International Macro Lectures 6 and 7

Ethan Kaplan

Gourinchas-Rey I

- We start with a dynamic current account asset equation:

$$NA_{t+1} = R_{t+1}\left(NA_t + NX_t\right)$$

- Rewriting, we get: $\left(\frac{A_{t+1}}{W_{t+1}} - \frac{L_{t+1}}{W_{t+1}}\right)\frac{W_{t+1}}{W_t} = R_{t+1}\left(\frac{A_t}{W_t} - \frac{L_t}{W_t} + \frac{X_t}{W_t} - \frac{M_t}{W_t}\right)$

- Also, defining:

$$R_{t+1} = R\left(1 + r_{t+1}\right) \qquad \frac{W_{t+1}}{W_t} = \Gamma\left(1 + \varepsilon_{t+1}^{\Delta W}\right)$$

Gourinchas-Rey II

- Imposing a steady state condition:

$$\left(\mu_{t+1}^{aw}-\mu_{t+1}^{lw}\right)=\frac{R}{\Gamma}\left(\mu_{t}^{aw}-\mu_{t}^{lw}+\mu_{t}^{mw}-\mu_{t}^{mw}\right)$$

$$\frac{R}{\Gamma}>1$$

- Rewriting, we get:

$$\frac{NX}{NA}=\rho-1<0$$

- Log-linearizing around the steady state, we get:

$$na_{t+1}\approx\frac{1}{\rho_t}na_t+\left(\hat{r}_{t+1}-\Delta w_{t+1}\right)-\left(\frac{1}{\rho_t}-1\right)nx_t$$

Gourinchas-Rey III

- Finally, assuming a common trend in underlying varaibles (x, m, a, l), we get:

$$nx_{t+1} \approx \frac{1}{\rho} nxa_t + r_{t+1} + \Delta nx_{t+1}$$

- Where:

$$nxa_t = |\mu_a|\varepsilon_t^a - |\mu_l|\varepsilon_t^l + |\mu_x|\varepsilon_t^x - |\mu_m|\varepsilon_t^m$$

$$\Delta nx_t = |\mu_x|\Delta\varepsilon_{t+1}^x - |\mu_m|\Delta\varepsilon_{t+1}^m - \Delta w_{t+1}$$

$$r_{t+1} = |\mu^a| r_{t+1}^a - |\mu^l| r_{t+1}^l$$

Gourinchas-Rey IV

- Note that: $$nx_{t+1} \approx -\sum_{j=1}^{\infty} E_t\left[r_{t+1} + \Delta nx_{t+j}\right]$$

- To construct imbalances:

$$nxa_t = |\mu_a|\varepsilon_t^a - |\mu_l|\varepsilon_t^l + |\mu_x|\varepsilon_t^x - |\mu_m|\varepsilon_t^m$$

the authors decompose log exports, imports, gross foreign assets, and liabilities relative to wealth and filter out (using an HP filter) low frequency trends (including removing unit roots).

$$\mu_a = 8.4; \mu_l = 7.49; \mu_x = -9.98; \mu_m = -10.98; \rho = 0.95$$

$$0.85\varepsilon_t^a - 0.75\varepsilon_t^l + \varepsilon_t^x - 1.1\varepsilon_t^m$$

Table 1: Descriptive Statistics

	Summary Statistics								
	Δx_t	Δm_t	Δa_t	Δl_t	r_t	r_t^a	r_t^l	Δe_t	nxa_t
Mean (%)	0.82	1.11	1.11	1.87	0.72	0.78	0.78	-0.03	0
Standard deviation (%)	4.28	3.81	3.08	2.87	13.16	2.50	2.57	3.55	11.94
Autocorrelation	-0.08	0.04	0.06	0.13	0.16	0.12	0.19	0.05	0.92
	r_t^{ae}	r_t^{le}	r_t^{ad}	r_t^{ld}	r_t^{af}	r_t^{lf}	r_t^{ao}	r_t^{lo}	
Mean (%)	1.87	1.86	0.72	0.56	1.08	1.09	0.48	0.39	
Standard deviation (%)	7.19	8.02	2.94	3.17	5.93	5.81	0.76	0.53	
Autocorrelation	0.15	0.09	0.16	0.13	0.09	0.10	0.19	0.73	

Note: Sample period is 1952:1-2004:1, except for Δe, 1973:1-2004:1.

Table 2: Unconditional Variance Decomposition of nxa

		Discount factor ρ		
#	percent	0.96	0.95	0.94
1	$\beta_{\Delta nx}$	71.77	63.96	57.05
2	β_r	23.76	26.99	28.85
	of which:			
3	β_{ra}	19.91	20.78	20.65
4	β_{rl}	3.87	6.22	8.21
5	Total (lines 1+2)	95.53	90.95	85.89
6	μ_a	6.72	8.49	10.08

Note: The sum of coefficients $\beta_{ra} + \beta_{rl}$ is not exactly equal to β_r due to numerical rounding in the VAR estimation. Sample: 1952:1 to 2004:1.

Table 3: Forecasting Quarterly Returns

Column:	1	2	3	4	5	6	7	8
	Panel A: Returns							
	Total real return (r_{t+1})				Real Equity Differential (Δr^e_{t+1})			
z_t:		r_t	$\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$	xm_t		Δr^e_t	$\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$	xm_t
$\hat{\beta}$	**-0.36**	**-0.33**	**-0.46**	**-0.37**	**-0.13**	**-0.14**	**−0.17**	-0.07
(s.e.)	(0.07)	(0.07)	(0.08)	(0.16)	(0.03)	(0.03)	(0.03)	(-0.06)
$\hat{\delta}$		0.09	-1.43	0.01		-0.07	-0.63	-0.09
(s.e.)		(0.07)	(1.60)	(0.19)		(0.07)	(0.61)	(0.07)
$\bar{R}^2$	0.10	0.10	0.15	0.10	0.07	0.07	0.12	0.07
# obs	208	207	136	208	208	207	136	208
	Panel B: Depreciation Rates							
	FDI-weighted (Δe_{t+1})				Trade-weighted (Δe^T_{t+1})			
z_t :		Δe_t	xm_t	$i_t - i^*_t$		Δe^T_t	xm_{t-1}	$i_t - i^*_t$
$\hat{\beta}$	**-0.08**	**-0.09**	**-0.10**	**-0.09**	**-0.09**	**-0.09**	**-0.08**	**-0.08**
(s.e.)	(0.02)	(0.02)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)
$\hat{\delta}$		-0.04	0.02	0.32		0.02	-0.01	-0.67
(s.e.)		(0.07)	(0.05)	(0.32)		(0.07)	(0.05)	(0.34)
$\bar{R}^2$	0.09	0.08	0.08	0.08	0.11	0.10	0.10	0.13
#obs	125	124	125	125	124	123	124	124

Note: Regressions of the form: $y_{t+1} = \alpha + \beta nxa_t + \delta z_t + \epsilon_{t+1}$ where y_{t+1} is the total real return (r_{t+1}); the equity return differential ($\Delta r^e_{t+1} = r^{ae}_{t+1} - r^{le}_{t+1}$) (panel A); the FDI-weighted depreciation rate (Δe_{t+1}) or the trade weighted depreciation rate (Δe^T_{t+1}) (panel B). $\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$ is the relative dividend price ratio (available since 1970:1); $i_t - i^*_t$ is the short term interest rate differential; xm_t is the stationary component from the trade balance, defined as $\epsilon^x_t - \epsilon^m_t$. Sample: 1952:1 to 2004:1 for total returns and 1973:1 to 2004:1 for depreciation rates. Robust standard errors in parenthesis.

Table 4: Forecasting Quarterly Returns (cont'ed)

Column	1	2	3	4	5	6	7	8
	Panel A: Dollar return on US equities and US gross liabilities							
	US equity return (r^{le}_{t+1})				US liabilities return (r^{l}_{t+1})			
z_t:		r^{le}_t	d_t/p_t	cay_t		r^l_t	d_t/p_t	cay_t
$\hat{\beta}$	0.02	0.02	0.05	0.03	0.01	0.01	0.02	0.01
(s.e.)	(0.05)	(0.08)	(0.05)	(0.05)	(0.02)	(0.02)	(0.02)	(0.02)
$\hat{\delta}$		0.08	**1.28**	**2.03**		**0.19**	**0.38**	**0.69**
(s.e.)		(0.06)	(0.60)	(0.45)		(0.07)	(0.19)	(0.16)
$\bar{R}^2$	0.00	0.00	0.02	0.09	0.00	0.03	0.02	0.10
# obs	208	207	208	206	208	207	208	206
	Panel B: US gross assets return (dollar and local currency)							
	Dollar return (r^{a}_{t+1})				Local currency return (r^{*a}_{t+1})			
z_t:		r^{a}_t	d^*_t/p^*_t	xm_t		r^{*a}_t	d^*_t/p^*_t	xm_t
$\hat{\beta}$	-0.03	-0.03	-0.03	0.00	0.03	0.02	**0.05**	**0.08**
(s.e.)	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)	(0.02)	(0.02)	(0.04)
$\hat{\delta}$		0.11	-0.01	-0.05		**0.16**	0.31	-0.08
(s.e.)		(0.09)	(0.21)	(0.05)		0.08	(0.22)	0.05
$\bar{R}^2$	0.02	0.01	0.01	0.02	0.01	0.03	0.03	0.02
# obs	208	207	136	208	208	207	136	208
	Panel C: Return on foreign equities (dollar and local currency)							
	Dollar return (r^{ae}_{t+1})				Local currency return (r^{*ae}_{t+1})			
z_t:		r^{ae}_t	d^*_t/p^*_t	xm_t		r^{ae*}_t	d^*_t/p^*_t	xm_t
$\hat{\beta}$	**-0.12**	**-0.11**	**-0.11**	-0.00	-0.06	-0.06	-0.03	-0.08
(s.e.)	(0.04)	(0.04)	(0.07)	(0.08)	(0.05)	(0.04)	(0.06)	(0.11)
$\hat{\delta}$		0.12	0.37	-0.16		0.16	0.69	-0.19
(s.e.)		(0.08)	(0.59)	(0.09)		(0.08)	(0.57)	(0.13)
$\bar{R}^2$	0.04	0.05	0.02	0.05	0.01	0.03	0.00	0.02
# obs	208	208	136	208	208	207	136	208

Note: Regressions of the form: $y_{t+1} = \alpha + \beta nxa_t + \delta z_t + \epsilon_{t+1}$ where y_{t+1} is the dollar return on US equities (r^{le}_{t+1}), the dollar return on US liabilities (r^{l}_{t+1}) (panel A); the dollar return on US assets (r^{a}_{t+1}), the local currency return on US assets (r^{a*}_{t+1}) (panel B); the dollar return on foreign equities (r^{ae}_{t+1}), the local currency return on foreign equities (r^{ae*}_{t+1}) (Panel C). $\frac{d_t}{p_t}$ (resp. $\frac{d^*_t}{p^*_t}$) is the domestic (resp. foreign) dividend price ratio (available since 1970:1); cay_t is the Lettau and Ludvigson (2001)'s deviation of the consumption-wealth ratio from trend; xm_t is the stationary component from the trade balance, defined as $\epsilon^x_t - \epsilon^m_t$. Sample: 1952:1 to 2004:1. Robust standard errors in parenthesis.

Table 5: Forecasting Bilateral Quarterly Rates of Depreciation

Currency	nxa_{t-1}	$\bar{R}^2$	#obs
UK pound	**-0.15**	0.04	125
	(0.06)		
Canadian dollar	-0.02	0.01	125
	(0.01)		
Swiss franc	**-0.08**	0.05	125
	(0.03)		
Japanese yen	**-0.06**	0.02	125
	(0.03)		
Deutschmark (Euro)	**-0.07**	0.08	125
	(0.02)		

Note: Sample: 1973:1 to 2004:1. Robust standard errors in parenthesis.

Table 6: Long Horizon Regressions

	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
	Real Total Net Portfolio Return $r_{t,k}$							
nxa	**-0.36**	**-0.35**	**-0.35**	**-0.33**	**-0.22**	**-0.14**	**-0.09**	**-0.04**
	(0.07)	(0.05)	(0.04)	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)
$\bar{R}^2(1)$	[0.11]	[0.18]	[0.24]	[0.26]	[0.21]	[0.13]	[0.09]	[0.02]
$\bar{R}^2(2)$	[0.14]	[0.25]	[0.34]	[0.38]	[0.35]	[0.24]	[0.19]	[0.16]
	Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$							
nxa	**-0.14**	**-0.13**	**-0.12**	**-0.11**	**-0.06**	**-0.03**	-0.02	0.01
	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.07]	[0.13]	[0.17]	[0.18]	[0.10]	[0.03]	[0.01]	[0.00]
$\bar{R}^2$ (2)	[0.11]	[0.20]	[0.28]	[0.31]	[0.26]	[0.15]	[0.10]	[0.17]
	Net Export growth $\Delta nx_{t,k}$							
nxa	**-0.08**	**-0.08**	**-0.07**	**-0.07**	**-0.07**	**-0.06**	**-0.06**	**-0.04**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.05]	[0.10]	[0.13]	[0.17]	[0.31]	[0.44]	[0.53]	[0.58]
$\bar{R}^2$ (2)	[0.04]	[0.08]	[0.12]	[0.17]	[0.38]	[0.55]	[0.66]	[0.79]
	FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$							
nxa	**-0.08**	**-0.08**	**-0.08**	**-0.08**	**-0.07**	**-0.06**	**-0.04**	**-0.02**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.09]	[0.16]	[0.28]	[0.31]	[0.41]	[0.41]	[0.33]	[0.12]
$\bar{R}^2$ (2)	[0.10]	[0.21]	[0.35]	[0.40]	[0.52]	[0.55]	[0.55]	[0.38]

Note: Regressions of the form: $y_{t,k} = \alpha + \beta nxa_t + \epsilon_{t+k}$ where $y_{t,k}$ is the k-period real total net portfolio return ($r_{t,k}$); total excess equity return ($r^{ae}_{t,k} - r^{le}_{t,k}$); net export growth ($\Delta nx_{t,k}$) or the FDI-weighted depreciation rate ($\Delta e_{t,k}$). Newey-West robust standard errors in parenthesis with $k-1$ Bartlett window. Adjusted R^2 in brackets. $\bar{R}(1)$ reports the adjusted R-squared of the regression on nxa_t; $\bar{R}(2)$ reports the adjusted R-squared of the regression on ϵ^x_t, ϵ^m_t, ϵ^a_t and ϵ^l_t. Sample: 1952:1 to 2004:1 (1973:1 to 2004:1 for exchange rate).

Table 7: Forecasting Exchange Rates. Sample 1973:2004.

ADF-like Regressions	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
	FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$							
e_{t-1}	-0.052	**-0.050**	**-0.052**	**-0.058**	**-0.067**	**-0.067**	**-0.064**	**-0.056**
(s.e.)	(0.027)	(0.020)	(0.015)	(0.013)	(0.10)	(0.008)	(0.006)	(0.004)
Δe_{t-1}	0.072	-0.028	0.077	**0.113**	**0.076**	**0.049**	0.028	0.004
(s.e.)	(0.090)	(0.065)	(0.049)	(0.043)	(0.032)	(0.025)	(0.020)	(0.012)
$\bar{R}^2$	[0.01]	[0.04]	[0.08]	[0.15]	[0.28]	[0.39]	[0.48]	[0.65]
e_{t-1}	-0.031	-0.028	**-0.032**	**-0.040**	**-0.051**	**-0.054**	**-0.054**	**-0.052**
(s.e.)	(0.028)	(0.019)	(0.014)	(0.012)	(0.008)	(0.006)	(0.005)	(0.004)
Δe_{t-1}	-0.015	-0.123	-0.006	0.039	0.008	-0.005	-0.012	-0.009
(s.e.)	(0.091)	(0.062)	(0.045)	(0.039)	(0.026)	(0.019)	(0.016)	(0.013)
nxa_{t-1}	**-0.080**	**-0.086**	**-0.076**	**-0.069**	**-0.061**	**-0.049**	**-0.036**	**-0.011**
(s.e.)	(0.025)	(0.017)	(0.012)	(0.011)	(0.007)	(0.005)	(0.004)	(0.003)
$\bar{R}^2$	[0.08]	[0.20]	[0.30]	[0.37]	[0.57]	[0.68]	[0.70]	[0.68]
	IFS nominal effective rate of depreciation $\Delta e^{IFS}_{t,k}$							
e^{IFS}_{t-1}	-0.048	**-0.048**	**-0.051**	**-0.056**	**-0.063**	**-0.061**	**-0.056**	**-0.046**
(s.e.)	(0.027)	(0.020)	(0.016)	(0.014)	(0.010)	(0.008)	(0.006)	(0.004)
Δe^{IFS}_{t-1}	0.149	0.066	**0.137**	**0.131**	0.066	0.036	0.017	-0.001
(s.e.)	(0.090)	(0.068)	(0.054)	(0.048)	(0.035)	(0.027)	(0.021)	(0.015)
(i) $\bar{R}^2$	[0.03]	[0.03]	[0.10]	[0.14]	[0.25]	[0.35]	[0.43]	[0.55]
e^{IFS}_{t-1}	0.002	0.007	-0.005	-0.015	**-0.031**	**-0.039**	**-0.041**	**-0.047**
(s.e.)	(0.029)	(0.021)	(0.016)	(0.015)	(0.010)	(0.008)	(0.007)	(0.005)
Δe^{IFS}_{t-1}	0.011	-0.082	0.010	0.021	-0.017	-0.020	-0.020	-0.001
(s.e.)	(0.096)	(0.068)	(0.053)	(0.047)	(0.034)	(0.027)	(0.022)	(0.017)
nxa_{t-1}	**-0.097**	**-0.105**	**-0.088**	**-0.079**	**-0.060**	**-0.041**	**-0.027**	0.000
(s.e.)	(0.029)	(0.020)	(0.016)	(0.014)	(0.010)	(0.008)	(0.007)	(0.005)
(ii) $\bar{R}^2$	[0.10]	[0.20	[0.27]	[0.30]	[0.42]	[0.47]	[0.51]	[0.55]

Note: Runs regressions of the form $\Delta e_{t,k} = \alpha e_{t-1} + \beta \Delta e_{t-1} + \gamma nxa_{t-1} + c + \epsilon_{t,k}$. Sample 1973:2004.

Table 8: Out of Sample Tests for Exchange Rate Depreciation against the Martingale Hypothesis

Horizon: (quarters)	1	2	3	4	8	12	16
	FDI-weighted depreciation rate						
MSE_u/MSE_r	0.960	0.920	0.858	0.841	0.804	0.818	0.903
ΔMSE-adjusted ($MSE_r - MSE_u$-adj)	1.48	1.53	1.61	1.51	1.20	0.74	0.35
(s.e.)	(0.68)	(0.60)	(0.57)	(0.53)	(0.37)	(0.24)	(0.23)
p-val	[0.01]	[0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[0.06]
	Trade-weighted depreciation rate						
MSE_u/MSE_r	0.949	0.900	0.830	0.788	0.733	0.929	0.961
ΔMSE-adjusted ($MSE_r - MSE_u$-adj)	2.76	3.03	2.94	2.78	1.91	0.67	0.29
(s.e.)	(1.03)	(1.03)	(1.02)	(0.98)	(0.69)	(0.38)	(0.24)
p-val	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[0.03]	[0.11]

Note: $\Delta MSPE - adjusted$ is the Clark-West (2004) test-statistic based on the difference between the out of sample MSE of the driftless random-walk model and the out-of-sample MSE of a model that regresses the rate of depreciation $\Delta e_t + 1$ against nxa_t. Rolling regressions are used with a sample size of 105. t-statistic in parenthesis. p-value of the one-sided test using critical values from a standard normal distribution in brackets. Under the null, the random-walk encompasses the unrestricted model. Sample: 1952:1-2004:1. Cut-off: 1978:1.

Table 9: Out of Sample Tests for various nested models.

	ENC-NEW	MSE_u/MSE_r						
Horizon: (quarters)		1	2	3	4	8	12	16
Panel A: Real Total Net Portfolio Return $r_{t,k}$								
nxa vs $AR(1)$	9.46^{**}	0.970	0.903	0.843	0.785	0.758	0.868	0.968
nxa vs $AR(1)$, $\frac{d}{p}$ and $\frac{d^*}{p^*}$	20.91^{**}	0.970	0.862	0.779	0.671	0.610	0.542	0.626
Panel B: Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$								
nxa vs $AR(1)$	19.58^{**}	0.894	0.782	0.693	0.638	0.744	0.925	1.057
nxa vs $AR(1)$, $\frac{d}{p}$ and $\frac{d^*}{p^*}$	27.92^{**}	0.917	0.790	0.686	0.626	0.810	0.899	1.026
Panel C: FDI-weighted depreciation rate $\Delta e_{t,k}$								
nxa vs $AR(1)$	6.57^{**}	0.948	0.882	0.834	0.809	0.736	0.736	0.811
nxa vs $AR(1)$, $i_t - i^*_t$	6.78^{**}	0.951	0.878	0.824	0.805	0.735	0.748	0.828

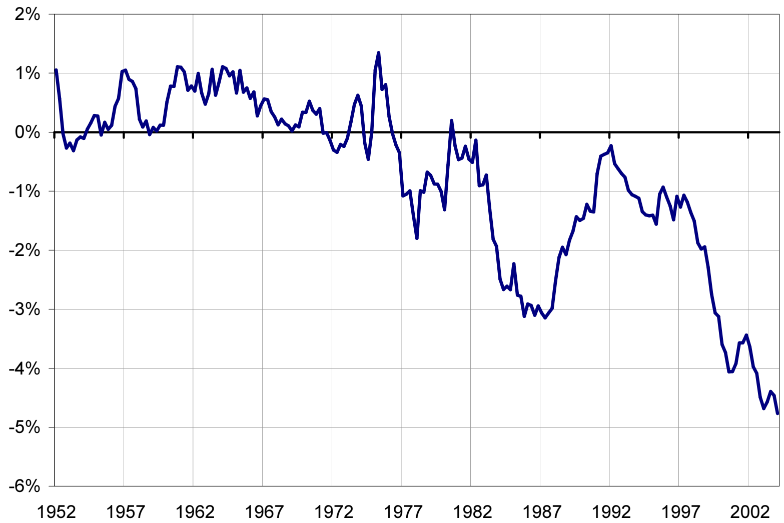
Note: MSE_u is the mean-squared forecasting error for an unrestricted model that includes the lagged dependent variable and lagged nxa (model 1); lagged d/p, d^*/p^* and lagged nxa (model 2). MSE_r is the mean-squared error for the restricted models which include the same variables as above but do not include lagged nxa. d/p (resp. d^*/p^*) is the US (resp. rest of the world) dividend price ratio. Each model is first estimated using the sample 1952:1 1978:1. ENC-NEW is the modified Harvey, Leybourne and Newbold (1998) statistic, as proposed by Clark and McCracken (2001). Under the null, the restricted model encompasses the unrestricted one. Sample: 1952:1-2004:1. * (resp. **) significant at the five (resp. one) percent level.

Table 10: Unconditional Variance Decomposition for nxa, when mean returns on assets and liabilities differ.

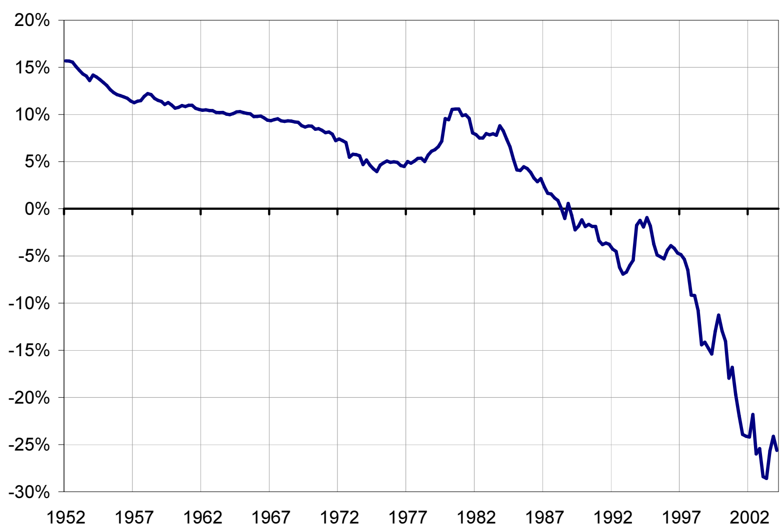
Variance Decomposition:		
#	percent	
1	$\beta_{\Delta nx}$	58
2	β_r	26
3	β_{cl}	12
5	Total	96

Note: Sample: 1952:1 to 2004:1.

Figure 1: US Net Exports and Net Foreign Assets (% of GDP, 1952-2004)



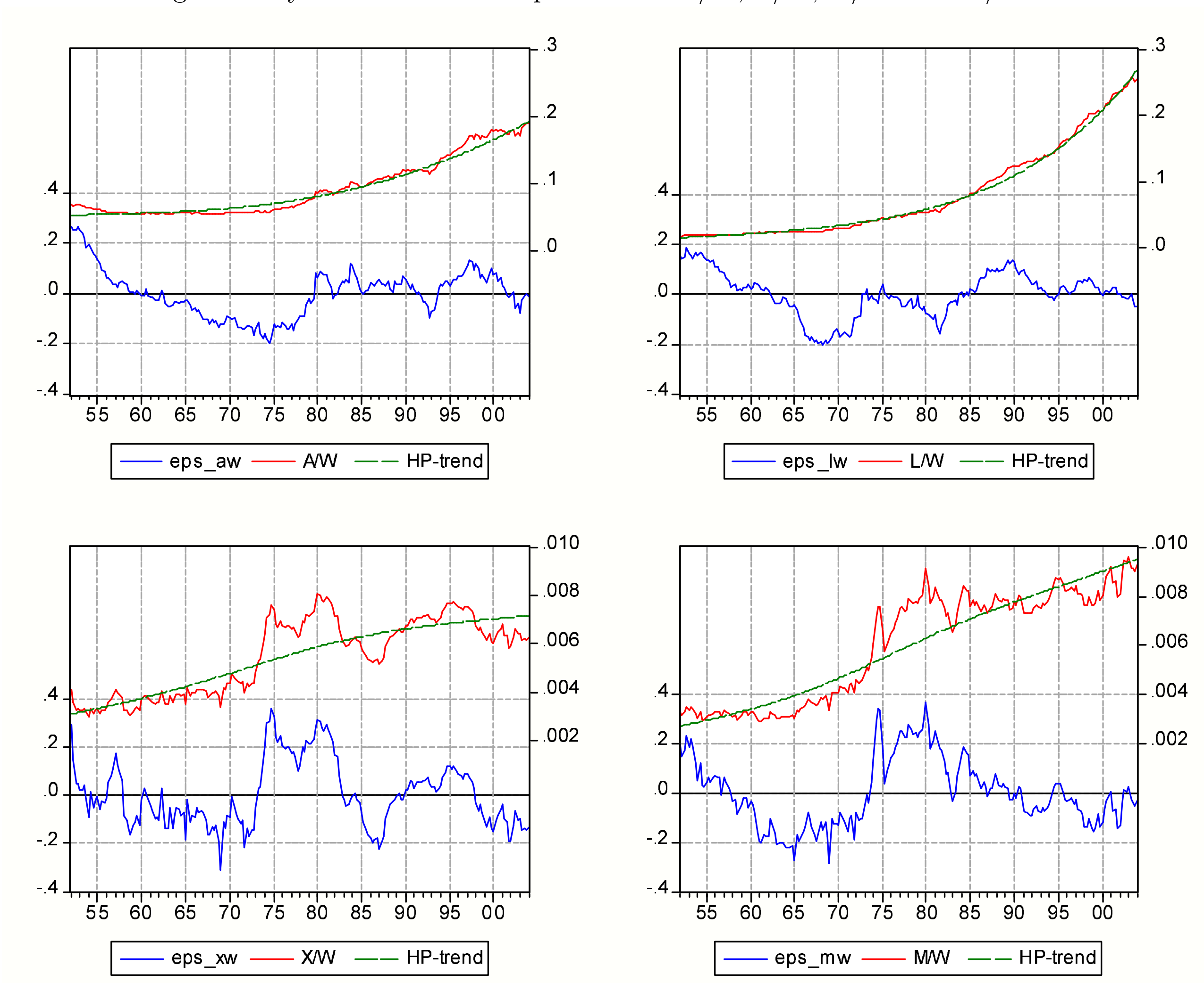
(a) Net Exports/GDP



(b) Net Foreign Assets/GDP

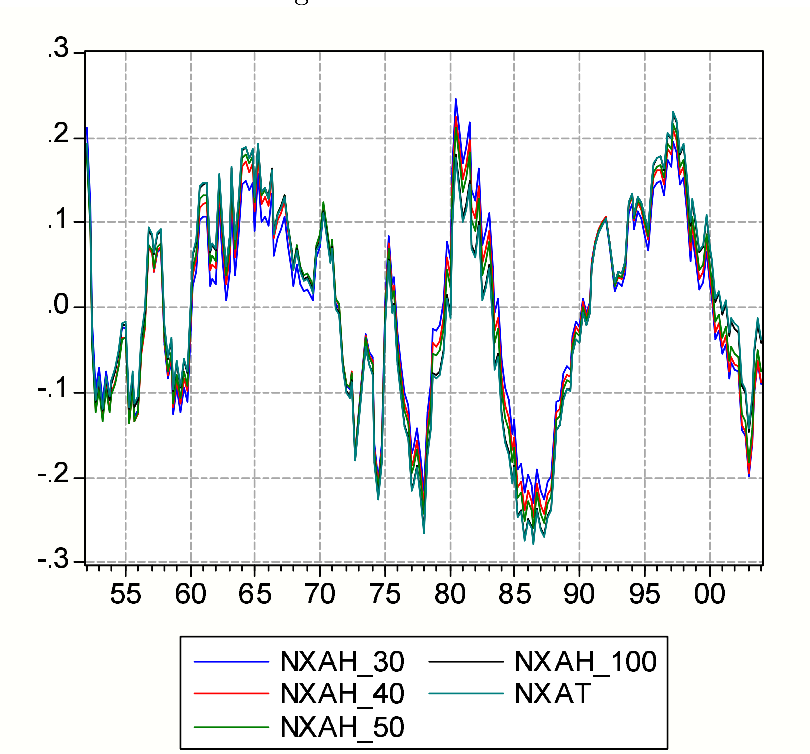
Note: The top panel shows the ratio of US net exports to US GDP. The bottom panel shows the ratio of US net foreign assets to US GDP. Sample: 1952:1-2004:1. Source: Bureau of Economic Analysis, Flow of Funds and Authors calculations.

Figure 2: Cycle and Trend Components for A/W, L/W, X/W and M/W.



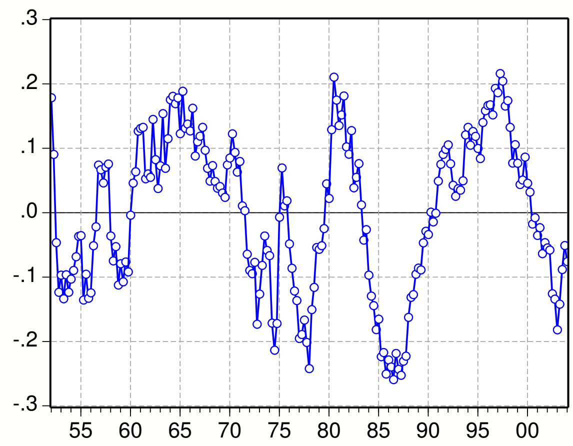
Note: Top two panels for US gross external assets A/W (left) and US gross external liabilities L/W (right); Bottom two panels for US exports X/W (left) and US imports M/W (right). Each panel reports the series Z/W (ratio to household wealth), the trend component μ_t^{zw}, labelled HP-trend, (right-axis) and the cyclical component ϵ_t^{zw} (left-axis). Sample: 1952:1-2004:1.

Figure 3: Various nxa

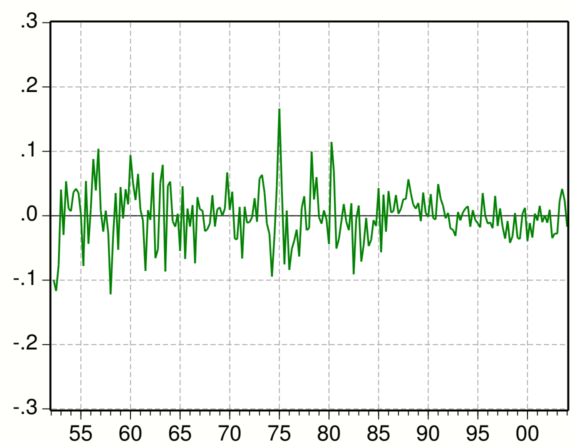


Note: nxa, constructed from various cut-offs (30, 40, 50, 100 years and linear filter). Sample: 1952:1-2004:1

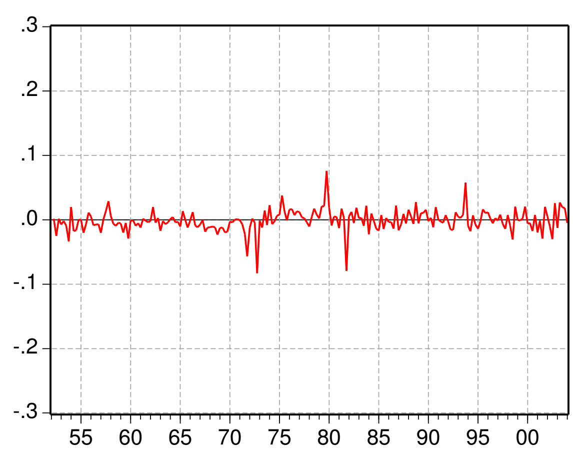
Figure 4: nxa, flow $r + \Delta nx$ and residual term ε from equation (6).



(a) nxa_t

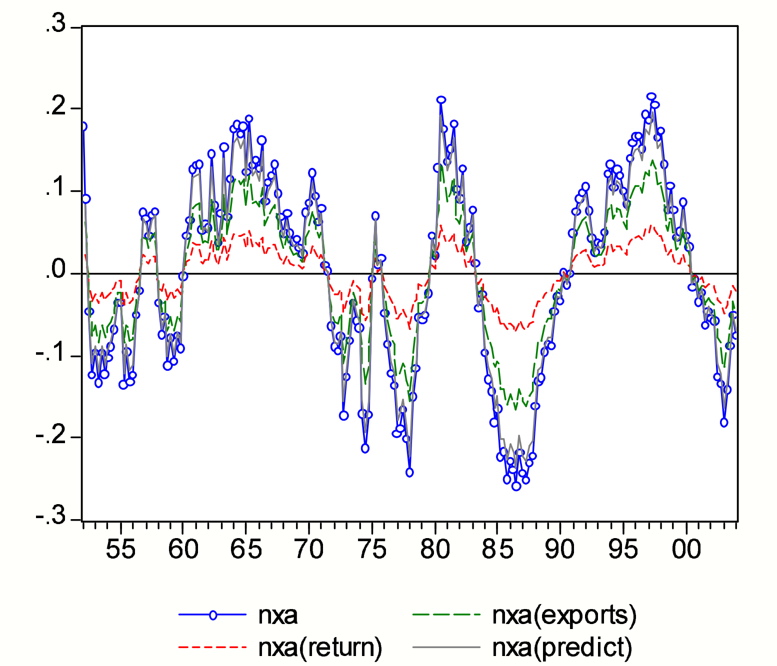


(b) flow $r_t + \Delta nx_t$

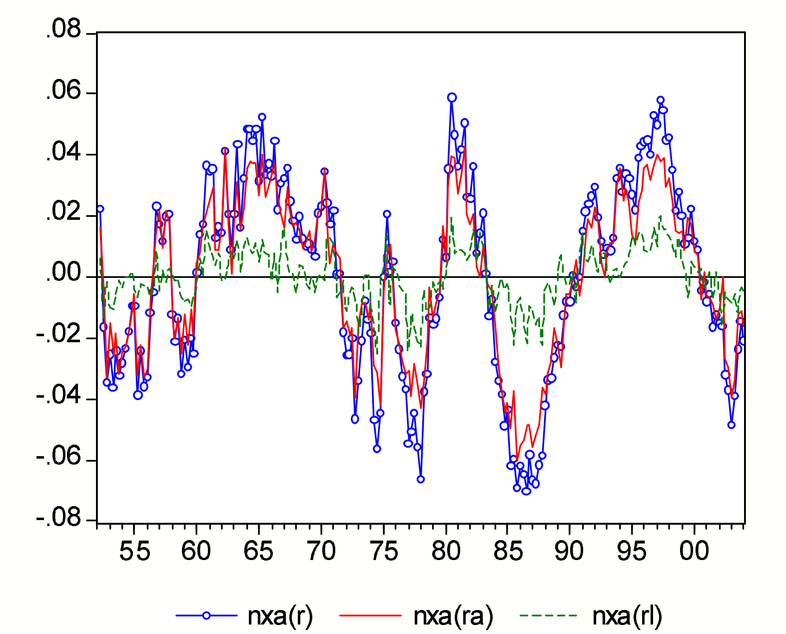


(c) residual ε_t

Figure 5: Decomposition of nxa into trade and valuation components.



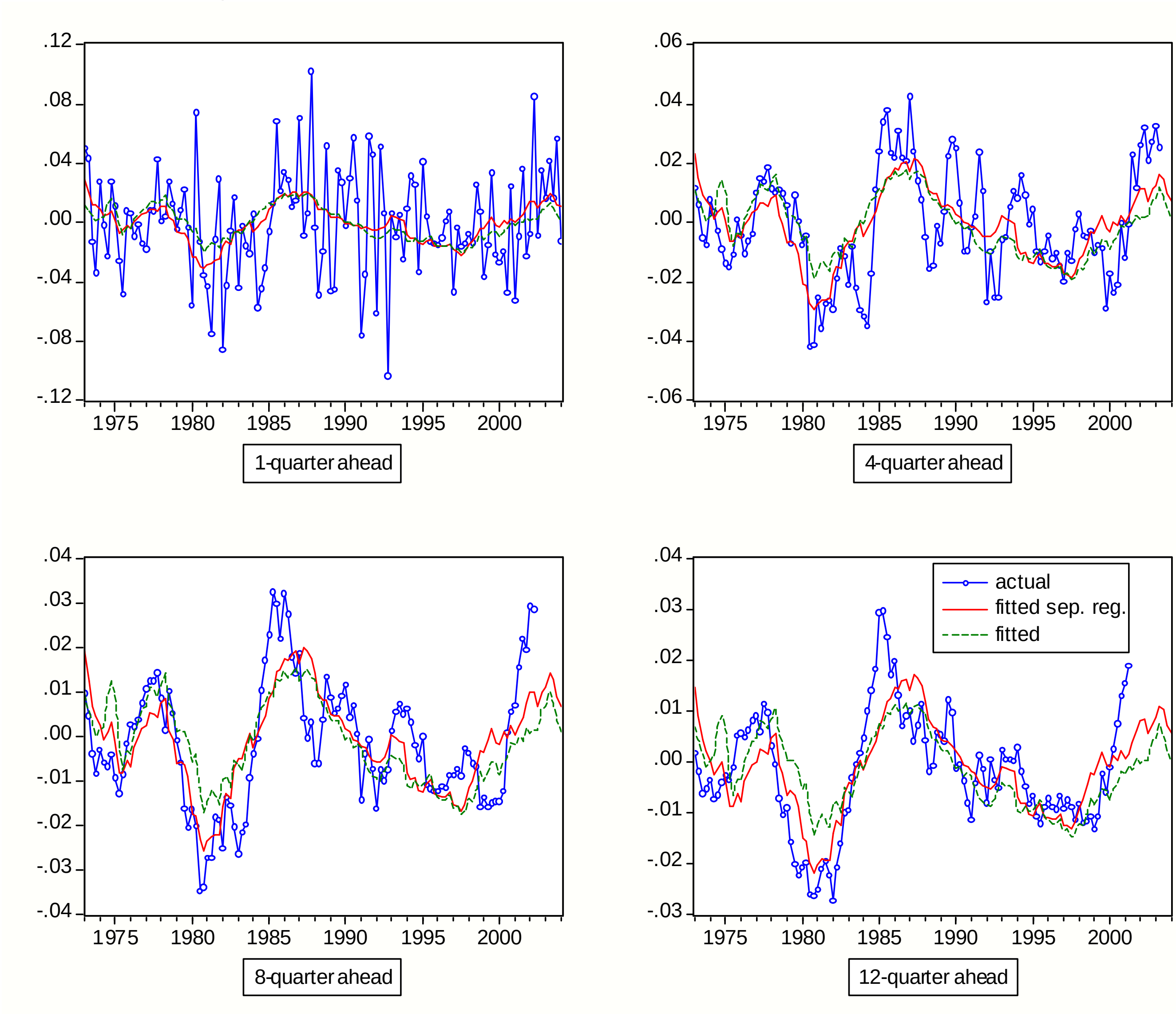
(a) return $nxa(return)$ and net exports $nxa(exports)$ components.



(b) asset return $nxa(ra)$ and liability return $nxa(rl)$ components.

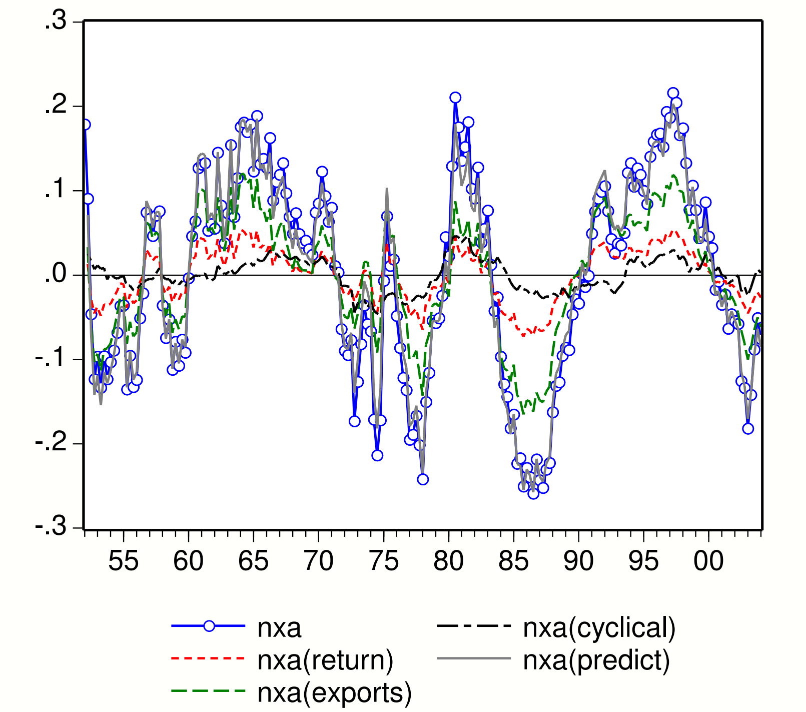
Note: The top panel reports the decomposition of nxa into its return ($nxa(return)$) and net exports ($nxa(exports)$) components. The bottom panel reports the decomposition of the return component ($nxa(return)$) into an asset return ($nxa(ra)$) and a liability return ($nxa(rl)$) components.

Figure 6: Predicted One to 12-quarter ahead depreciation rates.



Note: Each graph reports (a) the realized depreciation rate at 1 to 12 quarter horizon; (b) the fitted depreciation rate using nxa (fitted); (c) the fitted depreciation rate using ϵ^{xw}, ϵ^{mw}, ϵ^{aw} and ϵ^{lw} as separate regressors (fitted sep. reg.).

Figure 7: Decomposition of nxa into trade, valuation and cyclical components.



Note: The figure reports the decomposition of nxa into a return ($nxa(return)$), a net exports ($nxa(exports)$) and a cyclical ($nxa(\text{cyclical})$) components.

A Biased View of PPP

Demian Reidel and Jan Szilagyi

September 22, 2004

Road Map

- Current Debate
- Introduction to the PPP puzzle
- Theory: small sample and heterogeneity biases
- Empirical evidence
- Monte-Carlo analysis
- Conclusion

Current debate

- Imbs et al (2004): sectoral heterogeneity bias can explain the PPP puzzle
  - Use monthly disaggregated data at the sectoral level from Eurostat, 1981-1995.
  - Theoretical conditions for heterogeneity bias are present in the data.
  - Apply Pesaran (2002) estimator to deal with sectoral data and find that PPP puzzle disappears. Half-life is around 1 year.

Current debate

- Chen, Engel (2004): sectoral heterogeneity bias is irrelevant
  - Use similar data as Imbs et al (2004).
  - Use Monte Carlo simulations to show that in small samples heterogeneity bias is not enough to solve the PPP puzzle.
  - Their simulations focus on OLS time series estimates (not panel data).
  - Their sample sizes in the simulations run up only to around 40 years (500 monthly data points).
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

Current debate

- Choi, Mark, Sul (2003, 2004): country heterogeneity bias is irrelevant
  - Use annual data for 21 OECD countries.
  - Use the Recursive Demeaning Estimator to alleviate the small sample bias
  - Do not explore sectoral heterogeneity but find that country heterogeneity is irrelevant.
  - Their sample sizes in the simulations run up only to around 200 data points.
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

Our Methodology

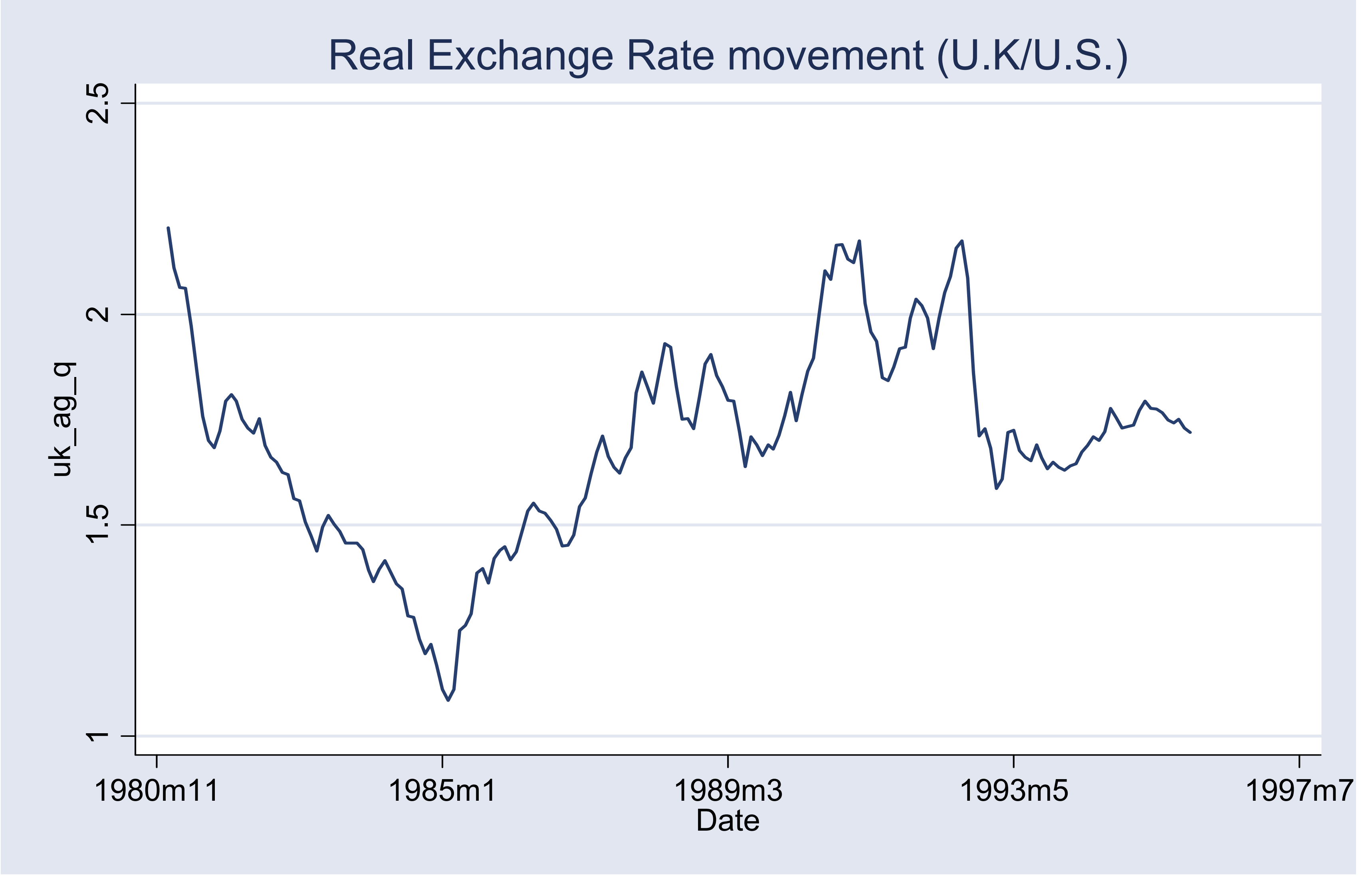
- Use disaggregated data as in Imbs et al (2004)
- Explore both time series and panel estimations
- Use OLS, SURE and CCE estimators
- Use improved Monte Carlo engine
  - non-diagonal covariance structure across sectors
  - calibrated for different data frequencies (monthly and annual)
  - short, medium and large samples (up to 10,000 points)
  - general structure of heterogeneity
  - Information-neutral initial conditions

Our results

We explore the interaction between small-sample and heterogeneity biases that plague estimates of half-lives of PPP deviations:

- in small samples heterogeneity bias can disappear completely.
- heterogeneity bias does not resolve the PPP puzzle.
- for longer series heterogeneity bias dominates small sample bias and results in upward-biased estimates.
- Imbs et al (2004) find low half-lives because they use Pesaran (2002) estimator which is not applicable to this problem.

Theory vs. Reality



Part I – Biases in estimation

Estimations of half-lives are particularly affected by two different biases:

1. Small Sample Bias (Orcutt (1948), Kendall (1954), …)

2. Heterogeneity Bias (Pesaran and Smith (1995), Imbs et al (2004))

Part I – Biases in estimation

Imbs et al (2004) actually claim that the effect of heterogeneity is large enough to solve the puzzle.

They find that in the data, heterogeneity creates an upward biased estimate of the half-lives of deviations from PPP.

Their bias-corrected estimates are close to one year, in contrast to the “consensus view” of 3-5 years.

Part II
Theory: Small Sample and Heterogeneity Biases

Part II – Heterogeneity bias

- Heterogeneity bias in time-series:

$$\textit{Sectoral real exchange rate}:$$

$$\begin{cases} q_{it} = c_i + \rho_i q_{it-1} + \varepsilon_{it} \\ \rho_i = \rho^{true} + \eta_i^{\rho}, c_i = c^{true} + \eta_i^{c} \end{cases}$$

$$\textit{Estimated model}:$$

$$\begin{cases} q_t = c + \rho q_{t-1} + \varepsilon_t \\ c = \sum_i \omega_i c_i,\ \varepsilon_t = \sum_i \omega_i \varepsilon_{it} + \sum_i \omega_i \eta_i^{\rho} q_{it-1} \end{cases}$$

$$\textit{Bias}: \tilde{\rho} = \rho + \Delta,\ \ \Delta = \sum_i (\rho_i - \rho)\alpha_i$$

$$\alpha_i = \left(\frac{\omega_i^2 \sigma_i^2}{1-\rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1-\rho_i \rho_j} \right) / \sum_i \left(\frac{\omega_i^2 \sigma_i^2}{1-\rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1-\rho_i \rho_j} \right)$$

Part II – Heterogeneity bias

- Key idea: the estimated ρ from aggregate series is a biased estimator of the average ρ in the economy (Pesaran, Smith (1995)). Therefore, we cannot interpret this number as the average speed of adjustment (which is the PPP puzzle)
- It does NOT mean that the estimates of ρ for the real exchange rate are wrong or biased.

Part II – Heterogeneity bias

- A simple illustration:

$$\text{Assume that } \sigma_{\mathrm{ij}} = 0, \sigma_i^2 = \sigma^2, \omega_i = \frac{1}{N}$$

$$\alpha_i = \frac{1/(1-\rho_i^2)}{\sum_j 1/(1-\rho_j^2)}, \text{positive and increasing in } \rho_i$$

$$\text{Therefore } \Delta = \sum_i (\rho_i - \rho)\alpha_i > 0$$

Part II – Small sample bias

- Small sample bias in OLS estimates:

Let $\vec{q}_T$ *be the real exchange rate vector and assume that*

$$\vec{q}_T = \rho\vec{Q}_{T-1} + \varepsilon_T$$

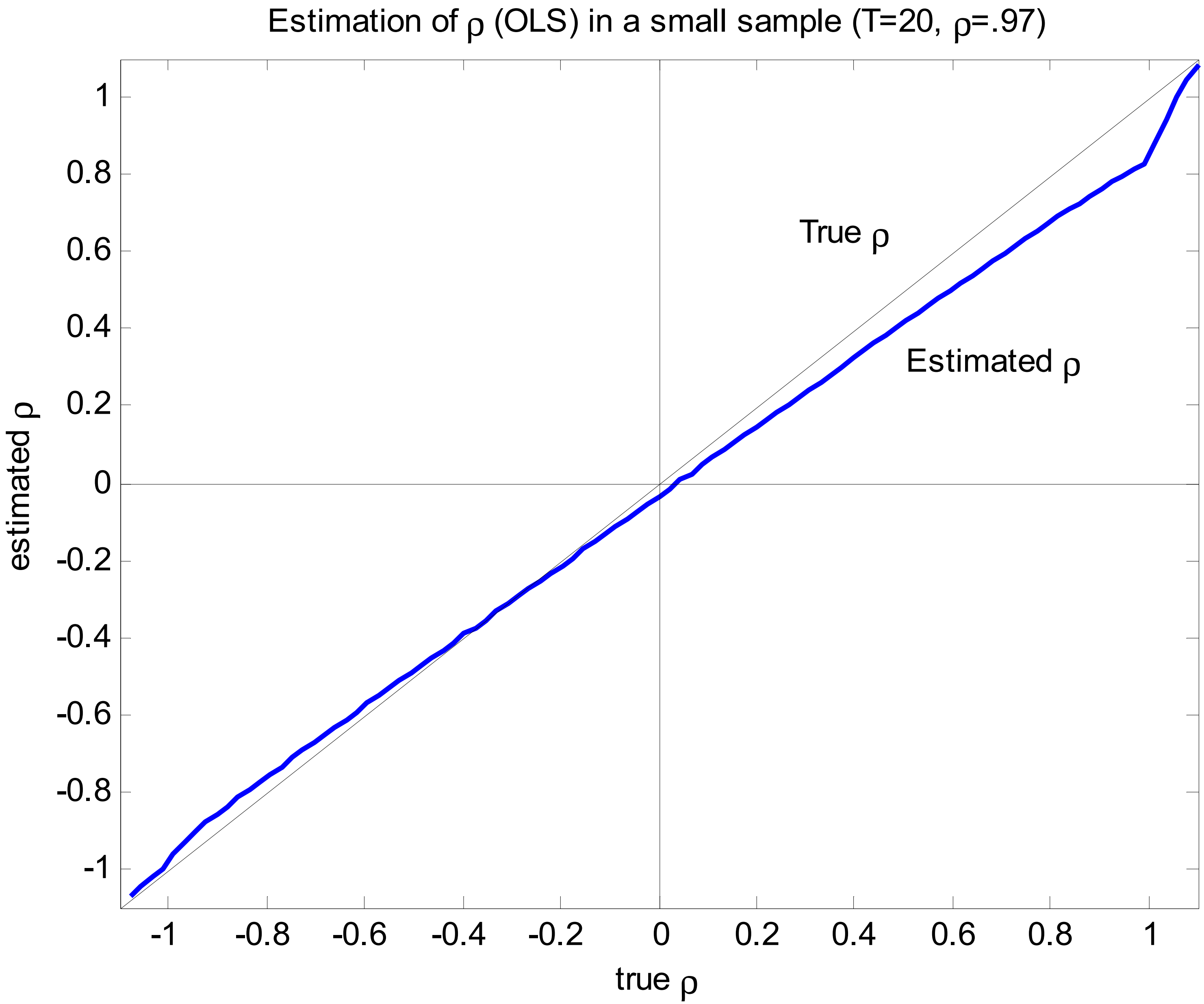
Then

$$\hat{\rho} = (\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\vec{q}_T$$

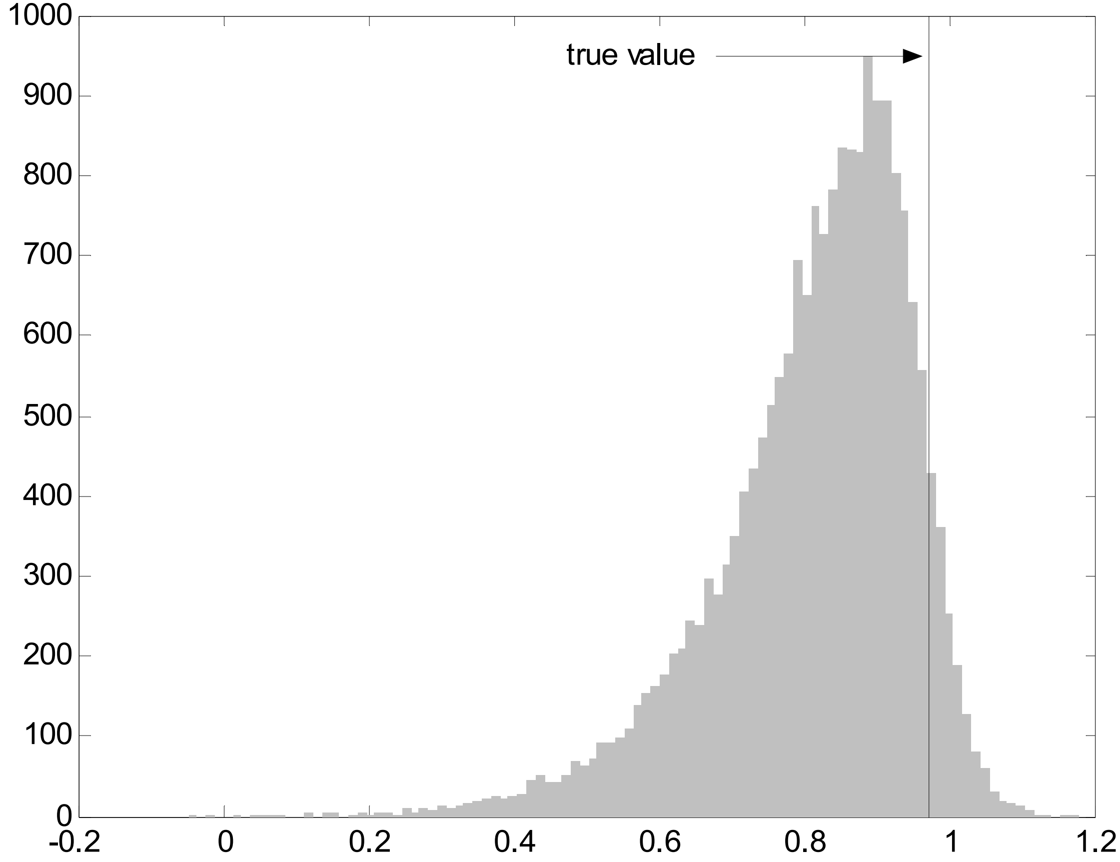
$$E[\hat{\rho}] = \rho + E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\varepsilon_T]$$

and

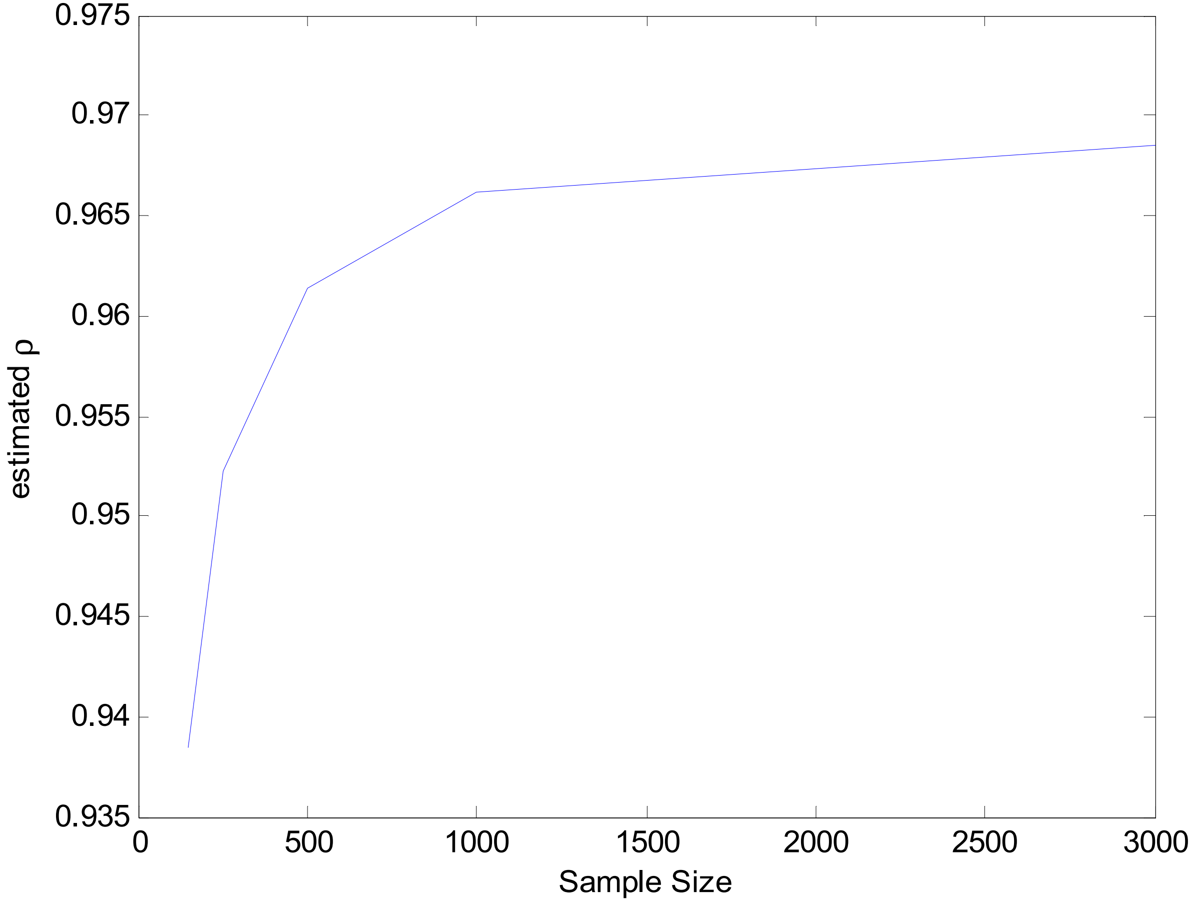
$$E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\varepsilon_T] \xrightarrow{p} E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}]E[\vec{Q}_{T-1}'\varepsilon_T] = 0$$


Estimation of ρ (OLS) in a small sample (T=20, ρ=.97)
True ρ
Estimated ρ
estimated ρ
true ρ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1

Histogram of OLS estimator
ρ_{true}=0.97, T=20



Small sample bias as a function of sample size ρ_{true}=0.97



Part II – Interaction

- Note that small-sample bias, by definition, appears in small samples.
- In contrast, the analytical results for heterogeneity bias are valid ONLY asymptotically.
- We do not have analytical solutions for their interaction in finite samples. In Part IV we use Monte Carlo simulations to explore this interaction.

Part III
Empirical Evidence

Part III – Empirical results

Dataset:

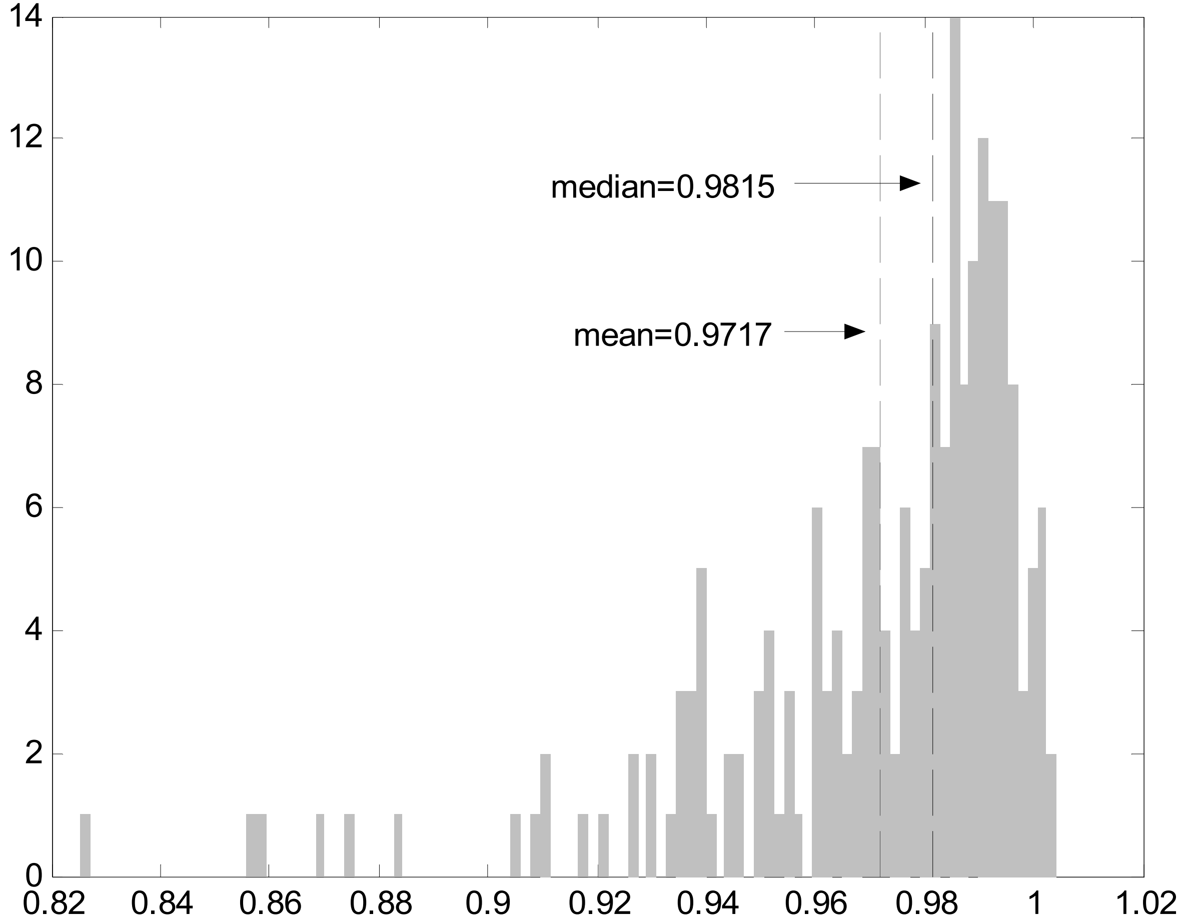
- Eurostat data for Belgium, Germany, Denmark, Finland, France, Greece, Italy, Netherlands, Portugal, Spain, U.K.
- 19 different sectors including “meat,” “dairy,” “clothes,” etc.
- Time range from 1981-1995, which gives them a maximum length of 180 datapoints

Part III – Empirical results

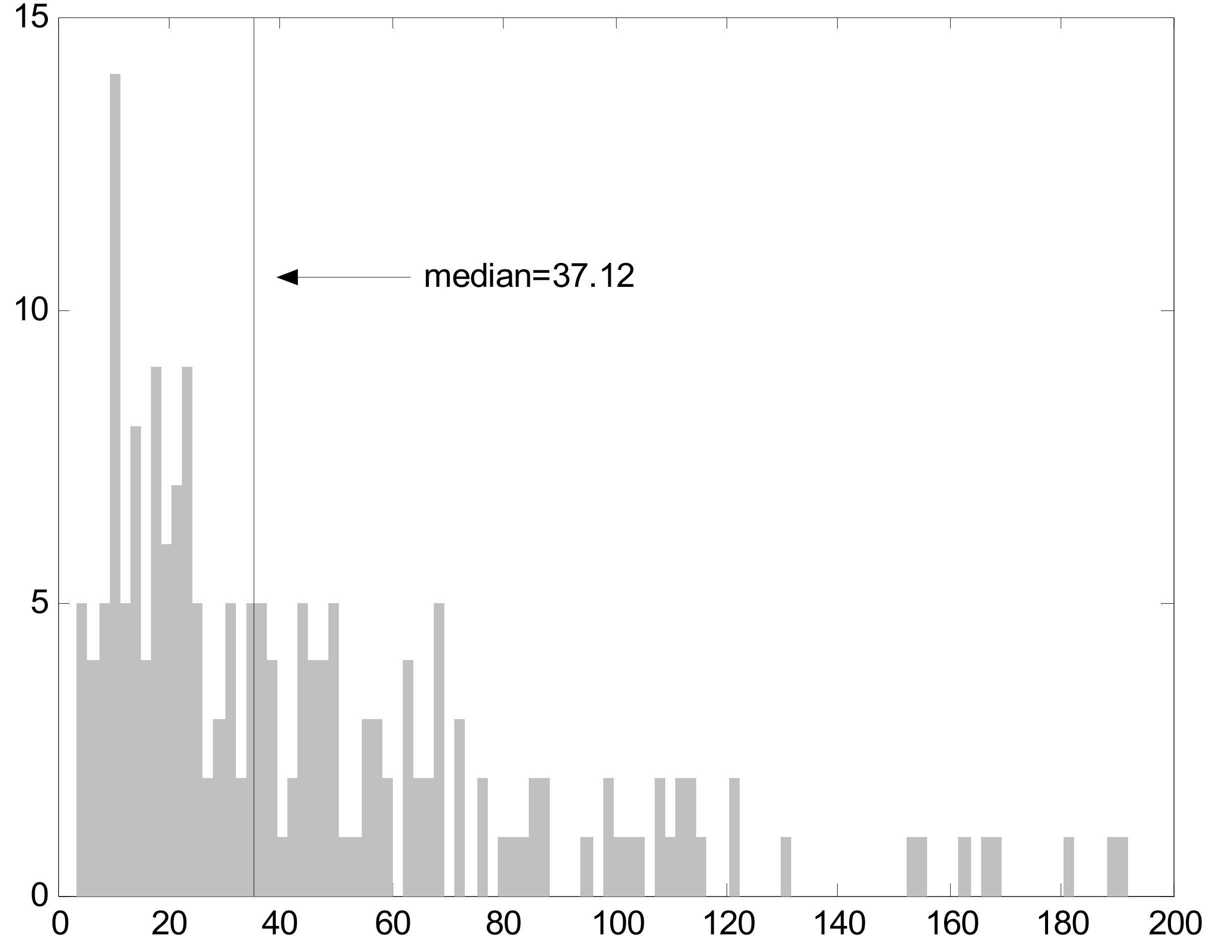
Main findings:

- There is significant sectoral heterogeneity
- Theoretical conditions for heterogeneity bias are satisfied in the dataset
- Individual sectors exhibit long half-lives (even before correcting for small-sample bias)

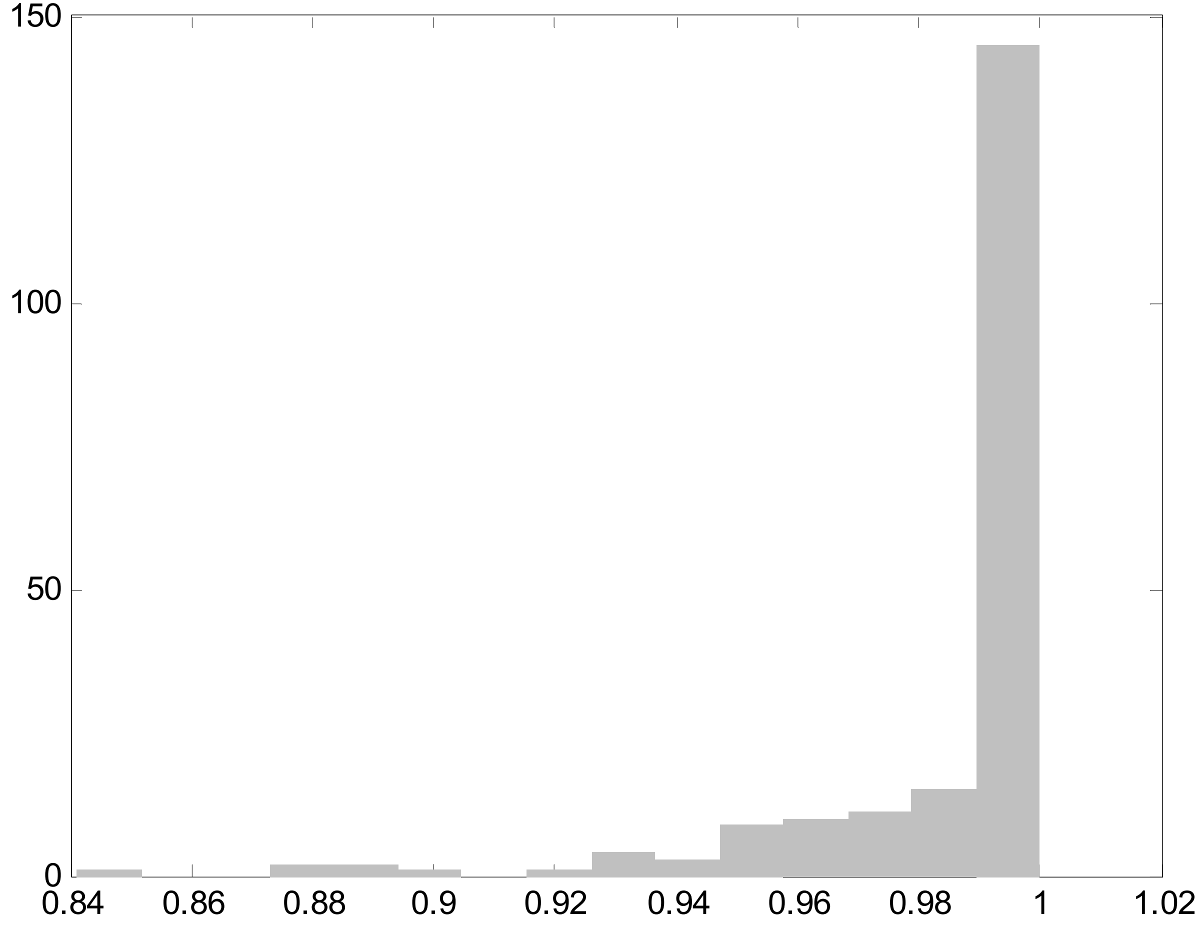
Distribution of ρ for different sectors and countries (without correcting for small sample bias)



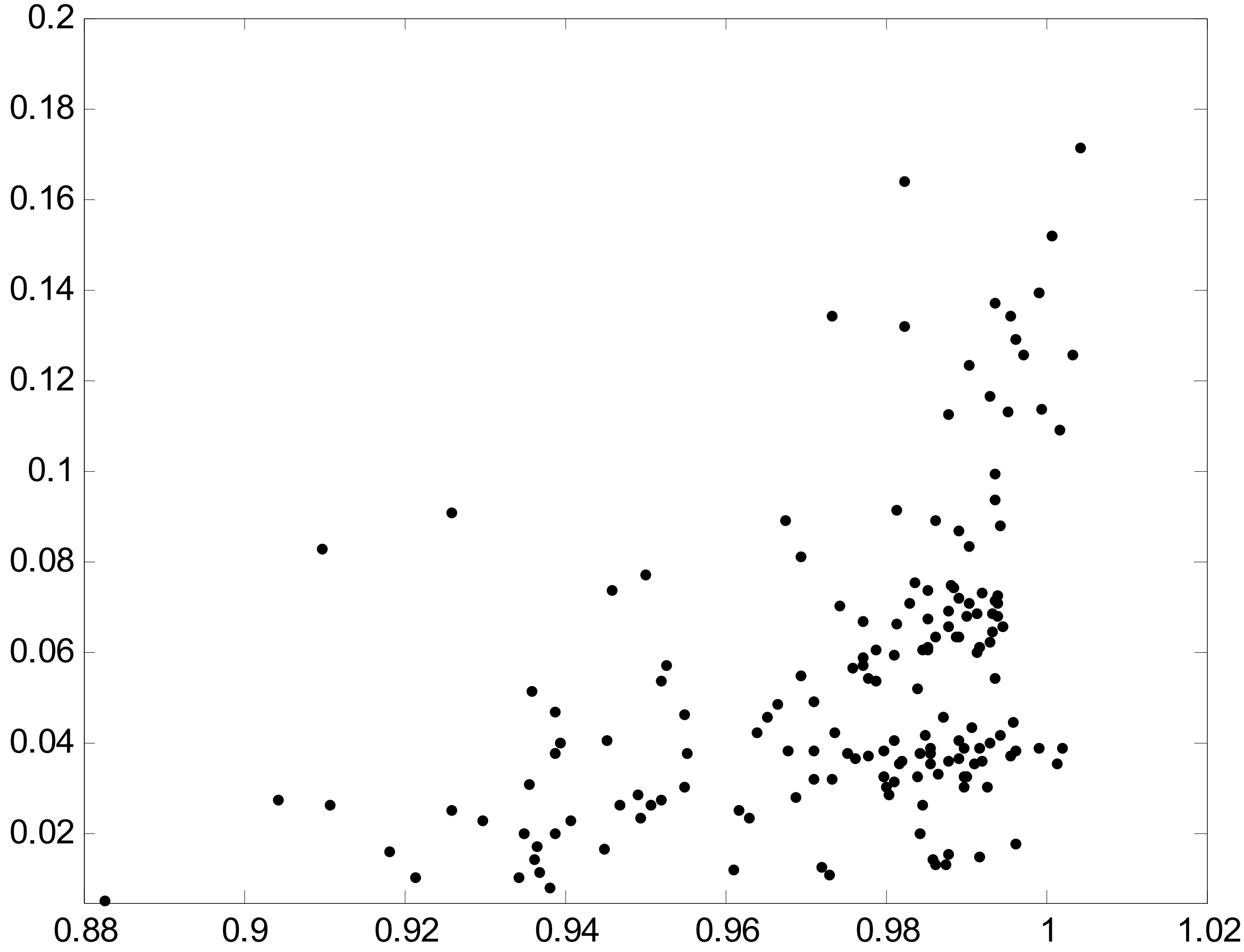
Distribution of half-lives for different sectors and countries (without correcting for small sample bias)



Distribution of ρ for different sectors and countries (correcting for small sample bias)



Scatter plot: α (weight) vs. ρ



Speculative Attacks – Krugman: I

- Krugman model shows that countries with a fixed exchange rate pursuing expansionary monetary policy will have an attack on reserves, forcing them to float, far before reserves run out.

- Start with money demand:

$$\frac{M_t}{P_t} = Ae^{-\eta i_t}$$

- And uncovered interest parity:

$$i_t = i_t^* + \frac{\dot{E}_t}{E_t}$$

Speculative Attacks – Krugman: II

- The central bank's balance sheet just says that money demand is equal to money supply. In other words, the amount of money in the economy is equal to domestic credit plus the value, in domestic currency of foreign reserves:

$$M_t = C_t + Ef_t$$

- Additionally, we assume that the government is expands domestic credit over time at a constant rate:

$$\frac{\dot{C}_t}{C_t} = \gamma$$

- Then we can show that foreign reserves decline over time.

Speculative Attacks – Krugman: III

- Deriving the time path of foreign reserves:

$$0 = \frac{\dot{M}_t}{C_t} = \frac{\dot{C}_t + \dot{f}_t}{C_t}$$

- We can solve for the rate of change in foreign reserves by solving (where omega(f) is the ratio of foreign reserves in high-powered money):

$$-\gamma = -\frac{\dot{C}_t}{C_t} = \frac{\dot{f}_t}{C_t}\frac{f_t}{f_t} = \frac{\dot{f}_t}{f_t}\frac{f_t}{C_t} \quad \Rightarrow \frac{\dot{f}_t}{f_t} = -\gamma\frac{f_t}{C_t} = -\gamma\frac{\frac{f_t}{M_t}}{\frac{C_t}{M_t}} = -\gamma\frac{1-\omega_f}{\omega_f}$$

Speculative Attacks – Krugman: IV

- So, eventually, the monetary authority will run out of foreign reserves. However, before that, it will experience an attack on the currency.

- First, we assume PPP. Therefore, the domestic price level is fixed with respect to the exchange rate. Normalizing the foreign price level to 1, we get:

$$P_t = E_t$$

- Now, we define the shadow exchange rate that would exist if the central bank sold all reserves and let the currency float. However, when reserves are gone money demand is equal to domestic credit:

$$M_t = C_t$$

- Taking logs of money demand and substituting in, we get:

$$\log C_t - \log \overline{E}_t = \log A - \eta i_t$$

Speculative Attacks – Krugman: V

- Also, we know that after the currency is allowed to float, given perfect price flexibility, the rate of depreciation of the exchange rate will be equal to the rate of domestic credit growth: $$\frac{\dot{C}_t}{C_t} = \gamma = \frac{\dot{E}_t}{E_t}$$
- Therefore, from uncovered interest parity: $$i_t = i^* + \gamma$$
- Thus, our shadow exchange rate equation is: $$\log C_t + \eta\left(i^* + \gamma\right) - \ln A = \log \overline{E}_t$$
-

Speculative Attacks – Krugman: VI

- Now, whenever the shadow exchange rate is depreciated relative to the fixed exchange rate, selling short domestic currency until the central bank floats brings an infinite rate of return. Therefore, this can not happen. Since the government will not float the currency until it gives up all its reserves, after the date when the shadow exchange rate equals the fixed exchange rate, the government can not hold any reserves. Also, before such time, speculators earn a negative return from selling the domestic currency.

- Therefore, there will be a speculative attack on the currency where by (1.) there will be a discrete drop in reserves with all remaining reserves dissappearing, (2.) the money supply will drop discretely, (3.) the exchange rate will float adjusting continuously starting from the date when the shadow exchange rate equals the fixed exchange rate.

Speculative Attacks: Obstfeld I

- In the Krugman model, (1.) government wasn’t a strategic actor and (2.) individual investors were not strategic actors.

- However, when government is a strategic actor and there are many speculators who are credit constrained, then, (1.) there is a possibility that even after the first date when an attack is viable, it does not happen because investors do not coordinate for it to happen even though if all investors coordinated, it would be profitable – i.e. multiple equilibria and (2.) whether or not multiple equilibria exist depends upon the state of the fundamentals (or reserves) of the economy.

Speculative Attacks: Obstfeld II

- Suppose there are a continuum of speculators with payoff equal to:

$$-t$$

- if there is an unsuccesful attack and:

$$\bar{e}-\theta-t$$

- if the attack is succesful. The government gets a payoff of zero if it does not defend the attack otherwise gets a payoff of:

$$v-\alpha\theta$$

Speculative Attacks: Obstfeld III

- Suppose there are a large finite number of speculators. If a speculator attacks, it must be a transactions fee whether or not the attack is succesful. Thus a specualtor gets a payoff equal to:

$$-t$$

- if there is an unsuccesful attack. However, if the attack is succesful, the investors get a benefit greater than t:

$$\bar{e}-t>0$$

- The government gets a payoff of zero if it does not defend the attack otherwise gets a payoff of:

$$\theta - n$$

- where theta is state of fundamentals and N is the number of potential specualtors and n is the number of actual ones.

Speculative Attacks: Obstfeld IV

- Note that it will only be worthwhile for a speculator to attack if the attack is succesful. Therefore, we can identify three regimes:

- (1.) $\theta > N$

  In this case, even if everyone attacks, the government will not give up the peg. Therefore, noone will ever attack.

- (2.) $\theta < 1$

  In this case, even if one person attacks, the government will give up the peg. Therefore, everyone will attack.

Speculative Attacks: Obstfeld V

- (3.) $N > \theta > 1$

  In this case, if everyone attacks, then it is rational for any speculator to attack. However, if noone attacks, then it is rational for any speculator to not attack. Therefore, there are multiple equilibria. There are, in fact, two pure strategy equilibra, one where everyone attacks and one where noone attacks. In addition, there is a symmetric equilibrium in mixed strategies where everyone attacks with the same probability and in equlibrium, the attack is succesful with a certain probability (generically not equal to the probability with which speculators mix). Notice that this equilibrium goes away with a continuum of speculators since then the outcome of the attack can not be stochastic and thus speculators can not be indifferent.

Speculative Attacks – Morris & Shin: I

- We now generalize the Obstfeld model both by allowing the costs of the government defending to vary with the state of fundamentals as well as the percentage of people attacking (α).

- Speculators have utility:

$$e^{*} - f\left(\theta\right) - t > 0$$

- The government has utility:

$$v - c\left(\alpha, \theta\right)$$

Speculative Attacks – Morris & Shin: II

- We make the following functional form and parameter assumptions:
  - (1.) Theta is bounded and transactions costs of speculation are positive

$$\theta \in [0,1],\ t > 0$$

  - (2.)The costs of defense are increasing in the percentage of attackers and decreasing in fundamentals:

$$\frac{\partial C}{\partial \alpha} > 0, \frac{\partial C}{\partial \theta} < 0,$$

  - (3.) In the worst state of fundamentals, the cost of defending the currency exceeds the value even if no speculators attack:

$$C(0,0) > v$$

  - (4.) If all speculators attack, the costs outweigh the values even in the best state:

$$C(1,1) > v$$

  - (5.) In the best state of fundamentals, the floating exchange rate is sufficiently close to the pegged level that it is not worth the transactions cost to speculate:

$$t > e^{*} - f(1)$$

  –

Speculative Attacks – Morris & Shin: III

- Then, as with the Obstfeld model, there is a tripartite division of fundamentals:
  - (1.) If fundamentals are sufficiently bad, then there will be no attack:

$$\exists \underline{\theta} \text{ s.t. } C(0, \underline{\theta}) = v \Rightarrow \forall \theta \leq \underline{\theta}, \text{ there is always an attack}$$

  - (2.)The costs of defense are increasing in the percentage of attackers and decreasing in fundamentals:

$$\exists \overline{\theta} \text{ s.t. } f(\overline{\theta}) = e^{*} - t, \forall \theta \geq \overline{\theta}, \text{ there is no attack}$$

  - (3.) For inbetween states of the economy, there are always multiple equilibria:

Speculative Attacks – Morris & Shin: IV

- Now we add that the state is observed with noise where every speculator observes the truth plus uniformly distributed error:

$$x \approx U\left[\theta - \varepsilon, \theta + \varepsilon\right]$$

- The government, which decides whether or not to defend, moves after attack decisions are made and thus observes both the state as well as the percentage who attack the currency.

Speculative Attacks – Morris & Shin: V

- We will now show that equilibria are unique with incomplete information. We solve by backward induction.

- Below $\underline{\theta}$, the government will not defend even if no speculator attacks. Denote by $a(\theta)$ the maximum percentage of attacking speculators for which the government will still be willing to defend as a function of the state. Note that $a(\theta) = \alpha \text{ s.t. } c(\alpha, \theta) = v$ so that a is strictly increasing for theta greater than $\underline{\theta}$.

- Also, define the set of combinations of percentage of attacking speculator as function of the state and an equilibrium strategy: $s(\theta, \pi)$

Speculative Attacks – Morris & Shin: VI

- The relation, s, can potentially be a correspondence.

- We can write s as: $s(\theta,\pi)=\frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta+\varepsilon}\pi(x)dx$

- Where $\pi(x)$ is the percentage of speculators who attack given signal x.

- Now we can define the set of combinations of percentage of attackers and fundamentals such that the government will not defend:

$$A(\pi)=\{\theta | s(\theta,\pi)\geq a(\theta)\}$$

Speculative Attacks – Morris & Shin: VII

- We want to show that s intersect a only once. In other words, there exists a value of theta, θ^*, such that the government abandons the peg if and only if: $\theta \leq \theta^*$

- We first show three lemmas and then prove the main theorem:
  - Utility is increasing in the aggressiveness of bidding
  - If individuals follow cutoff simple cutoff strategies, utility is decreasing in the cutoff
  - There is a unique cutoff such that, in any equilibrium, all investors attack if they get at or above the cutoff and do not attack otherwise

- Then we conclude with a proof of the main result, that the equilibrium is unique. Also, we prove that even in the limit as the variance of noise goes to zero, equilibria remain unique.

Speculative Attacks – Morris & Shin: VIII

- Lemma 1: $\pi(x) \geq \pi'(x) \Rightarrow U(x,\pi) \geq U(x,\pi')$
- Proof:
  - If the share of investors attacking at any given signal level is higher, then the share of investors attacking given any signal and any underlying state of the world is higher which means that the set under which the government abandons the peg is larger:

$$\pi(x) \geq \pi'(x) \Rightarrow s(\theta,\pi) \geq s(\theta,\pi') \Rightarrow A(\pi) \geq A(\pi')$$

  - Then the utility of a speculator is given by:

$$u(x,\pi) = \frac{1}{2\varepsilon}\left[\int_{A(\pi)\cap[x-\varepsilon,x+\varepsilon]} \left(e^{*} - f(\theta)\right) d\theta\right] - t \geq$$

  - $$\frac{1}{2\varepsilon}\left[\int_{A(\pi')\cap[x-\varepsilon,x+\varepsilon]} \left(e^{*} - f(\theta)\right) d\theta\right] - t = u(x,\pi')$$

Speculative Attacks – Morris & Shin: VIII

- Lemma 2: $U(k, I_k)$ is continuous and strictly decreasing in k where:

$$U(k, I_k) = \begin{cases} 1 & \mathrm{x} < \mathrm{k} \\ 0 & \mathrm{x} \geq \mathrm{k} \end{cases}$$

- Proof:
  - Given that all agents follow the above equilibrium strategy, then we can solve for s:

$$s(\theta, I_k) = \begin{cases} 1 & \theta \leq \mathrm{k} - \varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}(\theta - k) & \mathrm{k} - \varepsilon \leq \theta \leq \mathrm{k} + \varepsilon \\ 0 & \theta \geq \mathrm{k} + \varepsilon \end{cases}$$

  -

Speculative Attacks – Morris & Shin: IX

- Denote by $\psi(k) \ni \mathrm{s}(\mathrm{k}+\psi(k)) = a(k+\psi(k))$

- Then the government abandons the peg on the interval:

$$[0, k+\psi(k)]$$

- In which case, the payoff function for an attacking investor is given by:

$$u(k, I_k) = \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{k+\psi(k)} \left(e^* - f(\theta)\right) d\theta - t$$

  – Using Lebniz' rule, since we know that $e^* - f(k)$ is strictly decreasing in theta, we need only to show that phi(k) is weakly decreasing in k.

  – But $\psi(k) = \varepsilon \text{ if } \mathrm{k} \leq \underline{\theta} - \varepsilon, \text{-} \varepsilon < \psi(\mathrm{k}) < \varepsilon \text{ if } \mathrm{k} > \underline{\theta} - \varepsilon$

  –

Speculative Attacks – Morris & Shin: X

- Thus, in equilibrium:

$$\mathrm{s}\left(\mathrm{k}+\psi(k)\right)=a\left(k+\psi(k)\right)\Rightarrow\frac{1}{2}-\frac{\psi(k)}{2\varepsilon}$$

- Totally differentiating, we get:

$$a'\left(k+\psi(k)\right)\left(1+\psi'(k)\right)=\frac{-\psi'(k)}{2\varepsilon}$$

- Continuing to solve, we get:

$$a'(\theta)=-\frac{\left[1+2\varepsilon a'(\theta)\right]\psi'(k)}{2\varepsilon}$$

$$\psi'(k)=-\frac{2\varepsilon a'(\theta)}{\left[1+2\varepsilon a'(\theta)\right]}<0$$

–

Speculative Attacks – Morris & Shin: XI

- Thus we have shown the the utility of the cutoff strategy is decreasing in the cutoff.

- Moreover, since the utility is given by an integral, the utility function is differentiable and thus continuous. This concludes our proof of lemma 2.

- Lemma 3: There is a unique cutoff value of the signal such that all investors attack if they receive a value lower than the cutoff and do not attack otherwise.

  –

Speculative Attacks – Morris & Shin: XII

- Proof of lemma 3:

- First note that for small enough k, the utility of the cutoff strategy is positive and for large enough k, it is negative:

$$u(k, I_k) > 0,\ \mathrm{k} \in [0, \underline{\theta}],\ u(k, I_k) < 0,\ \mathrm{k} > \overline{\theta}$$

- By continuity of the utility function and the fact that it is strictly decreasing in k, we know that (1.) there exists a level of k such that the utility from following the cutoff strategy is equal to zero – i.e. equal to the utility of not attacking and (2.) that such a level of k is unique.

- Now we make the following definitions:

$$x^* = x \ni u(x, I_x) = 0,\ \underline{\mathrm{x}} = \inf\{x \mid \pi(x) < 1\},\ \overline{\mathrm{x}} = \inf\{x \mid \pi(x) > 0\},$$

–

Speculative Attacks – Morris & Shin: XIII

- We will show that:

$$x^* = \underline{x} = \overline{x}$$

- First note that:

$$\underline{x} \leq \overline{x}$$

- This is due to:

$$\overline{x} \geq \sup\{x \mid 0 < \pi(x) < 1\} \geq \inf\{x \mid 0 < \pi(x) < 1\} \geq \underline{x}$$

- Now it remains to show the reverse:

$$\underline{\mathrm{x}} \geq x^* \geq \overline{\mathrm{x}}$$

–

Speculative Attacks – Morris & Shin: XIV

- When pi(x)<1, then there are at least some investors who weakly prefer not to attack. Taking the limit, we get:

$$u(x,\pi(x)) \le 0 \Rightarrow \lim_{x \to \underline{x}} u(x,\pi(x)) \le 0 \Rightarrow u(\underline{x},\pi(x)) \le 0$$

- But: $I_{\underline{x}}(x) > \pi(x) \forall x$

- So, from lemma 1, we get:

$$\mathrm{u}(\underline{\mathrm{x}}, \mathrm{I}_{\underline{\mathrm{x}}}) \le \mathrm{u}(\underline{\mathrm{x}}, \mathrm{I}_{\underline{\mathrm{x}}}) \ge 0 \Rightarrow \underline{x} \ge x^*$$

Speculative Attacks – Morris & Shin: XV

- A symmetric argument gets us:

$$x^* \geq \overline{x}$$

- Thus, we have lemma 3. The unique strategy followed by investors is the cutoff strategy at x*:

$$I_{x^*}(x)$$

- We have figured out the unique strategy by investors and by the government. It remains to show that the equilibrium is unique.

Speculative Attacks – Morris & Shin: XVI

- The equilibrium strategy for investors is given by:

$$s\left(\theta, I_{x^*}\right) = \begin{cases} 1 & \theta \leq \mathrm{x}^* - \varepsilon \\ \dfrac{1}{2} - \dfrac{1}{2\varepsilon}\left(\theta - x^*\right) & \mathrm{x}^* - \varepsilon \leq \theta \leq \mathrm{x}^* + \varepsilon \\ 0 & \theta \geq \mathrm{x}^* + \varepsilon \end{cases}$$

- We know that:

$$\underline{\theta} + \varepsilon > x^* > \underline{\theta} - \varepsilon$$

- And s is strictly increasing over this range. Moreover, below this range, s is below a and above it, a is below s. This means that a and s cross precisely once.

Speculative Attacks – Morris & Shin: XVII

- Two additional notes:
  - There is a proof that uniqueness of equilibria remains even as epsilon (and thus the variance of the noise) goes to zero. There is a mistake in the proof by Morris and Shin in their original AER paper. The correct proof is a note in a subsequent AER edition.
  - Hellwig, Mukherji, and Tsyvinski have a paper which says that the Morris and Shin results on uniqueness are dependent upon whether or not the central bank has a discrete or continuous policy (like setting interest rates versus just defending) and whether or not the signals are public or private. This paper has a revise and resubmit right now at AER.

Speculative Attacks: Generation III

- Example: Aghion, Banerjee and Bachetta (JET)
  - Can explain why currency crises covary with recessions
  - Debt denominated in foreign currency
  - If speculators believe there will be an attack, then the value of debt will go up, making firms closer to insolvency.
  - Thus interest rates will rise and capital will flow out.
  - In order to restore low interest rates, central bank will let the currency float but this raises the value of external debt, causing inability of firms to borrow abroad and incompleted projects (recession).
  - However, if speculators did not believe there would be an attack, then none of this will happen – multiple equilibria (East Asian Currency Crisis).