1 matched the random draws for LAB1. A new set of random draws was used for the second session in each format, etc.

No subject participated in more than one session. Participants were seated in isolated booths. Each session lasted about 80 minutes. Bidder instructions for each treatment are in the Appendix. To test each subject’s understanding of the instructions, a subject had to answer a sequence of

multiple choice questions. The auctions in a session did not begin until all the subjects answered all of the multiple choice questions correctly.

In each session, each subject participated in 21 auctions. The first auction was a practice auction. Each auction had two bidders, selected at random among the subjects. Bidders were randomly rematched after each auction. All bidding was anonymous. Bids were entered via computer. The experiment is programmed in z-Tree (Fishbacher 2007). At the conclusion of each auction, the bidder learned whether she won and the price paid by the winning bidder.

Bidders had independent private values for a fictitious good with values drawn uniformly from 50 to 100. The number of rounds was selected to approximate those of actual auctions. For example in diamond auctions for large stones, typically there are 4-6 rounds and increments are 10%-20% of the price range. There were a maximum of five rounds in our discrete clock auctions and the price increments were 20% of the price range.

The winner in each auction earned her value minus the price paid in Experimental Currency Units (ECU). At the end of the experiment, total earnings were converted to US Dollars, at the conversion rate of 10 ECU = 1 US Dollar. Subjects also received a \$5 show-up fee. Cash payments were made at the conclusion of the experiment. The average subject payment was \$19.51.

## **4 Experimental results**

As in any dynamic auction, bidding strategies of all bidders are not observable. For example, in the Dutch auction only the bid of the winner, or in the English auction only the bids of the losers are observable. Nonetheless, it is still possible to make revenue and efficiency comparisons, which are important in auction design. Later, we will study the bidding strategies based on the observable bids. Among 920 auctions we conducted per treatment, we observed 1,092 and 1,103 exit bids in HRB and LAB, respectively.

Table 2 shows the actual (what we observed in the experiment) efficiency and revenue, and the theoretical revenue (equilibrium revenue given the realization of values) of each treatment for the aggregate data. Both treatments, HRB and LAB, yield the efficient allocation with high

frequency in each session (see Figure 1) and nearly 100% of the gains from trade are realized in both treatments. Using the 6 independent sessions per treatment, the Mann-Whitney rank test shows no significant difference in efficiencies ( $z=0.40$ ,  $p=0.69$ ), but it shows a significant difference in revenues between treatments ( $z = -2.88$ ,  $p = 0.00$ ) (see Figure 2). Furthermore, Mann-Whitney rank tests indicate that the actual revenue in LAB is significantly higher than the theoretical prediction ( $z = -2.88$ ,  $p = 0.00$ ) but there is no significant difference between the actual and theoretical revenues in HRB ( $z = 1.44$ ,  $p=0.15$ ).

-Table 2-

-Figure 1-

-Figure 2-

This evidence implies that subjects bid more aggressively than the equilibrium prediction in LAB while in HRB subjects bid close to the predictions of the theory. As a result the seller receives higher revenue in LAB. The bidding behaviors in HRB and LAB are investigated further in the next subsections.

One natural question is whether the relationship between actual and theoretical revenues in the two formats depends on the period or is persistent in the experiment. Figure 3 plots the average differences between the actual and theoretical revenues period by period. The actual revenues we observed in HRB are close to the prediction of the theory throughout the experiment. The regression of the difference between actual and theoretical revenues gives a significant coefficient for period and the constant is also significant. In HRB, actual revenue gets closer to the theoretical revenue: using the data from the six independent sessions of HRB, the Wilcoxon signed rank test shows that although the average difference between actual and theoretical revenue is significantly less than 0 in the first 10 periods ( $z=-0.99$ ,  $p<0.05$ ), this difference is not significantly different from 0 in the last 10 periods ( $z=0.11$ ,  $p=0.92$ ). The actual revenues in LAB are persistently higher than the theoretical ones as can be seen in Figure 3 and Table 3. In the regression for the difference between actual and theoretical revenues in LAB, only the constant was significant but not the coefficient of period. Moreover, the difference

between actual and theoretical revenues is significantly higher than 0 both in the first 10 ( $z=2.20$ ,  $p<0.05$ ) and last 10 periods ( $z=2.20$ ,  $p<0.05$ ).

-Figure 3-

-Table 3-

#### ***4.1 Bidding behavior in treatment HRB***

Subjects submitted a total of 1093 exit bids in 920 auctions. Figure 4 plots exit bids and corresponding values in HRB. The dashed line is the truthful bid function which is what the theory suggests. Although there were instances of bid shading and bidding above one's value, most of the exit bids were around the 45 degree line.

-Figure 4-

As shown in Figure 5, 83.9% of exit bids were submitted in the round that the theory predicts and 13.6% of exit bids were submitted earlier than predicted. Among those who exited in the predicted round, 64.9% submitted exit bids within one percent of the exit bids suggested by the theory; 26.1% underbid and 9% overbid.

-Figure 5-

The vast majority of subjects were successful in waiting until the right round to exit. Since the binary decision of staying in or out is similar to the strategic thinking of the English auction, this finding is in line with the success of value bidding in English auction experiments. Harstad (2000) finds that subjects who gain experience in auctions with this kind of binary decision making perform better in discovering the value bidding strategy in second-price sealed-bid auctions. Staying in the HRB auction until the price reaches the correct level provides a similar experience as participating in an English auction.

#### ***4.2 Bidding behavior in treatment LAB***

A total of 1,103 exit bids were submitted in 920 LAB auctions that are conducted. Figure 6 plots exit bids and corresponding values in LAB. The dashed line is the theoretical bid function as studied in Table 1.

-Figure 6-

Figure 7 shows that many subjects failed to exit in the round that the theory predicted. Only 58% exited in the correct round and 26% of the exit bids were submitted later than predicted. Even among the ones who exited in the predicted round, 44.4% overbid while 34.7% of those who exited in the predicted round did submit exit bids within one percent of the exit bids suggested by the theory and 20.9% underbid.

-Figure 7-

The high revenue in the LAB auction is caused by two types of upward deviations: (1) the winner submits a higher exit bid than predicted and (2) either bidder stays in too long. The first type involves a winner exiting at the round theory predicted and then submitting a too-high exit bid. The second type involves a winner, loser or both: a winner may drop out too late; or a loser who drops out too late extends the auction and thus raises the final price. Table 4 shows the difference between actual and theoretical revenues for each type of deviation. The relative effect of the second type of deviation is much stronger than the first. Furthermore, the second type of deviation was observed more frequently. These two types of deviation can take place in the same auction but there was only one auction with both types of deviation.

-Table 4-

## 5 Conclusion

The pricing rule is of fundamental importance in practical auction design. Pricing impacts both the efficiency and the revenues of the auction. Although there is an immense literature on the pricing rule in static (sealed-bid) auctions—first-price vs. second-price in single unit auctions and pay-as-bid vs. uniform-price in multi-unit auctions—little is known about alternative pricing rules in the dynamic auctions commonly used in practice. We show how different pricing rules influence bidding behavior in discrete clock auctions in a simple setting.

Based on the standard theory in which bidders seek to maximize profits, the highest-rejected-bid (HRB) and the lowest accepted-bid (LAB) auctions both seem attractive. They maximize revenues and are fully efficient in our unit-demand setting. Despite this theoretical

result, LAB pricing is often used in practice. In our experiments, both formats performed equally well and close to the theoretical predictions in terms of efficiency. However, the LAB auction yielded higher revenues than the HRB auction.

Although there is significant overbidding in LAB, this overbidding is not as severe as in LAB's limit case, the first-price sealed-bid auction. The dynamic auction, focusing initially on the binary in/out decision, appears to limit overbidding. This suggests an interesting tradeoff between price increments and revenues.

Developing strategies in HRB requires two things: when to exit as in an English auction, and what exit bid to submit as in a second-price sealed bid auction. Most bidders submit exit bids close to their values. The tendency of bidding above value found in second-price auctions is not observed here. We conjecture that this is due to the experience bidders gain in deciding whether to exit each round. That reasoning process is similar to decision making in an English auction. Harstad (2000) finds that practicing English auctions improves performance in second-price sealed-bid auctions. Similarly, our experiment shows that the multiple-round implementation of the second-price auction eliminates the tendency to overbid when faced with second-price incentives. The dynamic implementation also limits the spread between the highest bid and the price to at most one bid increment.

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**Table 1. Equilibrium Strategy for LAB Auction**

Value	Exit Round	Exit bid function
[50, 70)	1	$25 + \frac{1}{2}v_i$
[70, 90)	3	$35 + \frac{1}{2}v_i$
[90, 100]	5	$45 + \frac{1}{2}v_i$

**Table 2. Outcomes of Treatment HRB and LAB**

	<i>HRB</i>	<i>LAB</i>
Efficiency	91.09%	90.87%
	(0.01)	(0.01)
Revenue per auction	67.03	69.91
	(0.36)	(0.41)
Theoretical revenue	67.30	67.25
	(0.40)	(0.36)

Standard errors are shown in the parentheses.

**Table 3. Regression of the Difference between Actual and Theoretical Revenues in HRB and LAB Auctions**

Independent variables	(HRB)	(LAB)
Constant	-1.2***	1.98***
	(0.347)	(0.398)
Period	0.09***	0.06
	(0.029)	(0.033)
Session fixed effect	YES	YES
N	920	920

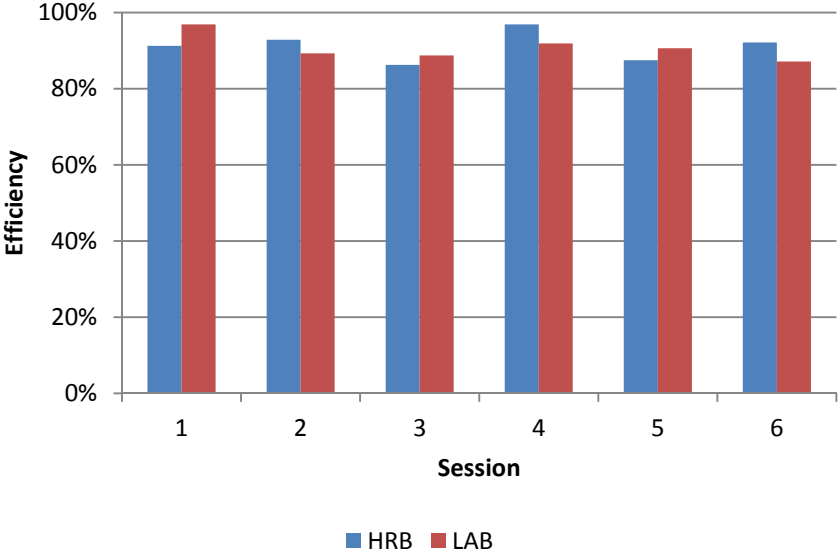
\*\*\* Significant at 99% confidence interval. Standard errors are shown in the parentheses.

**Table 4. Difference between actual and theoretical revenues by type of deviation**

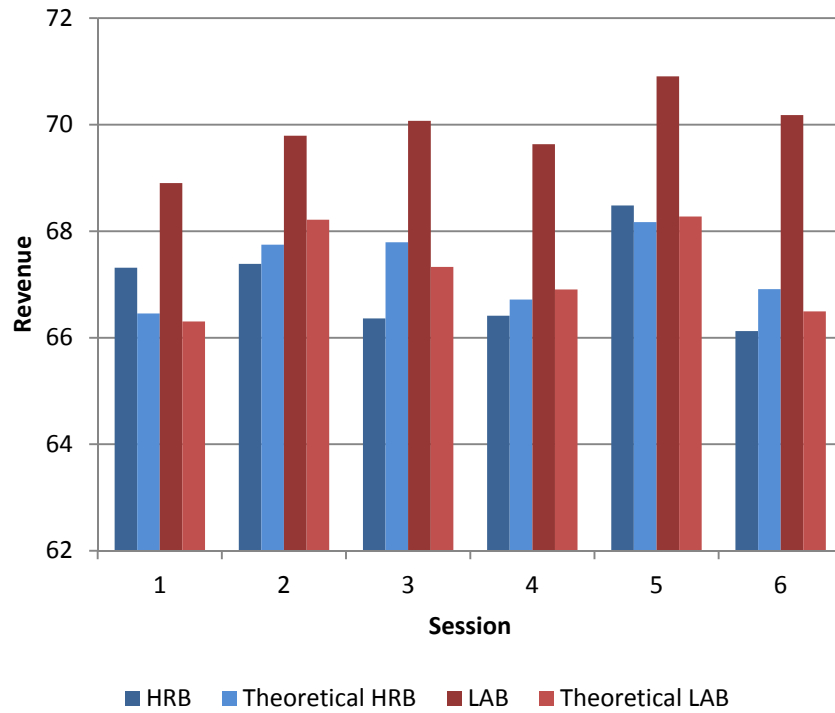
Type of upward deviation	Average difference between actual and theoretical revenues	Number of auctions
Winner exiting at the predicted round but submitting too high exit bid	1.44 (0.16)	71
Any bidder staying in for too long	9.85 (0.20)	260

Standard errors are shown in the parentheses.

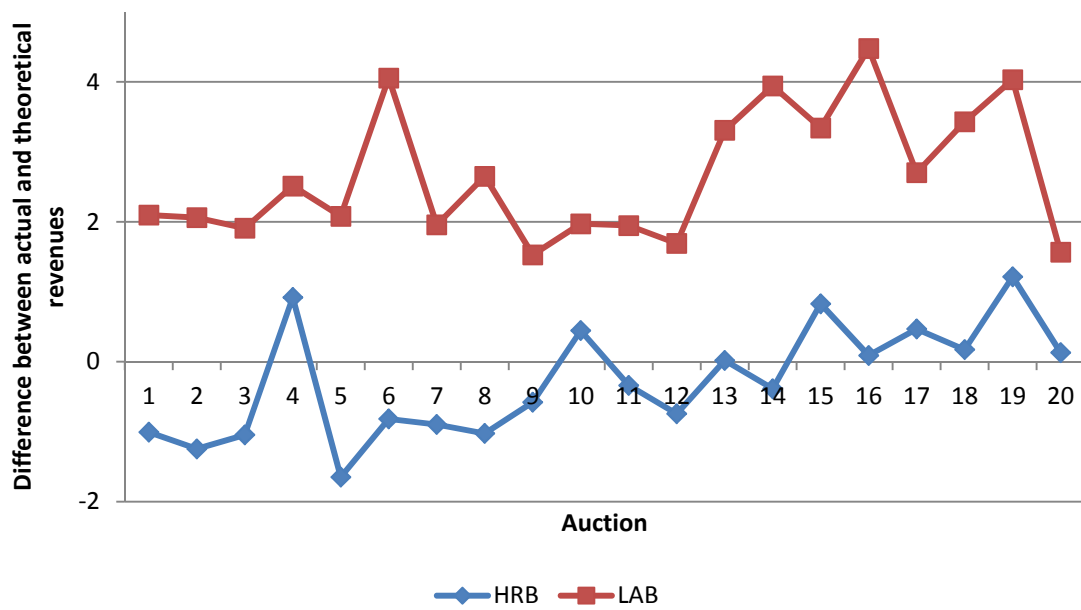
**Figure 1. Percentages of efficient allocations in each session**



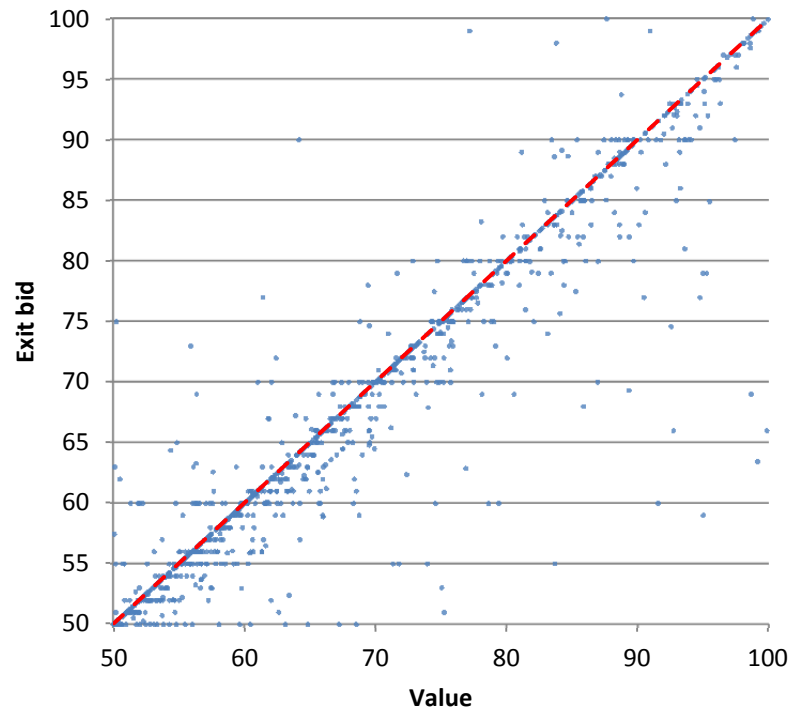
**Figure 2. Average actual and theoretical revenues in each session**



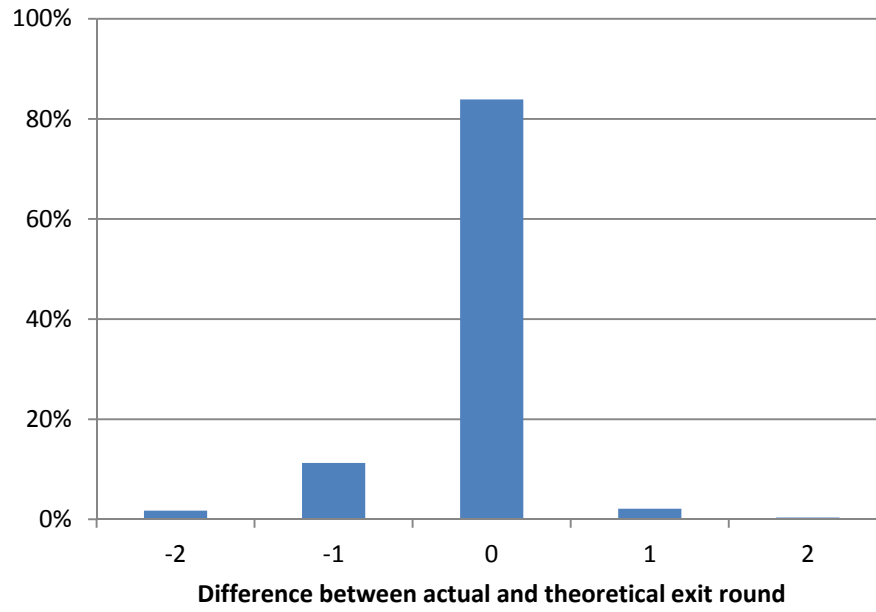
**Figure 3. Average differences between actual and theoretical revenues per period**



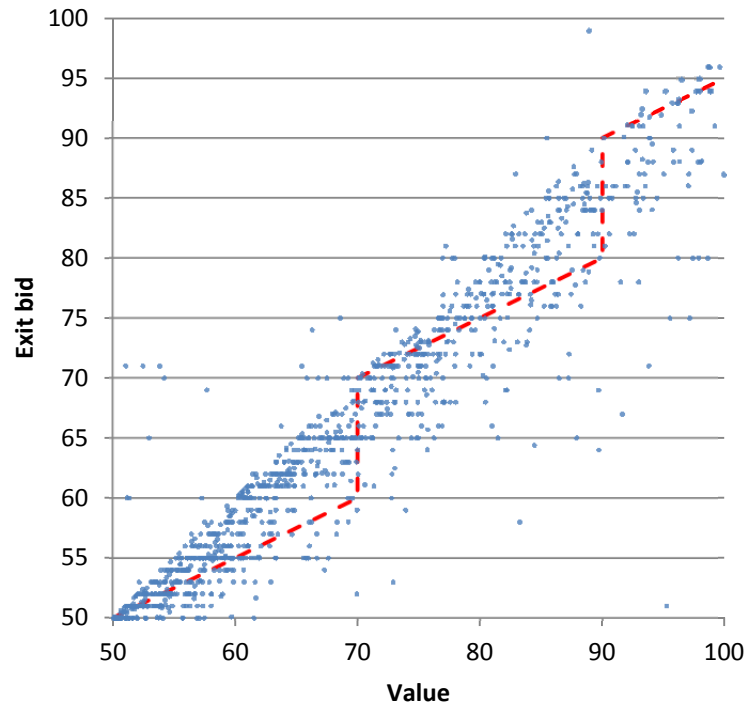
**Figure 4. Plot between values and exit bids in HRB (all data)**



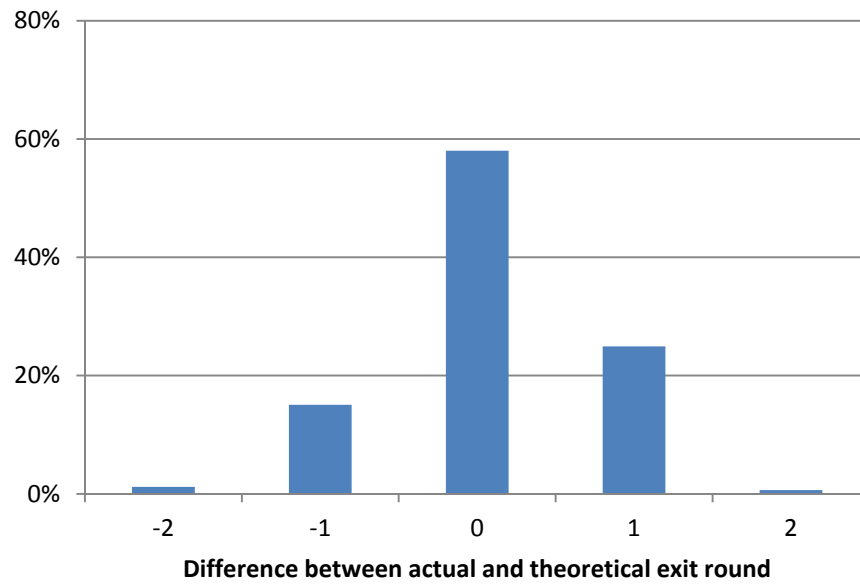
**Figure 5. Deviations from the predicted exit rounds in HRB**



**Figure 6. Plot between values and exit bids in LAB**



**Figure 7. Deviations from the predicted exit rounds in LAB.**



## **Appendix: Bidder Instructions**

*Treatment HRB*

### **Experiment Instructions**

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

### **Auction Rules and Calculation of Earnings**

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and your aim is to bid in order to win this good. When the first auction starts, you will observe your valuation of the fictitious good. Your valuation is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent valuation for the fictitious good and his or her valuation is also randomly selected from [50,100] interval. Each bidder will know only his or her own valuation.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask you if you would like to stay in the auction when the price increases to 60. You either stay IN

which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out for at a price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT with an exit bid of, say, 55: Then you win the fictitious good and pay 55, your opponent's exit bid.
- You stay OUT with exit bid of, say 57, and your opponent stays IN: Then your opponent wins the fictitious good and pays 57, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, and your opponent stays OUT with exit bid of, say, 65: Then you win the fictitious good and pay 65, your opponent's exit bid.
- You stay OUT with exit bid of, say 67, and your opponent stays IN: Then your opponent wins the fictitious good and pays 67, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.



Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out with an exit bid, the bidder who is in wins the good and pays the exit bid of the other bidder. Otherwise, if both bidders stay out then the one with the higher exit bid wins the good and pays the exit bid of the other bidder. For example, when the price level is 70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 63 (which is the smaller exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays the exit bid of the other.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your valuation and the price. For example, let us say you have a valuation of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

$$\text{Earning} = 82.55 - 61 = 21.55 \text{ ECU}$$

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this round. First, the computer will show you your new valuation for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own valuation for the good for this auction privately. The same auction rules as in the first auction will apply.

There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to the US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

**Questions for the test period** (*asked to the subjects during the test period by the computer*)

- 1) The price level just moved from 60 to 70 and the computer asks you if you would like to stay in at a price of 70. You said that you are IN and your opponent said that he or she is OUT with an exit bid of 65. Then what would be the outcome of the auction?
- a) You would win the good and pay 60.
  - b) Your opponent would win the good and pay 70.
  - c) You would win the good and pay 65.
  - d) Nobody would win the good and the price would move to 80.

Answer: (c)

- 2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?
- a) Any amount between 65 and 70.
  - b) Any amount between 70 and 80.
  - c) Any amount between 60 and 70.
  - d) Any amount between 80 and 90.

Answer: (b)

- 3) Let us say, your valuation of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?
- a) You would win the good and pay 77. You would earn 14 and your opponent would earn 0.
  - b) Your opponent would win the good and pay 74. You would earn 0 and you cannot know your opponent's earning without knowing his or her valuation.
  - c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.
  - d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (d)

**Experiment Instructions**

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

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The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask you if you would like to stay in the auction when the price increases to 60. You either stay IN which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out at a

price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT: Then you win the fictitious good and pay 60.
- You stay OUT, and your opponent stays IN: Then your opponent wins the fictitious good and pays 60.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays his or her exit bid.
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If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.

Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out, the bidder who is in wins the good and pays the price for which he or she indicated to stay in. Otherwise, if both bidders stay out then the one with the higher exit bid wins the good and pays his or her exit bid. For example, when the price level is

70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 67 (his or her exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays his or her exit bid.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your valuation and the price. For example, let us say you have a valuation of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

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- a) You would win the good and pay 70.
  - b) Your opponent would win the good and pay 70.
  - c) You would win the good and pay 65.
  - d) Nobody would win the good and the price would move to 80.

Answer: (a)

- 2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?
- a) Any amount between 65 and 70.
  - b) Any amount between 70 and 80.
  - c) Any amount between 60 and 70.
  - d) Any amount between 80 and 90.

Answer: (b)

- 3) Let us say, your valuation of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?
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  - c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.
  - d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (a)