Gourinchas-Rey I

- We start with a dynamic current account asset equation:

$$NA_{t+1} = R_{t+1}\left(NA_t + NX_t\right)$$

- Rewriting, we get: $\left(\frac{A_{t+1}}{W_{t+1}} - \frac{L_{t+1}}{W_{t+1}}\right)\frac{W_{t+1}}{W_t} = R_{t+1}\left(\frac{A_t}{W_t} - \frac{L_t}{W_t} + \frac{X_t}{W_t} - \frac{M_t}{W_t}\right)$

- Also, defining:

$$R_{t+1} = R\left(1 + r_{t+1}\right) \qquad \frac{W_{t+1}}{W_t} = \Gamma\left(1 + \varepsilon_{t+1}^{\Delta W}\right)$$

Gourinchas-Rey II

- Imposing a steady state condition:

$$\left(\mu_{t+1}^{aw} - \mu_{t+1}^{lw}\right) = \frac{R}{\Gamma}\left(\mu_t^{aw} - \mu_t^{lw} + \mu_t^{mw} - \mu_t^{mw}\right)$$

$$\frac{R}{\Gamma} > 1$$

- Rewriting, we get:

$$\frac{NX}{NA} = \rho - 1 < 0$$

- Log-linearizing around the steady state, we get:

$$na_{t+1} \approx \frac{1}{\rho_t} na_t + \left(\hat{r}_{t+1} - \Delta w_{t+1}\right) - \left(\frac{1}{\rho_t} - 1\right) nx_t$$

Gourinchas-Rey III

- Finally, assuming a common trend in underlying varaibles (x, m, a, l), we get:

$$nx_{t+1} \approx \frac{1}{\rho} nxa_t + r_{t+1} + \Delta nx_{t+1}$$

- Where:

$$nxa_t = |\mu_a|\varepsilon_t^a - |\mu_l|\varepsilon_t^l + |\mu_x|\varepsilon_t^x - |\mu_m|\varepsilon_t^m$$

$$\Delta nx_t = |\mu_x|\Delta\varepsilon_{t+1}^x - |\mu_m|\Delta\varepsilon_{t+1}^m - \Delta w_{t+1}$$

$$r_{t+1} = |\mu^a| r_{t+1}^a - |\mu^l| r_{t+1}^l$$

Gourinchas-Rey IV

- Note that: $nx_{t+1} \approx -\sum_{j=1}^{\infty} E_t \left[r_{t+1} + \Delta nx_{t+j} \right]$
- To construct imbalances:

$$nxa_t = |\mu_a| \varepsilon_t^a - |\mu_l| \varepsilon_t^l + |\mu_x| \varepsilon_t^x - |\mu_m| \varepsilon_t^m$$

the authors decompose log exports, imports, gross foreign assets, and liabilities relative to wealth and filter out (using an HP filter) low frequency trends (including removing unit roots).

$$\mu_a = 8.4; \mu_l = 7.49; \mu_x = -9.98; \mu_m = -10.98; \rho = 0.95$$

$$0.85\varepsilon_t^a - 0.75\varepsilon_t^l + \varepsilon_t^x - 1.1\varepsilon_t^m$$

Table 1: Descriptive Statistics

	Summary Statistics								
	Δx_t	Δm_t	Δa_t	Δl_t	r_t	r_t^a	r_t^l	Δe_t	nxa_t
Mean (%)	0.82	1.11	1.11	1.87	0.72	0.78	0.78	-0.03	0
Standard deviation (%)	4.28	3.81	3.08	2.87	13.16	2.50	2.57	3.55	11.94
Autocorrelation	-0.08	0.04	0.06	0.13	0.16	0.12	0.19	0.05	0.92
	r_t^{ae}	r_t^{le}	r_t^{ad}	r_t^{ld}	r_t^{af}	r_t^{lf}	r_t^{ao}	r_t^{lo}	
Mean (%)	1.87	1.86	0.72	0.56	1.08	1.09	0.48	0.39	
Standard deviation (%)	7.19	8.02	2.94	3.17	5.93	5.81	0.76	0.53	
Autocorrelation	0.15	0.09	0.16	0.13	0.09	0.10	0.19	0.73	

Note: Sample period is 1952:1-2004:1, except for Δe, 1973:1-2004:1.

Table 2: Unconditional Variance Decomposition of nxa

		Discount factor ρ		
#	percent	0.96	0.95	0.94
1	$\beta_{\Delta nx}$	71.77	63.96	57.05
2	β_r	23.76	26.99	28.85
	of which:			
3	β_{ra}	19.91	20.78	20.65
4	β_{rl}	3.87	6.22	8.21
5	Total (lines 1+2)	95.53	90.95	85.89
6	μ_a	6.72	8.49	10.08

Note: The sum of coefficients $\beta_{ra} + \beta_{rl}$ is not exactly equal to β_r due to numerical rounding in the VAR estimation. Sample: 1952:1 to 2004:1.

Table 3: Forecasting Quarterly Returns

Column:	1	2	3	4	5	6	7	8
			Panel A: Returns					
	Total real return (r_{t+1})				Real Equity Differential (Δr^e_{t+1})			
z_t:		r_t	$\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$	xm_t		Δr^e_t	$\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$	xm_t
$\hat{\beta}$	**-0.36**	**-0.33**	**-0.46**	**-0.37**	**-0.13**	**-0.14**	**−0.17**	-0.07
(s.e.)	(0.07)	(0.07)	(0.08)	(0.16)	(0.03)	(0.03)	(0.03)	(-0.06)
$\hat{\delta}$		0.09	-1.43	0.01		-0.07	-0.63	-0.09
(s.e.)		(0.07)	(1.60)	(0.19)		(0.07)	(0.61)	(0.07)
$\bar{R}^2$	0.10	0.10	0.15	0.10	0.07	0.07	0.12	0.07
# obs	208	207	136	208	208	207	136	208
			Panel B: Depreciation Rates					
	FDI-weighted (Δe_{t+1})				Trade-weighted (Δe^T_{t+1})			
z_t :		Δe_t	xm_t	$i_t - i^*_t$		Δe^T_t	xm_{t-1}	$i_t - i^*_t$
$\hat{\beta}$	**-0.08**	**-0.09**	**-0.10**	**-0.09**	**-0.09**	**-0.09**	**-0.08**	**-0.08**
(s.e.)	(0.02)	(0.02)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)
$\hat{\delta}$		-0.04	0.02	0.32		0.02	-0.01	-0.67
(s.e.)		(0.07)	(0.05)	(0.32)		(0.07)	(0.05)	(0.34)
$\bar{R}^2$	0.09	0.08	0.08	0.08	0.11	0.10	0.10	0.13
#obs	125	124	125	125	124	123	124	124

Note: Regressions of the form: $y_{t+1} = \alpha + \beta nxa_t + \delta z_t + \epsilon_{t+1}$ where y_{t+1} is the total real return (r_{t+1}); the equity return differential ($\Delta r^e_{t+1} = r^{ae}_{t+1} - r^{le}_{t+1}$) (panel A); the FDI-weighted depreciation rate (Δe_{t+1}) or the trade weighted depreciation rate (Δe^T_{t+1}) (panel B). $\frac{d_t}{p_t} - \frac{d^*_t}{p^*_t}$ is the relative dividend price ratio (available since 1970:1); $i_t - i^*_t$ is the short term interest rate differential; xm_t is the stationary component from the trade balance, defined as $\epsilon^x_t - \epsilon^m_t$. Sample: 1952:1 to 2004:1 for total returns and 1973:1 to 2004:1 for depreciation rates. Robust standard errors in parenthesis.

Table 4: Forecasting Quarterly Returns (cont'ed)

Column	1	2	3	4	5	6	7	8
Panel A: Dollar return on US equities and US gross liabilities								
	US equity return (r^{le}_{t+1})				US liabilities return (r^{l}_{t+1})			
z_t:		r^{le}_t	d_t/p_t	cay_t		r^l_t	d_t/p_t	cay_t
$\hat{\beta}$	0.02	0.02	0.05	0.03	0.01	0.01	0.02	0.01
(s.e.)	(0.05)	(0.08)	(0.05)	(0.05)	(0.02)	(0.02)	(0.02)	(0.02)
$\hat{\delta}$		0.08	**1.28**	**2.03**		**0.19**	**0.38**	**0.69**
(s.e.)		(0.06)	(0.60)	(0.45)		(0.07)	(0.19)	(0.16)
$\bar{R}^2$	0.00	0.00	0.02	0.09	0.00	0.03	0.02	0.10
# obs	208	207	208	206	208	207	208	206
Panel B: US gross assets return (dollar and local currency)								
	Dollar return (r^{a}_{t+1})				Local currency return (r^{*a}_{t+1})			
z_t:		r^a_t	d^*_t/p^*_t	xm_t		r^{*a}_t	d^*_t/p^*_t	xm_t
$\hat{\beta}$	-0.03	-0.03	-0.03	0.00	0.03	0.02	**0.05**	**0.08**
(s.e.)	(0.02)	(0.02)	(0.02)	(0.04)	(0.02)	(0.02)	(0.02)	(0.04)
$\hat{\delta}$		0.11	-0.01	-0.05		**0.16**	0.31	-0.08
(s.e.)		(0.09)	(0.21)	(0.05)		0.08	(0.22)	0.05
$\bar{R}^2$	0.02	0.01	0.01	0.02	0.01	0.03	0.03	0.02
# obs	208	207	136	208	208	207	136	208
Panel C: Return on foreign equities (dollar and local currency)								
	Dollar return (r^{ae}_{t+1})				Local currency return (r^{*ae}_{t+1})			
z_t:		r^{ae}_t	d^*_t/p^*_t	xm_t		r^{ae*}_t	d^*_t/p^*_t	xm_t
$\hat{\beta}$	**-0.12**	**-0.11**	**-0.11**	-0.00	-0.06	-0.06	-0.03	-0.08
(s.e.)	(0.04)	(0.04)	(0.07)	(0.08)	(0.05)	(0.04)	(0.06)	(0.11)
$\hat{\delta}$		0.12	0.37	-0.16		0.16	0.69	-0.19
(s.e.)		(0.08)	(0.59)	(0.09)		(0.08)	(0.57)	(0.13)
$\bar{R}^2$	0.04	0.05	0.02	0.05	0.01	0.03	0.00	0.02
# obs	208	208	136	208	208	207	136	208

Note: Regressions of the form: $y_{t+1} = \alpha + \beta nxa_t + \delta z_t + \epsilon_{t+1}$ where y_{t+1} is the dollar return on US equities (r^{le}_{t+1}), the dollar return on US liabilities (r^{l}_{t+1}) (panel A); the dollar return on US assets (r^{a}_{t+1}), the local currency return on US assets (r^{a*}_{t+1}) (panel B); the dollar return on foreign equities (r^{ae}_{t+1}), the local currency return on foreign equities (r^{ae*}_{t+1}) (Panel C). $\frac{d_t}{p_t}$ (resp. $\frac{d^*_t}{p^*_t}$) is the domestic (resp. foreign) dividend price ratio (available since 1970:1); cay_t is the Lettau and Ludvigson (2001)'s deviation of the consumption-wealth ratio from trend; xm_t is the stationary component from the trade balance, defined as $\epsilon^x_t - \epsilon^m_t$. Sample: 1952:1 to 2004:1. Robust standard errors in parenthesis.

Table 5: Forecasting Bilateral Quarterly Rates of Depreciation

Currency	nxa_{t-1}	$\bar{R}^2$	#obs
UK pound	**-0.15**	0.04	125
	(0.06)		
Canadian dollar	-0.02	0.01	125
	(0.01)		
Swiss franc	**-0.08**	0.05	125
	(0.03)		
Japanese yen	**-0.06**	0.02	125
	(0.03)		
Deutschmark (Euro)	**-0.07**	0.08	125
	(0.02)		

Note: Sample: 1973:1 to 2004:1. Robust standard errors in parenthesis.

Table 6: Long Horizon Regressions

	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
	Real Total Net Portfolio Return $r_{t,k}$							
nxa	**-0.36**	**-0.35**	**-0.35**	**-0.33**	**-0.22**	**-0.14**	**-0.09**	**-0.04**
	(0.07)	(0.05)	(0.04)	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)
$\bar{R}^2(1)$	[0.11]	[0.18]	[0.24]	[0.26]	[0.21]	[0.13]	[0.09]	[0.02]
$\bar{R}^2(2)$	[0.14]	[0.25]	[0.34]	[0.38]	[0.35]	[0.24]	[0.19]	[0.16]
	Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$							
nxa	**-0.14**	**-0.13**	**-0.12**	**-0.11**	**-0.06**	**-0.03**	-0.02	0.01
	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.07]	[0.13]	[0.17]	[0.18]	[0.10]	[0.03]	[0.01]	[0.00]
$\bar{R}^2$ (2)	[0.11]	[0.20]	[0.28]	[0.31]	[0.26]	[0.15]	[0.10]	[0.17]
	Net Export growth $\Delta nx_{t,k}$							
nxa	**-0.08**	**-0.08**	**-0.07**	**-0.07**	**-0.07**	**-0.06**	**-0.06**	**-0.04**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.05]	[0.10]	[0.13]	[0.17]	[0.31]	[0.44]	[0.53]	[0.58]
$\bar{R}^2$ (2)	[0.04]	[0.08]	[0.12]	[0.17]	[0.38]	[0.55]	[0.66]	[0.79]
	FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$							
nxa	**-0.08**	**-0.08**	**-0.08**	**-0.08**	**-0.07**	**-0.06**	**-0.04**	**-0.02**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.09]	[0.16]	[0.28]	[0.31]	[0.41]	[0.41]	[0.33]	[0.12]
$\bar{R}^2$ (2)	[0.10]	[0.21]	[0.35]	[0.40]	[0.52]	[0.55]	[0.55]	[0.38]

Note: Regressions of the form: $y_{t,k} = \alpha + \beta nxa_t + \epsilon_{t+k}$ where $y_{t,k}$ is the k-period real total net portfolio return ($r_{t,k}$); total excess equity return ($r^{ae}_{t,k} - r^{le}_{t,k}$); net export growth ($\Delta nx_{t,k}$) or the FDI-weighted depreciation rate ($\Delta e_{t,k}$). Newey-West robust standard errors in parenthesis with $k-1$ Bartlett window. Adjusted R^2 in brackets. $\bar{R}(1)$ reports the adjusted R-squared of the regression on nxa_t; $\bar{R}(2)$ reports the adjusted R-squared of the regression on ϵ^x_t, ϵ^m_t, ϵ^a_t and ϵ^l_t. Sample: 1952:1 to 2004:1 (1973:1 to 2004:1 for exchange rate).

Table 7: Forecasting Exchange Rates. Sample 1973:2004.

ADF-like Regressions	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$								
e_{t-1}	-0.052	**-0.050**	**-0.052**	**-0.058**	**-0.067**	**-0.067**	**-0.064**	**-0.056**
(s.e.)	(0.027)	(0.020)	(0.015)	(0.013)	(0.10)	(0.008)	(0.006)	(0.004)
Δe_{t-1}	0.072	-0.028	0.077	**0.113**	**0.076**	**0.049**	0.028	0.004
(s.e.)	(0.090)	(0.065)	(0.049)	(0.043)	(0.032)	(0.025)	(0.020)	(0.012)
$\bar{R}^2$	[0.01]	[0.04]	[0.08]	[0.15]	[0.28]	[0.39]	[0.48]	[0.65]
e_{t-1}	-0.031	-0.028	**-0.032**	**-0.040**	**-0.051**	**-0.054**	**-0.054**	**-0.052**
(s.e.)	(0.028)	(0.019)	(0.014)	(0.012)	(0.008)	(0.006)	(0.005)	(0.004)
Δe_{t-1}	-0.015	-0.123	-0.006	0.039	0.008	-0.005	-0.012	-0.009
(s.e.)	(0.091)	(0.062)	(0.045)	(0.039)	(0.026)	(0.019)	(0.016)	(0.013)
nxa_{t-1}	**-0.080**	**-0.086**	**-0.076**	**-0.069**	**-0.061**	**-0.049**	**-0.036**	**-0.011**
(s.e.)	(0.025)	(0.017)	(0.012)	(0.011)	(0.007)	(0.005)	(0.004)	(0.003)
$\bar{R}^2$	[0.08]	[0.20]	[0.30]	[0.37]	[0.57]	[0.68]	[0.70]	[0.68]
IFS nominal effective rate of depreciation $\Delta e^{IFS}_{t,k}$								
e^{IFS}_{t-1}	-0.048	**-0.048**	**-0.051**	**-0.056**	**-0.063**	**-0.061**	**-0.056**	**-0.046**
(s.e.)	(0.027)	(0.020)	(0.016)	(0.014)	(0.010)	(0.008)	(0.006)	(0.004)
Δe^{IFS}_{t-1}	0.149	0.066	**0.137**	**0.131**	0.066	0.036	0.017	-0.001
(s.e.)	(0.090)	(0.068)	(0.054)	(0.048)	(0.035)	(0.027)	(0.021)	(0.015)
(i) $\bar{R}^2$	[0.03]	[0.03]	[0.10]	[0.14]	[0.25]	[0.35]	[0.43]	[0.55]
e^{IFS}_{t-1}	0.002	0.007	-0.005	-0.015	**-0.031**	**-0.039**	**-0.041**	**-0.047**
(s.e.)	(0.029)	(0.021)	(0.016)	(0.015)	(0.010)	(0.008)	(0.007)	(0.005)
Δe^{IFS}_{t-1}	0.011	-0.082	0.010	0.021	-0.017	-0.020	-0.020	-0.001
(s.e.)	(0.096)	(0.068)	(0.053)	(0.047)	(0.034)	(0.027)	(0.022)	(0.017)
nxa_{t-1}	**-0.097**	**-0.105**	**-0.088**	**-0.079**	**-0.060**	**-0.041**	**-0.027**	0.000
(s.e.)	(0.029)	(0.020)	(0.016)	(0.014)	(0.010)	(0.008)	(0.007)	(0.005)
(ii) $\bar{R}^2$	[0.10]	[0.20	[0.27]	[0.30]	[0.42]	[0.47]	[0.51]	[0.55]

Note: Runs regressions of the form $\Delta e_{t,k} = \alpha e_{t-1} + \beta \Delta e_{t-1} + \gamma nxa_{t-1} + c + \epsilon_{t,k}$. Sample 1973:2004.

Table 8: Out of Sample Tests for Exchange Rate Depreciation against the Martingale Hypothesis

Horizon: (quarters)	1	2	3	4	8	12	16
FDI-weighted depreciation rate							
MSE_u/MSE_r	0.960	0.920	0.858	0.841	0.804	0.818	0.903
ΔMSE-adjusted ($MSE_r - MSE_u$-adj)	1.48	1.53	1.61	1.51	1.20	0.74	0.35
(s.e.)	(0.68)	(0.60)	(0.57)	(0.53)	(0.37)	(0.24)	(0.23)
p-val	[0.01]	[0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[0.06]
Trade-weighted depreciation rate							
MSE_u/MSE_r	0.949	0.900	0.830	0.788	0.733	0.929	0.961
ΔMSE-adjusted ($MSE_r - MSE_u$-adj)	2.76	3.03	2.94	2.78	1.91	0.67	0.29
(s.e.)	(1.03)	(1.03)	(1.02)	(0.98)	(0.69)	(0.38)	(0.24)
p-val	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[<0.01]	[0.03]	[0.11]

Note: $\Delta MSPE - adjusted$ is the Clark-West (2004) test-statistic based on the difference between the out of sample MSE of the driftless random-walk model and the out-of-sample MSE of a model that regresses the rate of depreciation $\Delta e_t + 1$ against nxa_t. Rolling regressions are used with a sample size of 105. t-statistic in parenthesis. p-value of the one-sided test using critical values from a standard normal distribution in brackets. Under the null, the random-walk encompasses the unrestricted model. Sample: 1952:1-2004:1. Cut-off: 1978:1.

Table 9: Out of Sample Tests for various nested models.

	ENC-NEW	MSE_u/MSE_r						
Horizon: (quarters)		1	2	3	4	8	12	16
Panel A: Real Total Net Portfolio Return $r_{t,k}$								
nxa *vs* $AR(1)$	9.46^{**}	0.970	0.903	0.843	0.785	0.758	0.868	0.968
nxa *vs* $AR(1)$, $\frac{d}{p}$ and $\frac{d^*}{p^*}$	20.91^{**}	0.970	0.862	0.779	0.671	0.610	0.542	0.626
Panel B: Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$								
nxa *vs* $AR(1)$	19.58^{**}	0.894	0.782	0.693	0.638	0.744	0.925	1.057
nxa *vs* $AR(1)$, $\frac{d}{p}$ and $\frac{d^*}{p^*}$	27.92^{**}	0.917	0.790	0.686	0.626	0.810	0.899	1.026
Panel C: FDI-weighted depreciation rate $\Delta e_{t,k}$								
nxa *vs* $AR(1)$	6.57^{**}	0.948	0.882	0.834	0.809	0.736	0.736	0.811
nxa *vs* $AR(1)$, $i_t - i^*_t$	6.78^{**}	0.951	0.878	0.824	0.805	0.735	0.748	0.828

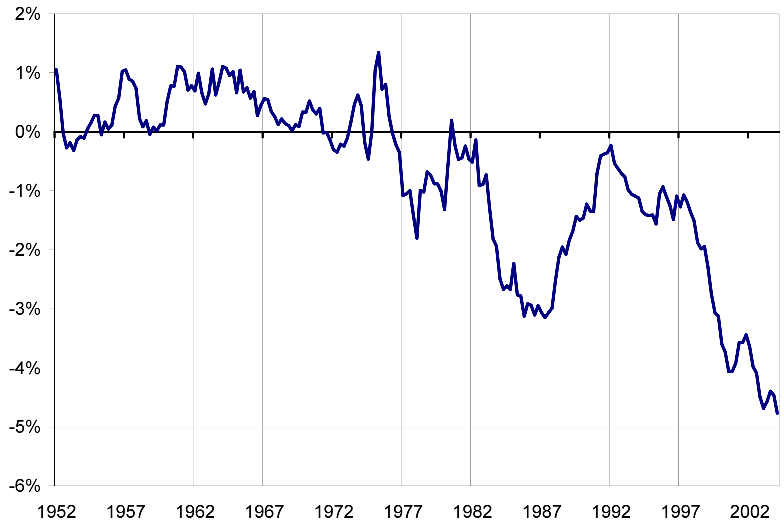
Note: MSE_u is the mean-squared forecasting error for an unrestricted model that includes the lagged dependent variable and lagged nxa (model 1); lagged d/p, d^*/p^* and lagged nxa (model 2). MSE_r is the mean-squared error for the restricted models which include the same variables as above but do not include lagged nxa. d/p (resp. d^*/p^*) is the US (resp. rest of the world) dividend price ratio. Each model is first estimated using the sample 1952:1 1978:1. ENC-NEW is the modified Harvey, Leybourne and Newbold (1998) statistic, as proposed by Clark and McCracken (2001). Under the null, the restricted model encompasses the unrestricted one. Sample: 1952:1-2004:1. * (resp. **) significant at the five (resp. one) percent level.

Table 10: Unconditional Variance Decomposition for nxa, when mean returns on assets and liabilities differ.

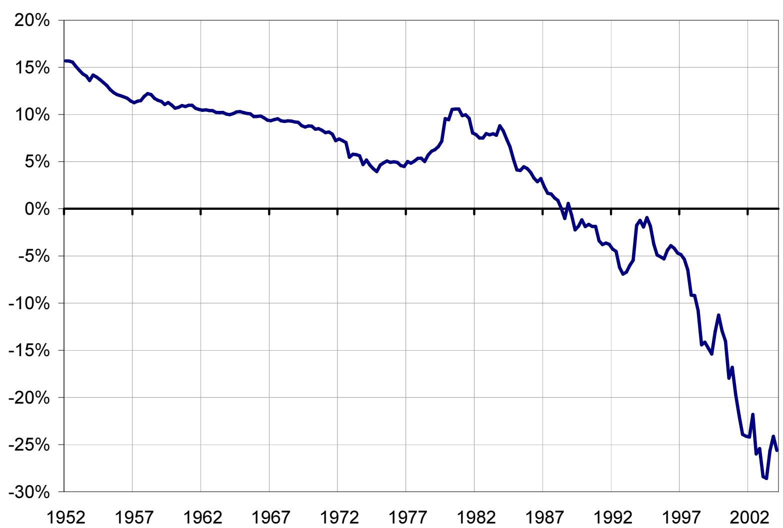
Variance Decomposition:		
#	percent	
1	$\beta_{\Delta nx}$	58
2	β_r	26
3	β_{cl}	12
5	Total	96

Note: Sample: 1952:1 to 2004:1.

Figure 1: US Net Exports and Net Foreign Assets (% of GDP, 1952-2004)



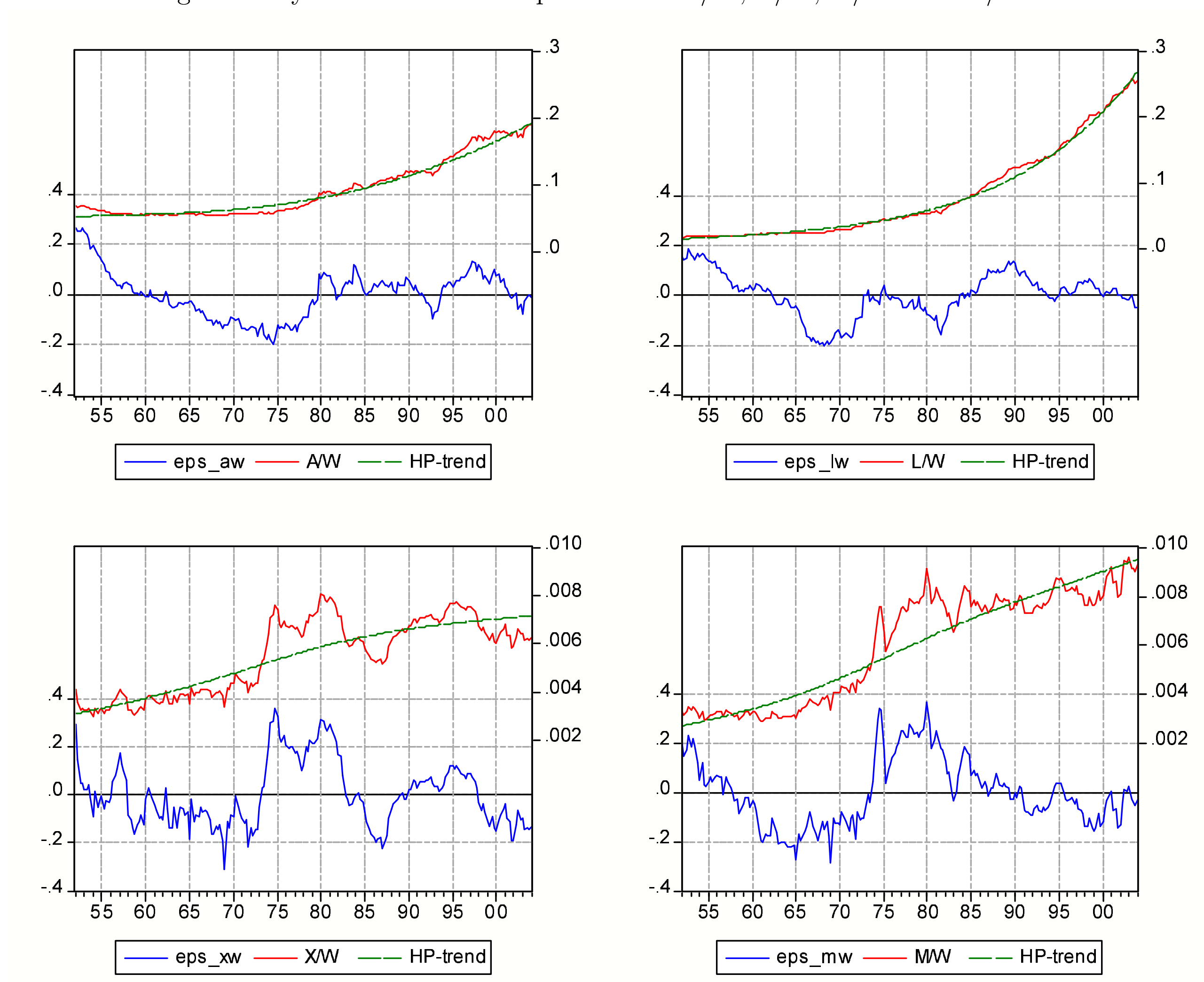
(a) Net Exports/GDP



(b) Net Foreign Assets/GDP

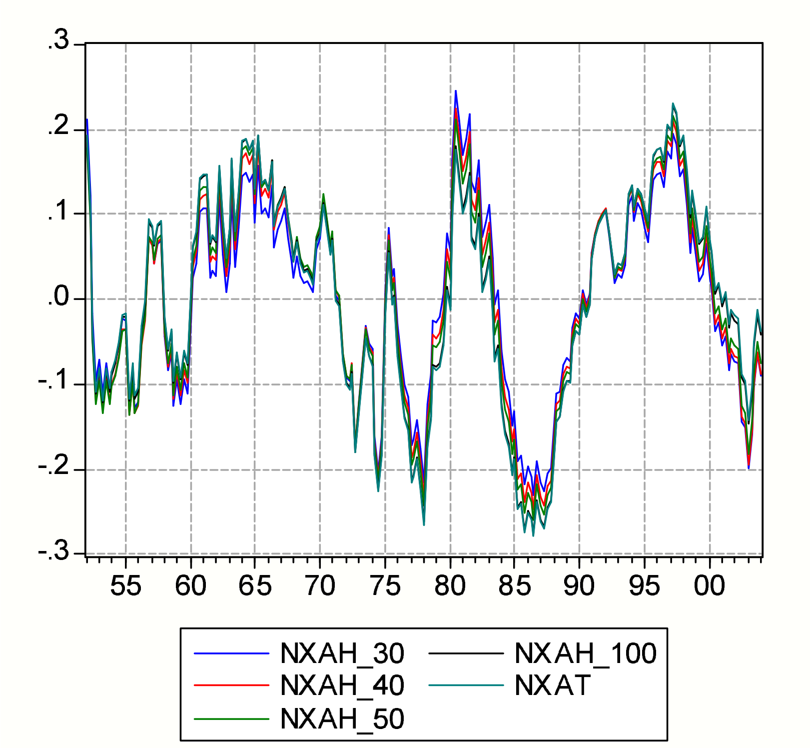
Note: The top panel shows the ratio of US net exports to US GDP. The bottom panel shows the ratio of US net foreign assets to US GDP. Sample: 1952:1-2004:1. Source: Bureau of Economic Analysis, Flow of Funds and Authors calculations.

Figure 2: Cycle and Trend Components for A/W, L/W, X/W and M/W.



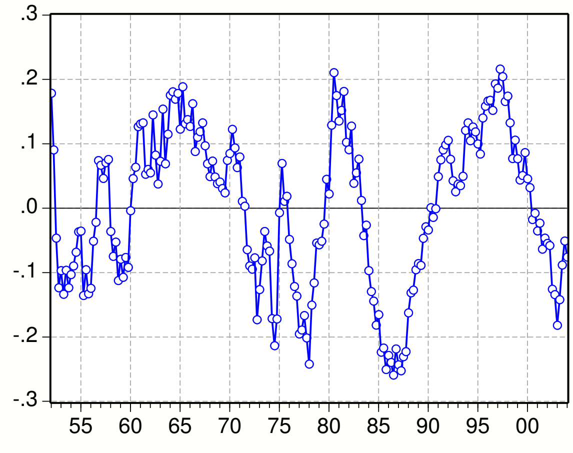
Note: Top two panels for US gross external assets A/W (left) and US gross external liabilities L/W (right); Bottom two panels for US exports X/W (left) and US imports M/W (right). Each panel reports the series Z/W (ratio to household wealth), the trend component μ_t^{zw}, labelled HP-trend, (right-axis) and the cyclical component ϵ_t^{zw} (left-axis). Sample: 1952:1-2004:1.

Figure 3: Various nxa

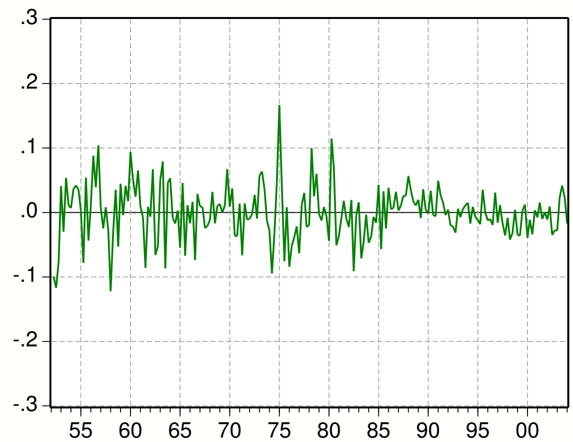


Note: nxa, constructed from various cut-offs (30, 40, 50, 100 years and linear filter). Sample: 1952:1-2004:1

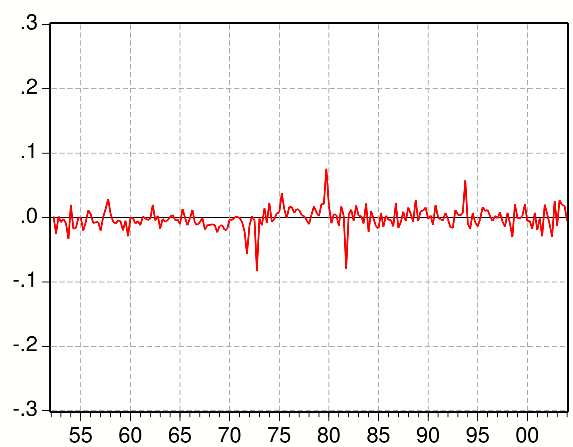
Figure 4: nxa, flow $r + \Delta nx$ and residual term ε from equation (6).



(a) nxa_t

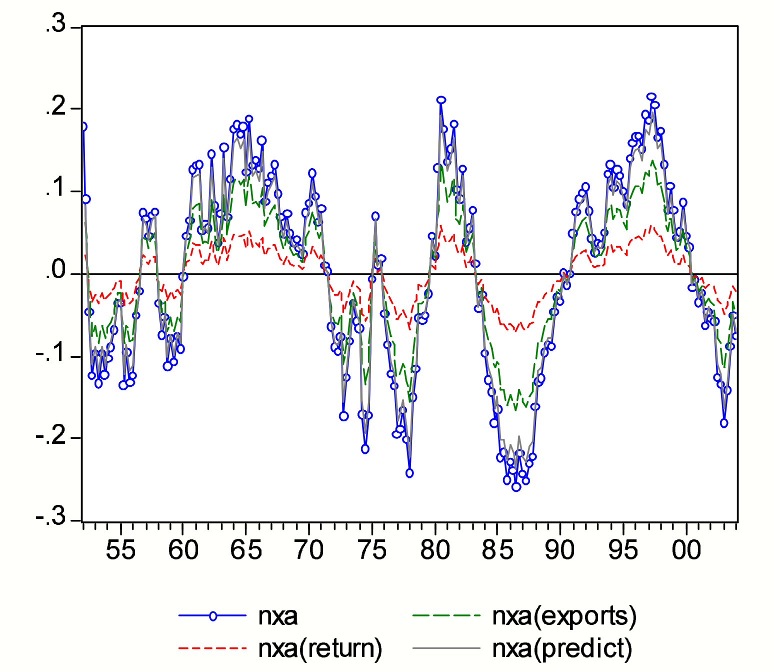


(b) flow $r_t + \Delta nx_t$

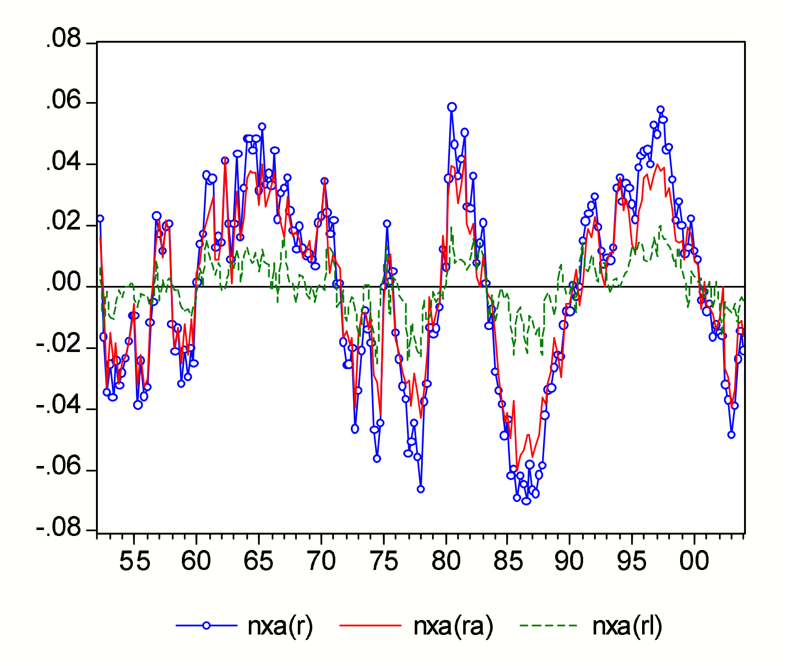


(c) residual ε_t

Figure 5: Decomposition of nxa into trade and valuation components.



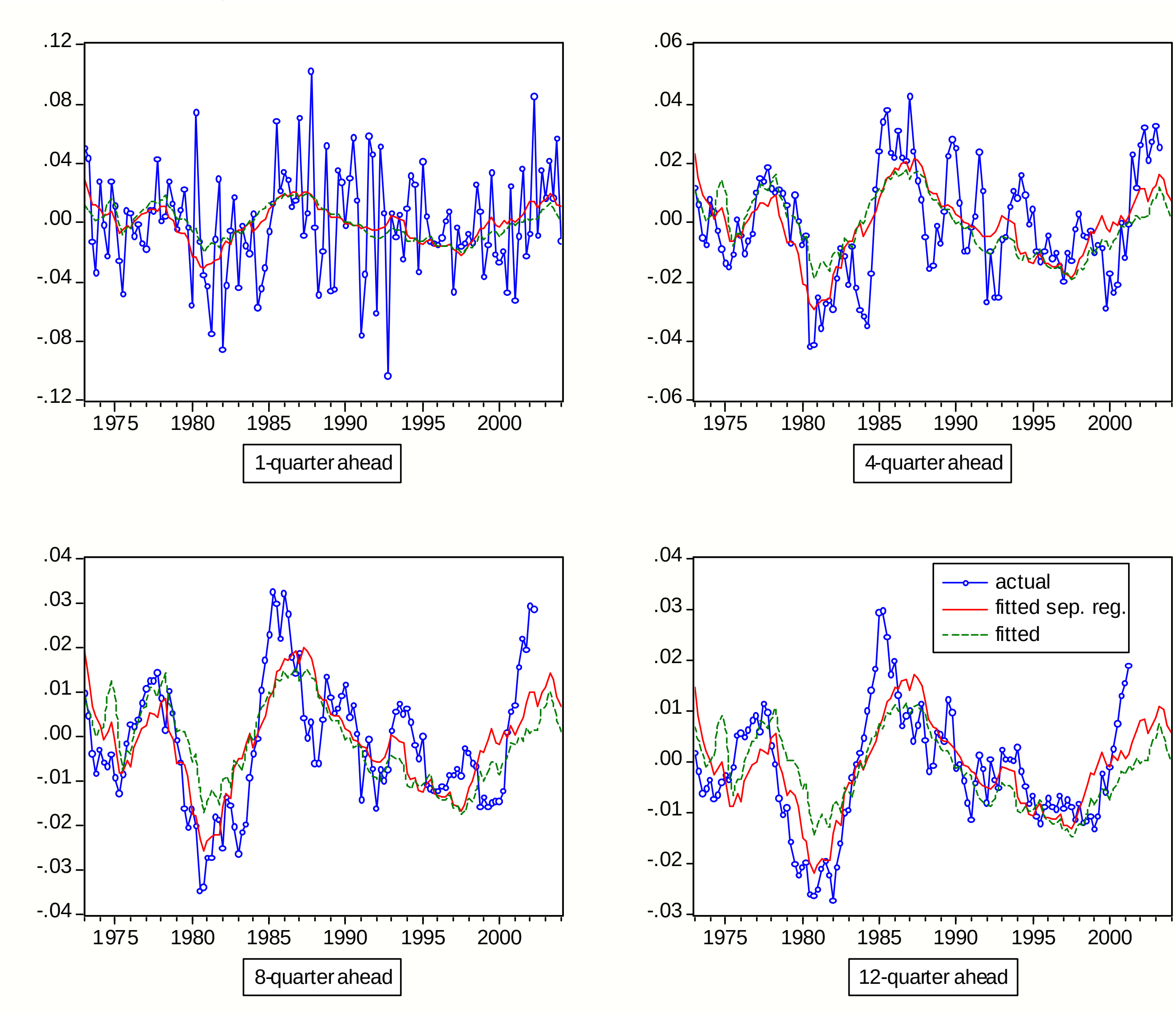
(a) return $nxa(return)$ and net exports $nxa(exports)$ components.



(b) asset return $nxa(ra)$ and liability return $nxa(rl)$ components.

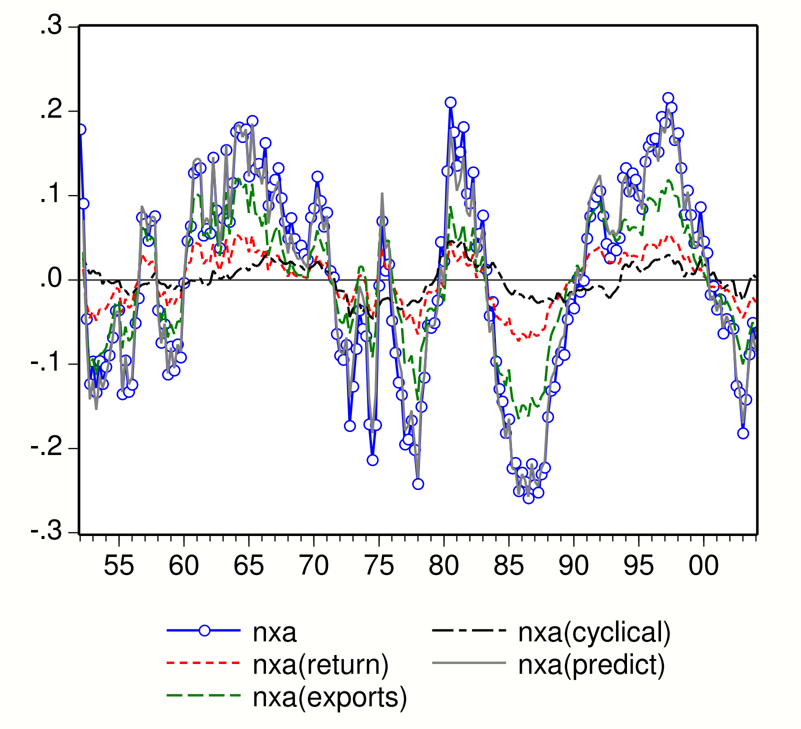
Note: The top panel reports the decomposition of nxa into its return ($nxa(return)$) and net exports ($nxa(exports)$) components. The bottom panel reports the decomposition of the return component ($nxa(return)$) into an asset return ($nxa(ra)$) and a liability return ($nxa(rl)$) components.

Figure 6: Predicted One to 12-quarter ahead depreciation rates.



Note: Each graph reports (a) the realized depreciation rate at 1 to 12 quarter horizon; (b) the fitted depreciation rate using nxa (fitted); (c) the fitted depreciation rate using ϵ^{xw}, ϵ^{mw}, ϵ^{aw} and ϵ^{lw} as separate regressors (fitted sep. reg.).

Figure 7: Decomposition of nxa into trade, valuation and cyclical components.



Note: The figure reports the decomposition of nxa into a return ($nxa(return)$), a net exports ($nxa(exports)$) and a cyclical (nxa(cyclical)) components.

A Biased View of PPP

Demian Reidel and Jan Szilagyi

September 22, 2004

Road Map

- Current Debate
- Introduction to the PPP puzzle
- Theory: small sample and heterogeneity biases
- Empirical evidence
- Monte-Carlo analysis
- Conclusion

Current debate

- Imbs et al (2004): sectoral heterogeneity bias can explain the PPP puzzle
  - Use monthly disaggregated data at the sectoral level from Eurostat, 1981-1995.
  - Theoretical conditions for heterogeneity bias are present in the data.
  - Apply Pesaran (2002) estimator to deal with sectoral data and find that PPP puzzle disappears. Half-life is around 1 year.

Current debate

- Chen, Engel (2004): sectoral heterogeneity bias is irrelevant
  - Use similar data as Imbs et al (2004).
  - Use Monte Carlo simulations to show that in small samples heterogeneity bias is not enough to solve the PPP puzzle.
  - Their simulations focus on OLS time series estimates (not panel data).
  - Their sample sizes in the simulations run up only to around 40 years (500 monthly data points).
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

Current debate

- Choi, Mark, Sul (2003, 2004): country heterogeneity bias is irrelevant
  - Use annual data for 21 OECD countries.
  - Use the Recursive Demeaning Estimator to alleviate the small sample bias
  - Do not explore sectoral heterogeneity but find that country heterogeneity is irrelevant.
  - Their sample sizes in the simulations run up only to around 200 data points.
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

Our Methodology

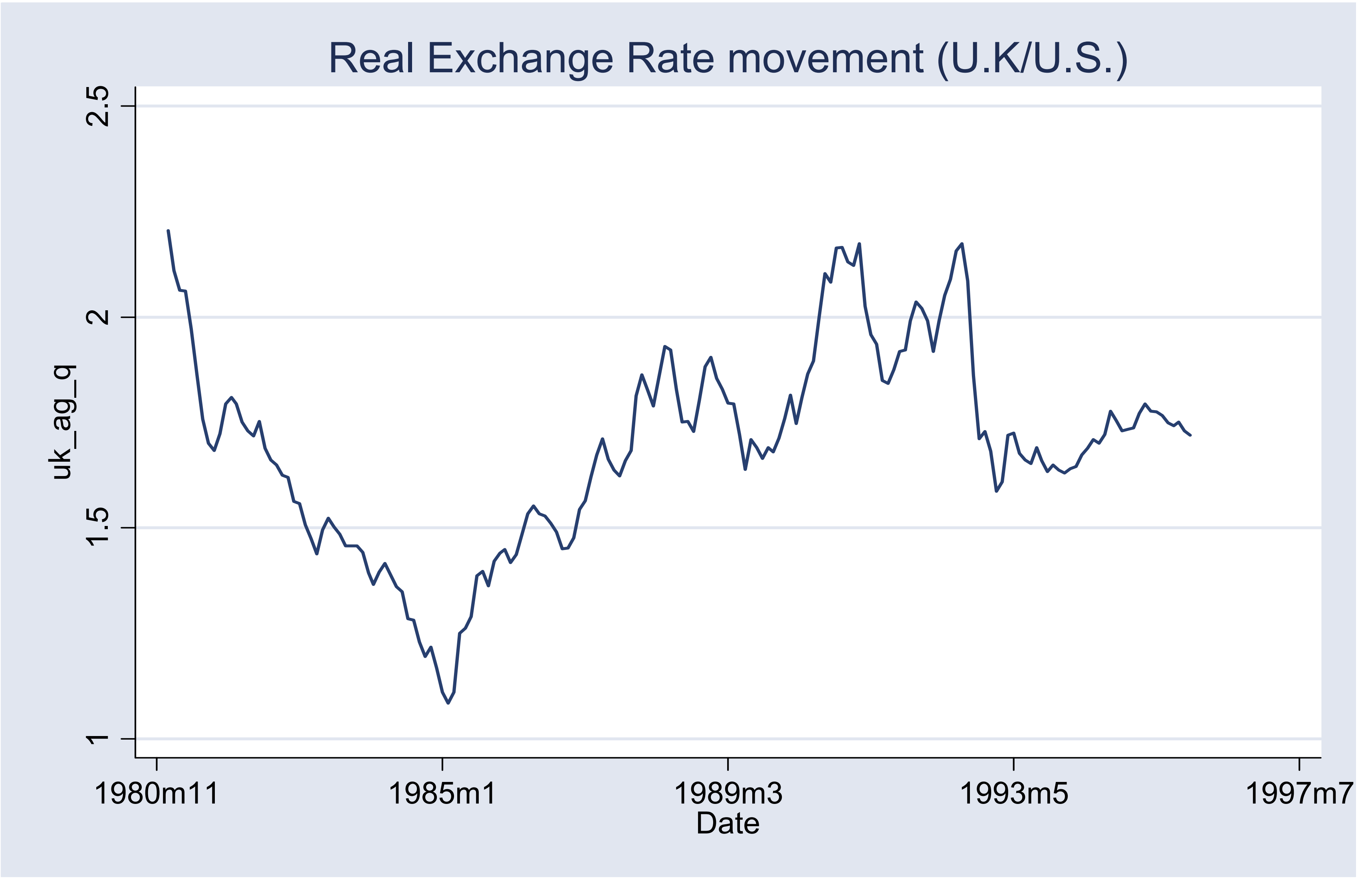
- Use disaggregated data as in Imbs et al (2004)
- Explore both time series and panel estimations
- Use OLS, SURE and CCE estimators
- Use improved Monte Carlo engine
  - non-diagonal covariance structure across sectors
  - calibrated for different data frequencies (monthly and annual)
  - short, medium and large samples (up to 10,000 points)
  - general structure of heterogeneity
  - Information-neutral initial conditions

Our results

We explore the interaction between small-sample and heterogeneity biases that plague estimates of half-lives of PPP deviations:

- in small samples heterogeneity bias can disappear completely.
- heterogeneity bias does not resolve the PPP puzzle.
- for longer series heterogeneity bias dominates small sample bias and results in upward-biased estimates.
- Imbs et al (2004) find low half-lives because they use Pesaran (2002) estimator which is not applicable to this problem.

Theory vs. Reality



Part I – Biases in estimation

Estimations of half-lives are particularly affected by two different biases:

1. Small Sample Bias (Orcutt (1948), Kendall (1954), …)

2. Heterogeneity Bias (Pesaran and Smith (1995), Imbs et al (2004))

Part I – Biases in estimation

Imbs et al (2004) actually claim that the effect of heterogeneity is large enough to solve the puzzle.

They find that in the data, heterogeneity creates an upward biased estimate of the half-lives of deviations from PPP.

Their bias-corrected estimates are close to one year, in contrast to the “consensus view” of 3-5 years.

Part II
Theory: Small Sample and Heterogeneity Biases

Part II – Heterogeneity bias

- Heterogeneity bias in time-series:

$$Sectoral\ real\ exchange\ rate:$$

$$\begin{cases} q_{it} = c_i + \rho_i q_{it-1} + \varepsilon_{it} \\ \rho_i = \rho^{true} + \eta_i^{\rho}, c_i = c^{true} + \eta_i^{c} \end{cases}$$

$$Estimated\ model:$$

$$\begin{cases} q_t = c + \rho q_{t-1} + \varepsilon_t \\ c = \sum_i \omega_i c_i,\ \varepsilon_t = \sum_i \omega_i \varepsilon_{it} + \sum_i \omega_i \eta_i^{\rho} q_{it-1} \end{cases}$$

$$Bias: \tilde{\rho} = \rho + \Delta,\ \ \Delta = \sum_i (\rho_i - \rho)\alpha_i$$

$$\alpha_i = \left(\frac{\omega_i^2 \sigma_i^2}{1-\rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1-\rho_i \rho_j} \right) \Big/ \sum_i \left(\frac{\omega_i^2 \sigma_i^2}{1-\rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1-\rho_i \rho_j} \right)$$

Part II – Heterogeneity bias

- Key idea: the estimated ρ from aggregate series is a biased estimator of the average ρ in the economy (Pesaran, Smith (1995)). Therefore, we cannot interpret this number as the average speed of adjustment (which is the PPP puzzle)
- It does NOT mean that the estimates of ρ for the real exchange rate are wrong or biased.

Part II – Heterogeneity bias

- A simple illustration:

$$\text{Assume that } \sigma_{\text{ij}} = 0, \sigma_i^2 = \sigma^2, \omega_i = \frac{1}{N}$$

$$\alpha_i = \frac{1/(1-\rho_i^2)}{\sum_j 1/(1-\rho_j^2)}, \text{positive and increasing in } \rho_i$$

$$\text{Therefore } \Delta = \sum_i (\rho_i - \rho)\alpha_i > 0$$

Part II – Small sample bias

- Small sample bias in OLS estimates:

$$Let\ \vec{q}_T\ be\ the\ real\ exchange\ rate\ vector\ and\ assume\ that$$

$$\vec{q}_T = \rho\vec{Q}_{T-1} + \varepsilon_T$$

$$Then$$

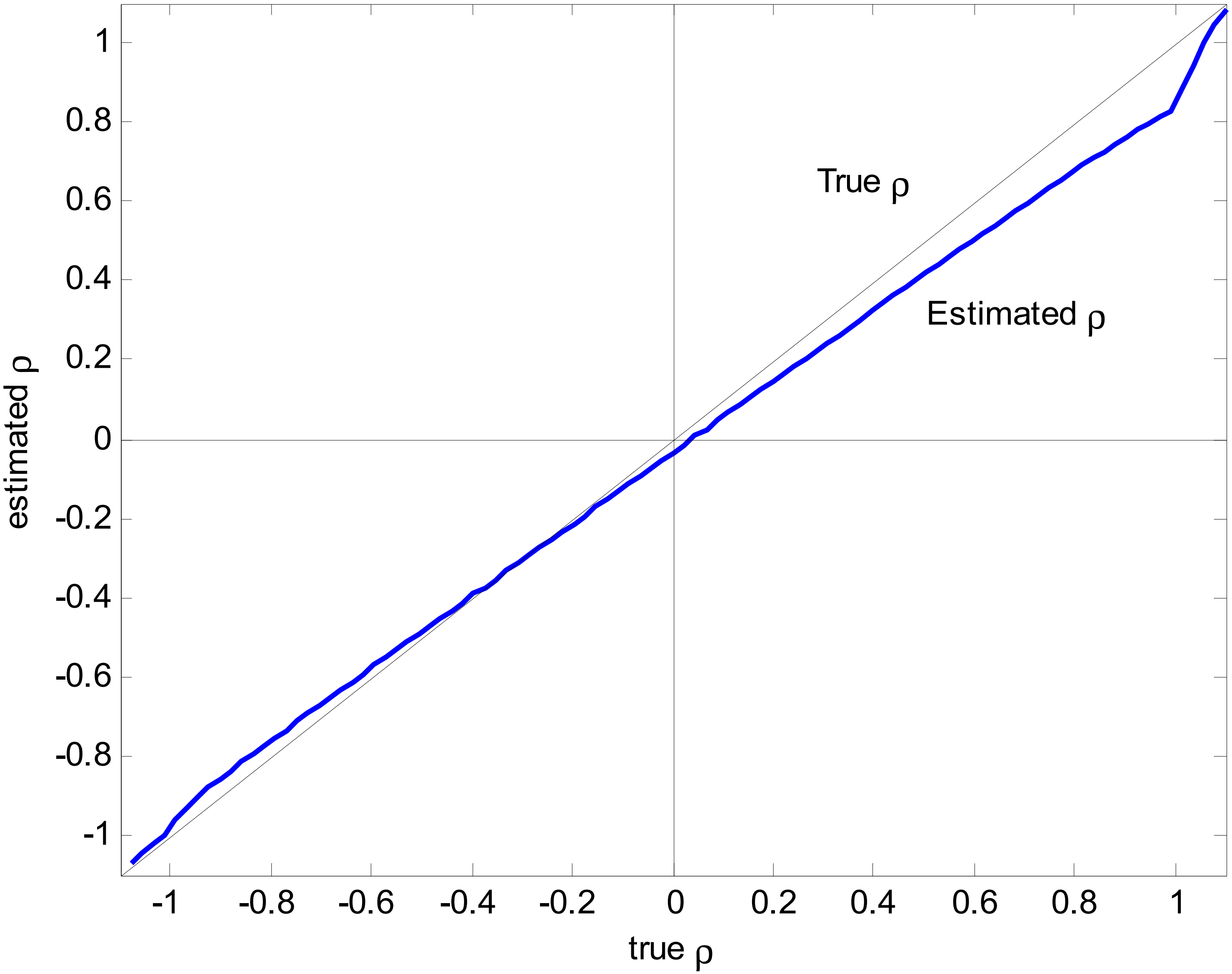
$$\hat{\rho} = (\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\vec{q}_T$$

$$E[\hat{\rho}] = \rho + E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\varepsilon_T]$$

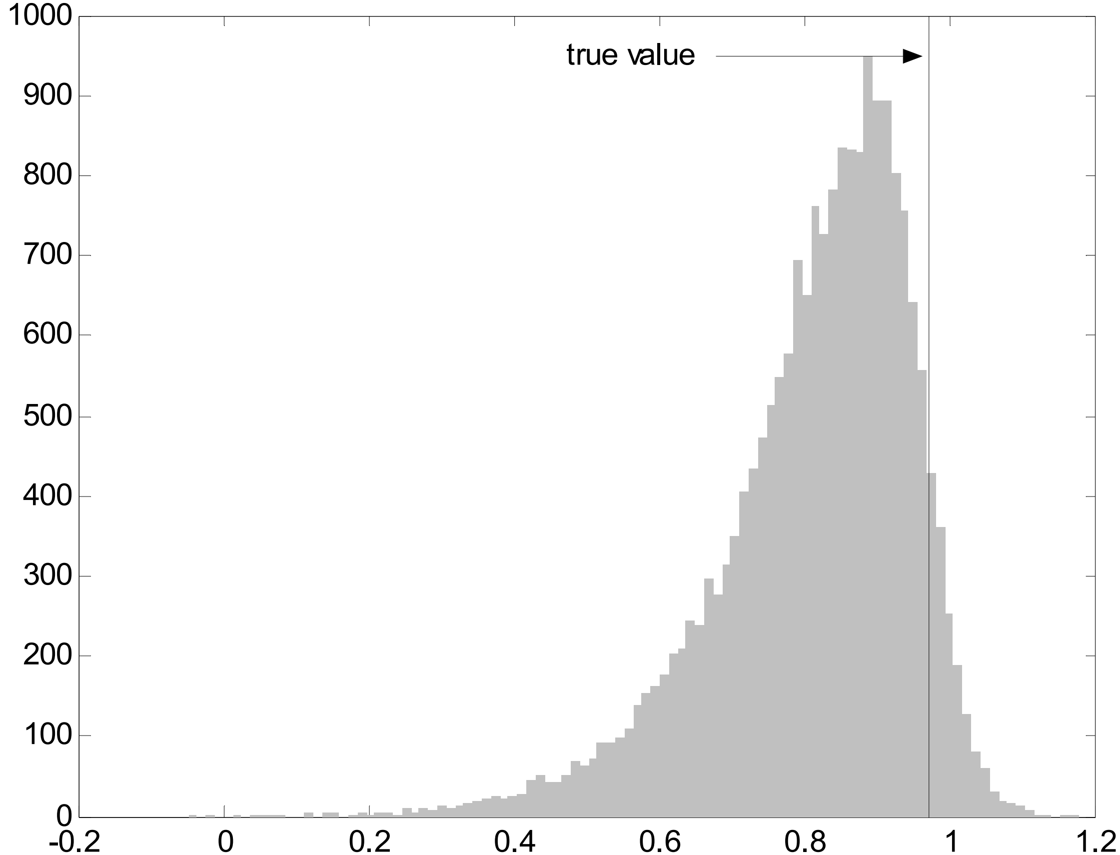
$$and$$

$$E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}\vec{Q}_{T-1}'\varepsilon_T] \xrightarrow{p} E[(\vec{Q}_{T-1}'\vec{Q}_{T-1})^{-1}]E[\vec{Q}_{T-1}'\varepsilon_T] = 0$$

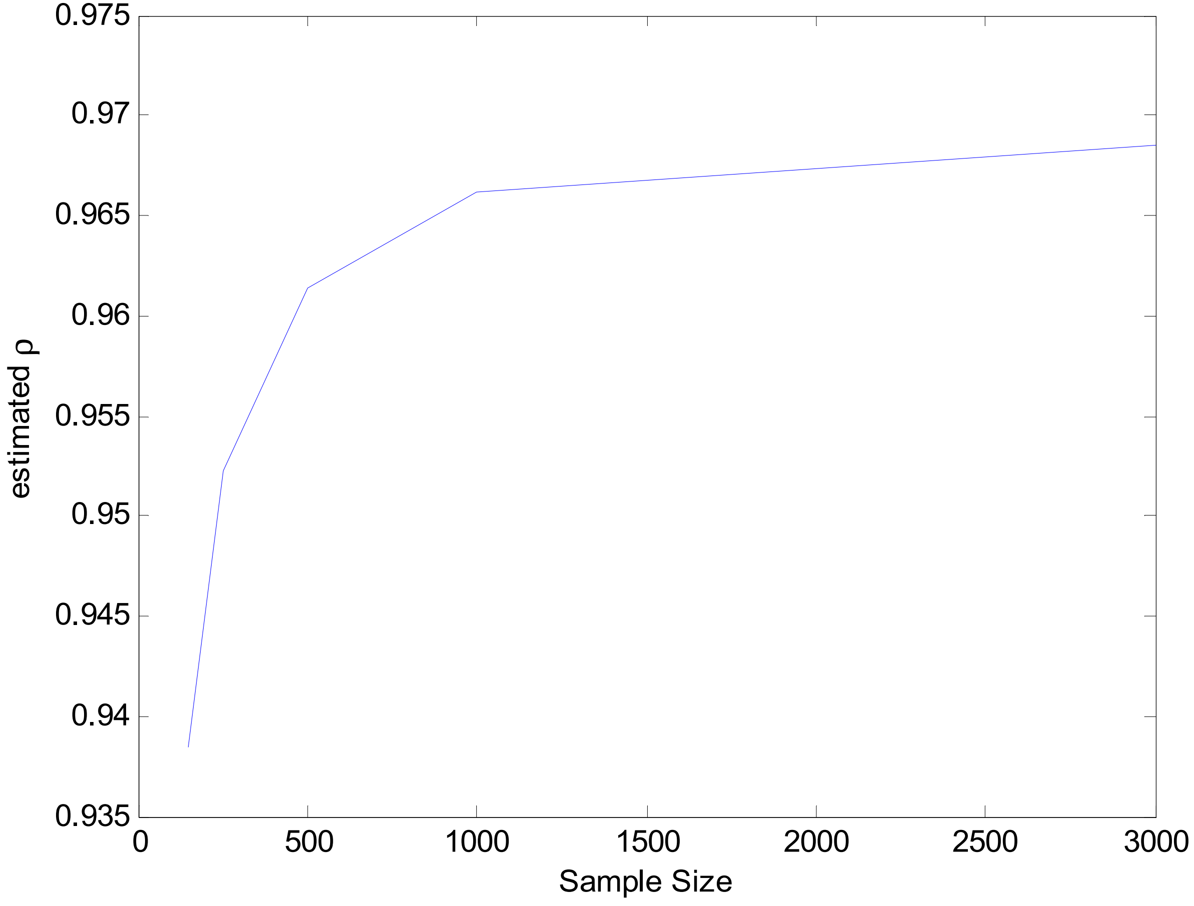
Estimation of ρ (OLS) in a small sample (T=20, ρ=.97)



Histogram of OLS estimator
ρ_{true}=0.97, T=20



Small sample bias as a function of sample size
ρ_{true}=0.97



Part II – Interaction

- Note that small-sample bias, by definition, appears in small samples.
- In contrast, the analytical results for heterogeneity bias are valid ONLY asymptotically.
- We do not have analytical solutions for their interaction in finite samples. In Part IV we use Monte Carlo simulations to explore this interaction.

Part III
Empirical Evidence

Part III – Empirical results

Dataset:

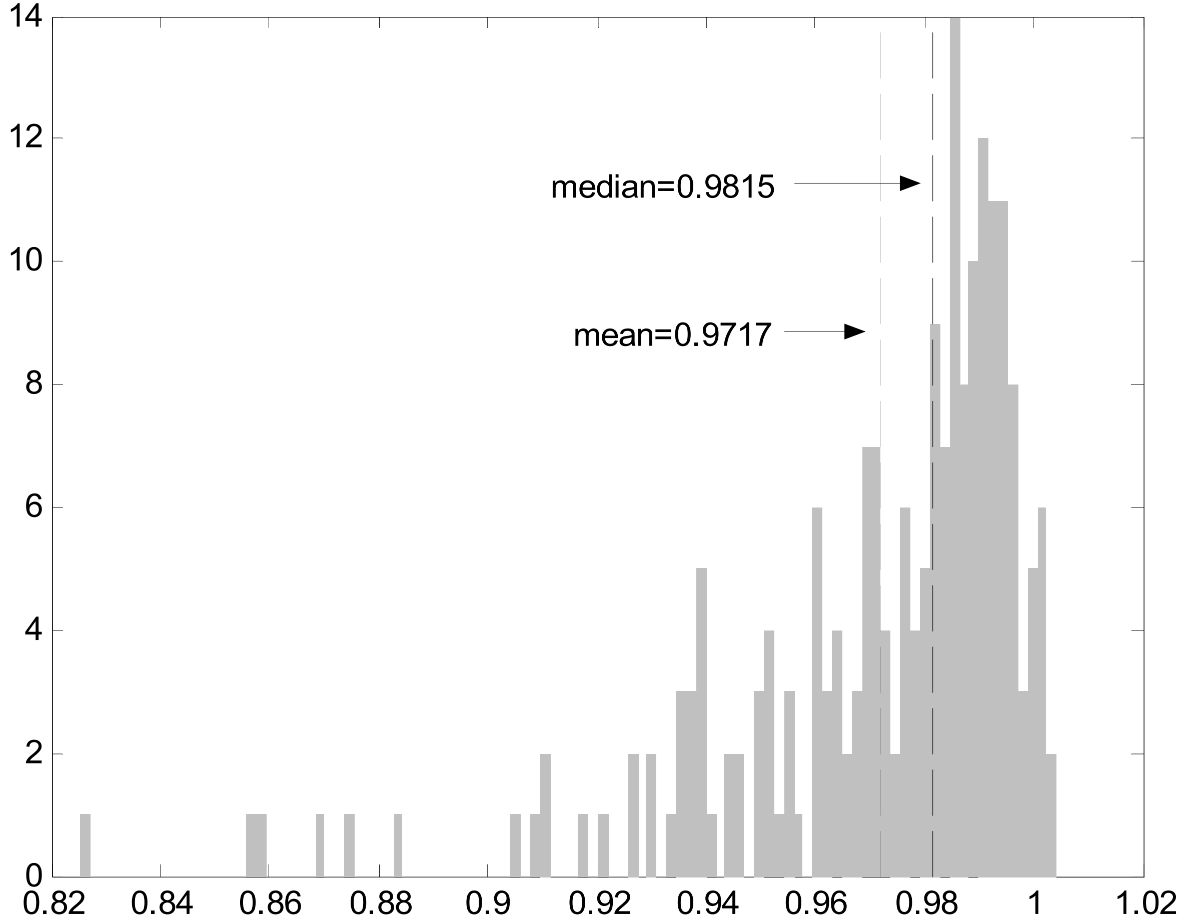
- Eurostat data for Belgium, Germany, Denmark, Finland, France, Greece, Italy, Netherlands, Portugal, Spain, U.K.
- 19 different sectors including “meat,” “dairy,” “clothes,” etc.
- Time range from 1981-1995, which gives them a maximum length of 180 datapoints

Part III – Empirical results

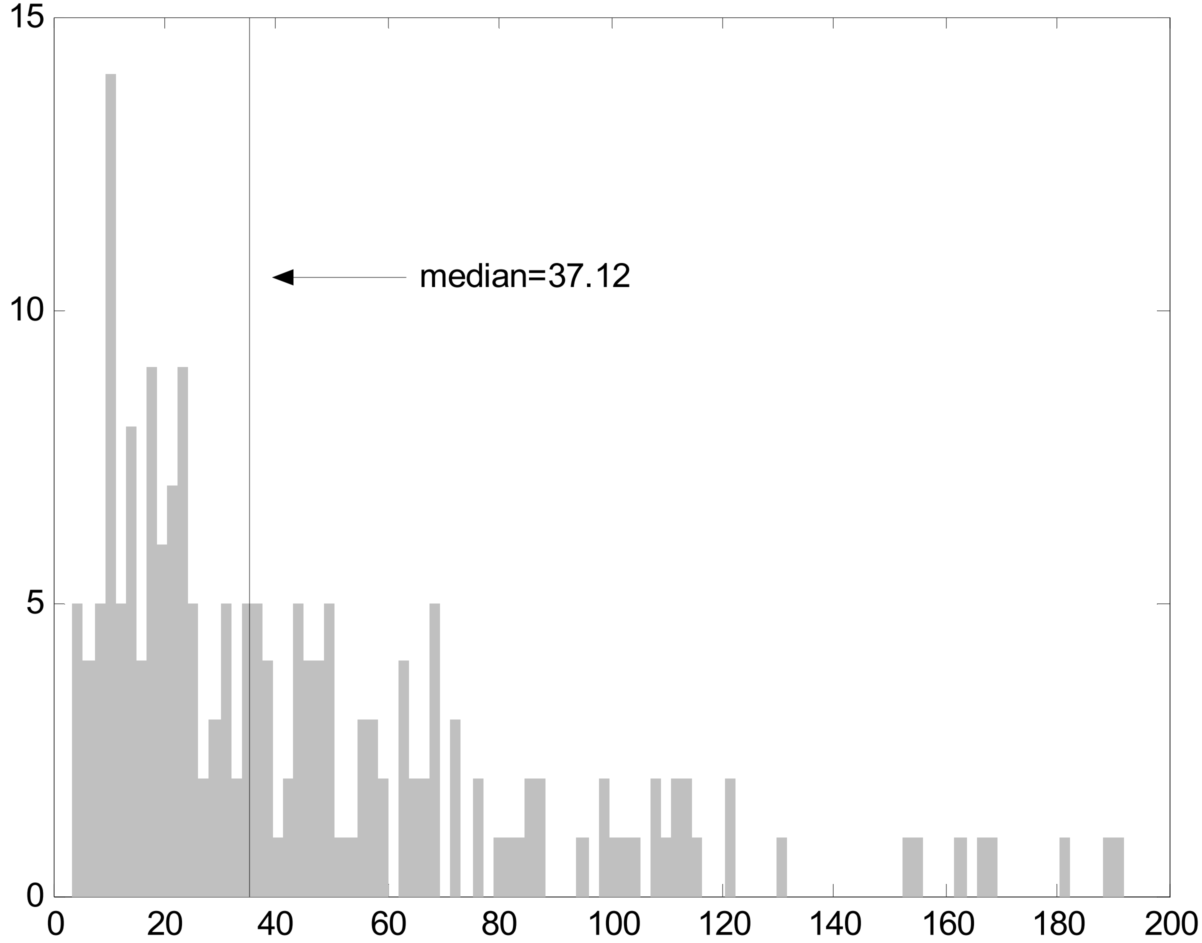
Main findings:

- There is significant sectoral heterogeneity
- Theoretical conditions for heterogeneity bias are satisfied in the dataset
- Individual sectors exhibit long half-lives (even before correcting for small-sample bias)

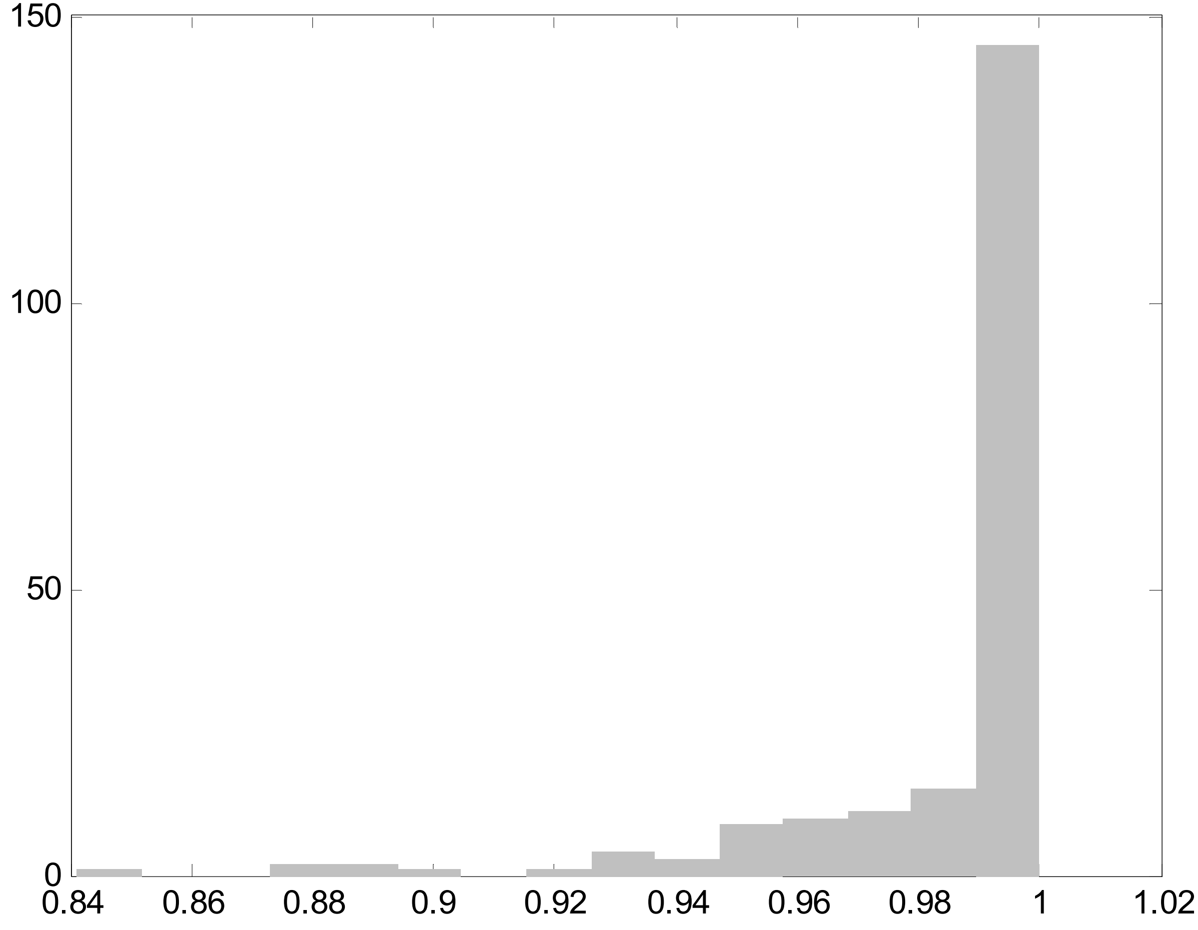
Distribution of ρ for different sectors and countries (without correcting for small sample bias)



Distribution of half-lives for different sectors and countries (without correcting for small sample bias)



Distribution of ρ for different sectors and countries (correcting for small sample bias)



Scatter plot: α (weight) vs. ρ