Incorporating Unawareness into Contract Theory[*]

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Abstract

Asymmetric awareness of the contracting parties regarding the uncertainty surrounding them is proposed as a reason for incompleteness in contractual forms. An insurance problem is studied between a risk neutral insurer, who has superior awareness regarding the nature of the uncertainty, and a risk averse insuree, who cannot foresee all the relevant contingencies. The insurer can mention in a contract some contingencies that the insuree was originally unaware of. It is shown that there are equilibria where the insurer strategically offers incomplete contracts. Next, equilibrium contracts are fully characterized for the case where the insuree is ambiguity averse and holds multiple beliefs when her awareness is extended. Competition among insurers who are symmetrically aware of the uncertainty promotes awareness of the insuree. [JEL Classification: D83, D86]

Keywords: Asymmetric Awareness, Insurance

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1 Introduction

In a world where insurance companies spend a lot of resources to compute the facts that are material to the risk, the relevant contingencies lie largely in the knowledge of the insurers. Insurance companies which have been in the industry for a long time may have a better understanding of the realities of nature than an insurance buyer. The buyers trust the insurance companies and proceed upon the confidence that the companies do not hold back any circumstances in their knowledge to mislead the judgement of the buyers. Moreover, policies are usually drafted by insurers, giving them a strong opportunity to manipulate (see Harnett, B. (1950)). This asymmetry between the insurance buyer and seller in foreseeing all the relevant contingencies is the key reason for expost conflicts. However, the standard contracting models do not allow for agents having asymmetric awareness regarding the nature of the uncertainty. This paper incorporates unawareness in contractual settings in order to understand how insurers use their superiority in terms of understanding the relevant contingencies against buyers. It questions whether such an insurer will mention in the contract those contingencies that the insuree does not foresee originally or he will remain silent on them. Moreover, if the insuree reads a clause about a contingency that did not cross her mind initially, how she evaluates this information is part of the solution concept we propose. Finally, we search for an instrument that leads to disclosure of all the unforeseen contingencies.

We address these questions by generalizing an insurance setting between an insurer (he) and an insuree (she) such that each agent may take into account a different set of contingencies. We call these subjective sets of contingencies awareness sets. When the insuree reads a contract offered by the insurer, she may become aware of some new aspects of the uncertainty and start taking them into account. For example, a home insurance buyer who has never thought about a tsunami before becomes aware of it

when the contract offers insurance against tsunami as well. Hence, the contract can be used as a communication device by the insurer in order to extend the awareness of the insuree.

If reading a contract adds new contingencies for the consideration of the insuree, the question is how she is going to assign probabilities to the new contingencies in order to evaluate them. In this study, a priori, there is no imposition on how the insuree generates a belief when her awareness is extended. Belief formation of the insuree is a part of the equilibrium concept. We require progressively more restrictions on belief formation. We start with compatible belief, then we will consider consistent beliefs. The definitions of these concepts will be given and discussed extensively in the paper. Roughly, we call a belief *compatible with a contract* if, with respect to this belief, the insuree thinks that the insurer is better off by making this offer rather than staying out of business. We require equilibrium beliefs to be compatible with the corresponding contracts whenever it is possible. Under this solution concept, we show that hiding some contingencies from the insuree is always part of some equilibria while mentioning all the possible contingencies may not be. Next, we refine this possibly large equilibrium set with a consistency requirement. A belief held after a contract is *consistent* if the contract is the best one for the insurer according to the insuree with respect to this belief.

In this setup, the contract that mentions an unforeseen contingency and promises zero coverage when it materializes and the contract that does not mention that contingency at all are different. Since the first one provides a complete list of relevant contingencies and the second one fails to do so, the second one is called incomplete. Complete and incomplete contracts correspond to different awareness sets of the insuree, and therefore, their subjective evaluations are not the same. If an incomplete contract is agreed to, then experiencing that contingency and learning that the dam-

age is not covered by the contract is an expost surprise for the insuree. In reality, in such situations insurees feel deceived and go to court. Although it is the role of the court to protect the deceived ones, and apply the doctrine of concealment, it still needs to be proved that the insurer intentionally left the contract incomplete. This is not an easy task since the subjective status of the insurer needs to be determined objectively. This is the main reason for the debate on the doctrine of concealment in law literature (see Harnett, B. (1950) and Brown, C. (2002)). We argue that this problem is due to monopolistic power of insurance provider and show that competition among insurance companies is an instrument to reach complete contracts in equilibrium. Even for the most severe type of incompleteness which arises under ambiguity aversion, competition promotes awareness.

The insurer would like to charge the highest possible premium with the least coverage in return. If the insurer announces a contingency which leads to a very costly damage on the good and if the insuree happens to believe that this is a very likely contingency, then she is willing to pay a high premium. Moreover, if this contingency is not that likely in reality, then providing coverage in such an event is not costly for the insurer in expectation. Such a contract generates the most severe type of incompleteness for the insuree. Intuitively, this type of evaluation by the insuree fits well into the behavior of a pessimist agent. Indeed, this intuition is verified when we use pessimism as a belief selection process among the compatible beliefs in Section 4 by modeling ambiguity averse insuree (see Gilboa, I. and Schmeidler, D. (1989)). Ambiguity is studied in the context of contractual incompleteness by Grant, S. H., Kline, J. and Quiggin, J. (2006) and Mukerji, S. (1998) and in the context of incomplete financial markets by Mukerji, S. and Tallon, J.-M. (2001). In this paper, we study ambiguity aversion in a contractual setting where unawareness is also present.

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Considering ambiguity aversion not only leads to a unique optimal contract but also seems in line with advertising strategies of some insurance providers. For example, Dell Inc. announces that its warranty will cover against costly accidental damages, such as liquid spill on computers or power surges, but does not say anything about some cheaper damages such as logical errors.

The rest of the paper is organized as follows: Next, we discuss the related literature. In Section 2, we introduce the one insurer-one insuree model and necessary notation. In Section 3, we give an equilibrium concept and study the form of equilibrium contracts that can arise in this setting. Then in Section 4, we model the insuree as a pessimist agent who is unable to hold single belief after each contract but instead exhibits an ambiguity averse behavior on a multiple belief set. We see that the equilibrium contracts in this case are always incomplete if the insuree is unaware of at least two contingencies. By introducing competition between insurers in Section 5, we show that the unawareness of the insuree can totally disappear under competition. In Section 6, we discuss some key points in the construction of our model: other forms of contracts besides the ones we study, the difference between being unaware of an event and assigning zero probability to that event, and robustness of the equilibrium concept under ambiguity aversion. We conclude in Section 7. All the proofs are presented in the Appendix.

Related Literature

Unawareness is first studied in economic theory by Modica, S. and Rustichini, A. (1994). In the literature, there are some recent developments in modeling unawareness. Unawareness models by Heifetz, A., Meier, M. and Schipper, B. C. (2006) and Li, J. (2006) are the basis of the unawareness concept we use in this paper (see Ozbay, E. Y. (2008) for more formal connection). In those models, each agent can

take into account a projection of the entire situation to the aspects that she is aware of. This set theoretic modeling of unawareness is incorporated into game theory by Halpern, J. Y. and Rego, L. C. (2006), Heifetz, A., Meier, M. and Schipper, B. C. (2007) and Ozbay, E. Y. (2008). As an application, our model is closer to Ozbay, E. Y. (2008), since we also have communication between agents regarding the nature of the uncertainty (through contracts in our case).

Standard economic theory has been developed within a paradigm that excludes unawareness. Recent studies addressed how accounting for unawareness changes the standard economic theory (see Modica, S., Rustichini, A. and Tallon, J.-M. (1998) and Kawamura, E. (2005) for applications in general equilibrium models).

Incomplete contracts are extensively studied in economics (see e.g. Hart, O. D. and Moore, J. (1990), Aghion, P. and Bolton, P. (1992), Grossman, S. J. and Hart, O. D. (1986), Bolton, P. and Whinston, M. D. (1993), Aghion, P. and Tirole, J. (1997), Hart, O. D. and Moore, J. (1998), Gertner, R. H., Scharfstein, D. S. and Stein, J. C. (1994), Hart, O. D. and Moore, J. (2005) and for a summary of this literature see Bolton, P. and Dewatripont, M. (2005) and Salanié, B. (2005). In this literature, the inability of contracting parties to foresee some aspects of the state of the world is frequently understood as a reason for the incompleteness of some contracts. However, this reasoning lead to well known discussions in the studies of Maskin, E. and Tirole, J. (1999), Tirole, J. (1999) and Maskin, E. (2002). They argued that in the models motivated by unforeseen contingencies, the parties are rational and able to understand the payoff related aspects of the state of the world, although they are unable to discuss the physical requirements leading to those payoffs. Our model is free from this inconsistency since here neither agents foresee some contingencies nor they are able to understand payoff related aspects of them. Tirole, J. (1999) states that the way *they currently stand*, unforeseen contingencies are not good motivation

for models of incomplete contracts, and he further notes that:

> "...there may be an interesting interaction between 'unforeseen contingencies' and asymmetric information. There is a serious issue as to how parties form probability distributions over payoffs when they cannot even conceptualize the contingencies..., and as to how they end up having common beliefs ex ante. ...[W]e should have some doubts about the validity of the common assumption that the parties to a contract have symmetric information when they sign the contract. ...Asymmetric information should therefore be the rule in such circumstances, and would be unlikely to disappear through bargaining and communication."

In line with the observation quoted above, in our model the agents cannot forecast the relevant contingencies symmetrically and they do not assign probabilities to those unforeseen contingencies. They are rational agents within their awareness set, but they are taking into account only the aspects of the uncertainty that they are able to conceptualize and ignore the rest.

Although the papers in the literature pertaining both to awareness and to incomplete contracts always refer to each other, there are not many studies that explicitly combine two strands of the theoretical literature.[1] Our study can be thought as one of the first attempts in contract theory which formally allows unawareness.

2 Model

There is a good owned by an agent. $v > 0$ is the value of the good for the agent. The good is subject to some uncertain future damages. The owner (insuree) wants

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[1]In an interesting study, Chung, K.-S. and Fortnow, L. (2006) model courts that make some "awareness check". Also Gabaix, X., and Laibson, D. (2006) provide contracting model for consumers who fail to anticipate certain future payments.

to be insured against realization of damages. Ω is the finite set of causes that lead to damages.[2] Elements of Ω are distributed according to μ. It is assumed that all the elements of Ω are possible, i.e. $\forall \omega \in \Omega,\ \mu(\omega) \neq 0$.

The insuree (she) is indexed by 0 and we assume that there is only one insurer (he) indexed by 1.[3] If a contingency is in an agent's state of mind while s/he is evaluating a situation, then we say that s/he is *aware* of that contingency. Otherwise, if the agent is unaware of a contingency, then s/he cannot take that contingency into account in the decision making process. The awareness structures of the insurer and the insuree are as follows:

- The insurer is aware of Ω and believes the distribution μ.

- The insuree is only aware of Ω', which is a proper subset of Ω such that $\mu(\Omega') > 0$. She believes the conditional distribution $\mu(.|\Omega')$.[4]

- The insuree is not aware of remaining realizations of damages in $\Omega \backslash \Omega'$ and she is not aware of the insurer's superior awareness. Therefore, initially the insuree believes that $(\Omega', \mu(.|\Omega'))$ describes the whole uncertainty that she and the insurer consider.

- The insurer knows that the insuree is considering only $(\Omega', \mu(.|\Omega'))$ and moreover, the insurer knows that the insuree is unaware that the insurer has superior awareness.

Damage levels are defined by a cost function $c : \Omega \longrightarrow \mathbb{R}_+$ where $c(\omega)$ is the damage level at $\omega \in \Omega$. Let S be the range of cost function, i.e. $c(\Omega) = S$. Although,

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[2]The tools we develop here can be easily modified for infinite Ω.

[3]Until Section 5 we assume that there is only one insurer. Then we will introduce competition in the model.

[4]The initial belief of the insuree does not have to be $\mu(.|\Omega')$ in order to derive our main points. If one starts with an arbitrary initial belief and defines the hierarchy of beliefs accordingly, the analysis can be carried over. In all the examples, we indeed consider singleton Ω'. Therefore, one can see that the nature of the results does not depend on initial belief being the true conditional distribution.

realistically contracts might be written on Ω, and the insuree might be unaware of some causes of damages but not the damage levels, which are nothing but real numbers, all that matters for agents are the damage levels, S. For notational simplicity we will consider the reduced form model and refer S and S' rather than Ω and Ω', where $S' = c(\Omega')$, as the relevant sets of contingencies and awareness sets.[5] With obvious abuse of notation we will use μ as the distribution on S.

Given this awareness structure, each party interprets the true problem as a projection of it onto the aspects of the uncertainty that s/he is aware of. Here S and S' are not state spaces in the sense of awareness literature and that is why we insist on calling them as sets of contingencies. In the literature a state also describes what a decision maker is aware of (or unaware of).[6] Here the insuree is unaware of some actions of the nature. However, it should be clear that what the insuree is unaware of is not just that. The insurer can come up with contracts that are based on the contingencies in $S\backslash S'$. The insuree is also unaware of those actions of the insurer. So if we think of the true game, the insuree is aware of only a part of this game that can be written only by referring to the contingencies in S'. This idea of being aware of a projection of the true game follows from Halpern, J. Y. and Rego, L. C. (2006), Heifetz, A., Meier, M. and Schipper, B. C. (2006), Heifetz, A., Meier, M. and Schipper, B. C. (2007) Li, J. (2006), Ozbay, E. Y. (2008).

The insurer offers a contract in order to insure the good against future damages. A typical contract is a specification of three objects:

(i) The contingencies on which a money transfer will be made from insurer to insuree;

(ii) The amount of transfer as a function of contingencies in (i);

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[5]The nature of the results of this paper would not change whether we work with Ω or S.

[6]The generalized state space for our model is not necessary for the analysis and therefore we do not define it.

(*iii*) The premium which is an in advance payment from insuree to insurer for the
agreement.

Definition 2.1. *A contract is a triplet $C = (t, A, k)$ where $A \subseteq S$, $t : A \longrightarrow \mathbb{R}_+$ is the transfer rule, and $k \in \mathbb{R}_+$ is the premium. The set of all contracts is denoted by $\mathbb{C}$.*

Note that Definition 2.1 does not restrict the set of contingencies that a contract can be written on. A can be any subset of S. If the contract is silent at some contingencies, it means that there will not be any transfer to the insuree when those contingencies are realized.[7] One critique to the use of incomplete contracts in the literature is that even if some contingencies are left open in the contract, each agent clearly knows what will happen if those contingencies realize. Therefore, in a sense, such contracts are still complete. In order to be free from this critique, an incomplete contract should leave some contingencies excluded in the evaluation of at least one agent. If initially foreseen contingencies are not specified in a contract, in this setup that contract does not qualify to be called incomplete. The insuree still knows the relevance of those contingencies and her utility if they realize. A contract is incomplete only if it leaves insuree unaware of some relevant contingencies. Therefore, the whole model needs to be known in order to call a contract incomplete.

Definition 2.2. *A contract $C = (t, A, k)$ is incomplete if $A \cup S' \neq S$.*

A contract may announce some contingencies that the insuree is not originally aware of, i.e. for a contract $C = (t, A, k)$, it can be the case that $A \backslash S' \neq \emptyset$. If such a contract is offered then the insuree becomes aware of those contingencies and her new understanding of the uncertainty enlarges to the aspects in $A \cup S'$. This means that there is no language barrier and the insuree is capable of understanding the content

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[7]We will discuss in Section 6 some other types of contracts that we did not consider here because they would not change the results.

of the offer. Since there are contingencies that the insuree is not aware of, unless a contract mentions them, the insuree will remain unaware of them and continue to omit these contingencies in her decision making process.

Consider a contract $C = (t, A, k)$ that offers transfer at some contingencies which the insuree is not originally aware of (i.e. $A \backslash S' \neq \emptyset$). In order to evaluate the transfer at those contingencies, she needs to extend her belief by assigning probabilities to the newly announced contingencies. When a contract C is offered, she holds a belief P_C which is a probability distribution on $A \cup S'$. The way beliefs are generated is a part of our solution concept and starting from Section 3, we will analyze the relationship between the formation of belief and the form of signed contracts. Here we will introduce the necessary notation for an arbitrary belief P_C.

After a contract C is offered, the insuree can either reject or take the offer. If she rejects the offer, then the negotiation stops at that point and she is not covered for any damage. The decision of the insuree on a contract is determined by a function $D : \mathbb{C} \rightarrow \{buy, reject\}$.

We assume that the insuree is a risk averse agent with an increasing and concave utility function u. Therefore, the expected utility of the insuree from contract $C = (t, A, k)$ with respect to distribution P_C can be written as

$$EU_0(C, D(C)|P_C) := \begin{cases} \sum\limits_{s \in A} u(v - s + t(s) - k)P_C(s) \\ \quad + \sum\limits_{s \in S' \backslash A} u(v - s - k)P_C(s) & \text{if } D(C) = buy \\ \sum\limits_{s \in A \cup S'} u(v - s)P_C(s) & \text{if } D(C) = reject \end{cases}$$

The expected utility of the risk neutral insurer from contract $C = (t, A, k)$ is

$$EU_1(C, D(C)) := \begin{cases} k - \sum\limits_{s \in A} t(s)\mu(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

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Observe that the expected utility of the insurer calculated by the insurer himself and the one calculated by the insuree under her belief P_C may not coincide in general. The insurer's expected utility from contract C according to the insuree with respect to her belief P_C is denoted by

$$EU_1^0(C, D(C)|P_C) := \begin{cases} k - \sum\limits_{s \in A} t(s)P_C(s) & \text{if } D(C) = buy \\ 0 & \text{if } D(C) = reject \end{cases}$$

Tirole, J. (1999) criticizes incomplete contract literature since, in that literature, agents are unable to conceptualize and write down the details of the nature although they are able to fully understand payoffs relevant to those aspects and consider them in their calculations. In our model, being unable to conceptualize a contingency also means that the agent cannot assign probability to that contingency and cannot take it into account in her evaluations. This intuition is expressed in definitions of EU_0 and EU_1^0 above.

3 Incompleteness in the Contractual Form

The crucial and non-standard point in our model is the following: although before anything is offered, the insuree is unaware of some relevant aspects of the uncertainty, once they are announced to her via a contract, she starts taking them into account. Her awareness evolves throughout the interaction. The contracts that extend the awareness set of the insuree do not inform her regarding the probability of those newly announced contingencies.[8]

However, the content of the contract might still be informative about the probability of contingencies it specifies. When contract $C = (t, A, k)$ is offered, the insuree

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[8]In Section 6, we discuss the contracts that also inform the insuree regarding the probabilities although we do not observe such contracts in reality.

needs to generate a belief which is a probability distribution on her extended awareness set, $A \cup S'$.

Definition 3.1. *A probability distribution $P_C \in \Delta(A \cup S')$ is compatible with contract $C = (t, A, k)$ if it satisfies:*

(i) $EU_1^0(C, buy|P_C) \geq 0$, *i.e.* $k \geq \sum\limits_{s \in A} t(s)P_C(s)$;

(ii) *For any $s \in A \cup S'$, $P_C(s) \neq 0$ and $P_C(.|S') = \mu(.|S')$*

The set of all probability distributions that are compatible with C is denoted by Π_C.

The insurer is a strategic agent and he always has an option of not participating in the negotiation and thereby guaranteeing himself zero profit. The insuree may reason that if a contract is offered, then it ought to be better for the insurer to make this offer rather than staying out of business. The first requirement in the above definition says that, with respect to a compatible belief, the expected gain of the insurer from a contract should be at least zero which is the outside option of the insurer. The second point in the above definition requires from a compatible belief that the newly announced contingencies does not alter the relative weights of the contingencies in S'. This makes the model close to Bayesian paradigm. As we noted in footnote 4, the model can be generalized easily for abitrary initial beliefs. Property (ii) in Definition 3.1 is not about making the belief formation agree with the true distribution conditionally, it is about making it agree with the initial belief, conditionally. According to a compatible belief, every contingency in the extended awareness set is possible. In the true model, all the relevant contingencies are possible. Therefore, the insurer cannot make up contingencies in a contract. In line with this, compatible belief assigns non-zero probability to every foreseen contingency.[9]

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[9]With a weaker definition of compatible belief without this assumption, we would get a larger

Compatible beliefs are candidates to be held by the insuree after an offer. The solution concept we introduce in this section requires belief formation to be part of an equilibrium. The insurer believes that the insuree will behave according to the beliefs that an equilibrium suggests under some rationality requirements and he responds to this belief. Equilibrium behavior of the insuree confirms this belief of the insurer as well.

Definition 3.2. *An equilibrium of this contractual model is a triplet* $(C^*, \ D^* : \mathbb{C} \to \{buy, reject\}, \ (P_C^*)_{C \in \mathbb{C}})$ *such that*

(i) $C^* \in \arg\max\limits_{C \in \mathbb{C}} EU_1(C, D^*(C))$;

(ii) *For any* $C \in \mathbb{C}$, *where* $C = (t, A, k)$,

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D^*(C) =
\begin{cases}
buy & \text{if } EU_0(C, buy|P_C^*) \geq EU_0(C, reject|P_C^*), \ \Pi_C \neq \emptyset \\
& \text{and } t(s) \leq s \text{ for any } s \in A \\
reject & \text{otherwise}
\end{cases}
$$

(iii) *For any* $C = (t, A, k)$, *and for any* $s \in A \cup S'$, $P_C^*(s) \neq 0$, $P_C^*(.|S') = \mu(.|S')$, *and* $P_C^* \in \Pi_C$ *whenever* $\Pi_C \neq \emptyset$.

In an equilibrium, given decision function D^* of the insuree, the insurer offers contract C^* that maximizes his expected utility. If there is any distribution that is compatible with a contract C then the equilibrium belief generated after that contract, P_C^*, has to be one of them. The insuree evaluates contract C by probability distribution P_C^*. She buys C if and only if the expected utility of buying it is higher than that of rejecting it, P_C^* is compatible with C, and offered transfers are less than the damage itself.

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equilibrium set. Therefore, our definition can at most make it more difficult to get incomplete contracts in equilibrium. Moreover, being non-zero does not prevent probability to be arbitrarily close to zero (see the proof of Theorem 3.1 for the formal argument). This assumption is introduced in order to keep our belief formation closer to Ozbay, E. Y. (2008).

The insuree is ready to update her awareness set according to the offered contract but she cannot put in her calculations anything more than that. In reality, agents might think that there may be something in the world that they are unable to name, especially after their awareness is extended. Unawareness is, by itself, the lack of ability to name, evaluate and estimate some aspects of the problem. At the given stage of the theoretical literature, we are bounded by modeling the economic agents as rational within their awareness unless we assume some exogenous evaluation of unforeseen world. In our model, one can think of two situations where the insuree might suspect that she might have been left unaware of some contingencies. First, imagine that the insurer makes an offer such that under any belief construction of the insuree, the insurer is making a loss by this offer. In reality, the insurer may not be making any loss if there are unmentioned contingencies in the contract. From the perspective of the insuree, this is a *too good to be true* offer. In the equilibrium, we require that the insuree rejects this kind of offers.[10] We are looking for equilibrium where the insuree will not be suspicious about her limited awareness. Another situation that may make the insuree suspicious is when the insurer offers a transfer more than the cost of a damage. Such an unrealistic contract might let the insurer make infinite amount of profit.[11] Hence, equilibrium requires that the insuree rejects transfers that exceed the cost of damage. These two rules out any outcome where the insuree signs a contract without understanding the rationale of the insurer in making this offer in any equilibrium.

Under this definition, equilibrium contracts induce non-empty set of compatible beliefs. To see this, consider the contract that is signed on S', and that fully insures

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[10]Even if we allow acceptance of *too good to be true* offers it can be shown that the set of equilibrium contracts would not change.

[11]For example, imagine a contract that is signed only on S', $C = (t, S', k)$. If the insuree accepts this contract, then for any constant $a > 0$, $\tilde{C} = (t + a, S', k + a)$ is also accepted. Observe that $\tilde{C}$ is $(1 - \mu(S'))a$ more profitable than C. By increasing a arbitrarily, the insurer can make an unbounded amount of profit.

the good against all the damages in S' and charges the premium which makes the insuree indifferent between buying and rejecting the offer. This is contract $C = (t(s) = s, S', k)$ where k solves $u(v - k) = \sum_{s \in S'} u(v - s)\mu(s|S')$. The insuree accepts this offer, and the insurer's expected utility from this contract is positive since u is concave. By existence of such an acceptable contract, the equilibrium contract has to be bought by the insuree. Hence the corresponding set of compatible beliefs for an equilibrium has to be non-empty as well because it is one of the conditions for buying a contract.

Theorem 3.1. *There always exists an equilibrium where the equilibrium contract does not extend the awareness of the insuree.*

The proof of existence of an equilibrium with an incomplete contract is constructive and given in the Appendix. It is shown that equilibrium beliefs can be constructed so that the best acceptable contract that the insurer can offer is signed on S'. The idea goes as follows: The contracts that lead to empty set of compatible beliefs are rejected, therefore any probability distribution can be equilibrium belief corresponding to them. For a contract that corresponds to a non-empty set of compatible beliefs, set the belief so that either the insuree rejects the offer or if she accepts it, then it is not beneficial for the insurer to offer this contract rather than the contract suggested by the theorem. By this belief construction, in equilibrium the insurer offers a contract on S' and provides full insurance on the elements of S' and sets the premium at the level which makes the insuree indifferent between buying or rejecting this offer.

The definition of equilibrium puts minimum restriction on the belief held after each contract. It only requires equilibrium beliefs to be compatible whenever it is possible. The insuree knows that the insurer is an expected utility maximizer. When a contract is offered in an equilibrium, the insuree may ask herself if this is the best offer for the insurer. The example below illustrates a situation where the insuree

cannot understand why the insurer offered the contract suggested by an equilibrium.

Example 3.1. *Let $S = \{100, 900\}$, $S' = \{100\}$, $v = 1000$, $u(x) = \sqrt{x}$, $\mu(\{100\}) = 0.99$, $\mu(\{900\}) = 0.01$.*

For contract $C^* = (t^*(s) = s, \{100, 900\}, k^* = 895.96)$ where $k^* = 895.96$ solves $u(v - k^*) = 0.01u(v - 100) + 0.99u(v - 900)$, define $P^*_{C^*}(\{100\}) = .01$ and $P^*_{C^*}(\{900\}) = .99$. Observe that $P^*_{C^*}$ is compatible with C^*. For any contract $C \neq C^*$, define P^*_C as in the construction of the proof of Theorem 3.1 so that $\tilde{C} = (t(s) = s, S', k = 100)$ is a better contract for the insurer than any $C \neq C^*$. Then there are two candidates for equilibrium contract under this belief construction: C^* and $\tilde{C}$.

$EU_1(\tilde{C}, buy) = 100 - .99(100) = 1$ and

$EU_1(C^*, buy) = k^* - .99(100) - .01(900) = k^* - 108 = 787.96$.

Therefore, the insurer will offer C^* in equilibrium and $(C^*, D^*, (P^*_C)_{C \in \mathbb{C}})$, where D^* is defined as in point (ii) of Definition 3.2, is an equilibrium of this problem.

In the example above, the equilibrium contract charges a high premium but makes small transfer in expectation since $\{900\}$ is a very unlikely event in reality. However, the belief that is held after the equilibrium contract assigns a high probability to event $\{900\}$ and hence, the insuree buys such a high premium offer. According to the insuree, the equilibrium contract promises a large transfer on a very likely event. Since this event was not conceptualized originally by the insuree, under her equilibrium belief she cannot reason why the insurer did not hide that event from her. According to the insuree, the expected utility of the insurer from the equilibrium contract is

$$EU_1^0(C^*, D^*(C^*)|P^*_{C^*}) = k^* - .01(100) - .99(900) = 3.96$$

However, after hearing the equilibrium offer, the insuree thinks that the insurer could

have made

$$EU_1^0(\tilde{C}, D^*(\tilde{C})|P_{C^*}^*) = 100 - .01(100) = 99$$

by hiding event $\{900\}$ and offering $\tilde{C}$. So, with respect to the insuree's belief, the insurer is not maximizing his expected utility at the equilibrium offer C^*.

The refinement introduced below eliminates this kind of equilibria. It imposes that with respect to the belief held by the insuree, the equilibrium contract should be the best one for the insurer among all the contracts that the insuree can think of. After hearing the equilibrium offer, the insuree can consider only the contracts that would extend her awareness less than the equilibrium contract.

Definition 3.3. *An equilibrium $(C^* = (t^*, A^*, k^*), D^* : \mathbb{C} \to \{buy, reject\}, (P^*)_{C \in \mathbb{C}})$ is consistent if $\forall C = (t, A, k) \in \mathbb{C}$ such that $A \cup S' \subseteq A^* \cup S'$*

$$EU_1^0(C^*, D^*(C^*)|P_{C^*}^*) \geq EU_1^0(C, D^*(C)|P_{C^*}^*).$$

Corollary 3.1. *There always exists a consistent equilibrium where the signed contract is incomplete.*

This corollary is an immediate implication of Theorem 3.1 because the theorem states that signing a contract only on S' is always a part of some equilibrium. Observe that if the equilibrium contract does not inform the insuree about any new contingencies, then that equilibrium is trivially consistent. Therefore, the equilibrium suggested by the statement of the theorem is consistent.

If the insuree is initially considering only the high cost contingencies, the insurer has no incentive to extend the insuree's awareness. So only incomplete contracts will be signed in the equilibrium. The following example points out a more interesting situation. It shows that even if the insuree is aware of the least costly damage initially, it is possible to have all the consistent contracts being incomplete.

Example 3.2. *Let $S = \{8.79, 9\}$, $S' = \{8.79\}$, $v = 10$, $u(x) = \sqrt{x}$, $\mu(\{8.79\}) = 0.01$, $\mu(\{9\}) = 0.99$. We show that a contract in the form of $C = (t, S, k)$ cannot be part of a consistent equilibrium. For contradiction assume that it can be. Let P_C^* be the equilibrium belief with $P_C^*(\{8.79\}) = p$ and $P_C^*(\{9\}) = 1 - p$, where $p \in (0, 1)$. Since C is bought, it needs to satisfy*

$$p\sqrt{10 - 8.79 - k + t(8.79)} + (1 - p)\sqrt{10 - 9 - k + t(9)}$$
$$\geq p\sqrt{10 - 8.79} + (1 - p)\sqrt{10 - 9}$$

Then, since u is concave and $p \in (0, 1)$, we have

$$.21p + 1 - k + pt(8.79) + (1 - p)t(9) > (1 + .1p)^2 \tag{1}$$

Consider $C' = (t'(8.79) = 8.79, S', k' = 8.79)$. If it was offered, the insuree would buy C' since she would be indifferent between buying or rejecting it.

Since C is assumed to be part of a consistent equilibrium, it has to be the case that

$$EU_1^0(C, buy | P_C^*) = k - pt(8.79) - (1 - p)t(9) \geq 8.79 - p8.79 = EU_1^0(C', buy | P_C^*) \tag{2}$$

Equations (1) and (2) implies that $p > 1$ and this contradicts with p being a probability. Hence a consistent equilibrium contract of this example cannot be complete.

In order to have a complete contract in a consistent equilibrium, the following two points should be satisfied at the same time: a) the belief should assign small enough probability to the less costly event for acceptance; b) the belief should assign large enough probability to the less costly event for consistency. These two points cannot happen simultaneously for the given parameters of the example.

In our setting, incompleteness in the contractual form arises as a result of a strategic decision process. Although both complete and incomplete contracts are feasible,

the incomplete ones are always signed in an equilibrium, but the complete ones may fail to arise in any equilibria.

4   Contracts Introducing Knightian Uncertainty

The equilibrium concept introduced in the previous section is based on the idea that the insuree's equilibrium belief after each contract supports the behavior of the insurer. Generally, each offer induces more than one compatible belief and the beliefs held in an equilibrium in the sense of Section 3 can be any of them. Among these multiple equilibria, the worst one for the insuree is where she pays the highest possible premium by minimally extending the awareness set, because then she is minimally covered by the contract but paying a lot of premium for nothing. In this section, we show that if the insurance company faces with a pessimist insuree, the above situation is indeed the equilibrium outcome. We use this analysis as a benchmark for the next section where, we will show that even this severe type of incompleteness can be removed via competition.

We consider a type of insuree who cannot pick an arbitrary belief from the set of compatible beliefs but instead holds multiple beliefs. If the insuree is unable to assign a single probability to the newly announced contingencies then the type of uncertainty that the insuree considers contains ambiguity. This means each newly announced contingency introduces Knightian uncertainty in the picture. In this section, we suppose that the insuree is uncertainty (risk and ambiguity) averse. The concave utility function u captures the risk aversion component of the uncertainty aversion. We assume that while evaluating a situation, the insuree uses the maxmin expected utility defined on her multiple belief set and this assumption captures the ambiguity aversion (for behavioral axiomatization of the maxmin expected utility model, see Gilboa, I. and Schmeidler, D. (1989)). In short, the maxmin expected utility model

says that the insuree evaluates an offer under every possible scenario that she can think of in her multiple belief set and considers the one with the smallest expected utility (the most pessimistic one) as the final evaluation of the offer.

The set of multiple beliefs collects all those probabilities that are compatible with the offer of the insurer and assign at least α probability to S' where $\alpha \in (0, \mu(S')]$. Previously we required that a compatible belief assigns non zero probability to every contingency in the extended awareness set. Here when we construct the set of multiple beliefs, we relax this assumption in order to make the multiple belief set closed. The same results would hold without this but in that case the minimum would not be attained in the multiple belief set when we calculate maxmin expected utilities. However, the equilibrium utilities of the insurer and the insuree and the form of the contract would remain the same.

The lower bound, α, on the probability of S' means that after learning about existence of some new contingencies, the insuree still thinks that S' is relevant and a non-zero probability event. This assumes that the insuree holds α-beliefs on S' (p-beliefs is the standard terminology, see e.g. Monderer, D. and Samet, D. (1989); and Ahn, D. (2007)). The constant α is assumed to be smaller than the true probability of S'. Although the whole analysis can be done without this assumption, we include it because it allows the insuree to consider the true distribution in the set of multiple beliefs when all the contingencies in S are revealed. Without this assumption, the insuree could be unable to take into account the correct distribution μ if she learned the true domain of uncertainty.[12] For any $C = (t, A, k) \in \mathbb{C}$, the set of multiple beliefs is defined by

$$
\Pi_C^* := \left\{ \begin{array}{l} P \in \Delta(A \cup S') \mid P(.|S') = \mu(.|S'), \ \alpha \leq P(S') \text{ and} \\ \qquad\qquad\qquad EU_1^0(C, buy|P) \geq 0 \end{array} \right\} \tag{3}
$$

---

[12]This point will be discussed further in Section 5.

The ambiguity averse insuree evaluates contract $C = (t, A, k)$ which leads to a non-empty Π_C^* by the following formula[13]:

$$EU_0(C, D(C)|\Pi_C^*) := \begin{cases} \min_{P \in \Pi_C^*} \left[\sum_{s \in A} u(v - s + t(s) - k)P(s) \right. \\ \qquad \left. + \sum_{s \in S' \setminus A} u(v - s - k)P(s) \right] & \text{if } D(C) = buy \\ \min_{P \in \Pi_C^*} \sum_{s \in A \cup S'} u(v - s)P(s) & \text{if } D(C) = reject \end{cases}$$

Observe that in the above formula, first the expected utility from offer C is calculated with respect to every compatible belief in Π_C^* and then the smallest of them is the final evaluation of the contract.

The multiple belief set Π_C^* in Equation (3) can be empty for some offer C. For example, if the corresponding set of compatible distributions is empty then Π_C^* is empty as well. In that case it really does not matter what kind of belief the insuree will hold after such contracts since those offers will not appear in equilibrium. For an offer C if no probability makes the expected utility of the insurer from C positive, then the insurer can always ask for higher premium or offer a smaller transfer to make Π_C^* non-empty, and thus guarantee himself some higher profit. In line with Section 3, we assume that in equilibrium the insuree rejects offer C if the corresponding Π_C^* is empty[14].

Definition 4.1. *An equilibrium under ambiguity aversion is a pair* $(C^*, D^* : \mathbb{C} \to \{buy, reject\})$ *such that*

(i) $C^* \in \arg\max_{C \in \mathbb{C}} EU_1(C, D^*(C))$

---

[13]Here, we use EU notation instead of MEU (maxmin expected utility) to keep the notation of the previous section.

[14]Alternatively, assuming that the insuree can accept offer C even if Π_C^* in Equation (3) is empty would not change the offer in equilibrium under ambiguity aversion.

(ii) For any $C = (t, A, k) \in \mathbb{C}$,

$$
D^*(C) = \begin{cases}
buy & \text{if } EU_0(C, buy|\Pi_C^*) \geq EU_0(C, reject|\Pi_C^*),\ \Pi_C^* \neq \emptyset \\
& \text{and } t(s) \leq s \text{ for any } s \in A \\
reject & \text{otherwise}
\end{cases}
$$

where Π_C^* is the multiple belief set defined in Equation (3).

Standard insurance models (where uncertainty is only risk rather than Knightian uncertainty) suggest that the risk neutral party takes all the risk and promises a constant wealth to the risk averse one at every realization of uncertainty. The following result shows that in one important respect, our non-standard insurance problem with a pessimist insuree does not differ from what the standard theory teaches us: the optimum contract in our setting provides full insurance on the set of contingencies that the contract mentions and this set includes every contingency that the insuree is originally aware of. This is a hard to achieve result in our setup since the belief formation is a function of contract. Hence, as the contract changes from partial coverage to full coverage, the insuree is free to form different beliefs and exhibit different behavior (for example, she may accept partial coverage but reject full coverage). However, under ambiguity aversion, we can achieve this desired result. The point is that if two contracts are not too different from each other then their corresponding compatible belief sets are similar. Once we fix the aggregation rule on the set of compatible beliefs (such as maxmin), then the behavior of the insuree does not change too much on these two contracts (see also the discussion at the end of Section 6).

Proposition 4.1. *If $C^* = (t^*, A^*, k^*)$ is the contract offered in an equilibrium under ambiguity aversion then $t^*(s) = s$ for any $s \in A^*$ and $S' \subseteq A^*$.*

This result is parallel to the standard theory and its proof is in the Appendix.

The only strategies the insuree has are "buy" and "reject". Therefore, while determining her best response, she only checks if the offered contract is better than

her outside option, which is shouldering the burden of the full uncertainty. Her understanding of the environment changes depending on the offered contract, so her evaluation of the outside option changes as well. In standard theory, this is not an issue as the uncertainty the agents face is known objectively. Hence the standard risk insurance contract between a risk averse insuree and a risk neutral insurer sets the premium so that the utility of the insuree with and without the contract are the same. Proposition 4.2 shows that this result still holds in our setting.

Proposition 4.2. *At an equilibrium under ambiguity aversion, the insuree is indifferent between buying and rejecting the offered contract.*

Proposition 4.1 and 4.2 state several necessary conditions that an equilibrium contract satisfies. In light of these results, we can fully characterize the equilibrium contract.

Theorem 4.1. *The equilibrium contract under ambiguity aversion is either signed only on S' or it announces one extra contingency besides S'. Moreover, if it announces a contingency, then the utility of the insuree at that contingency is lower than her expected utility on S' without any contract.*

Theorem 4.1 together with Propositions 4.1 and 4.2 concludes that a risk neutral insurer offers to an ambiguity averse insuree full insurance either on the already foreseen contingencies by the insuree or on those plus one more extra contingency. It is intuitive that the insurer announces at most one extra contingency. Otherwise, if more than one additional contingencies are mentioned in the contract, the insuree, as a pessimist agent, puts the highest weight to the most costly one. Lower cost contingencies play no role in determining the premium that the insuree is willing to pay. Therefore, it is better for the insurer to announce only the highest cost one among those contingencies and not to promise any transfer at those lower cost

ones. The newly announced contingency is not necessarily the worst damage among everything the insuree is unaware of.

In line with our intuition, Theorem 4.1 concludes that the most severe type of incompleteness arises when the insurance provider deals with a pessimist insuree. An equilibrium offer has one of the two forms stated in Theorem 4.1. The idea goes as follows: On the one hand, the insurer would like to inform the insuree about the worst possible contingency because by doing so he can benefit from the pessimism of the insuree. On the other hand, the worst contingency is also the most costly one for the insurer when he promises a transfer on it. Therefore, there is a trade-off between gaining over the premium and losing over the high transfer. If there are contingencies that the insuree is not aware of originally and announcing them makes her pessimistic enough to pay a high premium which compensates the extra transfer that the insurer promises on these contingencies, then the insurer would announce the most beneficial one of them. Otherwise, he will not inform the insuree regarding the unforeseen parts of the uncertainty.

If the contract offered in an equilibrium under ambiguity aversion does not extend the awareness of the insuree, then it is the best contract for the insurer also from the insuree's perspective (in the sense of consistency). However, if it announces one extra contingency, then in Knightian uncertainty setting, it is not immediate to conclude that this contract is the best for the insurer according to the insuree. First of all in this setting, we need to be precise with what we mean by consistency (the terminology defined in Section 3). In Section 3, the insuree held a single belief after each contract, so the insuree can calculate the insurer's expected utility with respect to this belief unambiguously. Here, the insuree holds multiple beliefs after the equilibrium offer if it extends the insuree's awareness set. We will check if the equilibrium contract is the best offer for the insurer from the insuree's perspective with respect to every belief

in the equilibrium multiple belief set.

Definition 4.2. *An equilibrium contract $C^* = (t^*, A^*, k^*)$ under ambiguity aversion is the best contract for the insurer according to the insuree if for any $C = (t, A, k) \in \mathbb{C}$ such that $A \cup S' \subseteq A^* \cup S'$*

$$EU_1^0(C^*, D^*(C^*)|P) \geq EU_1^0(C, D^*(C)|P) \text{ for any } P \in \Pi_{C^*}^*$$

where $\Pi_{C^*}^*$ is the set of multiple beliefs corresponding to C^*.

Proposition 4.3. *If an equilibrium contract under ambiguity aversion extends the insuree's awareness set, then it is the best contract for the insurer according to the insuree if α (and therefore $\mu(S')$) is sufficiently large.*

Sufficiently large α means that according to the insuree the newly announced contingency is unlikely. Therefore, she can reason that the insurer wanted to promise her transfer on this contingency because in expectation doing this was not very costly for the insurer.

There are two properties of the optimal contract we find in equilibrium under ambiguity aversion: a) it leaves no extra payoff to the insuree compared to the way she evaluates the situation without a contract, and b) if there are at least two contingencies unforeseen by the insuree, it hides some or all of them. The first property is a characteristic that carries over from the standard theory. However, the second property tells us that the optimal contract is silent on some contingencies. This is an appealing result that suggests that in addition to the arguments discussed extensively in the literature, asymmetry of awareness can be an underlying reason for incompleteness in contractual forms.

5 Competition Promotes Awareness

We saw in the previous section that a monopolistic insurer who has superior aware-
ness will possibly sign an incomplete contract with an ambiguity averse insuree. In
this section we study if the contracts offered by competing insurers reveal more con-
tingencies. The answer is affirmative and competition indeed promotes awareness.

In standard insurance settings where asymmetric awareness is not an issue, sym-
metric firms compete over premia. They offer a zero profit contract which is beneficial
for the insuree. In our setting, when we introduce competition on the insurers' side,
there are two dimensions that the insurers can compete over in their offers: premium
and awareness of the insuree. A competing insurer can make a counter offer by ei-
ther changing the premium or by further extending the awareness of the insuree. We
see that competition is an instrument under which not only the insuree can get the
cheapest offer but also her unawareness can totally disappear.

Assume there are N risk neutral insurers. All of them are aware of S and believe
μ. The ambiguity averse insuree (indexed by 0) is only aware of S', and she believes
$\mu(.|S')$ as before. The awareness structure between the insurers and the insuree is the
same as in the previous sections. The insuree knows that the insurers are symmetric
agents. The insurers make simultaneous offers denoted by $C_i = (t_i, A_i, k_i) \in \mathbb{C}$
for $i = 1, ..., N$. Vector $\mathbf{C} = (C_1, ..., C_N)$ is the collection of insurers' offers. The
collection of contracts offered by all insurers except insurer i is denoted by $\mathbf{C}_{-i} = (C_1, ..., C_{i-1}, C_{i+1}, ..., C_N)$.

The offers are exclusive and the insuree may accept, at most, one of the offers or
may reject all. The decision of the insuree is denoted by a function $D : \underset{i=1,...,N}{\times} \mathbb{C} \rightarrow \{buy_1, ..., buy_N, reject\}$.

For $i = 1, ..., N$, given the offers of other insurers, $\mathbf{C}_{-i}$, and the decision function

of the insuree, D, the expected utility of insurer i from contract C_i is:

$$EU_i(\mathbf{C}, D(\mathbf{C})) := \begin{cases} k_i - \sum\limits_{s \in A_i} t_i(s)\mu(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

The expected utility of insurer i according to the insuree with respect to a probability distribution P is given by

$$EU_i^0(\mathbf{C}, D(\mathbf{C})|P) := \begin{cases} k_i - \sum\limits_{s \in A_i} t_i(s)P(s) & \text{if } D(\mathbf{C}) = buy_i \\ 0 & \text{otherwise} \end{cases}$$

When $\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}$ is offered, the insuree aggregates the information from each contract in $\mathbf{C}$. She constructs her set of beliefs $\Pi_{\mathbf{C}}^* \subseteq \Delta((\underset{i=1,\dots,N}{\cup} A_i) \cup S')$ similar to the multiple belief set defined in Section 4. Here the multiple belief set is defined by

$$\Pi_{\mathbf{C}}^* := \left\{ \begin{array}{c} P \in \Delta((\underset{i=1,\dots,N}{\cup} A_i) \cup S') \mid P(.|S') = \mu(.|S'), \alpha \leq P(S'), \\ EU_i^0(\mathbf{C}, buy_i|P) \geq 0 \text{ for } i = 1, \dots, N \end{array} \right\} \tag{4}$$

The multiple belief set contains all the probability distributions that are compatible with the offer of each insurer and that assign at least α probability to S'. α is the exogenous lower bound that we defined in Section 4.

With respect to the construction of belief set in Equation (4), the expected utility of the ambiguity averse insuree from vector $\mathbf{C}$ is given by:

$$EU_0(\mathbf{C}, buy_i | \Pi_{\mathbf{C}}^*) := \min_{P \in \Pi_{\mathbf{C}}^*} \left(\sum_{s \in A_i} u(v - s + t_i(s) - k_i) P(s) \right.$$

$$\left. + \sum_{s \in ((\bigcup_{\substack{j=1,\dots,N \\ j \neq i}} A_j) \cup S') \backslash A_i} u(v - s - k_i) P(s) \right)$$

$$EU_0(\mathbf{C}, reject | \Pi_{\mathbf{C}}^*) := \min_{P \in \Pi_{\mathbf{C}}^*} \sum_{s \in (\bigcup_{i=1,\dots,N} A_i) \cup S'} u(v - s) P(s)$$

Definition 5.1. *An equilibrium under ambiguity aversion with competition is a pair*

$$(\mathbf{C}^*, D^* : \underset{i=1,\dots,N}{\times} \mathbb{C} \to \{buy_1, \dots, buy_N, reject\}) \text{ such that}$$

(i) $C_i^* \in \underset{C_i \in \mathbb{C}}{\arg\max} EU_i(C_i, \mathbf{C}_{-i}^*, D^*(C_i, \mathbf{C}_{-i}^*))$ *for* $i = 1, \dots, N$;

(ii) *For any* $\mathbf{C} \in \underset{i=1,\dots,N}{\times} \mathbb{C}$,

$$D^*(\mathbf{C}) = \begin{cases} buy_i & if \quad EU_0(\mathbf{C}, buy_i | \Pi_{\mathbf{C}}^*) \geq EU_0(\mathbf{C}, buy_j | \Pi_{\mathbf{C}}^*) \ \forall j \neq i \\ & \quad EU_0(\mathbf{C}, buy_i | \Pi_{\mathbf{C}}^*) \geq EU_0(\mathbf{C}, reject | \Pi_{\mathbf{C}}^*) \\ & \quad \Pi_{\mathbf{C}}^* \neq \emptyset \ and \ t_i(s) \leq s \ for \ any \ s \in A_i \\ reject & otherwise \end{cases}$$

where $\Pi_{\mathbf{C}}^*$ *is defined as in Equation (4).*

We assume that if the insuree is indifferent between two or more offers which are better than rejecting everything, then she picks one of them with equal probability. We did not put it in the definition formally to ease the notation.

The solution concept is analogous to the definition of equilibrium under ambiguity aversion for a single insurer case. Here, each insurer decides on his offer optimally. The insuree buys the offer of insurer i if it gives the highest expected utility among all the other offers she sees, and buying from insurer i is better than rejecting all the offers. Moreover, to avoid rejection, vector $\mathbf{C}$ should lead to a non-empty multiple

belief set in the sense of Equation (4). Again a vector of offers which lead to an empty set of multiple beliefs can be thought as *too good to be true offers*. If $\Pi_{\mathbf{C}}^{*}$ is empty then with respect to any belief the insuree can think of, some insurers would be better off by staying out of business rather than making these offers.[15]

Theorem 5.1. *There is a symmetric equilibrium under ambiguity aversion with competition where each insurer offers the same zero profit, full insurance, and complete contract, i.e. for any $i = 1, ..., N$, $C_i^* = (t^*(s) = s, S, k^*)$ such that $k^* = \sum_{s \in S} s\mu(s)$.*

The equilibrium given in Theorem 5.1 is a zero profit equilibrium which fully extends the awareness of the insuree. The first step in the proof shows that buying the offer that is suggested by Theorem 5.1 is better than rejecting all the offers. It is based on the observation that the multiple belief set induced by the equilibrium vector of contracts contains μ. Then we show that no insurer can benefit from deviating to another contract rather than C_i^*. The details of the proof are given in the Appendix.

Recall that according to Definition 5.1, one of the requirements for the insuree to buy a contract is that the offer should not lead to an empty multiple belief set. Although this assumption did not play a role in determining the equilibrium contract in a single insurer case, it has an impact on the multi-insurer case. Under this requirement, we can easily create examples which have incomplete contracts in equilibrium. Imagine that all the competing insurers offer the same incomplete contract which gives a high expected profit in reality but zero profit according to the insuree. Given the strategies of his opponents, insurer i can deviate to one of the following actions: a) he may announce some contingencies that are hidden by the others' offers, b) he may offer a different contract on the same contingencies. There are examples where none of these deviations is profitable. We provide a numeric example in the

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[15]Rejecting or accepting too good to be true offers determines only the size of competition that is needed for full awareness. Under each of the cases, we have the main result of this section: "Competition promotes awareness".

Appendix. The idea goes like this: the contingency that is hidden by the other offers is very costly and also very likely. Therefore, the first type of deviation is not profitable enough for insurer i. Alternatively, insurer i can make a different offer on the same set of contingencies as the one announced by his opponents (the second type of deviation). This offer may make the multiple belief set empty and it, as a result, may be rejected. Therefore, the best contract for insurer i is the same as the incomplete one offered by all of the other insurers.

We assume that when two or more insurers make the same offer that is good to buy for the insuree, she selects each insurer with an equal probability. For an insurer, the chance of attracting the insuree by offering the same contract as his competitors decreases with the number of insurers making this offer. Therefore, the expected gain of an insurer from following the strategies of the others decreases when the size of competition is large. So if the other insurers are offering an incomplete contract, it may be better for an insurer to further extend the insuree's awareness set . The next result is built on this observation. It shows that if the size of competition is large enough, then the insuree is offered a complete contract in all symmetric equilibria where the insuree accepts an offer.

Theorem 5.2. *If the number of insurers is large enough, then in any symmetric equilibrium where the insuree buys a contract, the offer is complete, zero profit and full insurance contract.*

6   Discussions

Form of Contracts: We focus on contracts that specify a premium and a transfer rule on the contingencies that the insurer announces. One may suggest two other types of contracts that are excluded in our set of feasible contracts. One of them is the type of contract which, in addition to a premium and a transfer rule, suggests a probability

distribution on the contingencies mentioned in the contract. The insurer, who offers the contract, has no incentive to announce the true probability. Therefore, he cannot convince the insuree to believe the suggested distribution. Hence, the insuree would behave as she would without the suggestion.

Another type of contract that one may think of can have a clause such as *anything not specified here is excluded by this contract*. Observe that in our setting the complement of a set of contingencies is not the same set for each agent. Therefore, the statement *anything not specified here* does not refer to the same set of contingencies by the insurer and by the insuree. Indeed, if the contract already mentions everything that the insuree is initially aware of -this is a property of equilibrium contracts we found- then according to the insuree there is nothing excluded in the contract. So, she will not take that clause into account in her evaluation process. Both the contracts which have this clause and the ones without it give the same payoff to the insurer since in our model no transfer takes place if some unspecified contingency is realized. Hence having these contracts in the feasible set would not change the results.

Zero probability: One general critique to awareness literature is about distinguishing unforeseen and zero probability events. Although this paper doesn't attempt to provide any concrete discussion on the problem, one can see that these two may lead to different behaviors in the insurance setup. For example, imagine that there are two damages, High (H) and Low (L). The model where insuree is unaware of H but aware of L can explain the following behavior: a) The insuree accepts the contract that covers only L and charges a premium equal to the cost of L; b) she is willing to pay more than L if the contract covers L and H. This behavior cannot be explained if the insuree is aware of everything but assigning zero probability to H. If the insuree is assigning zero probability then she is willing to pay at most L independent of whether H is covered or not. Here, when the initial asymmetry between insuree

and insurer is about holding non-common beliefs, there cannot be any updating after any offer because when the insurer makes an offer, he does not know the realization of the uncertainty. One may further argue that perhaps the asymmetry is due to incomplete information regarding the distribution and a contract signals something about the information of the insurer. Again, this alternative modeling cannot explain the above behavior. In order to behave as in (a), she has to assign zero probability to H according to all the distributions she has in mind, i.e. she must believe that with probability 1 the probabilities of L and H are 1 and 0, respectively. Hence, she cannot update her belief on distributions such that she will assign positive probability to H in order to present behavior (b). Another alternative model might have been the following: There are two types of insurers. If the insurer is type θ_1, then H is a zero probability event and if the insurer is type θ_2, then H is a positive probability event. Suppose the insuree's prior puts probability 1 on type θ_1. Assume there exists a separating equilibrium where θ_1 offers the contract in (a), and θ_2 offers the contract in (b). The insuree expects to see the contract in (a). If we use an equilibrium notion that has no restriction on how to generate the posterior belief on zero probability histories, can we explain a behavior as in (b)? The answer is "No". If this is the strategy of the insuree, such a separating equilibrium cannot exist since type θ_1 would mimic type θ_2. For type θ_1, charging a premium more than L (the premium of type θ_2) and paying only L when it realizes is better than charging only L (since L occurs with probability 1 in case of θ_1). Ozbay, E. Y. (2008) further discusses in detail why these situations cannot be studied by the tools of standard theory.

Dealing with Knightian Uncertainty: The solution concept introduced in Section 3 is not very restrictive on how the insuree picks her equilibrium belief among all the compatible ones after hearing an offer. In Section 4, we modeled an insuree who cannot hold a single belief but instead considers a subset of compatible beliefs. The

evaluation is done by the maxmin expected utility on the multiple belief set. Although this is an extreme form of pessimism, the main results derived under maxmin expected utility are robust. In order to see this, one may study an insuree who has a distribution $\mathbb{Q}^C$ over the set compatible beliefs corresponding to C. She calculates expected utilities with respect to each belief in the multiple belief set and then computes their mean with respect to $\mathbb{Q}^C$. Maxmin is a type of aggregation rule which puts the highest weight on the worst case scenario in the multiple belief set. It is thus a degenerate distribution on the multiple belief set generated after each contract.

Observe that each specification of $\mathbb{Q}^C$ generates a compatible belief since the set of compatible beliefs is convex and $\mathbb{Q}^C$ is a linear aggregation rule. Hence each correspondence $\mathbb{Q}$ picks one of the equilibria found in Section 3. The equilibrium contract under ambiguity aversion is also one of the contracts signed in equilibria in the sense of Section 3. Consider a sequence of $\{\mathbb{Q}_n\}_{n=1}^{\infty}$ that converges to the aggregation rule $\mathbb{Q}$ that corresponds to this equilibrium. Then the sequence of equilibrium generated by $\{\mathbb{Q}_n\}_{n=1}^{\infty}$ also converges to this equilibrium. Therefore, the equilibrium concept under ambiguity aversion is robust.

7 Conclusion

In this paper, we show that, if unawareness is an issue, then the insurance companies can use it to their advantage. We argue that, even if complete contracts are feasible, there are situations where only incomplete ones can emerge for strategic reasons. A severe type of incompleteness occurs when the insuree is pessimistic. Moreover, we show that equilibrium contracts extend awareness of the insurees on low probability but high cost damages. This approach is also often observed in insurance commercials.

Conflicts between contracting parties due to expost recognition of the incompleteness of contracts are difficult challenges for the courts. We offer competition as an

economic instrument to achieve complete contracts.

Our model is a starting point which relaxes a strong assumption in contract theory. It is a realistic exercise to allow for agents who take into account different aspects of an economic situation. The tools developed here can be used for models where the insuree has superior awareness. Additionally, modeling more complicated contractual situations where moral hazard or adverse selection is also an issue would be an insightful research question.

Appendix

Proof of Theorem 3.1. Consider $C^* = (t^*(s) = s, S', k^*)$, where k^* solves $u(v - k^*) = \sum_{s \in S'} u(v - s)\mu(s|S')$. Observe that C^* is accepted by the definition of equilibrium. The expected utility of the insurer from C^* is positive by concavity of u. Let $C = (t, A, k)$ be given. If $\Pi_C = \emptyset$, then independent of how we set the belief P_C^*, the definition of equilibrium makes the insuree reject contract C. So set P_C^* as an arbitrary distribution on $A \cup S'$ such that $P_C^*(s) \neq 0$ for any $s \in A \cup S'$ and $P_C^*(.|S') = \mu(.|S')$. Hence, C^* is a better offer than C for the insurer, since C^* but not C is accepted. If $\Pi_C \neq \emptyset$, then we have the following cases:

Case 1: If $k > \sum_{s \in S'} t(s)\mu(s|S')$[16] then there exists an $\varepsilon_1 \in (0, 1)$ such that

$$k > (1 - \varepsilon_1)\sum_{s \in S'} t(s)\mu(s|S') + \frac{\varepsilon_1}{m}\sum_{s \in A \backslash S'} t(s) \tag{5}$$

where m is the cardinality of $A \backslash S'$. Observe that for any $\varepsilon \in (0, \varepsilon_1]$, Inequality (5) holds.

Case 1.1: If $\sum_{s \in S'} u(v - s + t(s) - k)\mu(s|S') < \sum_{s \in S'} u(v - s)\mu(s|S')$

---

[16]Here, we abuse the notation and write it as if t is defined on $A \cup S'$ although it is only defined on A. However, since both agents are aware of S', they can interpret t as the transfer rule which transfers zero on $S' \backslash A$.

then there exists an $\varepsilon_2 \in (0,1)$ such that

$$(1-\varepsilon_2)\sum_{s\in S'}u(v-s+t(s)-k)\mu(s|S') + \frac{\varepsilon_2}{m}\sum_{s\in A\setminus S'}u(v-s+t(s)-k) \quad (6)$$

$$< \quad (1-\varepsilon_2)\sum_{s\in S'}u(v-s)\mu(s|S') + \frac{\varepsilon_2}{m}\sum_{s\in A\setminus S'}u(v-s)$$

Observe that for any $\varepsilon \in (0,\varepsilon_2]$, Inequality (6) holds. Then for $\varepsilon = \min\{\varepsilon_1,\varepsilon_2\}$, both Inequalities (5) and (6) hold. Define probability distribution $Q_C \in \Delta(A \cup S')$ as

$$Q_C(s) := \begin{cases} (1-\varepsilon)\mu(s|S') & \text{if } s \in S' \\[2mm] \frac{\varepsilon}{m} & \text{if } s \notin S' \end{cases}$$

Then from Equation (5) Q_C is compatible with C. Set $P_C^* := Q_C$. Then by Inequality (6), C is rejected therefore, it is worse than C^* for the insurer.

Case 1.2: If $\sum_{s\in S'}u(v-s+t(s)-k)\mu(s|S') \geq \sum_{s\in S'}u(v-s)\mu(s|S')$ then from concavity of u and definition of k^* we have

$$u\left[\sum_{s\in S'}(v-s+t(s)-k)\mu(s|S')\right] \geq \sum_{s\in S'}u(v-s+t(s)-k)\mu(s|S')$$

$$\geq \sum_{s\in S'}u(v-s)\mu(s|S') = u(v-k^*)$$

$$\sum_{s\in S'}(v-s+t(s)-k)\mu(s|S') \geq v-k^*$$

By rearranging the terms, we get $k^* - k \geq \sum_{s\in S'}(s-t(s))\mu(s|S') \geq \sum_{s\in S'}(s-t(s))\mu(s)$ or equivalently we have

$$EU_1(C^*,buy) = k^* - \sum_{s\in S'}s\mu(s) \geq k - \sum_{s\in S'}t(s)\mu(s) \geq k - \sum_{s\in A\cup S'}t(s)\mu(s)$$

$$= EU_1(C,buy)$$

Pick P_C^* an arbitrary probability distribution that is compatible with C. Then, C is

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either rejected or accepted under P_C^*. Either case, C^* (which is an accepted offer) is at least as profitable as C for the insurer.

Case 2: If $k < \sum_{s \in S'} t(s)\mu(s|S')$ then since $\Pi_C \neq \emptyset$, $\exists P_C \in \Pi_C$ such that

$$k \geq \sum_{s \in A \cup S'} t(s)P_C(s) = \sum_{s \in S'} t(s)P_C(s) + P_C(A \backslash S') \sum_{s \in A \backslash S'} t(s)P_C(s|A \backslash S') \qquad (7)$$

Inequality (7) and the assumption of Case (2) imply that

$$k > \sum_{s \in A \backslash S'} t(s)P_C(s|A \backslash S') \geq \min_{s \in A \backslash S'} t(s) =: t(\underline{s}) \qquad (8)$$

then $u(v - \underline{s} + t(\underline{s}) - k) < u(v - \underline{s})$. Then there exists an $\varepsilon_3 \in (0,1)$ such that

$$(1 - \varepsilon_3)u(v - \underline{s} + t(\underline{s}) - k) + \frac{\varepsilon_3}{m} \sum_{s \in A \backslash (S' \cup \{\underline{s}\})} u(v - s + t(s) - k) \qquad (9)$$

$$+ \frac{\varepsilon_3}{m} \sum_{s \in S'} u(v - s + t(s) - k)\mu(s|S')$$

$$< \quad (1 - \varepsilon_3)u(v - \underline{s}) + \frac{\varepsilon_3}{m} \sum_{s \in A \backslash (S' \cup \{\underline{s}\})} u(v - s) + \frac{\varepsilon_3}{m} \sum_{s \in S'} u(v - s)\mu(s|S')$$

Aside, Inequality (8) implies that for some $\varepsilon_4 \in (0,1)$, we have

$$k > (1 - \varepsilon_4)t(\underline{s}) + \frac{\varepsilon_4}{m} \sum_{s \in A \backslash (S' \cup \{\underline{s}\})} t(s) + \frac{\varepsilon_4}{m} \sum_{s \in S'} \mu(s|S') \qquad (10)$$

Set $\varepsilon = \min\{\varepsilon_3, \varepsilon_4\}$ and define a probability distribution $T_C \in \Delta(A \cup S')$ as

$$T_C(s) := \begin{cases} (1 - \varepsilon) & \text{if } s = \underline{s} \\ \frac{\varepsilon}{m}\mu(s|S') & \text{if } s \in S' \\ \frac{\varepsilon}{m} & \text{if } s \notin S' \cup \{\underline{s}\} \end{cases}$$

where m is the cardinality of $A \backslash S'$. Then by Inequality (10), T_C is compatible with

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C. Set $P_C^* := T_C$. From Inequality (9), C is rejected. Hence C^* is better than C for the insurer.

Case 3: If $k = \sum_{s \in S'} t(s)\mu(s|S')$ then since $\Pi_C \neq \emptyset$, pick P_C^* an arbitrary probability distribution from Π_C. Under P_C^*, if C is rejected then C^* (which is an accepted offer) is at least as profitable as C for the insurer. If C is accepted then

$$
\begin{aligned}
EU_1(C, buy) &= k - \sum_{s \in A \cup S'} t(s)\mu(s) \leq k - \sum_{s \in S'} t(s)\mu(s) = k(1 - \mu(S')) \\
&\leq k^*(1 - \mu(S')) \quad \text{since } k \leq k^* \text{ by assumption} \\
&\leq k^* - \mu(S') \sum_{s \in S'} s\mu(s|S') = EU_1(C^*, buy)
\end{aligned}
$$

So again C^* (which is an accepted offer) is at least as profitable as C for the insurer.

By following the construction suggested in above cases, $(C^*, (P_C^*)_{C \in \mathbb{C}})$ defines equilibrium which is incomplete. $\qquad \square$

Proof of Proposition 4.1. Let $C^* = (t^*, A^*, k^*)$ be an equilibrium contract such that either $t^*(s) \neq s$ for some $s \in A^*$ or $S' \not\subseteq A^*$.

Case 1: Let $A^* \backslash S' = \emptyset$. Then $\Pi_{C^*}^* = \mu(.|S')$. Now, consider $C = (t(s) = s, S', k)$ where k solves

$$ u(v - k) = \sum_{s \in A^*} u(v - s + t^*(s) - k^*)\mu(s|S') + \sum_{s \in S' \backslash A^*} u(v - s - k^*)\mu(s|S'). \tag{11} $$

The right and left hand sides of above equality are $EU_0(C^*|\mu(s|S'))$ and $EU_0(C|\mu(s|S'))$, respectively. As an equilibrium offer, since C^* is bought then so is C. Moreover, since u is concave and increasing Equation (11) implies that

$$ k - k^* > \sum_{s \in S'} s\mu(s|S') - \sum_{s \in A^*} t^*(s)\mu(s|S') \geq \mu(S')\left(\sum_{s \in S'} s\mu(s|S') - \sum_{s \in A^*} t^*(s)\mu(s|S')\right). $$

The last inequality holds since $1 \geq \mu(S') > 0$ and for any s, $s \geq t^*(s)$, then by

rearranging the terms we have

$$EU_1(C, D^*(C)) = k - \sum_{s \in S'} s\mu(s) > k^* - \sum_{s \in A^*} t^*(s)\mu(s) = EU_1(C^*, D^*(C^*))$$

This means that C gives higher utility to the insurer than C^* and it is also accepted by the insuree. This contradicts with optimality of C^*.

Case 2: Let $A^* \backslash S' \neq \emptyset$. Before proving the statement for this case we first need the following result.

Lemma 7.1. *Let $C^* = (t^*, A^*, k^*)$ be the contract offered at an equilibrium such that either $t^*(s) \neq s$ for some $s \in A^*$ or $S' \nsubseteq A^*$ and let $A^* \backslash S' \neq \emptyset$ then $P \in \Delta(A^* \cup S')$ defined as*

$$P(s) = \begin{cases} \mu(s) & \textit{if } s \in S' \\ 1 - \mu(S') & s = s_o \quad \textit{where } s_o \in \arg\max_{s \in A^* \backslash S'} [s - t^*(s)] \\ 0 & \textit{otherwise} \end{cases}$$

is an element of $\Pi^*_{C^*}$.

Proof. First observe that such s_o is well defined since we assumed $A^* \backslash S' \neq \emptyset$. $P \in \Pi^*_{C^*}$, if three conditions hold: (i) $P(.|S') = \mu(.|S')$ (this holds by definition), (ii) $\alpha \leq P(S')$ (this holds since $\alpha \leq \mu(S') = P(S')$), (iii) $k^* \geq \sum_{s \in A^*} t^*(s)P(s)$.

To prove point (iii), consider $C = (t(s) = s, A^* \cup S', k)$ where $k = \sum_{s \in A^* \cup S'} sP(s)$. Observe that by definition of k, $P \in \Pi^*_C$. Then from concavity of u

$$u(v - k) > \sum_{s \in A^* \cup S'} u(v - s)P(s) \geq \min_{R \in \Pi^*_C} \sum_{s \in A^* \cup S'} u(v - s)R(s) = EU_o(C, reject|\Pi^*_C)$$

Therefore the insuree buys C. Let us analyze the insurer's side. Since C^* is optimal

then

$$
\begin{aligned}
EU_1(C, D^*(C)) \quad &\leq EU_1(C^*, D^*(C^*)) \\
k - \sum_{s \in A^* \cup S'} s\mu(s) \quad &\leq k^* - \sum_{s \in A^*} t^*(s)\mu(s) \\
k \quad &\leq k^* + \sum_{s \in A^* \cup S'} (s - \tilde{t}(s))\mu(s) \text{ where } \tilde{t} = t^* \text{ on } A^* \text{ and } \tilde{t} = 0 \text{ on } S' \backslash A^* \\
&= k^* + \sum_{s \in S'} (s - \tilde{t}(s))\mu(s) + \sum_{s \in A^* \backslash S'} (s - \tilde{t}(s))\mu(s) \\
&\leq k^* + \sum_{s \in S'} (s - \tilde{t}(s))\mu(s) + \mu(A^* \backslash S')(s_o - \tilde{t}(s_o)) \\
&\leq k^* + \sum_{s \in S'} (s - \tilde{t}(s))\mu(s) + (1 - \mu(S'))(s_o - \tilde{t}(s_o))
\end{aligned}
$$

This implies by definition of k and $P(s)$ that

$$
\sum_{s \in A^* \cup S'} sP(s) \quad = \quad k \leq k^* + \sum_{s \in A^* \cup S'} (s - \tilde{t}(s))P(s)
$$

Therefore by definition of $\tilde{t}$, $\sum_{s \in A^*} t^*(s)P(s) \leq k^*$ and hence $P \in \Pi_{C^*}^*$.

$\square$

Now we can continue the proof of Case 2. Recall that so far we have shown P defined in Lemma 7.1 is in $\Pi_{C^*}^*$, we are in case $A^* \backslash S' \neq \emptyset$, and we assumed for contradiction that either $t^*(s) \neq s$ for some $s \in A^*$ or $S' \nsubseteq A^*$. Now consider $C = (t(s) = s, A^* \cup S', k)$ where k solves $u(v - k) = EU_0(C^*, D^*(C^*)|\Pi_{C^*}^*)$.

Let $Q := \arg\min_{R \in \Pi_{C^*}^*} \sum_{A^* \cup S'} u(v - s)R(s)$. Since $Q \in \Pi_{C^*}^*$ then

$$
\sum_{s \in A^*} u(v - s + t^*(s) - k^*)Q(s) + \sum_{s \in S' \backslash A^*} u(v - s - k^*)Q(s) \geq u(v - k)
$$

$$
u\left(\sum_{s \in A^*} (v - s + t^*(s) - k^*)Q(s) + \sum_{s \in S' \backslash A^*} (v - s - k^*)Q(s) \right) > u(v - k)
$$

$$
\sum_{s \in A^*} (v - s + t^*(s) - k^*)Q(s) + \sum_{s \in S' \backslash A^*} (v - s - k^*)Q(s) > v - k
$$

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$$k - \sum_{A^* \cup S'} sQ(s) > k^* - \sum_{A^*} t^*(s)Q(s) \geq 0.$$

The last inequality holds because $Q \in \Pi^*_{C^*}$ and this makes $Q \in \Pi^*_C$. Observe that

$$EU_0(C, buy|\Pi^*_C) = u(v - k) = EU_0(C^*, D^*(C^*)|\Pi^*_{C^*})$$

$$\geq \min_{R \in \Pi^*_{C^*}} \sum_{s \in A^* \cup S'} u(v - s)R(s) \text{ since } C^* \text{ is accepted}$$

$$= \sum_{s \in A^* \cup S'} u(v - s)Q(s) \text{ by definition of } Q$$

$$\geq \min_{R \in \Pi^*_C} \sum_{s \in A^* \cup S'} u(v - s)R(s) \text{ since } Q \in \Pi^*_C$$

$$= EU_0(C, reject|\Pi^*_C)$$

So the insuree buys C. Next we show the insurer prefers C to C^*.

$$EU_0(C^*, D^*(C^*)|\Pi^*_{C^*}) = u(v - k)$$

$$\leq \sum_{s \in A^*} u(v - s + t^*(s) - k^*)P(s) + \sum_{s \in S' \setminus A^*} u(v - s - k^*)P(s)$$

$$< u \left(\sum_{s \in A^*} (v - s + t^*(s) - k^*)P(s) + \sum_{s \in S' \setminus A^*} (v - s - k^*)P(s) \right)$$

The first inequality holds since $P \in \Pi^*_{C^*}$ by Lemma 7.1 and the last inequality holds since either $t^*(s) \neq s$ for some s or $S' \setminus A^* \neq \emptyset$ and u is concave. Then since u is increasing, if we rearrange the terms and plug P in, we get

$$k > k^* + \sum_{s \in A^* \cup S'} sP(s) - \sum_{s \in A^*} t^*(s)P(s)$$

$$k - \sum_{s \in S'} s\mu(s) > k^* - \sum_{s \in A^* \cap S'} t^*(s)\mu(s) + (1 - \mu(S'))(s_o - t^*(s_o))$$

$$\geq k^* - \sum_{s \in A^* \cap S'} t^*(s)\mu(s) + \mu(A^* \setminus S')(s_o - t^*(s_o))$$

$$\geq k^* - \sum_{s \in A^* \cap S'} t^*(s)\mu(s) + \sum_{s \in A^* \setminus S'} s - t^*(s)\mu(s) \text{ by definition of } s_o.$$

By rearranging the terms, $k - \sum_{A^* \cup S'} s\mu(s) > k^* - \sum_{A^*} t^*(s)\mu(s)$ which means the insurer prefers C to C^*. This contradicts with optimality of C^* and completes the proof of Case 2. $\qquad \square$

Proof of Proposition 4.2. Let $C^* = (t^*, A^*, k^*)$ be an equilibrium contract but the insuree strictly prefer the contract to her outside option, i.e. $EU_0(C^*, buy|\Pi^*_{C^*}) >$

$EU_0(C^*, reject|\Pi^*_{C^*})$. By Proposition 4.1, we know that $S' \subseteq A^*$ and $t^*(s) = s$. Then we have $u(v - k^*) = EU_0(C^*, buy|\Pi^*_{C^*}) > \min\limits_{P \in \Pi^*_{C^*}} \sum\limits_{s \in A^* \cup S'} u(v - s)P(s)$. Let $\varepsilon > 0$ be such that $u(v - k^* - \varepsilon) = \min\limits_{P \in \Pi^*_{C^*}} \sum\limits_{s \in A^* \cup S'} u(v - s)P(s)$, and consider the contract $C = (t(s) = s, A^*, k^* + \varepsilon)$. C is the contract which does not deliver information more than C^* but it charges higher premium. The insurer prefers C to C^* if it is accepted. For contradiction, it suffices to show that the insuree accepts C.

Let $R := \arg\min\limits_{P \in \Pi^*_{C^*}} \sum\limits_{s \in A^*} u(v - s)P(s)$. Since $R \in \Pi^*_{C^*}$, we have $R(.|S') = \mu(.|S')$, $\alpha \leq R(S')$, and $k^* - \sum\limits_{s \in A^*} sR(s) \geq 0$. Then $k^* + \varepsilon - \sum\limits_{A^*} sR(s) \geq 0$ and this makes $R \in \Pi^*_C$. Moreover,

$$EU_0(C, buy|\Pi^*_C) = u(v - k^* - \varepsilon) = \sum\limits_{s \in A^*} u(v - s)R(s) \geq \min\limits_{P \in \Pi^*_C} \sum\limits_{s \in A^* \cup S'} u(v - s)P(s).$$

So the insuree buys C which contradicts with optimality of C^*. $\qquad\square$

Proof of Theorem 4.1. The proof is divided into 6 steps. Let $C^* = (t^*, A^*, k^*)$ be the contract offered at an equilibrium. From Proposition 4.1, $t^*(s) = s$ for any $s \in A^*$ and $S' \subseteq A^*$. For Step 1 to 5, assume $A^* \backslash S' \neq \emptyset$.

Step 1: $(1 - \alpha) \max\limits_{s \in A^* \backslash S'} s + \alpha \sum\limits_{s \in S'} s\mu(s|S') \leq k^*$.

Proof: Assume for contradiction that the inequality above does not hold and consider $C = (t^*, A^*, k^* + \varepsilon)$ where $\varepsilon := (1 - \alpha) \max\limits_{s \in A^* \backslash S'} s + \alpha \sum\limits_{s \in S'} s\mu(s|S') - k^* > 0$. This contract obviously dominates C^* for the insurer as long as it is accepted by the insuree. Therefore, for contradiction, it suffices to show that C is accepted. First define $R \in \Delta(A^*)$ such that

$$R(s) := \begin{cases} (1 - \alpha) & \text{if } s = \max\limits_{s \in A^* \backslash S'} s \\ \alpha\mu(s|S') & \text{if } s \in S' \\ 0 & \text{otherwise} \end{cases}$$

Observe that $k^* + \varepsilon - \sum\limits_{s \in A^*} sR(s) \geq 0$ (indeed it is equality because of the definition

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of ε and R). So $R \in \Pi_C^*$. Next observe that

$$
\begin{aligned}
EU_0(C, buy|\Pi_C^*) &= u(v - k^* - \varepsilon) = u(v - \sum_{s \in A^*} sR(s)) > \sum_{s \in A^*} u(v - s)R(s) \\
&\geq \min_{P \in \Pi_C^*} \sum_{s \in A^*} u(v - s)P(s) = EU_0(C, reject|\Pi_C^*).
\end{aligned}
$$

The last inequality holds since $R \in \Pi_C^*$. Therefore, C is accepted.

Step 2: $\sum_{s \in S'} s\mu(s|S') \leq k^*$.

Proof: Assume not, then consider $C = (t^*, A^*, k^* + \varepsilon)$ where $\varepsilon := \sum_{s \in S'} s\mu(s|S') - k^* > 0$. This contract dominates C^* for the insurer as long as it is accepted. So we have contradiction if C is accepted. First define $Q \in \Delta(A^*)$:

$$
Q(s) := \begin{cases} \mu(s|S') & \text{if } s \in S' \\ 0 & \text{otherwise} \end{cases} \quad k^* + \varepsilon - \sum_{s \in S'} sQ(s) \geq 0 \text{ (indeed it is equality) so}
$$

$Q \in \Pi_C^*$. Then

$$
\begin{aligned}
EU_0(C, buy|\Pi_C^*) &= u(v - k^* - \varepsilon) = u(v - \sum_{s \in A^*} sQ(s)) > \sum_{s \in A^*} u(v - s)Q(s) \\
&\geq \min_{P \in \Pi_C^*} \sum_{s \in A^*} u(v - s)P(s) = EU_0(C, reject|\Pi_C^*).
\end{aligned}
$$

The last inequality holds since $Q \in \Pi_C^*$. Therefore, C is accepted.

Step 3: $\Pi_{C^*}^* = \mathcal{P} := \{P \in \Delta(A \cup S') \mid P(.|S') = \mu(.|S'), \alpha \leq P(S')\}$.

Proof: By definition $\Pi_{C^*}^* \subseteq \mathcal{P}$. Let $P \in \mathcal{P}$. From Step 1 and 2, we know that $k^* \geq \max\{\sum_{s \in S'} s\mu(s|S'), (1-\alpha) \max_{s \in A^* \backslash S'} s + \alpha \sum_{s \in S'} s\mu(s|S')\}$. Since $P \in \mathcal{P}$, $k^* \geq \sum_{s \in A^*} sP(s)$, i.e. $P \in \Pi_{C^*}^*$. Therefore, $\mathcal{P} \subseteq \Pi_{C^*}^*$.

Step 4: $A^* \backslash S'$ is singleton.

Proof: Assume $A^* \backslash S'$ is not singleton. By Proposition 4.2 and Step 3, we have $u(v - k^*) = \min_{P \in \mathcal{P}} \sum_{s \in A^*} u(v - s)P(s)$. Since distributions R and Q defined in Step 1 and 2 respectively are in $\mathcal{P}$, this minimization is attained either at R or at Q. Therefore, k^* only depends on the most costly state in $A^* \backslash S'$ and call this state $\hat{s} = \max_{s \in A^* \backslash S'} s$. If

C^* is accepted so is $C = (t^*, S' \cup \{\widehat{s}\}, k^*)$. If $A^* \backslash S'$ is not singleton then obviously the insurer would prefer C to C^*, and this would contradict with optimality of C^*. Hence, $A^* \backslash S' = \widehat{s}$.

Step 5: $u(v - k^*) = (1 - \alpha) \max\limits_{s \in A^* \backslash S'} s + \alpha \sum\limits_{s \in S'} s\mu(s|S')$.

Proof: From Proposition 4.2, Step 3 and Step 4 we have

$$u(v - k^*) = \min_{1 \geq \beta \geq \alpha} \left[(1 - \beta) u(v - \widehat{s}) + \beta \sum_{s \in S'} u(v - s) \mu(s|S') \right] \tag{12}$$

If $u(v - \widehat{s}) > \sum\limits_{s \in S'} u(v - s) \mu(s|S')$ then the minimum in Equation 12 is attained at $\beta = 1$ and $u(v - k^*) = \sum\limits_{s \in S'} u(v - s) \mu(s|S')$. Then consider $C = (t(s) = s, S', k^*)$. By definition of k^*, such a contract would be accepted by the insuree and the insurer prefers C to C^*. This contradicts with the optimality of C^*. Therefore, it has to be the case that $u(v - \widehat{s}) \leq \sum\limits_{s \in S'} u(v - s) \mu(s|S')$. Then the minimum in Equation 12 is attained at $\beta = \alpha$.

Step 6: If $\max\limits_{s \in S \backslash S'} k_s^* - s\mu(s) > k_{S'}$ where $\forall s \in S \backslash S'$, k_s^* and $k_{S'}^*$ are defined respectively:

$$u(v - k_s^*) = (1 - \alpha) u(v - s) + \alpha \sum_{\widetilde{s} \in S'} u(v - \widetilde{s}) \mu(\widetilde{s}|S') \tag{13}$$

$$u(v - k_{S'}^*) = \sum_{\widetilde{s} \in S'} u(v - \widetilde{s}) \mu(\widetilde{s}|S') \tag{14}$$

then $C^* = (t^*(s) = s, S' \cup \{\widehat{s}\}, k_{\widehat{s}}^*)$ where $\widehat{s} \in \arg\max\limits_{s \in S \backslash S'} (k_s^* - s\mu(s))$. Otherwise, $C^* = (t^*(s) = s, S', k_{S'}^*)$.

Proof: From Step 1 to 5 we have that for the insurer the best acceptable contract that announces some new contingencies is $C = (t^*(s) = s, S' \cup \{\widehat{s}\}, k_{\widehat{s}}^*)$. Rather than C, the insurer may also offer the best contract that does not announce any new contingency, $C' = (t^*(s) = s, S', k_{S'}^*)$ where $k_{S'}^*$ as in Equation (14). If $EU_1(C, buy) = k_{\widehat{s}}^* - \sum\limits_{s \in S'} s\mu(s) - \widehat{s}\mu(\widehat{s}) > k_{S'}^* - \sum\limits_{s \in S'} s\mu(s) = EU_1(C', buy)$ then she offers $C^* = C$,

otherwise the equilibrium contract is $C^* = C'$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Proof of Proposition 4.3. Let $C^* = (t^*(s) = s, A^*, k^*)$ be the equilibrium contract characterized in Theorem 4.1. Then A^* is either S' or $S' \cup \{\widehat{s}\}$. If $A^* = S'$ then C^* is clearly the best contract for the insurer according to the insuree. Consider $A^* = S' \cup \{\widehat{s}\}$. Then the only alternative contracts that the insuree can think of are on either S' or A^*.

$\quad$ *Case 1:* Let $C = (t(s) = s, S', k)$ where $u(v - k) = \sum\limits_{s \in S'} u(v - s)\mu(s|S')$.[17]

$\quad$ By Theorem 4.1, and definition of k:

$$u(v - k^*) = (1 - \alpha)u(v - \widehat{s}) + \alpha \sum\limits_{s \in S'} u(v - s)\mu(s|S') = (1 - \alpha)u(v - \widehat{s}) + \alpha u(v - k)$$

Then there exists a $\delta > 0$ such that $v - k^* + \delta = (1 - \alpha)(v - \widehat{s}) + \alpha(v - k)$.

$\quad$ By rearranging the terms we get $\delta + (1 - \alpha)\widehat{s} - (1 - \alpha)k = k^* - k$. For α large enough $(1 - \alpha)\widehat{s} \le k^* - k$. Then for all $\gamma \in [\alpha, 1]$, $k - \sum\limits_{s \in S'} sP(s) \le k^* - \sum\limits_{s \in S'} sP(s) - \widehat{s}(1 - \gamma)$ for any $P \in \Pi_{C^*}^*$. Since for any $P \in \Pi_{C^*}^*$, $P(\widehat{s}) = 1 - \gamma$ for some $\gamma \in [\alpha, 1]$, we can rewrite the last inequality as

$$EU_1^0(C, D^*(C)|P) = k - \sum\limits_{s \in S'} sP(s) \le k^* - \sum\limits_{s \in S'} sP(s) - \widehat{s}P(\widehat{s}) = EU_1^0(C^*, D^*(C^*)|P).$$

$\quad$ *Case 2:* Let $\tilde{C} = (t(s), S', \tilde{k})$ and $\sum\limits_{s \in S'} u(v - s + t(s) - \tilde{k})\mu(s|S') \ge \sum\limits_{s \in S'} u(v - s)\mu(s|S')$[18]

Then from concavity of u and definition of k in case 1:

$$u\left(\sum\limits_{s \in S'} (v - s + t(s) - \tilde{k})\mu(s|S') \right) \ge u(v - k)$$

$\forall \gamma \in [\alpha, 1]$ $k - \gamma \sum\limits_{s \in S'} s\mu(s|S') \ge \tilde{k} - \gamma \sum\limits_{s \in S'} t(s)\mu(s|S')$. This gives $EU_1^0(C^*, D^*(C^*)|P) \ge EU_1^0(C, D^*(C)|P) \ge EU_1^0(\tilde{C}, D^*(\tilde{C})|P)$ where C is the contract considered in Case 1 and the first inequality comes from the proof of Case

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[17] We show it only for k that is the premium which makes the insuree indifferent between accepting and rejecting the offer. The comparison with the rejected offers will be analyzed in Case 4.

[18] This is again assumed to make $\tilde{C}$ an accepted offer, $\tilde{C}$s that are not accepted will be addressed in Case 4. Observe that Case 2 is a generalization of Case 1 but we will use the proof of Case 1 here.

1.

Case 3: Let $\tilde{C} = (t(s), A^*, \tilde{k})$ be an accepted offer.[19] For contradiction assume that $\exists R \in \Pi^*_{C^*}$ such that $\tilde{k} - \sum\limits_{s \in A^*} t(s)R(s) > k^* - \sum\limits_{s \in A^*} sR(s) \geq 0$. The last inequality comes from $R \in \Pi^*_{C^*}$, therefore $R \in \Pi^*_{\tilde{C}}$. Since $\tilde{C}$ is bought, u is concave, and $R \in \Pi^*_{\tilde{C}}$ we have

$$u\left(\sum\limits_{s \in A^*} (v - s + t(s) - \tilde{k})R(s)\right) > \min\limits_{P \in \Pi^*_{\tilde{C}}} \sum\limits_{s \in A^*} u(v - s)P(s)$$
$$\geq \min\limits_{P \in \Pi^*_{C^*}} \sum\limits_{s \in A^*} u(v - s)P(s)$$

The last inequality holds since $\Pi^*_{\tilde{C}} \subseteq \Pi^*_{C^*}$ from the observation we made in Step 3 in the proof of Theorem 4.1. The right hand side is $u(v - k^*)$. Since u is increasing, we have

$$k^* - \sum\limits_{s \in A^*} sR(s) > \tilde{k} - \sum\limits_{s \in A^*} t(s)R(s) \text{ which is a contradiction.}$$

Case 4: Let $\tilde{C}$ be a rejected offer which specifies transfers on a subset of A^*, then $EU_1^0(\tilde{C}, D^*(\tilde{C})|P) = 0$ for any $P \in \Pi^*_{C^*}$. On the other hand $EU_1^0(C^*, D^*(C^*)|P) = k^* - \sum\limits_{s \in A^*} sP(s) \geq 0$ by definition of $P \in \Pi^*_{C^*}$. Hence $EU_1^0(C^*, D^*(C)|P) \geq EU_1^0(\tilde{C}, D^*(\tilde{C})|P)$.

$\square$

Proof of Theorem 5.1. Let any insurer $j \neq i$ offers $C^*_j = (t^*(s) = s, S, k^*)$ where $k^* = \sum\limits_{s \in S} s\mu(s)$. We show that $C^*_i = (t^*(s) = s, S, k^*)$ is the best contract insurer i can make.

Assume not then $\exists C_i = (t(s), A, k')$ such that

1) $k' > \sum\limits_{s \in A} t(s)\mu(s)$

2) $\min\limits_{P \in \Pi^*_{(C_i, C^*_{-i})}} \left(\sum\limits_{s \in A} u(v - s + t(s) - k')P(s) + \sum\limits_{s \in S \setminus A} u(v - s - k')P(s)\right) \geq u(v - k^*)$

3) $\min\limits_{P \in \Pi^*_{(C_i, C^*_{-i})}} \left(\sum\limits_{s \in A} u(v - s + t(s) - k')P(s) + \sum\limits_{s \in S \setminus A} u(v - s - k')P(s)\right)$
$$\geq \min\limits_{P \in \Pi^*_{(C_i, C^*_{-i})}} \sum\limits_{S} u(v - s)P(s).$$

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[19]Note that here there is room for $t(s) = 0$ for some s. Therefore, the deviation contracts considered in this case may not make transfers at every contingency in A^*. Proving the statement for such general transfer rules will be enough.

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(1) guarantees that the deviation is profitable. (2) shows that the insuree weakly prefers C_i to any C_j^*, therefore the offer of insurer i is attractive. (3) makes the insuree weakly prefer C_i to rejecting C_i and any C_j^*.

By (1) and the definition of k^*, we have $\mu \in \Pi^*_{(C_i, C^*_{-i})}$. Then

$$u\left(\sum_{s\in A}(v - s + t(s) - k')\mu(s) + \sum_{s\in S\backslash A}(v - s - k')\mu(s)\right)$$

$$\geq \min_{P\in\Pi^*_{(C_i,C^*_{-i})}}\left(\sum_{s\in A}u(v - s + t(s) - k')P(s) + \sum_{s\in S\backslash A}u(v - s - k')P(s)\right) \text{ since } u$$

is concave $\mu \in \Pi^*_{(C_i,C^*_{-i})}$

$\geq u(v - k^*)$ by point (2) above. Then since u is increasing, by rearranging the terms and using point (1) we get the following contradiction:

$$k^* \geq k' + \sum_{s\in S}s\mu(s) - \sum_{s\in A}t(s)\mu(s) > \sum_{s\in S}s\mu(s) = k^*$$

Observe that $\mu \in \Pi^*_{C^*}$, therefore $\Pi^*_{C^*} \neq \emptyset$. And since by definition k^* is a smaller premium than the certainty equivalent of rejecting both offers, C^* is accepted. $\qquad\square$

Proof of Theorem 5.2. Let there be N competing insurers. We will show that, besides the one in Theorem 5.1, there is no other symmetric equilibrium where the insuree buys a contract. Assume for contradiction that in a symmetric equilibrium each insurer offers contract $C_i^* = (t^*, A^*, k^*)$ with $A^* \cup S' \subsetneq S$, and the insuree buys one of them randomly. Since this is an equilibrium contract, $EU_i(C_1^*, ..., C_N^*, buy_i) = \frac{1}{N}(k^* - \sum_{s\in A^*}t^*(s)\mu(s)) \geq 0$. Define $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$u(v - \sum_{s\in S}s\mu(s) - \varepsilon_1) = \sum_{s\in S}u(v - s)\mu(s)$$

$$u(v - k^* + \sum_{s\in A^*}t^*(s)\mu(s) - \sum_{s\in S}s\mu(s) - \varepsilon_2)$$

$$= \sum_{s\in A^*}u(v - s - k^* + t^*(s))\mu(s) + \sum_{s\in S\backslash A^*}u(v - s - k^*)\mu(s)$$

Then define $\varepsilon := \frac{1}{2}\min\{\varepsilon_1, \varepsilon_2\}$. We claim that if all the insurers except insurer i offer

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contract C_j^* then for N large enough, insurer i can profitably deviate to $C_i = (t(s) = s, S, k)$ where $k = \sum_{s \in S} s\mu(s) + \varepsilon$. Then $\mu \in \Pi^*_{(C_i, C^*_{-i})}$. Since $k < \sum_{s \in S} s\mu(s) + \varepsilon_1$ then

$$EU_0(C_i, C^*_{-i}, buy_i | \Pi^*_{(C_i, C^*_{-i})}) = u(v - k)$$

$$> u(v - \sum_{s \in S} s\mu(s) - \varepsilon_1) = \sum_{s \in S} u(v - s)\mu(s)$$

$$k < \sum_{s \in S} s\mu(s) + \varepsilon_2 \leq \sum_{s \in S} s\mu(s) + \varepsilon_2 + k^* - \sum_{s \in A^*} t^*(s)\mu(s)$$

$$EU_0(C_i, C^*_{-i}, buy_i | \Pi^*_{(C_i, C^*_{-i})}) > u(v - \sum_{s \in S} s\mu(s) - \varepsilon_2 - k^* + \sum_{s \in A^*} t^*(s)\mu(s))$$

$$= \sum_{s \in A^*} u(v - s - k^* + t^*(s))\mu(s) + \sum_{s \in S \setminus A^*} u(v - s - k^*)\mu(s) \text{ by definition}$$

of ε_2

$$\geq \min_{P \in \Pi^*_{(C_i, C^*_{-i})}} \sum_{s \in A^*} u(v - s - k^* + t^*(s))P(s) + \sum_{s \in S \setminus A^*} u(v - s - k^*)P(s)$$

since $\mu \in \Pi^*_{(C_i, C^*_{-i})}$

$$= EU_0(C_i, C^*_{-i}, buy_j | \Pi^*_{(C_i, C^*_{-i})}) \text{ for any } j \neq i.$$

Therefore, the insuree prefers C_i to any C_j^* and rejection. Expected utility of insurer i from offering the same contract with his competitors is $\frac{1}{N}(k^* - \sum_{s \in A^*} t^*(s)\mu(s))$ and his expected utility from deviating to C_i is $k - \sum_{s \in S} s\mu(s) = \varepsilon$. Observe that for N sufficiently large $EU_i(C_i^*, C^*_{-i}, D^*(C_i^*, C^*_{-i}))$

$< EU_i(C_i, C^*_{-i}, D^*(C_i^*, C^*_{-i}))$ and therefore the deviation is beneficial which is a contradiction. $\square$

Example 7.1 (Incomplete Contract Equilibrium under Competition). *Let $S = \{1, 14.6, 15\}$, $S' = \{15\}, v = 25, u(x) = \sqrt{x}, \mu(\{1\}) = .01/3, \mu(\{14.6\}) = 1.99/3, \mu(\{15\}) = 1/3, \alpha = 1/3$ and $N = 2$.*

We claim that offering incomplete contracts $C_1^* = C_2^* = (t^*(s) = s, \{1, 15\}, k^* = 17/3)$ is part of an equilibrium under ambiguity aversion with competition. The multiple belief set C_1^* and C_2^* generate is

$$\Pi^*_{(C_1^*, C_2^*)} = \{p | \alpha \leq p \leq 1, \frac{17}{3} - p15 - (1 - p)1 \geq 0\} = \{\frac{1}{3}\} \tag{15}$$

$$EU_0(C_1^*, C_2^*, buy_i | \Pi^*_{(C_1^*, C_2^*)}) = \sqrt{25 - \tfrac{17}{3}} \geq \tfrac{1}{3}\sqrt{25 - 15} + \tfrac{2}{3}\sqrt{25 - 1}$$

$$EU_i(C_1^*, C_2^*, D^*(C_1^*, C_2^*)) = \frac{1}{2}(\frac{17}{3} - \frac{.01}{3}1 - \frac{1}{3}15) = \frac{1.99}{6} > 0$$

for $i = 1, 2$. Therefore, the insuree buys either C_1 or C_2^* which have positive expected utility for each insurer. For a contradiction, assume that there is a profitable deviation to complete contract $C = (t(s), S, k)$. To be a profitable deviation this contract needs to be accepted by the insuree, so the following inequality should hold:

$$\min_{(p,q) \in \Pi^*_{(C,C_2^*)}} [pu(v - 15 + t(15) - k) + qu(v - 14.6 + t(14.6) - k)$$

$$+ (1 - p - q)u(v - 1 + t(1) - k)] \tag{16}$$

$$\geq \min_{(p,q) \in \Pi^*_{(C,C_2^*)}} [pu(v - 15) + qu(v - 14.6) + (1 - p - q)u(v - 1)]$$

$k - \mu(15)t(15) - \mu(14.6)t(14.6) - \mu(1)t(1) \geq 0$ since C is a profitable deviation. So, $\mu \in \Pi^*_{(C,C_2^*)}$ and this implies that the left hand side of Inequality (16) is smaller than or equal to

$$\mu(15)u(v - 15 + t(15) - k) + \mu(14.6)u(v - 14.6 + t(14.6) - k) + \mu(1)u(v - 1 + t(1)) - k]$$

The right hand side of Inequality (16) is bigger than or equal to $u(v-15)$ (because it is a convex combination). By putting these two observations together and by concavity of u, we have

$$\sum_{s \in S}(v - s + t(s) - k)\mu(s) > v - 15$$

$$15 - \sum_{s \in S} s\mu(s) \geq k - \sum_{s \in S} t(s)\mu(s) > EU_1(C_1^*, C_2^*, D^*(C_1^*, C_2^*)) = \frac{1.99}{6}$$

The last inequality above holds since C is assumed to be a profitable deviation. By

plugging values of μ and s, we get a contradiction. So deviations that extend the awareness of the insuree further cannot be profitable.

Insurer 1 can also offer a contract that does not extend the awareness set of the insuree more. For any such contract $C = (t, \{1, 15\}, k)$ the belief set $\Pi^*_{(C, C^*_2)}$ is either empty or singleton which assigns probability α to event $\{15\}$ and $(1 - \alpha)$ to event $\{1\}$. This is observed from Equation (15). The contracts that make the corresponding belief set empty are rejected by definition of equilibrium strategy of the insuree, and among the contracts that make the belief set singleton will make the offer of insurer 2 more attractive. So in any case such contracts cannot be profitable.

This shows that offering C^*_1 is the best contract insurer 1 can offer given that insurer 2 is offering C^*_2. Therefore, the exercise have an equilibrium with incomplete contracts.

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