Economics 326
(Utility, Marginal Utility, MRS, Substitutes and Complements)

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1 Utility

- From last lecture: a utility function $U(x, y)$ is said to represent preferences $\succeq$ if for any bundles x_1 and x_2, the utility function is higher for bundle x_1 relative to x_2 when x_1 is preferred to x_2:

$$U(x_1) \geq U(x_2) \implies x_1 \succeq x_2$$

- What is utility intuitively? It is numerical score representing the preference for (or satisfaction from) a bundle of commodities.

- Suppose I get utility of 5 from consuming an apple and utility of 10 from consuming a mango. Does that mean that I'm twice as happy with a mango as with an apple?

- In class example. Write a utility function representing

$$mango \succ orange \succ apple$$

- Utility is said to have **ORDINAL** properties only: no **CARDINAL** properties.

- What does this mean? It means that the particular number that the utility function takes on is meaningless but the order of the bundles in the utility function does have meaning.

- In other words, if $U(x)$ represents preferences $\succeq$, then $5U(x)$ also represents it. What else represents the same preferences?

  - For any utility function:

    - $[U(x)]^3$

    - $12U(x) - 53$

  - If $U(x)$ is never negative:

    - $[U(x)]^2$

    * $LN\left[U\left(x\right)\right]$

  – What about $-U\left(x\right)$?

- Any positive monotonic transformation of $U\left(x\right)$ preserves the order of preferences:

  – Let f be a positive monotonic transformation. This means that $x_1 > x_2 \implies f\left(x_1\right) > f\left(x_2\right)$ or $\frac{df(x)}{dx} > 0$ for all x

  – Then $G\left(x\right) = f\left(U\left(x\right)\right)$ represents the same preferences as $U\left(x\right)$

  – Which representation is preferred?

    * None

    * Mathematically most convenient

2 Marginal Utility

- The change in utility from a one unit change in consumption of a good or service:
  - One variable: $\frac{dU(x)}{dx}$
  - Two variables: $\frac{\partial U(x,y)}{\partial x}$
- What does marginal utility mean?

3 Graphing Utility: Indifference Curves

- With one good, a utility function isn't that interesting: it can just tell us when more is preferred to less.

- With two goods, the graphing is quite difficult: 3 dimensional graph.

  - $U(x, y)$ - dimensions: U, x, y

- Our usual examples are from the simplest case with tradeoffs between commodities: two goods. In this case, we can graph indifference curves

  - An indifference curve is a set of commodity bundles (x, y) which are all equally preferred i.e. such that $U(x, y) = \bar{U}$. Another way of stating what an indifference curve is: a set of commodity bundles which give the same level of utility to the consumer.

- Indifference curves tell us alot about the relationship between two goods. They tell us how a consumer is willing to trade off one good for another good.

- How do we graph indifference curves?

  1. Choose a utility level: $U(x, y) = \bar{U}$ (a number)

  2. Choose one of the variables (we'll choose x) and vary it.

  3. Solve for the other variable (in this case y) for each x given $\bar{U}$.

  4. Go back to (1.) and choose another utility level $\widehat{U}$

- Three examples:

  1. $U(x, y) = x + y$

$x + y = 10$	x	y
$x + y = 10$	0	10
$x + y = 10$	2	8
$x + y = 10$	4	6
$x + y = 10$	6	4
$x + y = 10$	8	2
$x + y = 10$	10	0

2. $U(x,y) = \min[x,y]$

$\min[x,y] = 10$	x	y
$\min[x,y] = 10$	10	20
$\min[x,y] = 10$	10	15
$\min[x,y] = 10$	10	10
$\min[x,y] = 10$	15	10
$\min[x,y] = 10$	20	10

3. $U(x,y) = xy$

$xy = 10$	x	y
$xy = 10$	10	1
$xy = 10$	5	2
$xy = 10$	4	2.5
$xy = 10$	2.5	4
$xy = 10$	2	5
$xy = 10$	10	1

- Examples of indifference curves:
  - Bread and Butter
  - Car Bodies and Car Motors

  - Mandarin Oranges and Tangerines

  - Cigarettes and Cigarette Smoke

- Two other examples: Visual and Audial

  - DVDs and CDs

  - CDs and DVDs

- Terms: Complements, Substitutes, Economic Bads

4 Marginal Rate of Substitution

- How do we trade off consumption of one good and consumption of another?

  - Substitutes

- Complements

- Suppose we want to know how we trade off one good for another for very small changes in consumption?
  - Rephrased: How much of x will we have to give up in order to keep utility constant if we get one more unit of y?

- Total differentiate $U\left(x, y\right) = \bar{U}$

$$\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy = 0$$

$$\text{solve } for \frac{dx}{dy}$$

$$\frac{\partial U}{\partial x} dx = -\frac{\partial U}{\partial y} dy$$

$$\frac{dx}{dy} = -\frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}}$$

- Marginal Rate of Substitution

$$MRS = \frac{dx}{dy} = -\frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}}$$

- MRS will play a critical role in consumer theory.

- How do we compute the MRS: we use the formula (i.e. we compute marginal utilities).

- What is the MRS for:

1. $U(x, y) = x + y$
2. $U(x, y) = xy$
3. $U(x, y) = \ln x + \ln y$
4. $U(x, y) = x^2y^2$
5. $U(x, y) = x^3y$

6. $U(x, y) = 3 \ln x + \ln y$

- Note: a positive monotonic transformation of a utility function does not change the MRS! Again, if you can take a positive monotonic transformation of your utilty function and it makes the math simpler, do it!