Incorporating Unawareness into Contract Theory[*]

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Abstract

Asymmetric awareness of the contracting parties regarding the uncertainty surrounding them is proposed as a reason for incompleteness in contractual forms. An insurance problem is studied between a risk neutral insurer, who has superior awareness regarding the nature of the uncertainty, and a risk averse insuree, who cannot foresee all the relevant contingencies. The insurer can mention in a contract some contingencies that the insuree was originally unaware of. It is shown that there are equilibria where the insurer strategically offers incomplete contracts. Competition among insurers who are symmetrically aware of the uncertainty promotes awareness of the insuree. [JEL Classification: D83, D86]

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1 Introduction

In a world where insurance companies spend a lot of resources to compute the facts that are material to the risk, the relevant contingencies lie largely in the knowledge of the insurers. Insurance companies which have been in the industry for a long time may have a better understanding of the realities of nature than an insurance buyer. The buyers trust the insurance companies and proceed upon the confidence that the companies do not hold back any circumstances in their knowledge to mislead the judgement of the buyers. Moreover, policies are usually drafted by insurers, giving them a strong opportunity to manipulate (see Harnett (1950)). This asymmetry between the insurance buyer and seller in foreseeing all the relevant contingencies is the key reason for expost conflicts. However, the standard contracting models do not allow for agents having asymmetric awareness regarding the nature of the uncertainty. This paper incorporates unawareness in contractual settings in order to understand how insurers use their superiority in terms of understanding the relevant contingencies against buyers. It questions whether such an insurer will mention in the contract those contingencies that the insuree does not foresee originally or he will remain silent on them. Moreover, if the insuree reads a clause about a contingency that did not cross her mind initially, how she evaluates this information is part of the solution concept we propose. Finally, we search for an instrument that leads to disclosure of the unforeseen contingencies.

We address these questions by generalizing an insurance setting between an insurer (he) and an insuree (she) such that each agent may take into account a different set of contingencies. We call these subjective sets of contingencies awareness sets. When the insuree reads a contract offered by the insurer, she may become aware of some new aspects of the uncertainty and start taking them into account. For example, a home insurance buyer who has never thought about a tsunami before becomes aware of it

when the contract offers insurance against tsunami as well. Hence, the contract can be used as a communication device by the insurer in order to extend the awareness of the insuree.

If reading a contract adds new contingencies for the consideration of the insuree, the question is how she is going to assign probabilities to the new contingencies in order to evaluate them. In this study, a priori, there is no imposition on how the insuree generates a belief when her awareness is extended. Belief formation of the insuree is a part of the equilibrium concept. We require progressively more restrictions on belief formation. We start with compatible belief, then we will consider consistent beliefs. The definitions of these concepts will be given and discussed extensively in the paper. Roughly, we call a belief *compatible with a contract* if, with respect to this belief, the insuree thinks that the insurer is better off by making this offer rather than staying out of business. We require equilibrium beliefs to be compatible with the corresponding contracts whenever it is possible. Under this solution concept, we show that hiding some contingencies from the insuree is always part of some equilibria while mentioning all the possible contingencies may not be. Next, we refine this possibly large equilibrium set with a consistency requirement. A belief held after a contract is *consistent* if the contract is the best one for the insurer according to the insuree with respect to this belief. We show that consistent equilibrium always exists and there are situations where this refinement eliminates all complete contract equilibria.

In this setup, the contract that mentions an unforeseen contingency and promises zero coverage when it materializes and the contract that does not mention that contingency at all are different. Since the first one provides a complete list of relevant contingencies and the second one fails to do so, the second one is called incomplete. Complete and incomplete contracts correspond to different awareness sets of the insuree, and therefore, their subjective evaluations are not the same. If an incomplete

contract is agreed to, then experiencing that contingency and learning that the damage is not covered by the contract is an expost surprise for the insuree. In reality, in such situations insurees feel deceived and go to court. Although it is the role of the court to protect the deceived ones, and apply the doctrine of concealment, it still needs to be proved that the insurer intentionally left the contract incomplete. This is not an easy task since the subjective status of the insurer needs to be determined objectively. This is the main reason for the debate on the doctrine of concealment in law literature (see Harnett (1950) and Brown (2002)). We argue that this problem is due to monopolistic power of insurance provider and show that competition among insurance companies is an instrument to reach complete contracts in equilibrium. Competition promotes awareness of the insuree.

We model competing insurers as symmetric agents who can independently inform the insuree regarding the unforeseen events through their offers. We extend the solution concept that is studied for single-insurer case to this multi-insurer setting. In equilibrium it is required that the insuree holds beliefs that are compatible with all the offers that are made and she acts rationally based on her belief. It is shown that the zero profit offer that fully covers all the contingencies (complete) is always part of some equilibria. Therefore, the strength of competition in standard principal-agent models carries over when there is asymmetric awareness. We note that in some situations, it is still possible to have incomplete contracts in equilibrium. We provide a sufficient condition that leads to an extension of awareness in any equilibria.

The rest of the paper is organized as follows: Next, we discuss the related literature. In Section 2, we introduce the one insurer-one insuree model and necessary notation. In Section 3, we give an equilibrium concept and study the form of equilibrium contracts that can arise in this setting. By introducing competition between insurers in Section 4, we show that the unawareness of the insuree may totally dis-

4

appear under competition and study the setups where the awareness of insuree is at least partially extended in any equilibrium. In Section 5, we discuss some key points in the construction of our model: other forms of contracts besides the ones we study, the difference between being unaware of an event and assigning zero probability to that event, and robustness of the equilibrium concept under ambiguity aversion. We conclude in Section 6. All the proofs are presented in the Appendix.

Related Literature

Unawareness is first studied in economic theory by Modica and Rustichini (1994). In the literature, there are some recent developments in modeling unawareness. Unawareness models by Heifetz, Meier, and Schipper (2006) and Li (2009) are the basis of the unawareness concept we use in this paper (see Ozbay(2008) for more formal connection). In those models, each agent can take into account a projection of the entire situation to the aspects that she is aware of. This set theoretic modeling of unawareness is incorporated into game theory by Halpern and Rego (2006), Heifetz, Meier, and Schipper (2007) and Ozbay(2008).

Standard economic theory has been developed within a paradigm that excludes unawareness. Recent studies addressed how accounting for unawareness changes the standard economic theory (see Modica, Rustichini, and Tallon (1998) and Kawamura (2005) for applications in general equilibrium models, Tirole (2009) for inefficient investment on cognitive effort that may extend awareness in a contractual problem, and Masatlioglu, Nakajima, and Ozbay (2011) for choice theoretical foundation for modelling agents who do not pay attention to all the possibilities in the presented decision problem).

Incomplete contracts are extensively studied in economics (see e.g. Hart and Moore (1990), Aghion and Bolton (1992), Grossman and Hart (1986), Bolton and

Whinston (1993), Aghion and Tirole (1997), Hart and Moore (1998), Gertner, Scharfstein, and Stein (1994), Hart and Moore (2005) and for a summary of this literature see Bolton and Dewatripont (2005) and Salanie (2005). In this literature, the inability of contracting parties to foresee some aspects of the state of the world is frequently understood as a reason for the incompleteness of some contracts. However, this reasoning lead to well known discussions in the studies of Maskin and Tirole (1999), Tirole (1999) and Maskin (2002). They argued that in the models motivated by unforeseen contingencies, the parties are rational and able to understand the payoff related aspects of the state of the world, although they are unable to discuss the physical requirements leading to those payoffs. Our model is free from this inconsistency since here neither the agents foresee all contingencies nor they are able to understand payoff related aspects of the unforeseen ones. Tirole (1999) states that the way *they currently stand*, unforeseen contingencies are not good motivation for models of incomplete contracts, and he further notes that:

> "...there may be an interesting interaction between 'unforeseen contingencies' and asymmetric information. There is a serious issue as to how parties form probability distributions over payoffs when they cannot even conceptualize the contingencies..., and as to how they end up having common beliefs ex ante. ...[W]e should have some doubts about the validity of the common assumption that the parties to a contract have symmetric information when they sign the contract. ...Asymmetric information should therefore be the rule in such circumstances, and would be unlikely to disappear through bargaining and communication."

In line with the observation quoted above, in our model the agents cannot forecast the relevant contingencies symmetrically and they do not assign probabilities to those unforeseen contingencies. They are rational agents within their awareness sets, but

they are taking into account only the aspects of the uncertainty that they are able to conceptualize and ignore the rest.

Although the papers in the literature pertaining both to awareness and to incomplete contracts always refer to each other, there are not many studies that explicitly combine two strands of the theoretical literature.[1] Our study can be thought as one of the first attempts in contract theory which formally allows unawareness.

2   Model

There is a good owned by an agent. $v > 0$ is the value of the good for the agent. The good is subject to some uncertain future damages. The owner (insuree) wants to be insured against realization of damages. Ω is the finite set of causes that lead to damages. Elements of Ω are distributed according to μ. It is assumed that all the elements of Ω are possible, i.e. $\forall \omega \in \Omega$, $\mu(\omega) \neq 0$.

The insuree (she) is indexed by 0 and we assume that there is only one insurer (he) indexed by 1.[2] If a contingency is in an agent's state of mind while s/he is evaluating a situation, then we say that s/he is *aware* of that contingency. Otherwise, if the agent is unaware of a contingency, then s/he cannot take that contingency into account in the decision making process. The awareness structures of the insurer and the insuree are as follows:

- The insurer is aware of Ω and believes the distribution μ.

- The insuree is only aware of Ω', which is a non-empty proper subset of Ω. She

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[1]In an interesting study, Chung and Fortnow (2006) model courts that make some "awareness check". Gabaix and Laibson (2006) provide contracting model for consumers who fail to anticipate certain future payments. Tirole (2009) studies the cognitive efforts of contracting parties who need to figure out some relevant new technologies that they cannot describe initially.

[2]Until Section 4 we assume that there is only one insurer. Then we will introduce competition in the model.

believes the conditional distribution $\mu(.|\Omega').^{3}$

- The insuree is not aware of the remaining realizations of damages in $\Omega\backslash\Omega'$ and she is not aware of the insurer's superior awareness. Therefore, initially the insuree believes that $(\Omega', \mu(.|\Omega'))$ describes the whole uncertainty that she and the insurer consider.

- The insurer knows that the insuree is considering only $(\Omega', \mu(.|\Omega'))$ and moreover, the insurer knows that the insuree is unaware that the insurer has superior awareness.

Damage levels are defined by a cost function $c : \Omega \longrightarrow \mathbb{R}_+$ where $c(\omega)$ is the damage level at $\omega \in \Omega$. Let S be the range of cost function, i.e. $c(\Omega) = S$. Although, realistically contracts might be written on Ω, and the insuree might be unaware of some causes of damages but not the damage levels, which are nothing but real numbers, all that matters for agents are the damage levels, S. For notational simplicity we will consider the reduced form model and one to one cost functions, and refer to S and S' rather than Ω and Ω', where $S' = c(\Omega')$, as the relevant sets of contingencies and awareness sets.[4] It is assumed that there exists a non-zero damage level $s \in S'$. This means that the insuree initially understands that the good she owns is subject to some risk.[5] With obvious abuse of notation we will use μ as a distribution on S.

Given this awareness structure, each party interprets the true problem as a projection of it onto the aspects of the uncertainty that s/he is aware of. Here S and S' are not state spaces in the sense of awareness literature and that is why we insist

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[3]The initial belief of the insuree does not have to be $\mu(.|\Omega')$ in order to derive our main points. If one starts with an arbitrary initial belief and defines the hierarchy of beliefs accordingly, the analysis can be carried over. In all the examples, we indeed consider singleton Ω'. Therefore, one can see that the nature of the results does not depend on initial belief being the true conditional distribution.

[4]The nature of the results of this paper would not change whether we work with Ω or S.

[5]This assumption is needed for the result in Theorem 4.2. I appreciate John Quiggin for pointing this out.

on calling them as sets of contingencies. In the literature a state also describes what a decision maker is aware of (or unaware of).[6] Here the insuree is unaware of some actions of the nature. However, it should be clear that what the insuree is unaware of is not just that. The insurer can come up with contracts that are based on the contingencies in $S \backslash S'$. The insuree is also unaware of those actions of the insurer. So if we think of the true game, the insuree is aware of only a part of this game that can be written only by referring to the contingencies in S'. This idea of being aware of a projection of the true game follows from Halpern and Rego (2006), Heifetz, Meier, and Schipper (2006), Heifetz, Meier, and Schipper (2007), Li (2009), Ozbay (2008).

The insurer offers a contract in order to insure the good against future damages. A typical contract is a specification of three objects:

(*i*) The contingencies on which a money transfer will be made from insurer to insuree;

(*ii*) The amount of transfer as a function of contingencies in (*i*);

(*iii*) The premium which is an in advance payment from insuree to insurer for the agreement.

Definition 2.1. *A contract is a triplet $C = (t, A, k)$ where $A \subseteq S$, $t : A \longrightarrow \mathbb{R}_+$ such that $t(s) \leq s$ for any $s \in S$ is the transfer rule, and $k \in \mathbb{R}_+$ is the premium. The set of all contracts is denoted by $\mathbb{C}$.*

A feasible contract provides transfers that cover less than or equal to the cost of any damage. This is the case in reality as well and no insurer covers more than the damage level. In our asymmetric awareness model, this assumption is a necessary requirement for the existence of equilibrium. This will be clearer when we formally

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[6]The generalized state space for our model is not necessary for the analysis and therefore we do not define it.

define equilibrium but too see the intuition here, imagine a contract that is signed only on S', $C = (t, S', k)$. If the insuree accepts this contract, and if transfers are unbounded, then for any constant $a > 0$, $\tilde{C} = (t + a, S', k + a)$ is also accepted. Observe that $\tilde{C}$ is $(1 - \mu(S'))a$ more profitable than C. By increasing a arbitrarily, the insurer can make an unbounded amount of profit on the events that are remain to be unforeseen by the insuree. Therefore, not ruling out transfers that cover more than the damage level would lead to non-existence of a solution.

Note that Definition 2.1 does not restrict the set of contingencies that a contract can be written on. A can be any subset of S. If the contract is silent at some contingencies, it means that there will not be any transfer to the insuree when those contingencies are realized.[7] One critique to the use of incomplete contracts in the literature is that even if some contingencies are left open in the contract, each agent clearly knows what will happen if those contingencies realize. Therefore, in a sense, such contracts are still complete. In order to be free from this critique, an incomplete contract should leave some contingencies excluded in the evaluation of at least one agent. If initially foreseen contingencies are not specified in a contract, in this setup that contract does not qualify to be called incomplete. The insuree still knows the relevance of those contingencies and her utility if they realize. A contract is incomplete only if it leaves insuree unaware of some relevant contingencies. Therefore, the whole model needs to be known in order to call a contract incomplete.

Definition 2.2. *A contract $C = (t, A, k)$ is incomplete if $A \cup S' \neq S$.*

A contract may announce some contingencies that the insuree is not originally aware of, i.e. for a contract $C = (t, A, k)$, it can be the case that $A \backslash S' \neq \emptyset$. If such a contract is offered then the insuree becomes aware of those contingencies and her new understanding of the uncertainty enlarges to the aspects in $A \cup S'$. This means that

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[7]We will discuss in Section 5 some other types of contracts that we did not consider here because they would not change the results.

there is no language barrier and the insuree is capable of understanding the content of the offer. Since there are contingencies that the insuree is not aware of, unless a contract mentions them, the insuree will remain unaware of them and continue to omit these contingencies in her decision making process.

Consider a contract $C = (t, A, k)$ that offers transfer at some contingencies which the insuree is not originally aware of (i.e. $A \backslash S' \neq \emptyset$). In order to evaluate the transfer at those contingencies, she needs to extend her belief by assigning probabilities to the newly announced contingencies. When a contract C is offered, she holds a belief P_C which is a probability distribution on $A \cup S'$. The insuree is ready to update her awareness set according to the offered contract but she cannot put in her calculations anything more than that. In reality, agents might think that there may be something in the world that they are unable to name, especially after their awareness is extended. Unawareness is, by itself, the lack of ability to name, evaluate and estimate some aspects of the problem. At the given stage of the theoretical literature, we are bounded by modeling the economic agents as rational within their awareness unless we assume some exogenous evaluation of unforeseen world.

The way beliefs are generated is a part of our solution concept and starting from Section 3, we will analyze the relationship between the formation of belief and the form of signed contracts. Here we will introduce the necessary notation for an arbitrary belief P_C.

After a contract C is offered, the insuree can either reject or take the offer. If she rejects the offer, then the negotiation stops at that point and she is not covered for any damage. The decision of the insuree on a contract is determined by a function $D : \mathbb{C} \rightarrow \{buy, reject\}$.

We assume that the insuree is a risk averse agent with an increasing and concave utility function u. Therefore, the expected utility of the insuree from contract $C =$

(t, A, k) with respect to distribution P_C can be written as

$$EU_0(C, D(C)|P_C) := \begin{cases} \sum_{s \in A} u(v - s + t(s) - k)P_C(s) \\ \quad + \sum_{s \in S' \setminus A} u(v - s - k)P_C(s) & \text{if } D(C) = buy \\ \sum_{s \in A \cup S'} u(v - s)P_C(s) & \text{if } D(C) = reject \end{cases}$$

The expected utility of the risk neutral insurer from contract $C = (t, A, k)$ is

$$EU_1(C, D(C)) := \begin{cases} k - \sum_{s \in A} t(s)\mu(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

Observe that the expected utility of the insurer calculated by the insurer himself and the one calculated by the insuree under her belief P_C may not coincide in general. The insurer's expected utility from contract C according to the insuree with respect to her belief P_C is denoted by

$$EU_1^0(C, D(C)|P_C) := \begin{cases} k - \sum_{s \in A} t(s)P_C(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

Tirole (1999) criticizes incomplete contract literature since, in that literature, agents are unable to conceptualize and write down the details of the nature although they are able to fully understand payoffs relevant to those aspects and consider them in their calculations. In our model, being unable to conceptualize a contingency also means that the agent cannot assign probability to that contingency and cannot take it into account in her evaluations. This intuition is expressed in definitions of EU_0 and EU_1^0 above.

3 Incompleteness in the Contractual Form

The crucial and non-standard point in our model is the following: although before anything is offered, the insuree is unaware of some relevant aspects of the uncertainty, once they are announced to her via a contract, she starts taking them into account. Her awareness evolves throughout the interaction. The contracts that extend the awareness set of the insuree do not inform her regarding the probability of those newly announced contingencies.[8]

However, the content of the contract might still be informative about the probability of contingencies it specifies. When contract $C = (t, A, k)$ is offered, the insuree needs to generate a belief which is a probability distribution on her extended awareness set, $A \cup S'$.

Definition 3.1. *A probability distribution $P_C \in \Delta(A \cup S')$ is compatible with contract $C = (t, A, k)$ if it satisfies:*

(i) $EU_1^0(C, buy | P_C) \geq 0$, *i.e.* $k \geq \sum\limits_{s \in A} t(s) P_C(s)$;

(ii) *For any $s \in A \cup S'$, $P_C(s) \neq 0$ and $P_C(. | S') = \mu(. | S')$*

The set of all probability distributions that are compatible with C is denoted by Π_C.

The insurer is a strategic agent and he always has an option of not participating in the negotiation and thereby guaranteeing himself zero profit. The insuree may reason that if a contract is offered, then it ought to be better for the insurer to make this offer rather than staying out of business. The first requirement in the above definition says that, with respect to a compatible belief, the expected gain of the insurer from a contract should be at least zero which is his outside option. The second point

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[8]In Section 5, we discuss the contracts that also inform the insuree regarding the probabilities although we do not observe such contracts in reality.

in the above definition requires from a compatible belief that the newly announced contingencies does not alter the relative weights of the contingencies in S'. This makes the model close to Bayesian paradigm. As we noted in footnote 3, the model can be generalized easily for arbitrary initial beliefs. Property (ii) in Definition 3.1 is not about making the belief formation agree with the true distribution conditionally, it is about making it agree with the initial belief, conditionally. According to a compatible belief, every contingency in the extended awareness set is possible. In the true model, all the relevant contingencies are possible. Therefore, the insurer cannot make up contingencies in a contract. In line with this, a compatible belief assigns non-zero probability to every foreseen contingency.[9]

Compatible beliefs are candidates to be held by the insuree after an offer. The solution concept we introduce in this section requires belief formation to be part of an equilibrium. The insurer believes that the insuree will behave according to the beliefs that an equilibrium suggests under some rationality requirements and he responds to this belief. Equilibrium behavior of the insuree confirms this belief of the insurer as well.

Definition 3.2. *An equilibrium of this contractual model is a triplet* $(C^*,\ D^* : \mathbb{C} \to \{buy, reject\},\ (P^*_C)_{C \in \mathbb{C}})$ *such that*

(i) $C^* \in \underset{C \in \mathbb{C}}{\arg\max} EU_1(C, D^*(C))$;

(ii) *For any* $C \in \mathbb{C}$, *where* $C = (t, A, k)$,

$$D^*(C) = \begin{cases} buy & \text{if } EU_0(C, buy | P^*_C) \geq EU_0(C, reject | P^*_C) \\ reject & \text{otherwise} \end{cases}$$

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[9]With a weaker definition of compatible belief without this assumption, we would get a larger equilibrium set. Therefore, our definition can at most make it more difficult to get incomplete contracts in equilibrium. Moreover, being non-zero does not prevent probability to be arbitrarily close to zero (see the proof of Theorem 3.1 for the formal argument).

(iii) *For any* $C = (t, A, k)$, *and for any* $s \in A \cup S'$, $P_C^*(s) \neq 0$, $P_C^*(.|S') = \mu(.|S')$,

   and $P_C^* \in \Pi_C$ *whenever* $\Pi_C \neq \emptyset$.

In an equilibrium, given decision function D^* of the insuree, the insurer offers contract C^* that maximizes his expected utility. If there is any distribution that is compatible with a contract C then the equilibrium belief generated after that contract, P_C^*, has to be one of them. If there is no belief that is compatible with an offer, then the insuree is free to hold any belief. Lemma 6.1 in the Appendix shows that that kind of contracts will not be offered in equilibrium. There are always other contracts with compatible beliefs and they are more profitable for the seller. The insuree evaluates contract C by probability distribution P_C^*. She buys C if the expected utility of buying it is higher than that of rejecting it.

Under this definition, the insuree buys some contract in any equilibrium. To see this, consider the contract that is signed on S', and that fully insures the good against all the damages in S' and charges the premium which makes the insuree indifferent between buying and rejecting the offer. This is contract $C = (t(s) = s, S', k)$ where k solves $u(v - k) = \sum_{s \in S'} u(v - s)\mu(s|S')$. The insuree accepts this offer, and the insurer's expected utility from this contract is positive since u is concave. By existence of such an acceptable and profitable contract, the equilibrium contract has to be bought by the insuree.

Theorem 3.1. *There always exists an equilibrium where the equilibrium contract does not extend the awareness of the insuree.*

The proof of existence of an equilibrium with an incomplete contract is constructive and given in the Appendix. It is shown that equilibrium beliefs can be constructed so that the best acceptable contract that the insurer can offer is signed on S'. The idea goes as follows: For any contract that corresponds to a non-empty set of compatible beliefs, set the belief so that either the insuree rejects the offer or if she accepts it,

then it is not beneficial for the insurer to offer this contract rather than the contract suggested by the theorem. For any contract C that leads to empty set of compatible beliefs, there is always a contract that has non-empty compatible belief set and dominates C. Therefore, any probability distribution can be equilibrium belief to those off equilibrium offers. By this belief construction, in equilibrium the insurer offers a contract on S' and provides full insurance on the elements of S' and sets the premium at the level which makes the insuree indifferent between buying or rejecting this offer.

The definition of equilibrium puts minimum restriction on the belief held after each contract. It only requires equilibrium beliefs to be compatible whenever it is possible. The insuree knows that the insurer is an expected utility maximizer. When a contract is offered in an equilibrium, the insuree may ask herself if this is the best offer for the insurer. The example below illustrates a situation where the insuree cannot understand why the insurer offered the contract suggested by an equilibrium.

Example 3.1. *Let $S = \{100, 900\}$, $S' = \{100\}$, $v = 1000$, $u(x) = \sqrt{x}$, $\mu(\{100\}) = 0.99$, $\mu(\{900\}) = 0.01$.*

For contract $C^* = (t^*(s) = s, \{100, 900\}, k^* = 895.96)$ where $k^* = 895.96$ solves $u(v-k^*) = 0.01u(v-100)+0.99u(v-900)$, define $P^*_{C^*}(\{100\}) = .01$ and $P^*_{C^*}(\{900\}) = .99$. Observe that $P^*_{C^*}$ is compatible with C^*. For any contract $C \neq C^*$, define P^*_C as in the construction of the proof of Theorem 3.1 so that $\tilde{C} = (t(s) = s, S', k = 100)$ is a better contract for the insurer than any $C \neq C^*$. Then there are two candidates for equilibrium contract under this belief construction: C^* and $\tilde{C}$.

$EU_1(\tilde{C}, buy) = 100 - .99(100) = 1$ and

$EU_1(C^*, buy) = k^* - .99(100) - .01(900) = k^* - 108 = 787.96$.

Therefore, the insurer will offer C^* in equilibrium and $(C^*, D^*, (P^*_C)_{C \in \mathbb{C}})$, where D^* is defined as in point (ii) of Definition 3.2, is an equilibrium of this problem.

In the example above, the equilibrium contract charges a high premium but makes

16

small transfer in expectation since $\{900\}$ is a very unlikely event in reality. However, the belief that is held after the equilibrium contract assigns a high probability to event $\{900\}$ and hence, the insuree buys such a high premium offer. According to the insuree, the equilibrium contract promises a large transfer on a very likely event. Since this event was not conceptualized originally by the insuree, under her equilibrium belief she cannot reason why the insurer did not hide that event from her. According to the insuree, the expected utility of the insurer from the equilibrium contract is

$$EU_1^0(C^*, D^*(C^*)|P_{C^*}^*) = k^* - .01(100) - .99(900) = 3.96$$

However, after hearing the equilibrium offer, the insuree thinks that the insurer could have made

$$EU_1^0(\tilde{C}, D^*(\tilde{C})|P_{C^*}^*) = 100 - .01(100) = 99$$

by hiding event $\{900\}$ and offering $\tilde{C}$. So, with respect to the insuree's belief, the insurer is not maximizing his expected utility at the equilibrium offer C^*.

The refinement introduced below eliminates this kind of equilibria. It imposes that with respect to the belief held by the insuree, the equilibrium contract should be the best one for the insurer among all the contracts that the insuree can think of. After hearing the equilibrium offer, the insuree can consider only the contracts that would extend her awareness less than the equilibrium contract.

Definition 3.3. *An equilibrium* $(C^* = (t^*, A^*, k^*), D^* : \mathbb{C} \to \{buy, reject\}, (P^*)_{C \in \mathbb{C}})$ *is consistent if* $\forall C = (t, A, k) \in \mathbb{C}$ *such that* $A \cup S' \subseteq A^* \cup S'$

$$EU_1^0(C^*, D^*(C^*)|P_{C^*}^*) \geq EU_1^0(C, D^*(C)|P_{C^*}^*).$$

Corollary 3.1. *There always exists a consistent equilibrium where the signed contract is incomplete.*

This corollary is an immediate implication of Theorem 3.1 because the theorem states that signing a contract only on S' is always a part of some equilibrium. Observe that if the equilibrium contract does not inform the insuree about any new contingencies, then that equilibrium is trivially consistent. Therefore, the equilibrium suggested by the statement of the theorem is consistent.

If the insuree is initially considering only the high cost contingencies, the insurer has no incentive to extend the insuree's awareness. So only incomplete contracts will be signed in the equilibrium. The following example points out a more interesting situation. It shows that even if the insuree is aware of the least costly damage initially, it is possible to have all the consistent contracts being incomplete.

Example 3.2. *Let $S = \{8.79, 9\}$, $S' = \{8.79\}$, $v = 10$, $u(x) = \sqrt{x}$, $\mu(\{8.79\}) = 0.01$, $\mu(\{9\}) = 0.99$. We show that a contract in the form of $C = (t, S, k)$ cannot be part of a consistent equilibrium. For contradiction assume that it can be. Let P_C^* be the equilibrium belief with $P_C^*(\{8.79\}) = p$ and $P_C^*(\{9\}) = 1 - p$, where $p \in (0, 1)$. Since C is bought, it needs to satisfy*

$$p\sqrt{10 - 8.79 - k + t(8.79)} + (1 - p)\sqrt{10 - 9 - k + t(9)}$$
$$\geq p\sqrt{10 - 8.79} + (1 - p)\sqrt{10 - 9}$$

Then, since u is concave and $p \in (0, 1)$, we have

$$.21p + 1 - k + pt(8.79) + (1 - p)t(9) > (1 + .1p)^2 \qquad (1)$$

Consider $C' = (t'(8.79) = 8.79, S', k' = 8.79)$. If it was offered, the insuree would buy C' since she would be indifferent between buying or rejecting it.

Since C is assumed to be part of a consistent equilibrium, it has to be the case that

$$EU_1^0(C, buy|P_C^*) = k - pt(8.79) - (1 - p)t(9) \geq 8.79 - p8.79 = EU_1^0(C', buy|P_C^*) \quad (2)$$

Equations (1) and (2) implies that $p > 1$ and this contradicts with p being a probability. Hence a consistent equilibrium contract of this example cannot be complete.

In order to have a complete contract in a consistent equilibrium, the following two points should be satisfied at the same time: a) the updated belief should assign small enough probability to the less costly event for acceptance; b) the updated belief should assign large enough probability to the less costly event for consistency. These two points cannot happen simultaneously for the given parameters of the example.

In our setting, incompleteness in the contractual form arises as a result of a strategic decision process. Although both complete and incomplete contracts are feasible, the incomplete ones are always signed in an equilibrium, but the complete ones may fail to arise in any equilibria.

4   Competition Promotes Awareness

We saw in the previous section that a monopolistic insurer who has superior awareness will possibly sign an incomplete contract. In this section we study if the contracts offered by competing insurers reveal more contingencies. The answer is affirmative and competition indeed promotes awareness.

In standard insurance settings where asymmetric awareness is not an issue, symmetric firms compete over premia. They offer a zero profit contract which is beneficial for the insuree. In our setting, when we introduce competition on the insurers' side, there are two dimensions that the insurers can compete over in their offers: premium and awareness of the insuree. A competing insurer can make a counter offer by either decreasing the premium or by further extending the awareness of the insuree. We see that competition is an instrument under which not only the insuree can get the cheaper offer but also her unawareness may totally disappear.

Assume there are N risk neutral insurers. All of them are aware of S and believe μ. The risk averse insuree (indexed by 0) is only aware of S', and she believes $\mu(.|S')$ as before. The awareness structure between the insurers and the insuree is the same as in the previous sections. The insuree knows that the insurers are symmetric agents. The insurers make simultaneous offers denoted by $C_i = (t_i, A_i, k_i) \in \mathbb{C}$ for $i = 1, ..., N$. Vector $\mathbf{C} = (C_1, ..., C_N)$ is the collection of insurers' offers. The collection of contracts offered by all insurers except insurer i is denoted by $\mathbf{C}_{-i} = (C_1, ..., C_{i-1}, C_{i+1}, ..., C_N)$.

The offers are exclusive and the insuree may accept, at most, one of the offers or may reject all. The decision of the insuree is denoted by a function $D : \underset{i=1,...,N}{\times} \mathbb{C} \to \{buy_1, ..., buy_N, reject\}$.

For $i = 1, ..., N$, and given the decision function, D, of the insuree, the expected utility of insurer i when the vector of offers is $\mathbf{C}$ is:

$$EU_i(\mathbf{C}, D(\mathbf{C})) := \begin{cases} k_i - \sum_{s \in A_i} t_i(s)\mu(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

When $\mathbf{C} \in \underset{i=1,...,N}{\times} \mathbb{C}$ is offered, the insuree aggregates the information from each contract in $\mathbf{C}$. Then she generates her belief which is a probability distribution on her extended awareness set, $(\underset{i=1,...,N}{\cup} A_i) \cup S'$.

The expected utility of insurer i according to the insuree with respect to belief $P_\mathbf{C}$ is given by

$$EU_i^0(\mathbf{C}, D(\mathbf{C})|P_\mathbf{C}) := \begin{cases} k_i - \sum_{s \in A_i} t_i(s)P_\mathbf{C}(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

and the expected utility of the insuree from vector of offers $\mathbf{C}$ with respect to $P_\mathbf{C}$ is:

$$EU_0(\mathbf{C}, D(\mathbf{C})|P_\mathbf{C}) := \begin{cases} \sum\limits_{s \in A_i} u(v - s + t_i(s) - k_i)P_\mathbf{C}(s) \\ \qquad + \sum\limits_{\substack{s \in ((\underset{\substack{j=1,\dots,N \\ j \neq i}}{\cup} A_j) \cup S') \setminus A_i}} u(v - s - k_i)P_\mathbf{C}(s) & \text{if } D(\mathbf{C}) = buy_i \\ \\ \sum\limits_{s \in (\underset{i=1,\dots,N}{\cup} A_i) \cup S'} u(v - s)P_\mathbf{C}(s) & \text{if } D(\mathbf{C}) = reject \end{cases}$$

We will build on the belief formation idea discussed in Section 3 and focus on compatible beliefs when awareness is extended.

Definition 4.1. *A probability distribution $P_\mathbf{C} \in \Delta((\underset{i=1,\dots,N}{\cup} A_i) \cup S')$ is compatible with the vector of contracts $\mathbf{C} = (C_1, \dots, C_N)$ where $C_i = (t_i, A_i, k_i) \in \mathbb{C}$ for $i = 1, \dots, N$, if it satisfies:*

(i) $EU_i^0(\mathbf{C}, buy_i|P_\mathbf{C}) \geq 0$, *i.e.* $k_i \geq \sum\limits_{s \in A_i} t_i(s)P_\mathbf{C}(s)$;

(ii) *For any $s \in (\underset{i=1,\dots,N}{\cup} A_i) \cup S'$, $P_\mathbf{C}(s) \neq 0$ and $P_\mathbf{C}(.|S') = \mu(.|S')$*

   The set of all probability distributions that are compatible with $\mathbf{C}$ is denoted by $\Pi_\mathbf{C}$.

Definition 4.2. *An equilibrium of this contractual model with competition is a triplet*

$$(\mathbf{C}^*, D^* : \underset{i=1,\dots,N}{\times} \mathbb{C} \to \{buy_1, \dots, buy_N, reject\}, (P_\mathbf{C}^*)_{\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}}) \text{ such that}$$

(i) $C_i^* \in \underset{C_i \in \mathbb{C}}{\arg\max} EU_i(C_i, \mathbf{C}_{-i}^*, D^*(C_i, \mathbf{C}_{-i}^*))$ *for $i = 1, \dots, N$;*

(ii) *For any $\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}$,*

$$D^*(\mathbf{C}) = \begin{cases} buy_i & \text{if } EU_0(\mathbf{C}, buy_i|P_\mathbf{C}^*) \geq EU_0(\mathbf{C}, buy_j|P_\mathbf{C}^*) \ \forall j \neq i \\ & \qquad EU_0(\mathbf{C}, buy_i|P_\mathbf{C}^*) \geq EU_0(\mathbf{C}, reject|P_\mathbf{C}^*) \\ reject & otherwise \end{cases}$$

   where $\Pi_\mathbf{C}$ is defined as in Definition (4.1).

(iii) *For any* $s \in (\underset{i=1,...,N}{\cup} A_i) \cup S'$, $P_{\mathbf{C}}^*(s) \neq 0$, $P_{\mathbf{C}}^*(.|S') = \mu(.|S')$, *and* $P_{\mathbf{C}}^* \in \Pi_{\mathbf{C}}$
whenever $\Pi_{\mathbf{C}} \neq \emptyset$.

We focus on equilibria such that if the insuree is indifferent between two or more offers which are better than rejecting everything, then she picks one of them with equal probability. If she is indifferent between accepting the best offer or rejecting all the offers, she accepts it. We did not put this in the definition formally to ease the notation.

The solution concept is analogous to the definition of equilibrium of a single insurer case. Here, each insurer decides on his offer optimally. The insuree buys the offer of insurer i if it gives the highest expected utility among all the other offers she sees, and if buying from insurer i is better than rejecting all the offers. The insuree holds compatible beliefs whenever it is possible.

Theorem 4.1. *There is a symmetric equilibrium with competition where each insurer offers the same zero profit, full insurance, and complete contract, i.e. for any* $i = 1, ..., N$, $C_i^* = (t^*(s) = s, S, k^*)$ *such that* $k^* = \sum\limits_{s \in S} s\mu(s)$.

The equilibrium given in Theorem 4.1 is a zero profit equilibrium which fully extends the awareness of the insuree. This result shows that competition is an instrument that can achieve complete contracts in equilibrium. Competition can also increase the surplus of the buyer from trade by not only decreasing the premium but also extending her awareness. The first step in the proof shows that while all the insurers except i is offering the suggested contract, for any offer with positive profit that i may make, the true distribution μ is compatible. Then we show that no insurer can benefit from deviating to another contract rather than offering C_i^*. The details of the proof are given in the Appendix.

Remark: *The equilibrium described in Theorem 4.1 together with the beliefs that are defined in the proof to support it is consistent. This means that, according to the*

insuree, no insurer could do better by offering another contract given the offers of the others. The reason is that after hearing the equilibrium offers, the insuree learns all the contingencies and μ is her belief. Therefore, she can make the same reasoning as any of the insurers regarding the best response function of insurers.

A natural question at this point is whether the initial awareness of the insuree is always extended in any equilibrium under competition. The answer depends on the relationship between the contingencies that the insuree foresees and unforesees initially. The next result, Theorem 4.2, provides a sufficient condition in order to have extension of awareness of the insuree at any equilibrium.

Theorem 4.2. *The initial awareness of the insuree is extended in any symmetric equilibrium if there exists a contingency $\widetilde{s} \in S \backslash S'$ such that $u(v - \widetilde{s}) \leq \sum_{s \in S'} u(v - s)\mu(s|S')$.*

In order to prove this, we need to show that strategies where all the insurers are only covering the contingencies on S' cannot be part of any equilibrium as long as there is an unforeseen and relatively high damage contingency. If each insurer is choosing not to extend the awareness of the insuree, then this can only be a zero profit contract (otherwise, one would decrease the premium a little without extending the awareness of the buyer and attract the insuree fully). While others are offering such a zero profit contract, one insurer, say i, will always prefer to announce an unforeseen and relatively high damage contingency. By setting a premium that makes the offer not only profitable but also acceptable, such an insurer can be better off. Such an offer will be accepted by the buyer because she will assign even higher weight than the true weight to the new contingency to justify that announcement. Such a high weight on the new contingency will make the insuree prefer a coverage at that possible event and reject the offers of the other insurers.

5   Discussions

Form of Contracts: We focus on contracts that specify a premium and a transfer rule on the contingencies that the insurer announces. One may suggest two other types of contracts that are excluded in our set of feasible contracts. One of them is the type of contract which, in addition to a premium and a transfer rule, suggests a probability distribution on the contingencies mentioned in the contract. The insurer, who offers the contract, has no incentive to announce the true probability. Therefore, he cannot convince the insuree to believe the suggested distribution. Hence, the insuree would behave as she would without the suggestion.

Another type of contract that one may think of can have a clause such as *anything not specified here is excluded by this contract.* Observe that in our setting the complement of a set of contingencies is not the same set for each agent. Therefore, the statement *anything not specified here* does not refer to the same set of contingencies by the insurer and by the insuree. Indeed, if the contract already mentions everything that the insuree is initially aware of then according to the insuree there is nothing excluded in the contract. So, she will not take that clause into account in her evaluation process. Both the contracts which have this clause and the ones without it give the same payoff to the insurer since in our model no transfer takes place if some unspecified contingency is realized. Hence having these contracts in the feasible set would not change the results.

Zero probability: One general critique to awareness literature is about distinguishing unforeseen and zero probability events. Note that in the current model, the insuree's initial expected utility calculation would be the same if we assumed that she believed that anything in $S \setminus S'$ is zero probability. The main point is that if the insuree discovers that she was wrong about the support of uncertainty initially and the source of this information is a profit maximizing insurer, how updating of the

belief should take place. Our definitions of compatible and consistent beliefs carry over the idea of forward induction in the standard theory.

Although this paper doesn't attempt to provide any concrete discussion on zero probability vs unawareness problem, one can see that these two may lead to different behaviors in our setup. For example, imagine that there are two damages, High (H) and Low (L). The model where insuree is unaware of H but aware of L can explain the following behavior: a) The insuree accepts the contract that covers only L and charges a premium equal to the cost of L; b) she is willing to pay more than L if the contract covers L and H. This behavior cannot be explained if the insuree is aware of everything but assigning zero probability to H. If the insuree is assigning zero probability then she is willing to pay at most L independent of whether H is covered or not. Here, when the initial asymmetry between insuree and insurer is about holding non-common beliefs, there cannot be any updating after any offer because when the insurer makes an offer, he does not know the realization of the uncertainty. One may further argue that perhaps the asymmetry is due to incomplete information regarding the distribution and a contract signals something about the information of the insurer. Again, this alternative modeling cannot explain the above behavior. In order to behave as in (a), she has to assign zero probability to H according to all the distributions she has in mind, i.e. she must believe that with probability 1 the probabilities of L and H are 1 and 0, respectively. Hence, she cannot update her belief on distributions such that she will assign positive probability to H in order to present behavior (b).

Another alternative model might have been the following: There are two types of insurers. If the insurer is type θ_1, then H is a zero probability event and if the insurer is type θ_2, then H is a positive probability event. Suppose the insuree's prior puts probability 1 on type θ_1. Assume there exists a separating equilibrium where θ_1

offers the contract in (a), and θ_2 offers the contract in (b). The insuree expects to see the contract in (a). If we use an equilibrium notion that has no restriction on how to generate the posterior belief on zero probability histories, can we explain a behavior as in (b)? The answer is "No". If this is the strategy of the insuree, such a separating equilibrium cannot exist since type θ_1 would mimic type θ_2. For type θ_1, charging a premium more than L (the premium of type θ_2) and paying only L when it realizes is better than charging only L (since L occurs with probability 1 in case of θ_1).

Dealing with Knightian Uncertainty: The solution concept introduced in Section 3 leaves the insuree free in picking her equilibrium belief among all the compatible ones after hearing an offer. One may argue that the insuree is facing with Knightian Uncertainty, a type of uncertainty where the decision maker is unable to assign probability. In the literature, typically multiple belief models are used to study Knightian Uncertainty and ambiguity aversion or maxmin expected utility of Gilboa and Schmeidler (1989) are among the most celebrated ones. The idea is that the decision maker evaluates a situation by the worst scenario she can think of.

If we adopt this form of extreme pessimism while modeling our insuree, then we can fully characterize the set of equilibrium contracts in monopoly and competitive cases. A single insurer either does not extend the awareness of the insuree at all, or he informs the insuree only about a relatively costly damage. The intuition is simple, if there is an initially unforeseen and costly contingency and if it is not very likely, then the insurer would like to announce it. When the insuree hears about it, she will think that it is very probable (because she is pessimistic) and will be willing to pay high premium to be covered at that contingency. Since that contingency is not likely in reality, the insurer will make profit from this strategy. If such a contingency does not exist, then the equilibrium contract will not deliver any new information.

The unique equilibrium under ambiguity aversion is mostly an incomplete con-

tract and it has the most severe form of incompleteness, i.e. it either does not extend the awareness of the insuree at all or it does it over a single contingency. However, introducing competition in that set up eliminates any incomplete contract equilibrium. If the size of competition is large enough and the insuree is assigning at least the true probability to the initial awareness set in her updating, then complete contract is the unique equilibrium outcome under competition and ambiguity aversion. An earlier version of the current study (Filiz-Ozbay (2008)) has a section on modelling the insuree as an ambiguity averse agent and states the aforementioned results more formally.

6   Conclusion

In this paper, we show that, if unawareness of buyers is an issue, then the insurance companies can use it to their advantage. We argue that, even if complete contracts are feasible, there are situations where only incomplete ones can emerge for strategic reasons.

Conflicts between contracting parties due to expost recognition of the incompleteness of contracts are difficult challenges for the courts. It is hard to prove ex-post that some party left the contract incomplete intentionally. We offer competition as an instrument that does not require such a super power to achieve complete contracts. Competition among insurers with superior awareness can achieve complete contracts in equilibrium. Moreover, if there exists an unforeseen and relatively costly damage, the awareness of buyers is always extended in any equilibrium with competition.

Our model is a starting point which relaxes a strong assumption in contract theory. It is a realistic exercise to allow for agents who take into account different aspects of an economic situation. The tools developed here can be used for models where the insuree has superior awareness. Additionally, modeling more complicated contrac-

tual situations where moral hazard or adverse selection is also an issue would be an insightful research question.

Appendix

Lemma 6.1. *Any equilibrium offer C^* in the sense of Definition 3.2 has non-empty compatible belief set.*

Proof of Lemma 6.1. There are two possibilities: An equilibrium offer may or may not extend the awareness of the insuree. If the equilibrium offer is on S', then it has to be $C^* = (t^*(s) = s, S', k^*)$ where $u(v - k^*) = \sum_{s \in S'} u(v - s)\mu(s|S')$ because this is the most profitable contract among the contracts that does not extend the awareness of the insuree. Note that this contract has a non-empty compatible belief set, indeed $\mu(.|S')$ will be the belief held.

Next assume that an equilibrium offer $C^* = (t^*, A^*, k^*)$ is extending the awareness of the insuree but $\Pi_{C^*} = \emptyset$. Then for any $P \in \Delta(A^* \cup S')$,

$$k^* < P(S') \left[\sum_{s \in S'} t^*(s)\mu(s|S') \right] + (1 - P(S')) \left[\sum_{s \in A^* \setminus S'} t^*(s)P(s|A^* \setminus S') \right]$$

This implies that $k^* \leq \sum_{s \in S'} t^*(s)\mu(s|S')$. Consider $\widetilde{C} = \left(\widetilde{t}(s) = s, S', \widetilde{k} = k^* + \sum_{s \in S'} (s - t^*(s))\mu(s) \right)$. Observe that

$$\widetilde{k} = k^* + \sum_{s \in S'} (s - t^*(s))\mu(s) \leq k^* + \sum_{s \in S'} (s - t^*(s))\mu(s|S') \leq \sum_{s \in S'} s\mu(s|S')$$

and this implies that

$$EU_0(\widetilde{C}, buy) = u(v - \widetilde{k}) \geq u\left(\sum_{s \in S'}(v - s)\mu(s|S')\right) \geq \sum_{s \in S'} u(v-s)\mu(s|S') = EU_0(\widetilde{C}, reject)$$

Therefore, $\widetilde{C}$ is accepted. Moreover, $\widetilde{C}$ is more profitable than C^* since $\widetilde{C}$ covers less contingencies and although it provides more transfer on those contingencies it's premium is high enough to compensate that additional cost. To see it mathematically:

$$
\begin{aligned}
EU_1(\widetilde{C}, buy) &= \widetilde{k} - \sum_{s \in S'} s\mu(s) = k^* + \sum_{s \in S'}(s - t^*(s))\mu(s) - \sum_{s \in S'} s\mu(s) \\
&= k^* - \sum_{s \in S'} t^*(s)\mu(s) > k^* - \sum_{s \in A^* \cup S'} t^*(s)\mu(s) = EU_1(C^*, buy)
\end{aligned}
$$

This contradicts with C^* being an equilibrium offer. Hence $\Pi_{C^*} \neq \emptyset$ in any equilibrium. $\square$

Proof of Theorem 3.1. Consider $C^* = (t^*(s) = s, S', k^*)$, where k^* solves $u(v - k^*) = \sum_{s \in S'} u(v - s)\mu(s|S')$. Observe that C^* is accepted by the definition of equilibrium. The expected utility of the insurer from C^* is positive by concavity of u. Moreover, C^* is the best offer for the seller among the contracts that do not extend the awareness of the buyer. Next, we will form equilibrium beliefs so that any contract that extends the awareness of the buyer will be either rejected or not preferred by the seller since it will be less profitable than C^*.

Consider $C = (t, A, k)$ such that $A \backslash S' \neq \emptyset$. If $\Pi_C \neq \emptyset$, then we have the following cases:

Case 1: If $k > \sum\limits_{s \in S'} t(s)\mu(s|S')$[10] then there exists an $\varepsilon_1 \in (0,1)$ such that

$$k > (1 - \varepsilon_1)\sum_{s \in S'} t(s)\mu(s|S') + \frac{\varepsilon_1}{m}\sum_{s \in A \setminus S'} t(s) \tag{3}$$

where m is the cardinality of $A \setminus S'$. Observe that for any $\varepsilon \in (0, \varepsilon_1]$, Inequality (3) holds.

Case 1.1: If $\sum\limits_{s \in S'} u(v - s + t(s) - k)\mu(s|S') < \sum\limits_{s \in S'} u(v - s)\mu(s|S')$
then there exists an $\varepsilon_2 \in (0,1)$ such that

$$(1 - \varepsilon_2)\sum_{s \in S'} u(v - s + t(s) - k)\mu(s|S') + \frac{\varepsilon_2}{m}\sum_{s \in A \setminus S'} u(v - s + t(s) - k) \tag{4}$$

$$< (1 - \varepsilon_2)\sum_{s \in S'} u(v - s)\mu(s|S') + \frac{\varepsilon_2}{m}\sum_{s \in A \setminus S'} u(v - s)$$

Observe that for any $\varepsilon \in (0, \varepsilon_2]$, Inequality (4) holds. Then for $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, both Inequalities (3) and (4) hold. Define probability distribution $Q_C \in \Delta(A \cup S')$ as

$$Q_C(s) := \begin{cases} (1 - \varepsilon)\mu(s|S') & \text{if } s \in S' \\ \frac{\varepsilon}{m} & \text{if } s \notin S' \end{cases}$$

From Inequality (3), Q_C is compatible with C. Set $P_C^* := Q_C$. By Inequality (4), C is rejected therefore, it is worse than C^* for the insurer.

Case 1.2: If $\sum\limits_{s \in S'} u(v - s + t(s) - k)\mu(s|S') \geq \sum\limits_{s \in S'} u(v - s)\mu(s|S')$ then from concavity

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[10]Here, we abuse the notation and write it as if t is defined on $A \cup S'$ although it is only defined on A. However, since both agents are aware of S', they can interpret t as the transfer rule which transfers zero on $S' \setminus A$.

of u and definition of k^* we have

$$u\left[\sum_{s\in S'}(v-s+t(s)-k)\mu(s|S')\right] \geq \sum_{s\in S'}u(v-s+t(s)-k)\mu(s|S')$$

$$\geq \sum_{s\in S'}u(v-s)\mu(s|S') = u(v-k^*)$$

$$\sum_{s\in S'}(v-s+t(s)-k)\mu(s|S') \geq v-k^*$$

By rearranging the terms, we get $k^* - k \geq \sum_{s\in S'}(s-t(s))\mu(s|S') \geq \sum_{s\in S'}(s-t(s))\mu(s)$ or equivalently we have

$$EU_1(C^*, buy) = k^* - \sum_{s\in S'}s\mu(s) \geq k - \sum_{s\in S'}t(s)\mu(s) \geq k - \sum_{s\in A\cup S'}t(s)\mu(s)$$

$$= EU_1(C, buy)$$

Pick P_C^* as an arbitrary probability distribution that is compatible with C. Then, C is either rejected or accepted under P_C^*. Either case, C^* (which is an accepted offer) is at least as profitable as C for the insurer.

Case 2: If $k \leq \sum_{s\in S'}t(s)\mu(s|S')$ then observe that even if contract C is accepted by the insuree we have

31

$$
\begin{aligned}
EU_1(C, buy) &= k - \sum_{s \in S'} t(s)\mu(s) - \sum_{s \in A \setminus S'} t(s)\mu(s) \\
&\leq k - \sum_{s \in S'} t(s)\mu(s) \\
&\leq \sum_{s \in S'} t(s)\mu(s|S') - \sum_{s \in S'} t(s)\mu(s) \\
&= (1 - \mu(S')) \sum_{s \in S'} t(s)\mu(s|S') \\
&\leq (1 - \mu(S')) \sum_{s \in S'} s\mu(s|S') \\
&= \sum_{s \in S'} s\mu(s|S') - \sum_{s \in S'} s\mu(s) \\
&\leq k^* - \sum_{s \in S'} s\mu(s) = k^* - \sum_{s \in S'} t^*(s)\mu(s) \\
&= EU_1(C^*, buy)
\end{aligned}
$$

Therefore, the insurer cannot be better off by offering C rather than C^* independent of the belief held after offer C. So pick P_C^* as an arbitrary probability distribution that is compatible with C.

If $\Pi_C = \emptyset$, we know from the proof of Lemma 6.1 that there exists a $\widetilde{C}$ written on S' and $\widetilde{C}$ is strictly more profitable than C for the insurer. Since C^* is the best offer for the seller among the contracts that do not extend the awareness of the buyer, C cannot be a profitable deviation from C^* under any belief. Hence set P_C^* arbitrarily so that $P_C^*(.|S') = \mu(.|S')$ and $P_C^*(s) \neq 0$ for any $s \in A$.

By following the construction suggested in above cases, $(C^*, (P_C^*)_{C \in \mathbb{C}})$ defines equilibrium which is incomplete. □

Proof of Theorem 4.1. Let us assume that all the insurers except i offer the contract suggested by the theorem, i.e. $C_j^* = (t_j^*(s) = s, A_j^* = S, k_j^* = \sum_{s \in S} s\mu(s)) \ \forall j \neq i$. Next we will show that we can construct the equilibrium beliefs so that the best response

of i is to offer what others are offering which pays off zero to insurer i. For any offer, $C = (A, k, t) \neq C_j^*$, which gives positive profit to insurer i when it is accepted, set the belief $P_{(C, C_{-i}^*)}^* := \mu$. This is a possible belief because μ is compatible with (C, C_{-i}^*). Under this belief if such an offer is rejected then it cannot be a profitable deviation. If it is accepted then it has to be the case that

$$u(v - k_j^*) \leq \sum_{s \in A} u(v - k - s + t(s))\mu(s) + \sum_{s \in S \setminus A} u(v - k - s)\mu(s)$$

$$< u\left(v - k + \sum_{s \in A}(-s + t(s))\mu(s) + \sum_{s \in S \setminus A}(-s)\mu(s)\right)$$

then

$$k_j^* > k - \sum_{s \in A} t(s)\mu(s) + \sum_{s \in S} s\mu(s) > \sum_{s \in S} s\mu(s) = k_j^*$$

The last inequality above holds because C is assumed to be a positive profit offer if it is accepted. However, this leads to a contradiction. Therefore, under this off equilibrium belief formation, insurer i does not have any offer that is accepted and that provides positive profit. Hence, a best response of i is to follow the strategy of the others and to offer the complete, full insurance contract that makes zero profit.

In order to support this as an equilibrium, set $P_{\mathbf{C}}^* = \mu$ for any $\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}$ such that $\mu \in \Pi_{\mathbf{C}}$. For $\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}$ such that $\mu \notin \Pi_{\mathbf{C}}$ set $P_{\mathbf{C}}^*$ arbitrarily to be in line with Definition 4.2.

$\square$

Proof of Theorem 4.2. Let there be a contingency $\tilde{s} \in S \setminus S'$ such that $u(v - \tilde{s}) \leq \sum_{s \in S'} u(v - s)\mu(s|S')$. Next we will show that in any symmetric equilibrium, the awareness of the insuree is extended.

For contradiction, assume there is a symmetric equilibrium where $C_j^* = (t^*, S', k^*)$ for any $j = 1, \dots, N$. As it is well known from the standard insurance problem between a risk averse buyer and competing risk neutral sellers, this offer should be zero-profit and full coverage contract on S', i.e. $t^*(s) = s$ for any $s \in S'$ and $k^* = \sum_{s \in S'} t^*(s)\mu(s)$. Otherwise, if such an incomplete equilibrium leads to positive profit or provide partial

33

coverage, then one insurer can deviate to a full coverage contract on S' with some profitable premium. The deviation premium level can be set so that the risk averse insuree accepts it.

Define k that solves

$$u(v - k) = \max\{\mu(S')u(v - k^*) + (1 - \mu(S'))u(v - k^* - \widetilde{s}) \, ,$$
$$\mu(S') \sum_{s \in S'} u(v - s)\mu(s|S') + (1 - \mu(S'))u(v - \widetilde{s})\}$$

Now consider $C_i = (t(s) = s, S' \cup \{\widetilde{s}\}, k)$, and show that offering C_i is a profitable deviation for insurer i independent from the belief held in an equilibrium. Note that if it is accepted by the insuree, C_i is a positive profit contract and therefore a profitable deviation. Too see it formally,

$$u(v - k) < \max\{u\left(v - k^* - (1 - \mu(S'))\widetilde{s}\right) \, , \, u(v - \sum_{s \in S'} s\mu(s) + (1 - \mu(S'))\widetilde{s})\}$$
$$= u\left(v - \sum_{s \in S'} s\mu(s) - (1 - \mu(S'))\widetilde{s}\right) \tag{5}$$

and this implies that $k > \sum_{s \in S' \cup \{\widetilde{s}\}} s\mu(s)$ and hence

$$EU_i(C_i, \mathbf{C}^*_{-i}, buy_i) = k - \sum_{s \in S' \cup \{\widetilde{s}\}} s\mu(s) > 0 = EU_i(C^*_i, \mathbf{C}^*_{-i}, buy_i)$$

Next, we show that under any equilibrium belief construction, C_i is accepted. A belief $P \in \Delta(S' \cup \{\widetilde{s}\})$ is compatible with C^*_j for $j \neq i$, if

$$k^* = \mu(S') \sum_{s \in S'} t^*(s)\mu(s|S') \geq P(S') \sum_{s \in S'} t^*(s)\mu(s|S')$$

Since $\exists s \in S'$ such that $s \neq 0$, $\sum_{s \in S'} t^*(s)\mu(s|S') \neq 0$. Hence, by the inequality above, for any $P \in \Pi_{(C_i, \mathbf{C}^*_{-i})}$ $P(S') \leq \mu(S')$. Aside, observe that $Q \in \Delta(S' \cup \{\widetilde{s}\})$

defined by

$$Q(s) = \begin{cases} \mu(s) & if \ s \neq \widetilde{s} \\ 1 - \mu(S') & if \ s = \widetilde{s} \end{cases}$$

is compatible with (C_i, C^*_{-i}) since $Q(S') \leq \mu(S')$ and by Inequality (5), $k \geq \sum\limits_{s \in S' \cup \{\widetilde{s}\}} sQ(s)$.
Therefore, $\Pi_{(C_i, \mathbf{C}^*_{-i})} \neq \emptyset$. Then for any $P \in \Pi_{(C_i, \mathbf{C}^*_{-i})}$,

$$
\begin{aligned}
EU_0(C_i, \mathbf{C}^*_{-i}, buy_i | P) &= u(v - k) \\
&\geq \mu(S') \sum_{s \in S'} u(v - s)\mu(s|S') + (1 - \mu(S'))u(v - \widetilde{s}) \\
&\geq P(S') \sum_{s \in S'} u(v - s)\mu(s|S') + (1 - P(S'))u(v - \widetilde{s}) \\
&= EU_0(C_i, \mathbf{C}^*_{-i}, reject | P)
\end{aligned}
$$

The second inequality above holds since $\sum\limits_{s \in S'} u(v - s)\mu(s|S') \geq u(v - \widetilde{s})$ by assumption
and $P(S') \leq \mu(S')$. Aside,

$$
\begin{aligned}
EU_0(C_i, \mathbf{C}^*_{-i}, buy_i | P) &= u(v - k) \\
&\geq \mu(S')u(v - k^*) + (1 - \mu(S'))u(v - k^* - \widetilde{s}) \\
&\geq P(S')u(v - k^*) + (1 - P(S'))u(v - k^* - \widetilde{s}) \\
&= EU_0(C_i, \mathbf{C}^*_{-i}, buy_j | P) \quad for \ any \ j \neq i
\end{aligned}
$$

The second inequality above holds since $u(v - k^*) \geq u(v - k^* - \widetilde{s})$ and $P(S') \leq \mu(S')$.

Therefore, for any belief that insuree holds in any equilibrium, she will buy insurance C_i. Recall that we have already showed above that C_i is a positive profit contract when it is accepted although C^*_i is a zero-profit offer. Therefore, if no insurer extends the awareness of the insuree, there exists a profitable deviation from that strategy. Hence there cannot be any symmetric equilibrium with no announcement. $\qquad\square$

7 References

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