Auxiliary Appendix: Kernel Weighted GMM Estimators for Linear Time Series Models

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This Version: December 2011

Abstract

This Appendix provides additional Lemmas and proofs for the results of the main paper.

* *Georgetown University, Department of Economics, Washington D.C.* This paper is a revised version of a manuscript circulated under the title "Mean Squared Error Reduction For GMM Estimators of Linear Time Series Models". I wish to thank Xiaohong Chen, Ronald Gallant, Jinyong Hahn, Jerry Hausman, Whitney Newey and Ken West for helpful comments. Stavros Panageas and Sungju Chun provided excellent research assistance. The comments of three anonymous referees and a Co-editor have greatly improved the exposition. Financial support from NSF grant SES-0095132 and SES-0523186 is gratefully acknowledged.

1. Introduction

This appendix contains auxiliary lemmas as well as some of the proofs for the main paper. It also contains a complete description of the algorithm to compute the data-dependent moment selection parameter and the kernel weighted GMM estimators. The appendix is organized as follows: Section 1.1 contains variable definitions used throughout this document, Section 1.2 lists the assumptions made in the main paper for ease of reference. Auxiliary Lemmas are provided in Section 2 for subsequent results in this document, in Section 3 for the proof of Lemma A.11 in the main text and in Section 4 for the proof of Theorem 4.1 in the main text. Theoretical calculations of the approximate bias and MSE of the GMM estimator used in the Monte Carlo design are reported in Section 5. Additional Monte Carlo results are contained in Section 6. A detailed description of the algorithm is contained in Section 7.

1.1. Definitions

Throughout the appendix, C is used to denote a generic bounded constant. The exact nature of C may change depending on where C appears. For ease of reference the following definitions are repeated from the main paper.

Definition 1.1. *Let $u_t \in \mathbb{R}^p$ be a strictly stationary vector process with elements u_t^j such that $E\left[u_t^j\right] = 0$ and $E\left[\left(u_t^j\right)^k\right] < \infty$ for some fixed integer $k > 0$. Let $\varsigma = (\varsigma_1, ..., \varsigma_k) \in \mathbb{R}^k$ and $u = (u_{t_1}^{j_1}, ..., u_{t_k}^{j_k})$ then $\phi_{j_1,...j_k,t_1,...,t_k}(\varsigma) = E\left[e^{i\varsigma' u}\right]$ is the joint characteristic function of u. The joint k-th order cumulant is*

$$\operatorname{cum}^*_{j_1,...,j_k}(t_1, ..., t_k) = \frac{\partial^k}{\partial \varsigma_1 \cdots \partial \varsigma_k}|_{\varsigma=0} \ln \phi_{j_1,...j_k,t_1,...,t_k}(\varsigma).$$

Alternatively the notation $\operatorname{cum}^*(u_{t_1}^{j_1}, ..., u_{t_k}^{j_k})$ is used where more convenient. By stationarity it is enough to define

$$\operatorname{cum}_{j_1,...,j_k}(t_1, ..., t_{k-1}) = \operatorname{cum}^*_{j_1,...,j_k}(t_1, ..., t_{k-1}, 0).$$

Definition 1.2. *Let $\mu_x = E[x_t]$. Define $w_{t,i} = (x_{t+m} - \mu_x)\left(y_{t-i+1} - \mu_y\right)'$, $\Gamma_i^{xy} = E[w_{t,i}]$ and $\Gamma_{-i}^{yx} = E\left[w_{t,i}'\right]$ and let $\breve{w}_{t,i} = w_{t,i} - \Gamma_i^{xy}$. Next define $w_{t,j}^y = \left(y_t - \mu_y\right)\left(y_{t-j} - \mu_y\right)'$ with $E\left[w_{t,j}^y\right] = \Gamma_j^{yy}$. Let $\breve{w}_{t,j}^y = w_{t,j}^y - \Gamma_j^{yy}$. Define $v_{t,i} = \varepsilon_{t+m}(y_{t-i+1} - \mu_y)$, $E[\varepsilon_{t+m} y_{s+1}] = \Gamma_{t-s}^{\varepsilon y}$ and $\Gamma_{t-s}^{\varepsilon x} = E[\varepsilon_t x_s]$. Define the infinite dimensional instrument vector $\tilde{z}_{t,\infty} = (y_t', y_{t-1}', ...)'$ and let $P' = \operatorname{Cov}(x_{t+m}, \tilde{z}_{t,\infty})'$. Define the infinite dimensional matrix $\Omega = \sum_{l=-m+1}^{m-1} \gamma_l^\varepsilon \operatorname{Cov}\left[\tilde{z}_{t,\infty}, \tilde{z}_{t+l,\infty}'\right]$ by it's typical j,k-th block $\omega_{j,k}$ where*

$$\omega_{j,k} = \textstyle\sum_{l=-m+1}^{m-1} \gamma^\varepsilon(l)\, \Gamma_{k-j-l}^{yy}.$$

Denote by $\vartheta_{j,k}$ and $\vartheta^M_{j,k}$ the j,k-th block of Ω^{-1} and Ω^{-1}_M. Let $D = P'\Omega^{-1}P$ and $d_0 = P'\Omega^{-1}V$ where $V = n^{-1/2}\sum_{t=1}^{n-m}\varepsilon_{t+m}(z_{t,\infty} - \mathbf{1}_\infty \otimes \mu_y)$. Define the instrument vector $\tilde{z}_{t,M} = (y'_t, y'_{t-1}, ..., y'_{t-M+1})'$. An instrument selection matrix $S_M(t) = \operatorname{diag}(\mathbf{1}\{t \geq 1\}, ..., \mathbf{1}\{t \geq M\})$, where $\mathbf{1}\{.\}$ is the indicator function, is introduced to exclude instruments for which there is no data in the sample. The vector of available instruments is denoted by $z_{t,M} = (S_M(t) \otimes I_p)(\tilde{z}_{t,M} - \mathbf{1}_M \otimes \bar{y})$ where $\bar{y} = n^{-1}\sum_{t=1}^n y_t$, I_p is the p-dimensional identity matrix and $\mathbf{1}_M = (1, ..., 1)'$ is a vector of length M with all elements equal to 1. Let $H_n = [h_{\min}, h_{\max}]$ where $h_{\min} = c\log(\log(n))\log(n)$ and $h_{\max} = \log(n)^2$.

Definition 1.3. *Let $f_\Omega(\lambda) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty}\sum_{l=-m+1}^{m-1}\gamma^\varepsilon_l\Gamma^{yy}_{j-l}e^{-i\lambda j}$ which can be represented as $f_\Omega(\lambda) = 2\pi f_\varepsilon(\lambda)f_y(\lambda)$.*

Definition 1.4. *For a matrix A, $\|A\|^2 = \operatorname{tr} AA'$. The matrix norm $\|A\|_2^2$ is given by*

$$\|A\|_2^2 = \sup_{x \neq 0} x'A'Ax/x'x.$$

The $p^2 \times p^2$ commutation matrix $K_{pp} = \sum_{i,j=1}^p e_ie'_j \otimes e_je'_i$ where $\otimes$ is the Kronecker product and e_i is the i-th unit p-vector; see Magnus and Neudecker (1979).

Let $\hat{\gamma}^\varepsilon(l) = \frac{1}{n}\sum_{t=r+m+1}^n \hat{\varepsilon}_t\hat{\varepsilon}_{t-l}$ with $\hat{\varepsilon}_t = a(L, \tilde{\beta}_{n,M})(y_t - \bar{y})$ for some consistent first stage estimator $\tilde{\beta}_{n,M}$ for $l \geq 0$ and $\hat{\gamma}^\varepsilon(l) = \hat{\gamma}^\varepsilon(-l)$ for $l < 0$. Define[1] $\hat{\Omega}_M(l) = \frac{1}{n}\sum_{t=1}^n z_{t,M}z'_{t-l,M}$ for $l \geq 0$, $\hat{\Omega}_M(l) = \hat{\Omega}_M(-l)'$ for $l < 0$ and

$$\hat{\Omega}^*_M = \sum_{l=-m+1}^{m-1}\hat{\gamma}^\varepsilon(l)\hat{\Omega}_M(l). \tag{1.1}$$

Then,

$$\hat{\Omega}_M = \hat{\Omega}^*_M\mathbf{1}\left\{\min\hat{\xi}_\Omega \geq 0\right\} + \left(1 - \mathbf{1}\left\{\min\hat{\xi}_\Omega \geq 0\right\}\right)\sum_{l=-m+1}^{m-1}\left(1 - \frac{|l|}{m}\right)\hat{\gamma}^\varepsilon(l)\hat{\Omega}_M(l) \tag{1.2}$$

where $\min\hat{\xi}_\Omega$ is the smallest eigenvalue of $\hat{\Omega}^*_M$.

1.2. Assumptions

The following assumptions are repeated from the main paper and are assumed to hold throughout this appendix.

[1] Note that the effective summation limits in $\hat{\Omega}_M$ vary by element because of the definition of $z_{t,M}$ which is padded with zeros for periods with missing observations for a particular lag.

Assumption A. *Let* $\mathcal{K} = \{k(.)|k(.) : \mathbb{R} \to [-1,1], k(0) = 1, k(x) = 0$ *for* $|x| > 1, k(x) = k(-x), k(.)$ *is continuous except at a countable number of points*$\}$. *Define* $k_q = \lim_{x\to 0}(1-k(x))/|x|^q$. *In addition* $k(.) \in \mathcal{K}$ *satisfies one of the following two assumptions:*

A1 (Truncated Kernel). *For all* $q \in [0,\infty)$ *it follows that* $k_q = 0$.

A2 (Smooth Kernel). *There exists a smallest number* $q \in (0,\infty)$ *such that* $0 < k_q < \infty$.

Assumption B. *Let* $u_t \in \mathbb{R}^p$ *be strictly stationary and ergodic, with* $E[u_t] = 0$, $E(u_t u_t' | \tilde{z}_{t-1,M}) = \Sigma$ *for some positive definite and nonrandom matrix* Σ *and* $E(u_t u_s' | \tilde{z}_{t-1,M}) = 0$ *for* $t > s$. *Let* u_t^i *be the* i*-th element of* u_t *and* $\mathrm{cum}_{i_1,...,i_k}(t_1,...,t_{k-1})$ *the* k*-th order cross cumulant of* $u_{t_1}^{i_1},...,u_{t_k}^{i_k}$. *Assume that*

$$\sum_{t_1=-\infty}^{\infty} \cdots \sum_{t_{k-1}=-\infty}^{\infty} |\mathrm{cum}_{i_1,...,i_k}(t_1,...,t_{k-1})| < \infty \text{ for } k \le 12.$$

Assumption C. *The lag polynomial* $B(z)$ *with coefficient matrices* B_j *satisfies* $\det B(z) \neq 0$ *for* $|z| \le 1$. *Define* $B(z)^{-1} = \pi(z) = I - \sum_{j=1}^{\infty} \pi_j z^j$. *Moreover, let* $b(z) = \alpha(z,\beta) B(z)$ *and assume that* $b(z) = \sum_{j=0}^{m-1} b_j z^j$ *with* $b(z) \neq 0$ *for* $|z| \le 1$. *Let* $f_\varepsilon(\lambda) = (2\pi)^{-1} b(e^{i\lambda})' \Sigma b(e^{-i\lambda})$ *and assume that there exists a constant* σ_ε^2 *and lag polynomial* $\theta(z) = 1 - \theta_1 z - ... - \theta_{m-1} z^{m-1}$ *such that* $f_\varepsilon(\lambda)$ *can be represented as* $f_\varepsilon(\lambda) = (2\pi)^{-1} \sigma_\varepsilon^2 \left|\theta(e^{i\lambda})\right|^2$. *Let* $\theta(z)^{-1} = \sum_{j=0}^{\infty} \zeta_j^\theta z^j$. *For some* ν *with* $\nu \in (0,1)$ *and some generic constant* C, *assume that* $\sum_{j=k}^{\infty} \|\pi_j\| \le C\nu^k$, $\sum_{j=k}^{\infty} \zeta_j^\theta \le C\nu^k$ *and* $\sum_{j=k}^{\infty} \|B_j\| \le C\nu^k$ *uniformly in* $k = 1, 2, ...$. *Assume that* P *has full column rank.*

Assumption D. *Assume that*

$$\left\|\Gamma\left(j, \hat{\pi}_{\hat{h}}(z)\right) - \Gamma(j, \pi(z))\right\| = j\nu^j O_p\left(\sup_{|z|\le 1} \left\|\hat{\pi}_{\hat{h}}(z) - \pi(z)\right\|\right) + j^3 \nu_*^j O_p\left(\sup_{|z|\le 1} \left\|\hat{\pi}_{\hat{h}}(z) - \pi(z)\right\|^3\right)$$

uniformly in j *for* $\nu < \nu_* < 1$. *Let* $K_{\hat{h}}^y\left(e^{i\lambda}\right)$ *be the spectral density with Fourier coefficients* $\Gamma\left(j, \hat{\pi}_{\hat{h}}(z)\right)$ *and assume that* $K_{\hat{h}}^y\left(e^{i\lambda}\right) = \tilde{K}_{\hat{h}}^y\left(e^{i\lambda}\right) \tilde{K}_{\hat{h}}^y\left(e^{-i\lambda}\right)'$ *where* $\tilde{K}_{\hat{h}}^y(z)$ *is a matrix valued (infinite order) polynomial in* z.

Remark 1. *Kuersteiner (2005) requires that* $\sum_{j=0}^{\infty} j^2 \|B_j\| < \infty$. *This is implied by Assumption C because* $\sum_{j=0}^{\infty} j^2 \|B_j\| \le \sum_{j=0}^{\infty} j^2 \sum_{k=j}^{\infty} \|B_k\| \le C \sum_{j=0}^{\infty} j^2 v^j < \infty$.

2. Auxiliary Lemmas

Lemma 2.1. *Let* X, Y, W, Z *random matrices with all elements* $x_{ij}, y_{ij}, w_{ij}, z_{ij}$ *such that* $E[x_{ij}] = ... = E[z_{ij}] = 0$ *and* $E\left[|x_{ij}|^4\right] < \infty, ..., E\left[|z_{ij}|^4\right] < \infty$. *Let* A, B, C, D *matrices with fixed coefficient* *such that the matrix products $CWAXY'BZD$ as well as DC are well defined. Then*

$$\begin{aligned} E\left[\operatorname{tr} CWAXY'BZD\right] &= \left(\operatorname{vec} B'\right)' E\left[Y \otimes Z\right]\left(I \otimes DC\right) E\left[X' \otimes W\right] \operatorname{vec} A \\ &\quad + \operatorname{tr}\left(E\left[D'Z' \otimes AX\right] E\left[\operatorname{vec}(Y'B) \operatorname{vec}(W'C')'\right]\right) \\ &\quad + \operatorname{tr}\left[(E\left[AXY'B\right])(E\left[ZDCW\right])\right] + \mathcal{K}_4 \end{aligned}$$

where $\mathcal{K}_4 = \sum_k \sum_{j_1,\dots,j_7} \dots \sum a_{j_2,j_3} b_{j_5,j_6} c_{k,j_1} d_{j_7,k} \operatorname{cum}^*(w_{j_1,j_2}, x_{j_3,j_4}, y_{j_4,j_5}, z_{j_6,j_7})$.

Proof. Note that

$$\begin{aligned} \operatorname{tr} CWAXY'BZD &= \operatorname{tr}\left(\left[Y \otimes Z\right]\left(I \otimes DC\right)\left[X' \otimes W\right]\left(\operatorname{vec} A\right)\left(\operatorname{vec} B'\right)'\right) \\ &= \operatorname{tr}\left(\operatorname{vec}(W'C')'\left(D'Z' \otimes AX\right)\operatorname{vec}(Y'B)\right) \\ &= \operatorname{tr}(AXY'B)(ZDCW). \end{aligned} \tag{2.1}$$

The result then follows from applying

$$\begin{aligned} E\left[w_{i_1 j_1} x_{i_2} y_{i_3} z_{i_4 j_4}\right] &= E\left[w_{i_1 j_1} x_{i_2}\right] E\left[y_{i_3} z_{i_4 j_4}\right] + E\left[x_{i_2} y_{i_3}\right] E\left[w_{i_1 j_1} z_{i_4 j_4}\right] + E\left[x_{i_2} z_{i_4 j_4}\right] E\left[w_{i_1 j_1} y_{i_3}\right] \\ &\quad + \operatorname{cum}^*(w_{i_1 j_1}, x_{i_2}, y_{i_3}, z_{i_4 j_4}). \end{aligned}$$

element by element to the expressions in (2.1). ■

Lemma 2.2. *If $v_{t,i} = \varepsilon_{t+m}(y_{t-i+1} - \mu_y)$ and $w_{t,i} = (x_{t+m} - \mu_x)\left(y_{t-i+1} - \mu_y\right)'$ and $\ell \in \mathbb{R}^d$ is a vector of constants such that $\ell'\ell = 1$ then*
i) $E\left[v_{t,i} \otimes \check{w}'_{s,j}\ell\right] = ((\operatorname{vec}(\Gamma^{yy}_{s-t+i-j}) \otimes \left(\Gamma^{\varepsilon x}_{t-s}\right)') + K_{pp}(\Gamma^{\varepsilon y}_{t-s+j} \otimes \Gamma^{yx}_{t-i-s}) + \mathcal{K}^1_4)(I \otimes \ell)$ where $\mathcal{K}^1_4$ is a $p^2 \times d$ matrix with typical element (a, b) equal to

$$\left[\mathcal{K}^1_4\right]_{a,b} = \operatorname{cum}^*(\varepsilon_{t+m}, y^{[(a-1)/p]+1}_{t-i+1}, y^{a \bmod p-1}_{s-j+1}, x^b_{s+m}),$$

where $[a]$ is the largest integer smaller than a, $a \bmod p-1$ is the remainder on division of a by $p-1$, and K_{pp} is defined in (1.4).
ii) $E\left[v_{t,i}\ell' w_{s,j}\right] = (\ell'\Gamma^{\varepsilon x}_{t-s})\Gamma^{yy}_{t-s+j-i} + (\ell'\Gamma^{xy}_{s-t+i})'\Gamma^{\varepsilon y\prime}_{t-s+j} + \mathcal{K}^2_4$ where $\mathcal{K}^2_4$ is a $p \times p$ matrix with typical element (a, b)

$$\left[\mathcal{K}^2_4\right]_{a,b} = \operatorname{cum}^*(\varepsilon_{t+m}, y^b_{t-i+1}, y^a_{s-j+1}, \ell' x_{s+m}),$$

iii) $E\left[v_{t,i} v'_{s,j}\right] = \gamma^{\varepsilon}_{t-s}\Gamma^{yy}_{t-i+j-s}$, *iv)* $E\left[w_{t,i}w'_{s,j}\right] = \Gamma_i^{xy}\Gamma_{-j}^{yx} + \gamma_{t-i+j-s}^{yy}\Gamma_{t-s}^{xx} + \Gamma_{t+j-s}^{xy}\Gamma_{t-s-i}^{yx} + \mathcal{K}_4^4$ *where* $\mathcal{K}_4^4$ *is a* $p \times p$ *matrix with typical element* (a,b)

$$\left[\mathcal{K}_4^4(t,s,i,j)\right]_{a,b} = \sum_{l=1}^{p} \text{cum}^*(x_{s+m}^a, x_{t+m}^b, y_{t-i+1}^l, y_{s-j+1}^l)$$

and $\gamma_{s-t}^{yy} = E\left[(y_{t-j} - \mu_y)'(y_{s-j} - \mu_y)\right]$ *and* $\gamma_{s-t}^{yy} = \gamma_{t-s}^{yy}$.

Proof. Assume that w, x, y, z are zero mean random variables. Then the results are easily shown by applying $E[wxyz] = E[wx]E[yz] + E[wy]E[xz] + E[wz]E[xy]$ +cumulant to each element of the respective random matrix or vector and expressing the result in matrix notation. ∎

Lemma 2.3. *Assume that Conditions B and C hold. Then, for any integer* $M > 0$ *and some constant* C_y*,*
i) $\sum_{j=0}^{\infty} \left\|\Gamma_{j+M}^{yy}\right\| \leq C_y \nu^M$,
ii) $\sum_{j=0}^{\infty} \left\|\Gamma_{j+M}^{xy}\right\| \leq C_y \nu^M$,
iii) $\left\|\Gamma_j^{yy}\right\| \leq C\nu^j$.

Proof. For i) note that $\Gamma_j^{yy} = \sum_{l=0}^{\infty} B_{l+j}\Sigma B_l'$ such that

$$\textstyle\sum_{j=0}^{\infty} \left\|\Gamma_{j+M}^{yy}\right\| \leq \displaystyle\sum_{l=0}^{\infty} \|B_l\| \|\Sigma\| \sum_{j=0}^{\infty} \|B_{j+l+M}\| \leq C\nu^M \sum_{l=0}^{\infty} \nu^l \|B_l\| \|\Sigma\| \leq C_y \nu^M.$$

Then, ii) follows from i) by noting that x_t is composed of at most a finite number of lagged elements of y_t.

For iii) use

$$\left\|\Gamma_j^{yy}\right\| \leq \sum_{l=0}^{\infty} \|B_l\| \|\Sigma\| \|B_{j+l}\| \leq C\nu^j \sum_{l=0}^{\infty} \nu^l \|B_l\| \|\Sigma\| \leq C\nu^j.$$

∎

Lemma 2.4. *Assume that Conditions B and C hold. Then,*
i) the j,k*-th element of* Ω *is* $\omega_{j,k} = 2\pi \int_{-\pi}^{\pi} f_\varepsilon(\lambda) f_y(\lambda) e^{i\lambda(k-j)} d\lambda = \sum_{l=-m+1}^{m-1} \gamma^\varepsilon(l) \Gamma_{k-j-l}^{yy}$,
ii) for any $k \leq M$ *and* $0 \leq q < \infty$, $\sum_{j=1}^{\infty} |j|^q \|\omega_{j+M,k}\| < \infty$ *uniformly in* k *for* $1 \leq k \leq M$ *and* $M > 0$.

Proof. The statement in i) follows directly from the definitions of $f_\varepsilon(\lambda)$ and $f_y(\lambda)$. Note that $\sum_{j=1}^{\infty} |j|^q \|\omega_{j+M,k}\| \leq \sum_{l=-m+1}^{m-1} |\gamma^\varepsilon(l)| \sum_{j=1}^{\infty} |j|^q \left\|\Gamma_{j+M+l-k}^{yy}\right\| < \infty$ where the last inequality follows from Lemma 2.3iii). ∎

Lemma 2.5. *Let $\theta(L)$ be as defined in Assumption C and $f_\varepsilon(\lambda) = (2\pi)^{-1}\sigma_\varepsilon^2 \left|\theta(e^{i\lambda})\right|^2$. Define $\tilde{\pi}(L) = \theta(L)^{-1}\pi(L)$ where $\tilde{\pi}(L) = \sum_{j=0}^{\infty}\tilde{\pi}_j L^j$. The j,k-th block element of Ω^{-1} is denoted by $\vartheta_{j,k}$ where Ω is defined in Definition 1.2. Then*

i)

$$\vartheta_{j,k} = \sigma_\varepsilon^{-2}\sum_{l=0}^{j-1}\tilde{\pi}_l'\Sigma^{-1}\tilde{\pi}_{l+k-j} = \sigma_\varepsilon^{-2}\sum_{l=0}^{k-1}\tilde{\pi}_{l+j-k}'\Sigma^{-1}\tilde{\pi}_l \tag{2.2}$$

where $\tilde{\pi}_j = 0$ for $j < 0$,

ii) $\|\vartheta_{j+M,k}\| \le CM\nu^M$.

Proof. For i) note that $f_\varepsilon(\lambda)f_y(\lambda) = \left(4\pi^2\right)^{-1}\sigma_\varepsilon^2\left|\theta(e^{-i\lambda})\right|^2\pi(e^{-i\lambda})^{-1}\Sigma\pi(e^{i\lambda})'^{-1}$. Thus Ω is the infinite dimensional matrix of autocovariances of a vector valued infinite order VAR process with coefficients $\sqrt{2\pi}\sigma_\varepsilon\tilde{\pi}_j$. The result then follows from Lewis and Reinsel (1985, p.401).

For ii) note that for $j > k$

$$\begin{aligned} \|\vartheta_{j+M,k}\| &\le C\sum_{l=0}^{k-1}\sum_{s=0}^{l+M+j-k}\left|\zeta_{l+M+j-k-s}^{\theta}\right|\|\pi_s\|\|\tilde{\pi}_l\| \\ &\le C\nu^{M+j-k}(M+j-k)\sum_{l=0}^{k-1}|l|\nu^l\|\tilde{\pi}_l\| = O\left(Mv^M\right) \end{aligned} \tag{2.3}$$

and for $k < j$ use the first term on the RHS in (2.2) in the above argument to obtain the same conclusion. ■

Lemma 2.6. *Let $\vartheta_{j,k}$ be the j,k-th block element of Ω^{-1} for Ω defined in Definition 1.2. Define $\vartheta_j^\infty = (2\pi)^{-2}\int_{-\pi}^{\pi} f_\Omega(\lambda)^{-1}e^{i\lambda j}d\lambda = \sigma_\varepsilon^{-2}\sum_{l=0}^{\infty}\tilde{\pi}_l'\Sigma^{-1}\tilde{\pi}_{l+j}$ for $j \ge 0$, $f_\Omega(\lambda)$ is given in Definition 1.3 and $\vartheta_j^\infty = \vartheta_{-j}^{\infty\prime}$ for $j < 0$. Let $q > 0$ be fixed. Then,*

i) $\vartheta_{j,k} = \vartheta_{k,j}'$,

ii) for $k \ge j$, $\left\|\vartheta_{j,k} - \vartheta_{k-j}^\infty\right\| < C_{B_{j-k}}$ where $C_{B_j} = \left|\sigma_\varepsilon^{-2}\right|\sum_{l=0}^{\infty}\left\|\tilde{\pi}_{l+|j|}\right\|\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_l\|$,

iii) for $k < j$, $\left\|\vartheta_{j,k} - \left(\vartheta_{k-j}^\infty\right)'\right\| \le C_{B_{j-k}}$,

iv) $\sum_{j=1}^{\infty}\left\|\vartheta_{j,k} - \vartheta_{k-j}^\infty\right\| < \infty$ uniformly in k,

v) when $k \ge j$, $\left\|\vartheta_{j+M,k+M} - \vartheta_{k-j}^\infty\right\| = O(M\nu^M)$ as $M \to \infty$, and the same bound holds for

$$\left\|\vartheta_{j+M,k+M} - \left(\vartheta_{k-j}^\infty\right)'\right\|$$

when $j > k$,

vi) $\left\|\vartheta_{[zM],[zM-k]} \to \vartheta_{-k}^\infty\right\| = o(1)$ uniformly in $z \in (0,1]$ and $k > 0$ as $M \to \infty$, where [.] denotes the largest integer smaller than the argument,

vii) $\sum_{j=1}^{\infty}|j|^q\|\vartheta_{j,k}\| < C|k|^q < \infty$ for $0 < q < \infty$ and k fixed, when $q = 0$ then $\sum_{j=1}^{\infty}\|\vartheta_{j,k}\| < C$ uniformly in k,

viii) $\sum_{j=1}^{\infty} \|\vartheta_{j+M,k}\| (M-k)^{-1} \nu^{-(M-k)} < \infty$ *uniformly in* $1 \leq k < M$ *and* $M > 0$,
ix) $\sum_{j_1=1}^{M} \sum_{j_2=1}^{\infty} \Gamma_{j_1}^{xy} \vartheta_{j_1,j_2+M} \Gamma_{-j_2-M}^{yx} = O\left(M^2 \nu^{2M}\right)$,
x) $\sum_{j_1,j_2=1}^{\infty} \Gamma_{j_1+M}^{xy} \vartheta_{j_1+M,j_2+M} \Gamma_{-j_2-M}^{yx} = O(\nu^{2M})$.

Proof. For i) note that from 2.2 it follows that for $k - j \geq 0$

$$\vartheta_{k,j}' = \sigma_\varepsilon^{-2} \left(\sum_{l=0}^{j-1} \tilde{\pi}_{l+k-j}' \Sigma^{-1} \tilde{\pi}_l \right)' = \sigma_\varepsilon^{-2} \sum_{l=0}^{j-1} \tilde{\pi}_l' \Sigma^{-1} \tilde{\pi}_{l+k-j} = \vartheta_{j,k}$$

and for $k - j < 0$

$$\vartheta_{k,j}' = \left(\sigma_\varepsilon^{-2} \sum_{l=0}^{k-1} \tilde{\pi}_l' \Sigma^{-1} \tilde{\pi}_{l+j-k} \right)' = \sigma_\varepsilon^{-2} \sum_{l=0}^{k-1} \tilde{\pi}_{l+j-k}' \Sigma^{-1} \tilde{\pi}_l = \vartheta_{j,k}.$$

For ii) note that for $k - j \geq 0$,

$$\left\| \vartheta_{j,k} - \vartheta_{k-j}^{\infty} \right\| = \left\| \sigma_\varepsilon^{-2} \sum_{l=j}^{\infty} \tilde{\pi}_l' \Sigma^{-1} \tilde{\pi}_{l+k-j} \right\| \leq \left| \sigma_\varepsilon^{-2} \right| \sum_{l=0}^{\infty} \|\tilde{\pi}_l\| \left\| \Sigma^{-1} \right\| \|\tilde{\pi}_{l+k-j}\|$$

and similarly for iii) where $k - j < 0$,

$$\left\| \vartheta_{j,k} - \left(\vartheta_{j-k}^{\infty} \right)' \right\| = \left\| \sigma_\varepsilon^{-2} \sum_{l=k}^{\infty} \tilde{\pi}_{l+j-k}' \Sigma^{-1} \tilde{\pi}_l \right\| \leq \left| \sigma_\varepsilon^{-2} \right| \sum_{l=0}^{\infty} \|\tilde{\pi}_{l+j-k}\| \left\| \Sigma^{-1} \right\| \|\tilde{\pi}_l\|.$$

Summability in iv) follows immediately from the properties of C_{B_k}.

To establish v) note that for $k \geq j$,

$$\left\| \vartheta_{j+M,k+M} - \vartheta_{k-j}^{\infty} \right\| = \sigma_\varepsilon^{-2} \left\| \sum_{l=j+M}^{\infty} \tilde{\pi}_l' \Sigma^{-1} \tilde{\pi}_{l+k-j} \right\| \leq \sigma_\varepsilon^{-2} \sum_{l=j}^{\infty} \|\tilde{\pi}_{l+M}\| \left\| \Sigma^{-1} \right\| \|\tilde{\pi}_{l+M+k-j}\| = O(M\nu^M)$$

where $\tilde{\pi}_k = \sum_{j=0}^{k} \zeta_{k-j}^{\theta} \pi_j$ such that

$$\|\tilde{\pi}_{l+M}\| \leq \sum_{j=0}^{l+M} \left| \zeta_{l+M-j}^{\theta} \right| \|\pi_j\| \leq (M+l) \nu^{M+l} C \leq CM \left(1 + \frac{l}{M} \right) \nu^{M+l} \leq CM\nu^M (1+l) \nu^l \tag{2.4}$$

for a bounded constant C by Assumption C. The result follows by noting that $(1+l)\nu^l$ is bounded uniformly in l. Using iii) the case for $k < j$ can be handled in the same way.

For vi) set $j = 0$ in v) and let $[zM] = M^*$. Then $[zM - k] = M^* - k$, $M^* \to \infty$ as $M \to \infty$ and

$$\left\| \vartheta_{M^*,M^*-k} - \vartheta_{-k}^{\infty} \right\| = O(M^* \nu^{M^*}) = o(1).$$

For vii) use (2.3) to bound

$$
\begin{aligned}
(2.5) \qquad \sum_{j=1}^{\infty}|j|^q\|\vartheta_{j,k}\| &\leq C\sigma_\varepsilon^{-2}\sum_{j=1}^{k}|j|^q\textstyle\sum_{l=0}^{j-1}\left\|\tilde{\pi}_l'\right\|\sum_{s=0}^{l+k-j}\left|\zeta_{l+k-j-s}^{\theta}\right|\|\pi_s\| \\
&\quad +C\sum_{j=k+1}^{\infty}|j|^q\textstyle\sum_{l=0}^{k-1}\sum_{s=0}^{l+j-k}\left|\zeta_{l+j-k-s}^{\theta}\right|\|\pi_s\|\|\tilde{\pi}_l\| \\
&\leq C\sum_{j=1}^{k}|j|^q\textstyle\sum_{l=0}^{k-1}|l+k-j|\,\nu^{l+k-j}\|\tilde{\pi}_l\| \\
&\quad +C\sum_{j=k+1}^{\infty}|j|^q\textstyle\sum_{l=0}^{k-1}|l+j-k|\,\nu^{l+j-k}\|\tilde{\pi}_l\| \\
&\leq C\sum_{j=1}^{k}|j|^q|k-j+1|\,\nu^{k-j}+C\sum_{j=k+1}^{\infty}|j|^q|j-k|\,\nu^{j-k} \\
&\leq C|k|^q\left(\sum_{l=0}^{k-1}\left|1-\frac{l}{k}\right|^q|l+1|\,\nu^l+\sum_{l=1}^{\infty}|l+1|\left|\frac{l}{k}+1\right|^q\nu^l\right) \\
&\leq C|k|^q
\end{aligned}
$$

where $|1-l/k|\leq 1$ for $l\leq k$ and $|l/k+1|^q\leq|l+1|^q$ for $q>0$ was used. For $q=0$ it follows that $\sum_{j=1}^{k}|k-j|\,\nu^{k-j}<\infty$ and $\sum_{j=k+1}^{\infty}|j-k|\,\nu^{j-k}<\infty$.

For viii) note that by (2.2) and (2.4) it follows that

$$
\begin{aligned}
\textstyle\sum_{j=1}^{\infty}\|\vartheta_{j+M,k}\|(M-k)^{-1}\nu^{-(M-k)} &\leq \sigma_\varepsilon^{-2}\textstyle\sum_{l=0}^{k-1}\sum_{j=1}^{\infty}\left\|\tilde{\pi}_{l+j+M-k}'\right\|(M-k)^{-1}\nu^{-(M-k)}\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_l\| \\
&\leq \sigma_\varepsilon^{-2}\textstyle\sum_{l=0}^{\infty}\sum_{j=1}^{\infty}\dfrac{(M+l+j-k)}{(M-k)}\nu^{l+j}C\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_l\| \\
&\leq \sigma_\varepsilon^{-2}\textstyle\sum_{l=0}^{\infty}\sum_{j=1}^{\infty}(1+l+j)\,\nu^{l+j}C\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_l\|<\infty
\end{aligned}
$$

where the bound is uniform because the expression after the third inequality does not depend on M and k.

For ix) note that by Lemmas 2.3 and 2.6vii) it follows that

$$\textstyle\sum_{j_1=1}^{M}\sum_{j_2=1}^{\infty}\left\|\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2+M}\Gamma_{-j_2-M}^{yx}\right\|\leq\sum_{j_1=1}^{M}\sum_{j_2=1}^{\infty}\left\|\Gamma_{j_1}^{xy}\right\|\|\vartheta_{j_1,j_2+M}\|\left\|\Gamma_{-j_2-M}^{yx}\right\|$$

where by Lemma 2.6viii),

$$\textstyle\sum_{j_1=1}^{M-1}\left\|\Gamma_{j_1}^{xy}\right\|\sum_{j_2=1}^{\infty}\|\vartheta_{j_1,j_2+M}\|\left\|\Gamma_{-j_2-M}^{yx}\right\|\leq C\nu^M\sum_{j_1=1}^{M}\left\|\Gamma_{j_1}^{xy}\right\|M\nu^{M-j_1}=O\left(M^2\nu^{2M}\right).$$

For x) note that in the same way as in ix),

$$\left\|\textstyle\sum_{j_1,j_2=1}^{\infty}\Gamma_{j_1+M}^{xy}\vartheta_{j_1+M,j_2+M}\Gamma_{-j_2-M}^{yx}\right\|\leq C\nu^{2M}\sup_{j_1,j_2}\|\vartheta_{j_1,j_2}\|\textstyle\sum_{j_1,j_2=1}^{\infty}\nu^{j_1+j_2}=O\left(\nu^{2M}\right).$$

■

Lemma 2.7. *For finite dimensional matrices Ω_M it follows that the j,k-th block element $\vartheta^M_{j,k}$ of Ω_M^{-1} is*

$$\vartheta^M_{j,k} = \sigma_\varepsilon^{-2} \sum_{l=0}^{j-1} \tilde{\pi}'_{l,l+M-j} \tilde{\Sigma}^{-1}_{l+M-j} \tilde{\pi}_{l+k-j,l+M-j}$$

where the coefficients $\tilde{\pi}_{l,j}$ are defined in the proof below. Moreover,
i) $\left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| = O\left((M-j)\,\nu^{M-j}\right)$ *uniformly in* $j, k < M$,
ii) $\left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| = O\left((M-k)\,\nu^{M-k}\right)$ *uniformly in* $k, j < M$,
iii) $\left\|\vartheta^M_{[zM],[zM]-k} - \vartheta_{[zM],[zM]-k}\right\| = o(1)$ *uniformly in* $z \in (0,1)$ *and* $k > 0$ *as* $M \to \infty$, *where* $[.]$ *denotes the largest integer smaller than the argument,*
iv) $\sum_{j=1}^M \left\|\vartheta^M_{j,k}\right\| \leq C < \infty$ *uniformly in* M *and* $k \leq M$,
v) $\sum_{j=1}^M |j|^q \left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| \leq C\,|k|^q$ *for* $\infty > q > 0$ *and* $k \leq M$, *and when* $q = 0$, $\sum_{j=1}^M |j|^q \left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| \leq C$ *uniformly in* k.

Proof. Let $\tilde{y}_t = \tilde{\pi}(L)^{-1} u_t$ for $\tilde{\pi}(L)$ defined in Lemma 2.5 and define

$$\tilde{\pi}^h_h = (\tilde{\pi}_{1,h}, ..., \tilde{\pi}_{h,h}) = \tilde{\Gamma}'_{1,h} \tilde{\Gamma}_h^{-1} \tag{2.6}$$

with $\tilde{\Gamma}_h$ the $hp \times hp$ matrix whose (m,n)th block is $\tilde{\Gamma}^{yy}_{n-m}$, $\tilde{\Gamma}'_{1,h} = \left[\tilde{\Gamma}^{yy}_1, ..., \tilde{\Gamma}^{yy}_h\right]$ and $\tilde{\Gamma}^{yy}_{j-i} = \mathrm{Cov}(\tilde{y}_{t-i}, \tilde{y}_{t-j})$. Let $\tilde{\pi}_\infty(h) = (\tilde{\pi}'_1, ..., \tilde{\pi}'_h)'$ and $\tilde{\pi}_h(k) = \left(\tilde{\pi}'_{1,h}, ..., \tilde{\pi}'_{k,h}\right)'$ for $k \leq h$ and set $\tilde{Y}_{t,h} = \left(\tilde{y}'_{t-1}, ..., \tilde{y}'_{t-h}\right)'$. Define the approximation $\tilde{y}_{t,h}$ of $\tilde{y}_t$ such that

$$\tilde{y}_{t,h} = \tilde{\pi}_h(h)' \tilde{Y}_{t,h}$$

and let $\tilde{\Sigma}_h = E\left[(\tilde{y}_t - \tilde{y}_{t,h})(\tilde{y}_t - \tilde{y}_{t,h})'\right]$. By noting that $\omega_{i,j} = \tilde{\Gamma}^{yy}_{j-i}$ where $\omega_{i,j}$ is the i,j-th block of Ω_M the representation for $\vartheta^M_{j,k}$ now follows from Lewis and Reinsel (1985, p.402).

For the second part, note that from Kuersteiner (2005, p.99) it follows that

$$\begin{aligned} \left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| \leq\; & \textstyle\sum_{i=0}^{j-1} \left\|\tilde{\pi}_{i,i+M-j} - \tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-j}\right\| \left\|\tilde{\pi}_{i+k-j,i+M-j}\right\| \\ & + \textstyle\sum_{i=0}^{j-1} \left\|\tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-j}\right\| \left\|\tilde{\pi}_{i+k-j,i+M-j} - \tilde{\pi}_{i+k-j}\right\| \\ & + \textstyle\sum_{i=0}^{j-1} \left\|\tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-j} - \Sigma^{-1}\right\| \left\|\tilde{\pi}_{i+k-j}\right\|. \end{aligned} \tag{2.7}$$

Similarly, one obtains, because $\vartheta^M_{j,k} = \left(\vartheta^M_{k,j}\right)'$ by symmetry, that

$$\begin{aligned} \left\|\vartheta^M_{j,k} - \vartheta_{j,k}\right\| \leq\; & \textstyle\sum_{i=0}^{k-1} \left\|\tilde{\pi}_{i,i+M-k} - \tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-k}\right\| \left\|\tilde{\pi}_{i+j-k,i+M-k}\right\| \\ & + \textstyle\sum_{i=0}^{k-1} \left\|\tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-k}\right\| \left\|\tilde{\pi}_{i+j-k,i+M-k} - \tilde{\pi}_{i+j-k}\right\| \\ & + \textstyle\sum_{i=0}^{k-1} \left\|\tilde{\pi}_i\right\| \left\|\Sigma^{-1}_{i+M-k} - \Sigma^{-1}\right\| \left\|\tilde{\pi}_{i+j-k}\right\|. \end{aligned} \tag{2.8}$$

By Hannan and Deistler (1988, Theorem 6.6.12) and (2.4)

$$(2.9)\qquad \begin{aligned}\sum_{s=1}^{i+M-j}\|\tilde{\pi}_{s,i+M-j}-\tilde{\pi}_s\| &\leq C\sum_{s=i+M-j}^{\infty}\|\tilde{\pi}_s\|\leq C\sum_{s=0}^{\infty}(s+i+M-j)\,\nu^{s+i+M-j}\\ &\leq C\,(i+M-j)\,\nu^{i+M-j}\sum_{s=0}^{\infty}(s+1)\,\nu^{s}\\ &\leq C\,(i+M-j)\,\nu^{i+M-j}.\end{aligned}$$

Also, for $u_{t,M}=\tilde{y}_t-\tilde{\pi}_M(M)'\,\tilde{Y}_{t-1,M}$,

$$(2.10)\qquad \Sigma_M-\Sigma=E\left[(u_{t,M}-u_t)\,u_{t,M}'\right]+E\left[u_{t,M}\,(u_{t,M}-u_t)'\right]-E\left[(u_{t,M}-u_t)\,(u_{t,M}-u_t)'\right]$$

where

$$(2.11)\qquad u_{t,M}-u_t=(\tilde{\pi}_M(M)-\tilde{\pi}_\infty(M))'\,\tilde{Y}_{t-1,M}-\sum_{j=M+1}^{\infty}\tilde{\pi}_j\tilde{y}_{t-j}.$$

Then

$$(2.12)\qquad \begin{aligned}&E\left\|(\tilde{\pi}_M(M)-\tilde{\pi}_\infty(M))'\,\tilde{Y}_{t-1,M}\right\|^2\\ =\;&\operatorname{tr}\left[(\tilde{\pi}_M(M)-\tilde{\pi}_\infty(M))'\,E\left[\tilde{Y}_{t-1,M}\tilde{Y}_{t-1,M}'\right](\tilde{\pi}_M(M)-\tilde{\pi}_\infty(M))\right]\\ =\;&\textstyle\sum_{j_1,j_2=1}^{M}\operatorname{tr}\left[\Gamma_{j_2-j_1}^{\tilde{y}\tilde{y}}\,(\tilde{\pi}_{j_2,M}-\tilde{\pi}_{j_2})\,(\tilde{\pi}_{j_1,M}-\tilde{\pi}_{j_1})'\right]\\ \leq\;&\sup_j\left\|\Gamma_j^{\tilde{y}\tilde{y}}\right\|\left(\textstyle\sum_{j=1}^{M}\|\tilde{\pi}_{j,M}-\tilde{\pi}_j\|\right)^2\\ \leq\;&C\textstyle\sum_{j=M+1}^{\infty}\|\tilde{\pi}_j\|\\ =\;&O\left(M^2\nu^{2M}\right)\end{aligned}$$

where the first inequality uses Magnus and Neudecker (1988, Theorem 2, p. 201) and the last inequality is based on $\sup_j\left\|\Gamma_j^{\tilde{y}\tilde{y}}\right\|=O(1)$ and $\sum_{j=1}^{M}\|\tilde{\pi}_{j,M}-\tilde{\pi}_j\|=O\left(M\nu^M\right)$ from Hannan and Deistler (1988, Theorem 6.6.12). Further,

$$(2.13)\qquad E\left\|\sum_{j=M+1}^{\infty}\tilde{\pi}_j\tilde{y}_{t-j}\right\|^2\leq\sup_j\left\|\Gamma_j^{\tilde{y}\tilde{y}}\right\|\left(\sum_{j=M+1}^{\infty}\|\tilde{\pi}_j\|\right)^2=O\left(M^2\nu^{2M}\right).$$

From (2.10) and (2.11) it follows that

$$(2.14)\qquad \begin{aligned} \|\Sigma_M - \Sigma\| &\leq 2\left(E\left\|(\tilde{\pi}_M(M) - \tilde{\pi}_\infty(M))'\tilde{Y}_{t-1,M}\right\|^2 E\left\|u'_{t,M}\right\|^2\right)^{1/2} \\ &\quad +2\left(E\left\|\sum_{j=M+1}^{\infty}\tilde{\pi}_j\tilde{y}_{t-j}\right\|^2 E\left\|u'_{t,M}\right\|^2\right)^{1/2} \\ &\quad +E\left\|(\tilde{\pi}_M(M) - \tilde{\pi}_\infty(M))'\tilde{Y}_{t-1,M}\right\|^2 \\ &\quad +2\left(E\left\|(\tilde{\pi}_M(M) - \tilde{\pi}_\infty(M))'\tilde{Y}_{t-1,M}\right\|^2 E\left\|\sum_{j=M+1}^{\infty}\tilde{\pi}_j\tilde{y}_{t-j}\right\|^2\right)^{1/2} \\ &\quad +E\left\|\sum_{j=M+1}^{\infty}\tilde{\pi}_j\tilde{y}_{t-j}\right\|^2 \\ &= O(M\nu^M) \end{aligned}$$

by (2.12) and (2.13). Substituting for the bounds in (2.9) and (2.14) and using (2.4) in (2.7) one obtains

$$(2.15)\qquad \begin{aligned} \left\|\vartheta_{j,k}^M - \vartheta_{j,k}\right\| &\leq C\sum_{i=0}^{j-1}\left\|\Sigma_{i+M-j}^{-1}\right\|\left\|\tilde{\pi}_{i+k-j,i+M-j}\right\|(i+M-j)\,\nu^{i+M-j} \\ &\quad +\sum_{i=0}^{j-1}\|\tilde{\pi}_i\|\left\|\Sigma_{v,i+M-j}^{-1}\right\|\sum_{s=i+M-j}^{\infty}\|\tilde{\pi}_s\| \\ &\quad +C\sum_{i=0}^{j-1} i\nu^i\left\|\Sigma_{i+M-j}^{-1} - \Sigma^{-1}\right\|\|\tilde{\pi}_i\| \\ &= O\left((M-j)\,\nu^{M-j}\right) \end{aligned}$$

by the properties of $\tilde{\pi}_j$, the fact that $\left\|\Sigma_M^{-1}\right\| \leq \left\|\Sigma^{-1}\right\|$ uniformly in M and $\sum_{i=1}^M |i|\,\|\tilde{\pi}_{i,M}\| \leq \sum_{i=1}^\infty |i|\,\|\tilde{\pi}_i\| + \sum_{i=1}^M |i|\,\|\tilde{\pi}_{i,M} - \tilde{\pi}_i\| \leq \sum_{i=1}^\infty |i|\,\|\tilde{\pi}_i\| + \sum_{i=M+1}^\infty |i|\,\|\tilde{\pi}_i\|$ is uniformly bounded in M by Hannan and Deistler (1988, Theorem 6.6.12).

For (ii) it follows by symmetry that $\vartheta_{k,j}^M = \vartheta_{k,j}^{M\prime}$ and $\vartheta_{k,j} = \vartheta'_{j,k}$ such that by analogy, one obtains $\left\|\vartheta_{j,k}^M - \vartheta_{j,k}\right\| = O\left((M-k)\,\nu^{M-k}\right).$

For (iii) note that

$$(2.16)\qquad \begin{aligned} &\left\|\vartheta_{[zM],[zM]-k}^M - \vartheta_{[zM],[zM]-k}\right\| \\ \leq\ &\sum_{i=0}^{[zM]-1}\left\|\tilde{\pi}_{i,i+M-[zM]} - \tilde{\pi}_i\right\|\left\|\Sigma_{i+M-[zM]}^{-1}\right\|\left\|\tilde{\pi}_{i-k,i+M-[zM]}\right\| \\ &+\sum_{i=0}^{[zM]-1}\|\tilde{\pi}_i\|\left\|\Sigma_{i+M-[zM]}^{-1}\right\|\left\|\tilde{\pi}_{i-k,i+M-[zM]} - \tilde{\pi}_{i-k}\right\| \\ &+\sum_{i=0}^{[zM]-1}\|\tilde{\pi}_i\|\left\|\Sigma_{i+M-[zM]}^{-1} - \Sigma^{-1}\right\|\|\tilde{\pi}_{i-k}\|. \end{aligned}$$

Because $M - [zM] \geq M(1-z)$ and $M(1-z) \to \infty$ for $z < 1$,it follows that $\left\|\Sigma_{i+M-[zM]}^{-1} - \Sigma^{-1}\right\| = O\left(M\nu^M\right) \to 0$. Then,

$$(2.17)\qquad \sum_{i=0}^{[zM]-1}\|\tilde{\pi}_i\|\left\|\Sigma_{i+M-[zM]}^{-1} - \Sigma^{-1}\right\|\|\tilde{\pi}_{i-k}\| \leq O\left(M\nu^M\right)\sum_{i=0}^{[zM]-1}\|\tilde{\pi}_i\|\,\|\tilde{\pi}_{i-k}\| = O\left(M\nu^M\right).$$

In the same way, it follows from Hannan and Deistler (1988, Theorem 6.6.12) that

$$\left\|\tilde{\pi}_{i-k,i+M-[zM]} - \tilde{\pi}_{i-k}\right\| \leq \sum_{s=1}^{i+M-[zM]} \left\|\tilde{\pi}_{s,i+M-[zM]} - \tilde{\pi}_s\right\| \leq \sum_{s=M-[zM]}^{\infty} \|\tilde{\pi}_s\| \leq O\left(M\nu^M\right). \tag{2.18}$$

Substitution of (2.17) and (2.18) in (2.16) then establishes (iii).

For iv) use (2.12), (2.13) and (2.15) to obtain

$$\begin{aligned}
\sum_{j=1}^{M} \left\|\vartheta_{j,k}^M\right\| &\leq \sum_{j=1}^{M} \|\vartheta_{j,k}\| + \sum_{j=1}^{M} \left\|\vartheta_{j,k}^M - \vartheta_{j,k}\right\| \\
&\leq \sum_{j=1}^{\infty} \|\vartheta_{j,k}\| + C\sum_{j=1}^{M} \textstyle\sum_{i=0}^{j-1} (i+M-j)\,\nu^{i+M-j} \\
&\quad + C\sum_{j=1}^{M} \textstyle\sum_{i=0}^{j-1} i\nu^i\,(i+M-j)\,\nu^{i+M-j} + C\sum_{j=1}^{M} \textstyle\sum_{i=0}^{j-1} i^2\nu^{2i}\,(i+M-j)\,\nu^{i+M-j}
\end{aligned}$$

where $\sum_{j=1}^{\infty} \|\vartheta_{j,k}\| < C$ by Lemma 2.6vii),

$$\begin{aligned}
\sum_{j=1}^{M} \textstyle\sum_{i=0}^{j-1} (i+M-j)\,\nu^{i+M-j} &\leq \sum_{j=1}^{M} |M-j|\,\nu^{M-j} \textstyle\sum_{i=0}^{j-1} (i+1)\,\nu^i \\
&\leq C\sum_{j=0}^{M-1} |j|^q\,\nu^j \leq C
\end{aligned}$$

where $i+M-j \leq (i/(M-j)+1)(M-j) \leq (i+1)(M-j)$ for $0 \leq i \leq j-1$ and $0 \leq j \leq M$ was used. Similar bounds can be obtained for the other terms.

For v) use the fact that for $j \leq k$ one has

$$\begin{aligned}
|j|^q\,\|\tilde{\pi}_{i+k-j,i+M-j} - \tilde{\pi}_{i+k-j}\| &\leq |k|^q\,|i+k-j|^q\,\|\tilde{\pi}_{i+k-j,i+M-j} - \tilde{\pi}_{i+k-j}\| \\
&\leq |k|^q \sum_{l=0}^{i+M-j} |l|^q\,\|\tilde{\pi}_{l,i+M-j} - \tilde{\pi}_l\| \\
&\leq |k|^q \sum_{l=i+M-j}^{\infty} |l|^q\,\|\tilde{\pi}_l\| \\
&= |k|^q\,|i+M-j|^q\,\nu^{i+M-j}
\end{aligned}$$

such that, using the convention that $\tilde{\pi}_i = 0$ for $i < 0$,

$$
\begin{aligned}
(2.19)\qquad & \sum_{j=1}^{M} |j|^q \left\|\vartheta_{j,k}^M - \vartheta_{j,k}\right\| \\
\leq\ & \textstyle\sum_{j=1}^M |j|^q \sum_{i=0}^{j-1} \|\tilde{\pi}_{i,i+M-j} - \tilde{\pi}_i\| \left\|\Sigma_{i+M-j}^{-1}\tilde{\pi}_{i+k-j,i+M-j} - \Sigma^{-1}\tilde{\pi}_{i+k-j}\right\| \\
& + \textstyle\sum_{j=1}^M |j|^q \sum_{i=0}^{j-1} \|\tilde{\pi}_{i,i+M-j} - \tilde{\pi}_i\| \left\|\Sigma^{-1}\tilde{\pi}_{i+k-j}\right\| \\
& + \textstyle\sum_{j=1}^M |j|^q \sum_{i=0}^{j-1} \|\tilde{\pi}_i\| \left\|\Sigma_{i+M-j}^{-1}\tilde{\pi}_{i+k-j,i+M-j} - \Sigma^{-1}\tilde{\pi}_{i+k-j}\right\| \\
\leq\ & C\textstyle\sum_{j=1}^M |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^{2(i+M-j)} |i+M-j|^q \\
& + C\textstyle\sum_{j=1}^M |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^{(i+M-j)} |i+M-j|^q \|\tilde{\pi}_{i+k-j}\| \\
& + \textstyle\sum_{j=1}^M |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^{i} |i+M-j|^q \nu^{i+M-j}.
\end{aligned}
$$

Now analyize the first term after the second ineqaulity in (2.19)

$$
\begin{aligned}
(2.20)\qquad & \textstyle\sum_{j=1}^M |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^{2(i+M-j)} |i+M-j|^q \\
\leq\ & \textstyle\sum_{j=1}^M |j|^q \nu^{2(M-j)} |M-j|^q \sum_{i=\max(j-k,0)}^{j-1} |i+1|^q \nu^{2i} \\
\leq\ & |k|^q \textstyle\sum_{j=1}^k \nu^{2(M-j)} |M-j|^q \sum_{i=0}^{j-1} |i+1|^q \nu^{2i} \\
& + \textstyle\sum_{j=k+1}^M |j|^q \nu^{2(M-j)} |M-j|^q \sum_{i=j-k}^{j-1} |i+1|^q \nu^{2i} \\
\leq\ & C|k|^q \textstyle\sum_{j=1}^M \nu^{2(M-j)} |M-j|^q \\
& + \textstyle\sum_{j=k+1}^M |j|^q \nu^{2(M-j)} |M-j|^q |j-k+1|^q \nu^{2(j-k)} \\
\leq\ & C|k|^q
\end{aligned}
$$

where the second inequality follows from

$$
\begin{aligned}
\textstyle\sum_{i=j-k}^{j-1} |i+1|^q \nu^{2i} \ \leq\ & \textstyle\sum_{i=0}^{j-1} |i+j-k+1|^q \nu^{2(i+j-k)} \\
\leq\ & |j-k+1|^q \nu^{2(j-k)} \textstyle\sum_{i=0}^{j-1} |i+1|^q \nu^{i} \leq C|j-k+1|^q \nu^{2(j-k)}
\end{aligned}
$$

and the last ineqaulity in (2.20) follows because $|j|^q = |j-k+k|^q \leq |k|^q |j-k+1|^q$ such that $|j|^q |j-k+1|^q \nu^{2(j-k)} \leq |k|^q |j-k+1|^{2q} \nu^{2(j-k)} \leq C|k|^q$. The second term in (2.19) is bounded by

$$
\begin{aligned}
(2.21)\qquad & \textstyle\sum_{j=1}^M |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^{(i+M-j)} |i+M-j|^q \|\tilde{\pi}_{i+k-j}\| \\
\leq\ & |k|^q \textstyle\sum_{j=1}^k \nu^{M-j} |M-j|^q \sum_{i=0}^{j-1} \nu^i |i+1|^q \|\tilde{\pi}_{i+k-j}\| \\
& + \textstyle\sum_{j=k+1}^M |j|^q |M-j|^q \nu^{M-j} \sum_{i=j-k}^{j-1} \nu^i |i+1|^q \|\tilde{\pi}_{i+k-j}\| \\
\leq\ & C|k|^q + C\textstyle\sum_{j=k+1}^M |j|^q |M-j|^q \nu^{M-j} \sum_{i=j-k}^{j-1} (i+k-j)\, \nu^{i+k-j}\nu^i |i+1|^q \\
\leq\ & C|k|^q + C\textstyle\sum_{j=k+1}^M |j|^q |j-k+1|^q \nu^{j-k} \\
\leq\ & C|k|^q + C|k|^q \textstyle\sum_{j=k+1}^M |j-k+1|^{2q} \nu^{j-k} \\
\leq\ & C|k|^q.
\end{aligned}
$$

by the same arguments as for (2.20). Finally, the last term after the second inequality in (2.19) is bounded by

$$
\begin{aligned}
(2.22) \qquad & \textstyle\sum_{j=1}^{M} |j|^q \sum_{i=\max(j-k,0)}^{j-1} \nu^i \, |i+M-j|^q \, \nu^{i+M-j} \\
\leq \ & |k|^q \textstyle\sum_{j=1}^{k} |M-j| \, \nu^{M-j} \sum_{i=0}^{j-1} |i+1|^q \, \nu^{2i} \\
& + \textstyle\sum_{j=k+1}^{M} |M-j| \, \nu^{M-j} \, |j|^q \sum_{i=j-k}^{j-1} |i+1|^q \, \nu^{2i} \\
\leq \ & C \, |k|^q + \textstyle\sum_{j=k+1}^{M} |M-j| \, \nu^{M-j} \, |k|^q \, |j-k+1|^{2q} \, \nu^{2(j-k)} \\
\leq \ & C \, |k|^q \, .
\end{aligned}
$$

Substituting (2.20), (2.21) and (2.22) in (2.19) then establishes the result. The proof for $q = 0$ follows from the same inequalities as above. ■

Lemma 2.8. *Let* $\check{\Gamma}_j^{xy} = n^{-1} \sum_{t=\max(r-m,j)+1}^{n-m} w_{t,j}$, *for* $|l| < m$, $\hat{\omega}_{j_1,j_2}(l) = \frac{1}{n} \sum_{t=r_1+1}^{r_2} (y_{t-j_1} - \bar{y})(y_{t-l-j_2} - \bar{y})'$, $\check{\omega}_{j_1,j_2}(l) = \frac{1}{n} \sum_{t=r_1+1}^{r_2} \left(y_{t-j_1} - \mu_y\right)\left(y_{t-l-j_2} - \mu_y\right)'$ *where* $r_1 = \max(j_1, j_2 + l)$ *and* $r_2 = n - |\min(0, l)|$ *and*

$$
(2.23) \qquad \check{P}_M = \left[\check{\Gamma}_1^{xy}, ..., \check{\Gamma}_M^{xy}\right]'
$$

Then
i) $E \left\| \hat{\Gamma}_j^{xy} - \check{\Gamma}_j^{xy} \right\|^2 = O(n^{-2})$ *uniformly in* $j \leq n$,
ii) $E \left\| \hat{\Gamma}_j^{xy} - \check{\Gamma}_j^{xy} \right\| = O(n^{-1})$ *uniformly in* $j \leq n$,
iii) $\left\| \hat{P}_M - \check{P}_M \right\| = O_p(M/n)$,
iv) $E \left\| \hat{\omega}_{j_1,j_2}(l) - \check{\omega}_{j_1,j_2}(l) \right\|^2 = O(n^{-2})$ *uniformly in* $k, j \leq M$ *and* $|l| < m$,
v) $E \left\| \check{\Gamma}_j^{xy} - \Gamma_j^{xy} \right\|^2 = O\left(n^{-1}\right)$ *uniformly in* $j \leq n$,
vi) $E \left\| \check{\Gamma}_j^{xy} \right\|^2 = \left\| \Gamma_j^{xy} \right\|^2 + O(n^{-1})$ *uniformly in* $j \leq M$,
vii) $\left\| \check{P}_M - P_M \right\| = O_p(M/n^{1/2})$,
viii) The same results as in (v)-(vi) hold for $\check{\omega}_{j_1,j_2}(l) - \Gamma_{j_2+l-j_1}^{yy}$ *and* $\check{\omega}_{j_1,j_2}(l) - \frac{r_2-r_1}{n} \Gamma_{j_2+l-j_1}^{yy}$ *uniformly in* $j_1, j_2 \leq M$,
ix) $E \left\| \check{\Gamma}_j^{xy} - \frac{r_2-r_1}{n} \Gamma_j^{xy} \right\|^4 = O\left(n^{-2}\right)$, $E \left\| \check{\Gamma}_j^{xy} - \Gamma_j^{xy} \right\|^4 = O\left(n^{-2}\right)$ *and similarly for* $\check{\omega}_{j_1,j_2}(l) - \Gamma_{j_2+l-j_1}^{yy}$.

Proof. For i) one obtains

$$
\begin{aligned}
(2.24) \quad \left\| \hat{\Gamma}_j^{xy} - \check{\Gamma}_j^{xy} \right\| \ \leq \ & \left\| \bar{x} - \mu_x \right\| \left\| n^{-1} \sum_{t=\max(r-m,j)+1}^{n-m} (y_{t+1-j} - \mu_y) \right\| \\
& + \left\| \bar{y} - \mu_y \right\| \left\| n^{-1} \sum_{t=\max(r-m,j)+1}^{n-m} (x_{t+m} - \mu_x) \right\| + \left\| \bar{y} - \mu_y \right\| \left\| \bar{x} - \mu_x \right\| .
\end{aligned}
$$

It then follows from the Cauchy-Schwartz inequality that $E\left\|\hat{\Gamma}_j^{xy}-\check{\Gamma}_j^{xy}\right\|^2$ depends on

$$\left(E\left\|\bar{x}-\mu_x\right\|^4\right)^{1/4}=O\left(n^{-1/2}\right),$$

$$\left(E\left\|\bar{y}-\mu_y\right\|^4\right)^{1/4}=O\left(n^{-1/2}\right),$$

$$\left(E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}(y_{t+1-j}-\mu_y)\right\|^4\right)^{1/4}=O\left(n^{-1/2}\right)$$

and

$$\left(E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}(x_{t+m}-\mu_x)\right\|^4\right)^{1/4}=O\left(n^{-1/2}\right)$$

which are established next. First,

$$\begin{aligned}E\left\|\bar{y}-\mu_y\right\|^4 &= \frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=1}^{n}E\left[\left(y_{t_1}-\mu_y\right)'\left(y_{t_2}-\mu_y\right)\left(y_{t_3}-\mu_y\right)'\left(y_{t_4}-\mu_y\right)\right]\\ &= \frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=1}^{n}\left(\gamma_{t_1-t_2}^{yy}\gamma_{t_3-t_4}^{yy}+\operatorname{tr}\left(\Gamma_{t_3-t_2}^{yy}\Gamma_{t_1-t_4}^{yy}\right)+\operatorname{tr}\left(\Gamma_{t_1-t_3}^{yy}\Gamma_{t_4-t_2}^{yy}\right)\right)\\ &\quad+\frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=1}^{n}\sum\nolimits_{q_1,q_2=1}^{p}\operatorname{cum}^*\left(y_{t_1}^{q_1},y_{t_2}^{q_1},y_{t_3}^{q_2},y_{t_4}^{q_2}\right)\\ &= O\left(n^{-2}\right).\end{aligned}$$

where the cumulant term is of lower order by Kuersteiner (2005, Lemma 4.2). By a similar argument $E\left\|\bar{x}-\mu_x\right\|^4=O(n^{-2})$. Further,

$$\begin{aligned}&E\left\|n^{-1}\sum\nolimits_{t=\max(r-m-j,0)+1}^{n-m-j}(y_t-\mu_y)\right\|^4 \qquad (2.25)\\ &= \frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=\max(r-m-j,0)+1}^{n-m-j}E\left[(y_{t_1}-\mu_y)'(y_{t_2}-\mu_y)(y_{t_3}-\mu_y)'(y_{t_4}-\mu_y)\right]\\ &\leq \frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=1}^{n}\left(\left|\gamma_{t_1-t_2}^{yy}\right|\left|\gamma_{t_3-t_4}^{yy}\right|+\left\|\Gamma_{t_3-t_2}^{yy}\right\|\left\|\Gamma_{t_1-t_4}^{yy}\right\|+\left\|\Gamma_{t_1-t_3}^{yy}\right\|\left\|\Gamma_{t_4-t_2}^{yy}\right\|\right)\\ &\quad+\frac{1}{n^4}\sum\nolimits_{t_1,\dots,t_4=1}^{n}\sum\nolimits_{q_1,q_2=1}^{p}\left|\operatorname{cum}^*\left(y_{t_1}^{q_1},y_{t_2}^{q_1},y_{t_3}^{q_2},y_{t_4}^{q_2}\right)\right|\\ &= O\left(n^{-2}\right)\end{aligned}$$

In the same way $E\left\|n^{-1}\sum_{t=\max(r-m,j)+1}^{n-m}(x_{t+m}-\mu_x)\right\|^4=O\left(n^{-2}\right)$ leading to $E\left\|\hat{\Gamma}_j^{xy}-\check{\Gamma}_j^{xy}\right\|^2=O(n^{-2})$.

For ii) we need to consider

$$\begin{aligned}E\left\|\bar{y}-\mu_y\right\|^2 &= \frac{1}{n^2}\sum\nolimits_{t_1,t_2=1}^{n}E\left[\left(y_{t_1}-\mu_y\right)'\left(y_{t_2}-\mu_y\right)\right]\\ &= \frac{1}{n^2}\sum\nolimits_{t_1,t_2=1}^{n}\left\|\Gamma_{t_1-t_2}^{yy}\right\|^2=O\left(n^{-1}\right).\end{aligned}$$

where a similar argument establishes $E\left\|n^{-1}\sum_{t=\max(r-m,0)+1}^{n-m-j}(y_t-\mu_y)\right\|^2=O\left(n^{-1}\right)$.

For iii),

$$E\left\|\hat{P}_M-\check{P}_M\right\|\leq\sum_{j_1=1}^{M}E\left\|\hat{\Gamma}^{xy}_{j_1}-\check{\Gamma}^{xy}_{j_1}\right\|=O\left(Mn^{-1}\right)$$

by the first result. The proof of iv) follows in the same way as the proof of i) where the difference between i) and iv) lies in the term (2.25) with the sums in iv) running over a more limited set of indices. However, inspection of (2.25) shows that this is without consequences for the $O\left(n^{-2}\right)$ rate.

For v) use Lemma (2.2iv) and consider

$$\begin{aligned} E\left\|\check{\Gamma}^{xy}_j-\Gamma^{xy}_j\right\|^2 \leq\; & E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^2+\frac{\left(j+1+\max\left(m,r\right)\right)^2}{n^2}\left\|\Gamma^{xy}_j\right\|^2 \\ & +2\left(\frac{j+1+\max\left(m,r\right)}{n}\right)\left\|\Gamma^{xy}_j\right\|E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\| \end{aligned}$$

where $\frac{j+1+\max(m,r)}{n}\left\|\Gamma^{xy}_j\right\|=O(n^{-1})$ uniformly in j because $|j|\left\|\Gamma^{xy}_j\right\|$ is uniformly bounded and (2.26)

$$E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^2=n^{-2}\sum\nolimits_{t,s=\max(r-m,j)+1}^{n-m}\operatorname{tr}(\Gamma^{xx}_{s-t}\gamma^{yy}_{s-t}+\Gamma^{xy}_{t-s+j}\Gamma^{yx}_{t-s-j}+\mathcal{K}^4_4)=O(n^{-1})$$

uniformly in j.

For vi) use Lemma 2.2iv) such that

$$\begin{aligned} E\left\|\check{\Gamma}^{xy}_j\right\|^2 &= n^{-2}\sum\nolimits_{t,s=\max(r-m,j)+1}^{n-m}\operatorname{tr}E\left[w_{t,j}w'_{s,j}\right] \\ &= \frac{\left(n-m-\left(\max(r-m,j)+1\right)\right)^2}{n^2}\left\|\Gamma^{xy}_j\right\|^2 \\ &\quad n^{-2}\sum\nolimits_{t,s=\max(r-m,j)+1}^{n-m}\operatorname{tr}(\Gamma^{xx}_{s-t}\gamma^{yy}_{s-t}+\Gamma^{xy}_{t-s+j}\Gamma^{yx}_{t-s-j}+\mathcal{K}^4_4) \\ &\leq \left\|\Gamma^{xy}_j\right\|^2+O(n^{-1}) \end{aligned} \tag{2.27}$$

because the elements of $\mathcal{K}^4_4$ appearing in $\operatorname{tr}E\left[w_{t,j}w'_{s,j}\right]$ are absolutely summable by Kuersteiner (2005, Lemma 4.2) and because

$$\frac{\left(n-m-\left(\max(r-m,j)+1\right)\right)}{n}\leq 1.$$

For vii) note that $E\left\|\hat{P}_M-P_M\right\|\leq\sum_{j_1=1}^{M}E\left\|\hat{\Gamma}^{xy}_{j_1}-\Gamma^{xy}_{j_1}\right\|=O\left(Mn^{-1/2}\right)$ by (i) and (ii) and the triangular inequality.

Finally, for viii) consider

$$\begin{aligned} & E\left\|\check{\omega}_{j_1,j_2}\left(l\right)-\Gamma^{yy}_{j_2+l-j_1}\right\|^2 \\ \leq\; & E\left\|n^{-1}\sum\nolimits_{t=r_1+1}^{r_2}\left(\left(y_{t-j_1}-\mu_y\right)\left(y_{t-l-j_2}-\mu_y\right)'-\Gamma^{yy}_{j_2+l-j_1}\right)\right\|^2+\frac{\left(n-r_2+r_1\right)^2}{n^2}\left\|\Gamma^{yy}_{j_2+l-j_1}\right\|^2 \\ & +2\left(\frac{n-r_2+r_1}{n}\right)\left\|\Gamma^{yy}_{j_2+l-j_1}\right\|E\left\|n^{-1}\sum\nolimits_{t=r_1+1}^{r_2}\left(\left(y_{t-j_1}-\mu_y\right)\left(y_{t-l-j_2}-\mu_y\right)'-\Gamma^{yy}_{j_2+l-j_1}\right)\right\| \end{aligned} \tag{2.28}$$

where, by Lemma 2.2iv)

$$(2.29)\qquad E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\left(\left(y_{t-j_1}-\mu_y\right)\left(y_{t-l-j_2}-\mu_y\right)'-\Gamma^{yy}_{j_2+l-j_1}\right)\right\|^2$$

$$= n^{-2}\sum\nolimits_{t,s=\max(r-m,j)+1}^{n-m}\operatorname{tr}(\Gamma^{yy}_{s-t}\gamma^{yy}_{s-t}+\Gamma^{yy}_{t-s-j_1+j_2+l}\Gamma^{yy}_{t-s-j_1+j_2+l}+\mathcal{K}_4)=O(n^{-1})$$

and the cumulant term is of lower order. Since

$$(2.30)\qquad \left|\frac{n-r_2+r_1}{n}\right|\left\|\Gamma^{yy}_{j_2+l-j_1}\right\|\leq\frac{\max\left(j_1,j_2+l\right)+\left|\min\left(0,l\right)\right|}{n}C\nu^{|j_2+l-j_1|}=O\left(\frac{M}{n}\right)$$

one has

$$E\left\|\check{\omega}_{j_1,j_2}\left(l\right)-\Gamma^{yy}_{j_2+l-j_1}\right\|^2=O\left(n^{-1}\right)+O\left(M^2n^{-2}\right)+O\left(Mn^{-3/2}\right)=O\left(n^{-1}\right)$$

when $M/n^{1/2}\to 0$. It then also follows that $E\left\|\check{\omega}_{j_1,j_2}\left(l\right)\right\|^2=\left\|\Gamma^{yy}_{j_2+l-j_1}\right\|^2+O\left(n^{-1}\right)$ in the same way as (2.27), using (2.28) and (2.29) to conclude that

$$E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\left(y_{t-j_1}-\mu_y\right)\left(y_{t-l-j_2}-\mu_y\right)'\right\|^2$$

$$= n^{-2}\sum\nolimits_{t,s=\max(r-m,j)+1}^{n-m}\operatorname{tr}(\Gamma^{yy}_{j_2+l-j_1}\Gamma^{yy\prime}_{j_2+l-j_1}+\Gamma^{yy}_{s-t}\gamma^{yy}_{s-t}+\Gamma^{yy}_{t-s-j_1+j_2+l}\Gamma^{yy}_{t-s-j_1+j_2+l}+\mathcal{K}_4)$$

$$= \left\|\Gamma^{yy}_{j_2+l-j_1}\right\|+O\left(n^{-1}\right).$$

The last statement in viii) follows because the term $\left(1-\frac{r_2-r_1}{n}\right)\Gamma^{yy}_{j_2+l-j_1}=O\left(M/n\right)$ can be dropped from (2.28) with only (2.29) remaining.

ix) Consider

$$(2.31)\qquad E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^4$$

$$= n^{-4}\sum\nolimits_{t_1,\ldots,t_4=\max(r-m,j)+1}^{n-m}E\left[\operatorname{tr}\check{w}_{t_1,j}\check{w}'_{t_2,j}\operatorname{tr}\check{w}_{t_3,j}\check{w}'_{t_4,j}\right]$$

$$= n^{-4}\sum\nolimits_{t_1,\ldots,t_4=\max(r-m,j)+1}^{n-m}\left(\operatorname{tr}E\left[\check{w}_{t_1,j}\check{w}'_{t_2,j}\right]\right)\left(\operatorname{tr}E\left[\check{w}_{t_3,j}\check{w}'_{t_4,j}\right]\right)$$

$$+\operatorname{tr}\left(E\left[\operatorname{vec}\check{w}'_{t_2,j}\left(\operatorname{vec}\check{w}'_{t_3,j}\right)'\right]E\left[\operatorname{vec}\check{w}'_{t_4,j}\left(\operatorname{vec}\check{w}'_{t_1,j}\right)'\right]\right)$$

$$+\operatorname{tr}\left(E\left[\operatorname{vec}\check{w}'_{t_1,j}\left(\operatorname{vec}\check{w}'_{t_3,j}\right)'\right]E\left[\operatorname{vec}\check{w}'_{t_4,j}\left(\operatorname{vec}\check{w}'_{t_2,j}\right)'\right]\right)+\mathcal{K}$$

$$= O\left(n^{-2}\right)$$

where $\mathcal{K}$ is an eigth order cumulant term of lower order and the other terms can be written as the sum of covariance and fourth order cumulant terms by Lemma 2.2iv). For $E\left\|\check{\Gamma}^{xy}_j-\Gamma^{xy}_j\right\|^4$ consider

$$(2.32)\qquad E\left\|\check{\Gamma}^{xy}_j-\Gamma^{xy}_j\right\|^4\leq E\left(\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|+\left\|\frac{j+1+\max\left(m,r\right)}{n}\Gamma^{xy}_j\right\|\right)^4$$

where $\left\|\frac{j+1+\max(m,r)}{n}\Gamma_j^{xy}\right\| = O\left(n^{-1}\right)$ uniformly in j and

$$E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\| = O\left(n^{-1/2}\right),$$

$E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^2 = O\left(n^{-1}\right)$ and

$$E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^3 \leq \left(E\left\|n^{-1}\sum\nolimits_{t=\max(r-m,j)+1}^{n-m}\check{w}_{t,j}\right\|^4\right)^{3/4} = O\left(n^{-3/2}\right)$$

by (2.31) such that $E\left\|\check{\Gamma}_j^{xy} - \Gamma_j^{xy}\right\|^4 = O\left(n^{-2}\right) + O\left(n^{-5/2}\right) + o\left(n^{-5/2}\right)$ by expanding (2.32) and using (2.31). ■

Lemma 2.9. *Let $\hat{\Omega}_M^*$ be defined as $\hat{\Omega}_M^* = \sum_{l=-m+1}^{m-1}\hat{\gamma}^{\varepsilon}(l)\hat{\Omega}_M(l)$ with typical i,j-th block $\hat{\omega}_{i,j}$ and $\sqrt{n}(\tilde{\beta}_n - \beta_0) = O_p(1)$. Let $M \to \infty$ such that $M/n^{1/2} \to 0$. Let r_1 and r_2 be as defined in Lemma (2.8) and let $\hat{\omega}_{j_1,j_2}^{\Delta}$ be as defined in (2.34) below such that the remainder term $\hat{\omega}_{j_1,j_2}^{R}$ is given by $\hat{\omega}_{j_1,j_2}^{R} = \omega_{j_1,j_2} - \hat{\omega}_{j_1,j_2} - \hat{\omega}_{j_1,j_2}^{\Delta}$. Let $\hat{\omega}_{j_1,j_2}^{R} = \sum_{j=1}^{4}\hat{\omega}_{j_1,j_2}^{R_j}$ where $\hat{\omega}_{j_1,j_2}^{R_j}$ are defined in (2.35), (2.36), (2.37) and (2.38) and $\hat{\omega}_{j_1,j_2}^{\bar{R}_2}$ is defined in (2.41). Let $\xi_n = \left(\sum_{l=-m+1}^{m-1}|\gamma_l^{\varepsilon} - \hat{\gamma}_l^{\varepsilon}|^2\right)^{1/2}$. Then*
i) $\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R}\right\| = O_p\left(n^{-1/2}\right) + O_p\left(M/n\right) = O_p\left(n^{-1/2}\right)$ *uniformly in* j_2,
ii) $\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_1}\right\| \leq C\left(M/n\right)$ *uniformly in* j_2,
iii) $\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_2}\right\| \leq \xi_n\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\|$ *with* $\xi_n = O_p\left(n^{-1/2}\right)$ *and* $\sum_{j_1=1}^{M}\left(E\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\|^2\right)^{1/2} = O\left(M/n^{1/2}\right)$,
iv) $\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_3}\right\| \leq C\xi_n = O_p\left(n^{-1/2}\right)$,
v) $\sum_{j_1=1}^{M}E\left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\| = O\left(M/n\right)$ *uniformly in* $j_2 \leq M$,
vi) $E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2\right] = O\left(n^{-1}\right)$ *uniformly in* $j_1, j_2 \leq M$,
vii) $E\left[\sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2\right] = O\left(M^2/n\right)$ *such that* $\left\|\hat{\Omega}_M^* - \Omega_M\right\| = O_p(M/n^{1/2})$.

Proof. First note that for $l > 0$,

$$\hat{\Omega}_M(l) = \frac{1}{n}\sum_{t=1}^{n} z_{t,M}z_{t-l,M}'$$

where the summation indecies effectively depend on the block (j_1, j_2) of $\hat{\Omega}_M(l)$ because of the definition of $z_{t,M}$ which padds up the vector of instruments with zeros for periods t where there is no data for a particular lag. Then, for a typical (j_1, j_2) block $\hat{\omega}_{j_1,j_2}(l)$ of $\hat{\Omega}_M(l)$ one obtains for $l \geq 0$

$$\hat{\omega}_{j_1,j_2}(l) = \frac{1}{n}\sum_{t=\max(j_1,j_2+l)+1}^{n}\left(y_{t+1-j_1} - \bar{y}\right)\left(y_{t+1-l-j_2} - \bar{y}\right)'$$

while for $l<0$, $\hat{\Omega}_M(l)=\hat{\Omega}_M(-l)'$ such that

$$\hat{\omega}_{j_1,j_2}(l)=\frac{1}{n}\sum_{t=\max(j_1+|l|,j_2)+1}^{n}\left(y_{t-j_1-|l|}-\bar{y}\right)\left(y_{t-j_2}-\bar{y}\right)'=\frac{1}{n}\sum_{t=\max(j_1,j_2-|l|)+1}^{n-|l|}\left(y_{t-j_1}-\bar{y}\right)\left(y_{t+|l|-j_2}-\bar{y}\right)'.$$

It follows that for all $|l|\leq m$, $\hat{\omega}_{j_1,j_2}(l)$ is as defined in Lemma 2.8. From $\hat{\Omega}_M^*$ defined in (1.1) and for a typical j_1,j_2 block of $\hat{\Omega}_M^*-\Omega_M$ it follows that

$$\begin{aligned}\hat{\omega}_{j_1,j_2}-\omega_{j_1,j_2} &= \textstyle\sum_{l=-m+1}^{m-1}\hat{\gamma}_l^{\varepsilon}\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}\\ &= \textstyle\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\left(\hat{\omega}_{j_1,j_2}(l)-\frac{r_2-r_1}{n}\Gamma_{j_2+l-j_1}^{yy}\right)\\ &\quad -\frac{n-r_2+r_1}{n}\textstyle\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\Gamma_{j_2+l-j_1}^{yy}\\ &\quad +\textstyle\sum_{l=-m+1}^{m-1}\left(\hat{\gamma}_l^{\varepsilon}-\gamma_l^{\varepsilon}\right)\hat{\omega}_{j_1,j_2}(l).\end{aligned}\tag{2.33}$$

For i)-v) define

$$\breve{\omega}_{j_1,j_2}(l)=\frac{1}{n}\sum_{t=\max(j_1,j_2+l)+1}^{n-|\min(0,l)|}\left(y_{t-j_1}-\mu_y\right)\left(y_{t-l-j_2}-\mu_y\right)'$$

and let $\tilde{\omega}_{j_1,j_2}(l)=\breve{\omega}_{j_1,j_2}(l)-\frac{r_2-r_1}{n}\Gamma_{j_2+l-j_1}^{yy}$. Then define

$$\hat{\omega}_{j_1,j_2}^{\Delta}=\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\tilde{\omega}_{j_1,j_2}(l)\tag{2.34}$$

$$\hat{\omega}_{j_1,j_2}^{R_1}=\frac{n-r_2+r_1}{n}\textstyle\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\Gamma_{j_2+l-j_1}^{yy}\tag{2.35}$$

$$\hat{\omega}_{j_1,j_2}^{R_2}=\textstyle\sum_{l=-m+1}^{m-1}\left(\gamma_l^{\varepsilon}-\hat{\gamma}_l^{\varepsilon}\right)\left(\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\right)\tag{2.36}$$

$$\hat{\omega}_{j_1,j_2}^{R_3}=\textstyle\sum_{l=-m+1}^{m-1}\left(\gamma_l^{\varepsilon}-\hat{\gamma}_l^{\varepsilon}\right)\omega_{j_1,j_2}(l)\tag{2.37}$$

$$\hat{\omega}_{j_1,j_2}^{R_4}=\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\left(\hat{\omega}_{j_1,j_2}(l)-\breve{\omega}_{j_1,j_2}(l)\right)\tag{2.38}$$

where $\omega_{j,k}(l)=\Gamma_{k-j+l}^{yy}$. Now, for ii)

$$\textstyle\sum_{j_1=1}^{\infty}\left\|\hat{\omega}_{j_1,j_2}^{R_1}\right\|\leq\frac{M}{n}\textstyle\sum_{l=-m+1}^{m-1}\left|\gamma_l^{\varepsilon}\right|\displaystyle\sum_{j=-\infty}^{\infty}\left\|\Gamma_j^{yy}\right\|=O\left(M/n\right).\tag{2.39}$$

For iii)

$$\begin{aligned}\textstyle\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_2}\right\| &\leq \textstyle\sum_{j_1=1}^{M}\left(\sum_{l=-m+1}^{m-1}\left|\gamma_l^{\varepsilon}-\hat{\gamma}_l^{\varepsilon}\right|\left\|\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\right\|\right)\\ &\leq \left(\textstyle\sum_{l=-m+1}^{m-1}\left|\gamma_l^{\varepsilon}-\hat{\gamma}_l^{\varepsilon}\right|^2\right)^{1/2}\textstyle\sum_{j_1=1}^{M}\sum_{l=-m+1}^{m-1}\left\|\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\right\|\\ &= O_p\left(n^{-1/2}\right)O_p\left(M/\sqrt{n}\right)=O_p\left(M/n\right)\end{aligned}\tag{2.40}$$

because $\left(\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon-\hat{\gamma}_l^\varepsilon|^2\right)^{1/2}=O_p\left(n^{-1/2}\right)$ and

$$E\left[\|\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\|\right]\leq\left(E\left[\|\breve{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\|^2\right]\right)^{1/2}+\left(E\left[\|\hat{\omega}_{j_1,j_2}(l)-\breve{\omega}_{j_1,j_2}(l)\|^2\right]\right)^{1/2}$$

such that $\left[\sum_{j_1=1}^{M}\sum_{l=-m+1}^{m-1}\|\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\|\right]=O(M/\sqrt{n})$ by Lemma 2.8iv) and viii) and the result holds for

$$\hat{\omega}_{j_1,j_2}^{\bar{R}_2}=\sum_{l=-m+1}^{m-1}\|\hat{\omega}_{j_1,j_2}(l)-\omega_{j_1,j_2}(l)\|. \tag{2.41}$$

For iv) use the bound

$$\begin{aligned}\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_3}\right\| &= \sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon-\hat{\gamma}_l^\varepsilon|\sum_{j_1=1}^{M}\|\omega_{j_1,j_2}(l)\| \\ &\leq \sum_{j_1=-\infty}^{\infty}\left\|\Gamma_{j_1}^{yy}\right\|\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon-\hat{\gamma}_l^\varepsilon| \\ &\leq \sum_{j_1=-\infty}^{\infty}\left\|\Gamma_{j_1}^{yy}\right\|(2m)^{1/2}\left(\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon-\hat{\gamma}_l^\varepsilon|^2\right)^{1/2}=O_p\left(n^{-1/2}\right).\end{aligned} \tag{2.42}$$

Finally, for v)

$$\begin{aligned}E\left[\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\|\right] &\leq \sum_{j_1=1}^{M}\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon|\left(E\left[\|\hat{\omega}_{j_1,j_2}(l)-\breve{\omega}_{j_1,j_2}(l)\|^2\right]\right)^{1/2} \\ &\leq C\frac{M}{n}\left(\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon|\right) \\ &= O(M/n).\end{aligned} \tag{2.43}$$

Together (2.39), (2.40), (2.42) and (2.43) imply that $\sum_{j_1=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R}\right\|=O_p\left(n^{-1/2}\right)+O_p(M/n)=O_p\left(n^{-1/2}\right)$.

For vi) consider

$$\begin{aligned}E\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2 &\leq E\left(\sum_{l=-m+1}^{m-1}|\gamma_l^\varepsilon|\,\|\tilde{\omega}_{j_1,j_2}(l)\|\right)^2 \\ &\leq \sum_{l_1,l_2=-m+1}^{m-1}|\gamma_{l_1}^\varepsilon|\,|\gamma_{l_2}^\varepsilon|\sqrt{E\left[\|\tilde{\omega}_{j_1,j_2}(l_1)\|^2\right]E\left[\|\tilde{\omega}_{j_1,j_2}(l_2)\|^2\right]}=O(n^{-1})\end{aligned} \tag{2.44}$$

which follows from Lemma 2.8viii).

For vii) consider

$$\begin{aligned}\left\|\hat{\Omega}_M-\Omega_M\right\|^2 &= \sum_{j_1,j_2=1}^{M}\|\hat{\omega}_{i,j}-\omega_{i,j}\|^2 \\ &\leq \sum_{j_1,j_2=1}^{M}\left(\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2+2\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\left\|\hat{\omega}_{j_1,j_2}^{R}\right\|+\left\|\hat{\omega}_{j_1,j_2}^{R}\right\|^2\right)\end{aligned}$$

where $\sum_{j_1,j_2=1}^{M}E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2\right]=O\left(M^2/n\right)$ by (2.44). Also note that

$$\left\|\hat{\omega}_{j_1,j_2}^{R}\right\|\leq C\frac{M}{n}+C\xi_n\left(1+\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\|\right)+\left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\|$$

such that

$$\sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\left\|\hat{\omega}_{j_1,j_2}^{R}\right\| \leq \left(C\frac{M}{n}+\xi_n\right)\sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\| + \sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\left(\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\| + \left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\|\right)$$

where

$$\sum_{j_1,j_2=1}^{M} E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\right] = O\left(M^2/n^{1/2}\right)$$

such that

$$\left(C\frac{M}{n}+\xi_n\right)\sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\| = O_p\left(\frac{M^3}{n^{3/2}}+\frac{M^2}{n}\right) = O_p\left(\frac{M^2}{n}\right)$$

while

$$\sum_{j_1,j_2=1}^{M} E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\|\right] \leq \sum_{j_1,j_2=1}^{M}\sqrt{E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2\right]E\left[\left\|\hat{\omega}_{j_1,j_2}^{\bar{R}_2}\right\|^2\right]} = O\left(M^2/n\right)$$

and

$$\sum_{j_1,j_2=1}^{M} E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|\left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\|\right] \leq \sum_{j_1,j_2=1}^{M}\sqrt{E\left[\left\|\hat{\omega}_{j_1,j_2}^{\Delta}\right\|^2\right]E\left[\left\|\hat{\omega}_{j_1,j_2}^{R_4}\right\|^2\right]} = O\left(M^2/n^{3/2}\right).$$

In the same way $\sum_{j_1,j_2=1}^{M}\left\|\hat{\omega}_{j_1,j_2}^{R}\right\|^2 = O_p\left(M^2/n\right)$. Together these results show that $\left\|\hat{\Omega}_M-\Omega_M\right\|^2 = O_p\left(M^2/n\right)$ and thus $\left\|\hat{\Omega}_M-\Omega_M\right\| = O_p\left(M/\sqrt{n}\right)$. ■

Lemma 2.10. *Let $\hat{\Omega}_M$ be defined in (1.2). Let $\bar{M}\to\infty$ such that $\bar{M}/n^{1/2}\to 0$. Then $\Pr\left(\min\hat{\xi}_\Omega \geq 0\right)\to 1$ and*

$$\Pr\left(\hat{\Omega}_M^* = \hat{\Omega}_M\right)\to 1$$

uniformly in $M\leq\bar{M}$. In addition $\left\|\hat{\Omega}_M-\Omega_M\right\| = O_p\left(M/\sqrt{n}\right)$ uniformly in $M\leq\bar{M}$.

Proof. Note that

$$\begin{aligned}
0 &< \inf_{a,\|a\|=1} a'\Omega_M a = \inf_{a,\|a\|=1}\left(a'\hat{\Omega}_M^* a + a'\left(\Omega_M-\hat{\Omega}_M^*\right)a\right) \\
&\leq \inf_{a,\|a\|=1}\left(a'\hat{\Omega}_M^* a + \|a\|^2\left\|\Omega_M-\hat{\Omega}_M^*\right\|\right) = \inf_{a,\|a\|=1} a'\hat{\Omega}_M^* a + \left\|\Omega_M-\hat{\Omega}_M^*\right\| \\
&= \min\hat{\xi}_\Omega + \left\|\Omega_M-\hat{\Omega}_M^*\right\|
\end{aligned}$$

such that $\min\hat{\xi}_\Omega \geq \inf_{a,\|a\|=1} a'\Omega_M a - \left\|\Omega_M-\hat{\Omega}_M^*\right\|$ and

$$\Pr\left(\min\hat{\xi}_\Omega\geq 0\right) \geq \Pr\left(\inf_{a,\|a\|=1} a'\Omega_M a - \left\|\Omega_M-\hat{\Omega}_M^*\right\| \geq 0\right)\to 1$$

by Lemma 2.9. Then,

$$\Pr\left(\hat{\Omega}_M^* = \hat{\Omega}_M\right) = \Pr\left(\min \hat{\xi}_\Omega \geq 0\right) \to 1. \tag{2.45}$$

For the last part note that for some $C > 0$, $\Pr\left(\sqrt{n}/M \left\|\hat{\Omega}_M - \Omega_M\right\| > C\right) \leq \Pr\left(\sqrt{n}/M \left\|\hat{\Omega}_M^* - \Omega_M\right\| > C\right) + \left(1 - \Pr\left(\hat{\Omega}_M^* = \hat{\Omega}_M\right)\right) \to 0$ by Lemma 2.9 and (2.45). ■

Lemma 2.11. *Let $\hat{\Omega}_M$ be defined in (1.2) and $\sqrt{n}(\tilde{\beta}_n - \beta_0) = O_p(1)$. Let $\bar{M} \to \infty$ such that $\bar{M}/n^{1/2} \to 0$. Then $\left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2 = O_p(M/n^{1/2})$ and $\left\|\hat{\Omega}_M^{-1}\right\|_2 = O_p(1)$ uniformly in $M \leq \bar{M}$.*

Proof. The proof follows the argument of Lewis and Reinsel (1985, p.397). Note that $\left\|\hat{\Omega}_M^{-1}\right\|_2 \leq \left\|\Omega_M^{-1}\right\|_2 + \left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2$. From Kuersteiner (2005, Lemma 4.5) and Lemma 2.7 it follows that $\left\|\Omega_M^{-1}\right\|_2 < C$ for some finite constant C is bounded uniformly in M. Then,

$$\left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2 / \left(C\left(\left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2 + C\right)\right) \leq \left\|\hat{\Omega}_M - \Omega_M\right\|$$

such that $\left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2 = O_p(M/n^{1/2})$ by Lemma 2.10. This now also implies $\left\|\hat{\Omega}_M^{-1}\right\|_2 = O_p(1)$. ■

3. Auxiliary Lemmas for Lemma A.11

Lemma 3.1. *Assume that Conditions B and C hold. Then*
i) $H_{11} = P_M'\Omega_M^{-1}P_M - P'\Omega P = -\sigma_{1M} = O\left(M^2\nu^M\right)$,
ii) $H_{12} \equiv P_M'(I - W_M)\Omega_M^{-1}(I - W_M)P_M = M^{-q2}k_q^2\mathcal{B}_0^{(q)} + o(M^{-2q}) = O(M^{-2q})$,
iii) $H_{13} \equiv -P_M'\Omega_M^{-1}(I - W_M)P_M = O(M^{-q})$,
iv) $H_{14} \equiv -P_M'(I - W_M)\Omega_M^{-1}P_M = O(M^{-q})$,
v) $H_{211} \equiv -\left(\hat{P}_M - \check{P}_M\right)' W_M\Omega_M^{-1}W_M(\hat{P}_M - \check{P}_M) = O_p(M/n)$,
vi) $H_{212} \equiv \hat{P}_M'W_M\Omega_M^{-1}W_M(\hat{P}_M - \check{P}_M) + (\hat{P}_M - \check{P}_M)'W_M\Omega_M^{-1}W_M\hat{P}_M = O_p\left(M/n^{3/2}\right) + O_p(n^{-1})$,
vii) $H_{221} \equiv -\left(\check{P}_M - P_M\right)' W_M\Omega_M^{-1}W_M\left(\check{P}_M - P_M\right) = O_p(M/n)$,
viii) $H_{222} \equiv \check{P}_M'W_M\Omega_M^{-1}W_M(\check{P}_M - P_M) + (\check{P}_M - P_M)'W_M\Omega_M^{-1}W_M\check{P}_M = O_p(M/n) + O_p\left(n^{-1/2}\right)$,
ix) $H_{31} \equiv (\hat{P}_M - P_M)'W_M\Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1}W_M(\hat{P}_M - P_M) = o_p(M/n^{1/2})$,
x) $H_{32} \equiv P_M'W_M\Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1}W_M(\hat{P}_M - P_M) = o_p(M/n^{1/2})$,
xi) $H_{33} \equiv (\hat{P}_M - P_M)'W_M\Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1}W_M P_M = o_p(M/n^{1/2})$,
xii) $H_{34} \equiv P_M'W_M\Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1}W_M P_M = O_p(n^{-1/2})$,
xiii) $H_4 \equiv \hat{P}_M'W_M(\hat{\Omega}_M^{-1} - \Omega_M^{-1} + \Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1})W_M\hat{P}_M = o_p(M/n^{1/2})$.

Proof. For i) Let $\vartheta_{i,j}^M$ be the i,j-th $p\times p$ block of Ω_M^{-1}. Let $\left[\Omega^{-1}\right]_{11}$ be the upper $pM\times pM$ block of the infinite dimensional matrix Ω^{-1}. Partition Ω and Ω^{-1} conformingly as

$$\Omega=\begin{bmatrix}\Omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{bmatrix},\ \Omega^{-1}=\begin{bmatrix}\left[\Omega^{-1}\right]_{11} & \left[\Omega^{-1}\right]_{12}\\ \left[\Omega^{-1}\right]_{21} & \left[\Omega^{-1}\right]_{22}\end{bmatrix}$$

such that $\Omega_{11}=\Omega_M$ and correspondingly, $P=[P_M',P_2']'$. Then

$$\begin{aligned}&P_M'\Omega_M^{-1}P_M-P'\Omega^{-1}P\\ =\ & P_M'\left(\Omega_M^{-1}-\left[\Omega^{-1}\right]_{11}\right)P_M+P_M'\left[\Omega^{-1}\right]_{12}P_2+P_2'\left[\Omega^{-1}\right]_{21}P_M+P_2'\left[\Omega^{-1}\right]_{22}P_2\end{aligned}$$

and expanding further

$$\begin{aligned}P_M'\Omega_M^{-1}P_M-P'\Omega^{-1}P\ =\ & \sum_{j_1,j_2}^{M}\Gamma_{j_1}^{xy}\left(\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right)\Gamma_{-j_2}^{yx}\\ & +\sum_{j_1=1}^{M}\sum_{j_2=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2+M}\Gamma_{-j_2-M}^{yx}+\sum_{j_1=1}^{\infty}\sum_{j_2=1}^{M}\Gamma_{j_1+M}^{xy}\vartheta_{j_1+M,j_2}\Gamma_{-j_2}^{yx}\\ & +\sum_{j_1,j_2=1}^{\infty}\Gamma_{j_1+M}^{xy}\vartheta_{j_1+M,j_2+M}\Gamma_{-j_2-M}^{yx}\end{aligned}$$

The term $P_M'\left(\Omega_M^{-1}-\left[\Omega^{-1}\right]_{11}\right)P_M$ depends on $\sum_{j_1=1}^M\Gamma_{j_1}^{xy}\left(\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right)$. By Lemma 2.7, for some bounded constant C,

$$\begin{aligned}\sum_{j_1=1}^{M}\left\|\Gamma_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right\| &\leq C\sum_{j_1=1}^{M}\nu_1^{j_1}\left(M-j_1\right)\nu^{M-j_1}\\ &= O\left(M^2\nu^M\right).\end{aligned}$$

such that

$$\begin{aligned}&\left\|\sum_{j_1,j_2}^{M}\Gamma_{j_1}^{xy}\left(\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right)\Gamma_{-j_2}^{yx}\right\|\leq CM^2\nu^M\sum_{j_2=1}^{M}\nu^{j_2}\\ =\ & O\left(M^2\nu^M\right).\end{aligned}$$

From Lemma 2.6ix) and x) it follows that $P_M'\left(\Omega_M^{-1}-\left[\Omega^{-1}\right]_{11}\right)P_M$ dominates the other terms.

For ii) write $H_{12}=M^{-q2}\sum_{j_1,j_2=1}^M\Gamma_{j_1}^{xy}\left|j_1\right|^q\frac{1-k(j_1/M)}{|j_1|^qM^{-q}}\vartheta_{j_1,j_2}^M\frac{1-k(j_2/M)}{|j_2|^qM^{-q}}\left|j_2\right|^q\Gamma_{-j_2}^{yx}$. Note that $\vartheta_{j_1,j_2}^M\to\vartheta_{j_1,j_2}$ as $M\to\infty$ for all j_1,j_2 fixed and finite by Lemma 2.7i) and ii). By the Dominated Convergence Theorem, upon using the dominating function $\Gamma_{j_1}^{xy}\left|j_1\right|^q\left(\left\|\vartheta_{j_1,j_2}\right\|+\left\|\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right\|\right)\left|j_2\right|^q\left\|\Gamma_{-j_2}^{yx}\right\|$ where $\left\|\vartheta_{j_1,j_2}^M-\vartheta_{j_1,j_2}\right\|$ can be uniformly bounded by (2.15), it follows that

$$\sum_{j_1,j_2=1}^{M}\Gamma_{j_1}^{xy}\left|j_1\right|^q\frac{1-k\left(j_1/M\right)}{\left|j_1\right|^qM^{-q}}\vartheta_{j_1,j_2}^M\frac{1-k\left(j_2/M\right)}{\left|j_2\right|^qM^{-q}}\left|j_2\right|^q\Gamma_{-j_2}^{yx}\to k_q^2\mathcal{B}_0^{(q)}\text{ as }M\to\infty$$

where Assumption A was used such that $H_{12} = M^{-2q} k_q^2 \mathcal{B}_0^{(q)} + o(M^{-2q}) = O(M^{-2q})$.

Results iii) and iv) follow from Lemma A.12viii) in the main paper.

For v) write $H_{211} = \sum_{j_1,j_2=1}^{M} \left(\hat{\Gamma}_{j_1}^{xy} - \check{\Gamma}_{j_1}^{xy} \right) k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) \left(\hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right)'$ where

$$\check{\Gamma}_{j}^{xy} = n^{-1} \sum_{t=j+1}^{n-m} w_{t,j}.$$

First note that

$$\|H_{211}\| \leq \sum_{j_1,j_2=1}^{M} \left\| \hat{\Gamma}_{j_1}^{xy} - \check{\Gamma}_{j_1}^{xy} \right\| \left\| \vartheta_{j_1,j_2}^{M} \right\| \left\| \hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right\|.$$

By Lemma 2.8i) H_{211} is bounded in expectation by $n^{-2} C \sum_{j_1,j_2=1}^{M} \left\| \vartheta_{j_1,j_2}^{M} \right\| = O(M/n^2)$ where the last equality follows because $\sum_{j_1=1}^{M} \left\| \vartheta_{j_1,j_2}^{M} \right\| < \infty$ uniformly in j_2 and M by Kuersteiner (2005, Lemma 4.6) and Lemma 2.7iv).

For vi) note that H_{212} can be written as

(3.1)

$$H_{212} = -\sum_{j_1,j_2=1}^{M} \left(\hat{\Gamma}_{j_1}^{xy} - \check{\Gamma}_{j_1}^{xy} \right) k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) \check{\Gamma}_{-j_2}^{yx} - \sum_{j_1,j_2=1}^{M} \check{\Gamma}_{j_1}^{xy} k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) \left(\hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right)'.$$

where in the first term $\hat{\Gamma}_{j_2}^{xy}$ was replaced by $\check{\Gamma}_{-j_2}^{yx}$ which adds a $O_p\left(M/n^2\right)$ error term by the results in (v). The same adjustment is made for the second term in (3.1). First note that

$$E\left[\|H_{212}\|\right] \leq \sum_{j_1,j_2=1}^{M} \left\| \vartheta_{j_1,j_2}^{M} \right\| E\left[\left\| \hat{\Gamma}_{j_1}^{xy} - \check{\Gamma}_{j_1}^{xy} \right\| \left\| \check{\Gamma}_{j_2}^{xy} \right\| \right] + \left\| \vartheta_{j_1,j_2}^{M} \right\| E\left[\left\| \check{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right\| \right].$$

Lemmas 2.8i) and vi) imply that

$$E\left[\left\| \check{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right\| \right] \leq \sqrt{ E\left[\left\| \check{\Gamma}_{j_1}^{xy} \right\|^2 \right] E\left[\left\| \hat{\Gamma}_{j_2}^{xy} - \check{\Gamma}_{j_2}^{xy} \right\|^2 \right] } = \left\| \Gamma_{j_1}^{xy} \right\| O\left(n^{-1}\right) + O\left(n^{-3/2}\right)$$

uniformly in $j_1, j_2 \leq M$ such that

$$E\left\|H_{212}\right\| \leq O\left(n^{-1}\right) \sum_{j_1,j_2=0}^{M} \left\| \vartheta_{j_1,j_2}^{M} \right\| \left\| \Gamma_{j_2}^{yx} \right\| + O(M/n^2) = O(Mn^{-3/2} + n^{-1}).$$

For vii) use the definition $H_{221} = \sum_{j_1,j_2=1}^{M} \left(\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy} \right) k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) \left(\check{\Gamma}_{j_2}^{yx} - \Gamma_{j_2}^{yx} \right)'$. Then, using Lemma 2.7iv) and 2.8v) one finds

$$\begin{aligned} E & \left\| \sum_{j_1,j_2=1}^{M} \left(\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy} \right) \vartheta_{j_1,j_2}^{M} \left(\check{\Gamma}_{-j_2}^{yx} - \Gamma_{-j_2}^{yx} \right)' \left|k(j_1/M)\right| \left|k(j_2/M)\right| \right\| \\ & \leq \sum_{j_1,j_2=1}^{M} \left(E \left\| \check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy} \right\|^2 \right)^{1/2} \left(E \left\| \check{\Gamma}_{-j_2}^{yx} - \Gamma_{-j_2}^{yx} \right\|^2 \right)^{1/2} \left\| \vartheta_{j_1,j_2}^{M} \right\| \\ & \leq C n^{-1} \sum_{j_1,j_2=1}^{M} \left\| \vartheta_{j_1,j_2}^{M} \right\| = O(M/n). \end{aligned}$$

where C is some constant.

For viii) write

$$\begin{aligned} H_{222} &= -\sum_{j_1,j_2=1}^{M} \check{\Gamma}_{j_1}^{xy} k(j_1/M)\, \vartheta_{j_1,j_2}^{M} k(j_2/M) \left(\check{\Gamma}_{-j_2}^{yx} - \Gamma_{-j_2}^{yx}\right)' \\ &\quad -\sum_{j_1,j_2=1}^{M} \left(\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right) k(j_1/M)\, \vartheta_{j_1,j_2}^{M} k(j_2/M)\, \check{\Gamma}_{-j_2}^{yx}. \end{aligned}$$

Use the same arguments as in the proof of vii) to obtain the following bound

$$\begin{aligned} & E\left\|\sum_{j_1,j_2=1}^{M} \check{\Gamma}_{j_1}^{xy} \vartheta_{j_1,j_2}^{M} \left(\check{\Gamma}_{-j_2}^{yx} - \Gamma_{-j_2}^{yx}\right)' k(j_1/M)\, k(j_2/M)\right\| \\ \leq\ & Cn^{-1/2} \sum_{j_1,j_2=1}^{M} \left(E\left\|\check{\Gamma}_{j_1}^{xy}\right\|^2\right)^{1/2} \left\|\vartheta_{j_1,j_2}^{M}\right\| = O(n^{-1/2}) + O(M/n) \end{aligned}$$

where C is a constant such that $\left(E\left\|\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right\|^2\right)^{1/2} < Cn^{-1/2}$ uniformly in j_1 and $E\left\|\check{\Gamma}_{j_2}^{xy}\right\|^2 = \left\|\Gamma_{j_2}^{xy}\right\|^2 + O(n^{-1})$ uniformly in j_2 by Lemma 2.8v) and vi). The result then follows from Markov's inequality.

For ix) expand $H_{31} = H_{311} + H_{312} + H_{313} + H_{314}$ where

$$\begin{aligned} H_{311} &= \left(\check{P}_M - P_M\right)' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\hat{P}_M - \check{P}_M) \\ H_{312} &= \left(\hat{P}_M - \check{P}_M\right)' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\hat{P}_M - \check{P}_M) \\ H_{313} &= \left(\check{P}_M - P_M\right)' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\check{P}_M - P_M) \\ H_{314} &= \left(\hat{P}_M - \check{P}_M\right)' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\check{P}_M - P_M). \end{aligned}$$

From

$$\|H_{311}\| \leq \|W_M\|_2^2 \left\|\check{P}_M - P_M\right\| \left\|\hat{\Omega}_M - \Omega_M\right\| \left\|\Omega_M^{-1}\right\|_2^2 \left\|\hat{P}_M - \check{P}_M\right\|$$

where $\|W_M\|_2^2 \leq 1$ it follows that $\|H_{311}\| = O_p\left(M^3/n^2\right) = o_p\left(M/n\right)$ by Lemmas 2.8iii), 2.8vii) and 2.10. Next, note that$\|H_{312}\| = o_p\left(\|H_{311}\|\right)$ because the leading term of H_{311} is of bigger order. Also, $\|H_{314}\| = O_p\left(\|H_{311}\|\right)$. Next,

$$\|H_{313}\| \leq \left\|\check{P}_M - P_M\right\| \left\|\Omega_M^{-1}\right\|_2^2 \left\|\hat{\Omega}_M - \Omega_M\right\| \left\|\check{P}_M - P_M\right\| = O_p\left(M^3/n^{3/2}\right) = o_p\left(M/n^{1/2}\right)$$

by Lemmas 2.8iii), 2.8vii) and 2.10.

For x) and xi) note that $H_{32} = H_{33}'$ such that it is enough to consider H_{32}. Write $H_{32} = H_{321} + H_{322}$ where H_{32j} for $j = 1, 2$ is defined as

$$\begin{aligned} H_{321} &= -P_M' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\hat{P}_M - \check{P}_M) \\ H_{322} &= -P_M' W_M \Omega_M^{-1} (\hat{\Omega}_M - \Omega_M) \Omega_M^{-1} W_M (\check{P}_M - P_M). \end{aligned}$$

Then $\|H_{321}\| \leq \|P_M' W_M \Omega_M^{-1}\| \left\|\hat{\Omega}_M - \Omega_M\right\| \|\Omega_M^{-1}\|_2 \left\|\hat{P}_M - \check{P}_M\right\|$ where $\|\Omega_M^{-1}\|_2 = O(1)$ uniformly in M by Lemma 2.7 and Lewis and Reinsel (1988, p.397),

$$\|P_M' W_M \Omega_M^{-1}\| \leq \sum_{j_1,j_2=1}^{\infty} \left\|\Gamma_{j_1}^{xy}\right\| \|\vartheta_{j_1,j_2}^M\| = O(1)$$

by Lemma 2.7iv) and Kuersteiner (2005, Lemma 4.5) and

$$\left\|\hat{P}_M - \check{P}_M\right\| \leq \sum_{j=1}^{M} \left\|\hat{\Gamma}_j^{xy} - \check{\Gamma}_j^{xy}\right\| = O_p\left(M/n\right)$$

by Lemma 2.8ii). Then by Lemma 2.10, $\|H_{321}\| = O_p\left(M^2/n^{3/2}\right) = o_p\left(M/n\right)$. Define

$$a_i^M = \sum_{j=1}^{M} \Gamma_j^{xy} k\left(j/M\right) \vartheta_{ji}^M$$

where$\|a_i^M\| \leq \sum_{j=1}^{M} \left\|\Gamma_j^{xy} \vartheta_{ji}^M\right\|$ and $\|P_M W_M \Omega_M^{-1}\| \leq \sum_{i=1}^{M} \|a_i^M\|$ is uniformly bounded in M by Lemma 2.7iv) such that

$$\begin{aligned} \|H_{322}\| &\leq \|P_M' W_M \Omega_M^{-1}\| \left\|\hat{\Omega}_M - \Omega_M\right\| \|\Omega_M^{-1}\|_2 \|W_M\|_2 \|\check{P}_M - P_M\| \\ &= O_p\left(M^2/n\right) = o_p\left(M/n^{1/2}\right). \end{aligned}$$

by Lemmas 2.8v) and 2.10.

For xii) note that H_{34} can be written as $\sum_{j_1,j_2=1}^{M} a_{j_1}^M (\omega_{j_1,j_2} - \hat{\omega}_{j_1,j_2}) a_{j_2}'^M$ with a_i^M defined as in the Proof of ix). Then

$$\|H_{34}\| \leq \sum_{j_1,j_2=1}^{M} \|a_{j_1}^M\| \|a_{j_2}^M\| \|\hat{\omega}_{j_2,j_3}^{\Delta}\| + \sum_{j_1,j_2=1}^{M} \|a_{j_1}^M\| \|\hat{\omega}_{j_1,j_2}^R\| \|a_{j_2}^M\| = O_p(n^{-1/2})$$

by Lemma 2.9 and the Cauchy-Schwartz inequality.

For xiii) use the matrix valued expansion $\hat{\Omega}_M^{-1} = \Omega_M^{-1} - \Omega_M^{-1}(\hat{\Omega}_M - \Omega_M)\Omega_M^{-1} + B$ where B can be expressed as $B = \hat{\Omega}_M^{-1}(\Omega_M - \hat{\Omega}_M)\Omega_M^{-1}(\Omega_M - \hat{\Omega}_M)\Omega_M^{-1}$. Using the fact that for two conformable matrices A and C, $\|AC\| \leq \|A\|_2 \|C\|$ (see Lewis and Reinsel, 1985, p. 396), it follows from Lemmas 2.9 and 2.11 that $\|B\| \leq \left\|\hat{\Omega}_M^{-1}\right\|_2 \left\|\Omega_M - \hat{\Omega}_M\right\|^2 \|\Omega_M^{-1}\|_2^2 = O_p(M^2/n)$ such that $\|B\| = o_p\left(\left\|\Omega_M - \hat{\Omega}_M\right\|\right)$. It then follows from the same arguments as in ix) and xii) that $H_4 = o_p\left(\|H_3\|\right) = o_p\left(M/n^{1/2}\right)$. ■

Lemma 3.2. *i)* $d_0 \equiv P'\Omega^{-1}V = O_p(1)$ *and* $\lim_n E\left[d_0 d_0'\right] = D$,
ii) $d_1 \equiv P_M'\Omega_M^{-1}V_M - P'\Omega^{-1}V = O_p(\|\sigma_{1M}\|)$ *and* $E\left[d_1 d_1'\right] = -H_{11} + O(n^{-1})$,
iii) $d_2 \equiv P_M'(I - W_M)\Omega_M^{-1}(I - W_M)V_M = O_p(M^{-2q})$,
iv) $d_3 \equiv -P_M'(I - W_M)\Omega_M^{-1}V_M - P_M'\Omega_M^{-1}(I - W_M)V_M = O_p(M^{-q})$,
v) $d_4 \equiv \left(\hat{P}_M - \check{P}_M\right)' W_M \Omega_M^{-1} W_M V_M = O_p(M/n)$,
vi) $d_5 \equiv \left(\check{P}_M - P_M\right)' W_M \Omega_M^{-1} W_M V_M = O_p(M/n^{1/2})$,

vii) $d_6 \equiv \left(\hat{P}_M - P_M\right)' W_M \Omega_M^{-1}(\Omega_M - \hat{\Omega}_M)\Omega_M^{-1} W_M V_M = o_p(M/n^{1/2})$,
viii) $d_7 \equiv P_M' W_M \Omega_M^{-1}(\Omega_M - \hat{\Omega}_M)\Omega_M^{-1} W_M V_M = O_p(M/n^{1/2})$,
ix) $d_8 \equiv \hat{P}_M' W_M B W_M V_M = o_p(M/n^{1/2})$,
x) $d_9 \equiv \hat{P}_M' W_M \hat{\Omega}_M^{-1} W_M \left(\tilde{V}_M - V_M\right) = o_p(M/n^{1/2})$ *where*

$$\tilde{v}_{t,i} = \begin{cases} \varepsilon_{t+m}\left(y_{t+1-i} - \bar{y}\right) & \text{if } i \leq t \\ 0 & \text{otherwise} \end{cases},$$

$\tilde{\psi}_{t,M} = \left(\tilde{v}_{t,1}', ..., \tilde{v}_{t,M}'\right)'$ *and* $\tilde{V}_M = n^{-1/2} \sum_{t=1}^{n-m} \tilde{\psi}_{t,M}$.

Proof. For i) note that $E\left[d_0\right] = 0$. Using Lemma (2.2iii)

$$\begin{aligned} E\left[d_0 d_0'\right] &= \frac{1}{n}\sum\nolimits_{t,s=1}^{n-m}\sum\nolimits_{j_1,\ldots,j_4=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2}\gamma_{t-s}^{\varepsilon}\Gamma_{t-s-j_2+j_3}^{yy}\vartheta_{j_3,j_4}\Gamma_{-j_4}^{yx} \\ &= \sum\nolimits_{l=-m+1}^{m-1}\frac{n-|l|}{n}\sum\nolimits_{j_1,\ldots,j_4=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2}\gamma_l^{\varepsilon}\Gamma_{l+j_3-j_2}^{yy}\vartheta_{j_3,j_4}\Gamma_{-j_4}^{yx} \\ &\to P'\Omega^{-1}P = D \text{ as } n\to\infty \end{aligned}$$

where the second line follows from the fact that $\gamma_l^{\varepsilon} = 0$ for $l \geq m$ and $\sum_{l=-m+1}^{m-1}\gamma_l^{\varepsilon}\Gamma_{l+j_3-j_2}^{yy} = \omega_{j_2,j_3}$.

For ii) write $d_1 = P_M'\Omega_M^{-1}V_M - P'\Omega^{-1}V$. Consider

$$\begin{aligned} E\left[d_1 d_1'\right] &= P_M'\Omega_M^{-1}E\left[V_M V_M'\right]\Omega_M^{-1}P_M - P'\Omega^{-1}E\left[VV_M'\right]\Omega_M^{-1}P_M \\ &\quad - P_M'\Omega_M^{-1}E\left[V_M V'\right]\Omega^{-1}P + P'\Omega^{-1}E\left[VV'\right]\Omega^{-1}P \end{aligned}$$

where the i,j-th $p\times p$ block of $E\left[V_M V_M'\right]$ is $n^{-1}\sum_{t,s=1}^{n-m}E\left[v_{t,i}v_{s,j}\right] = n^{-1}\sum_{t,s=1}^{n-m}\gamma_{t-s}^{\varepsilon}\Gamma_{t-s+i-j}^{yy} = \omega_{i,j} + O(n^{-1})$ where the $O\left(n^{-1}\right)$ term is uniform in i and j. The same argument shows that the i,j-th $p\times p$ block of the $\infty\times Mp$ matrix $E\left[VV_M'\right]$ is $\omega_{i,j}+o(n^{-1})$. It then follows that $E\left[V_M V_M'\right]\Omega_M^{-1} = I_{Mp}+O(n^{-1})$ and $E\left[V_M V'\right]\Omega^{-1} = [I_{Mp}, \mathbf{0}_{Mp\times\infty}] + O(n^{-1})$ with a similar expression for $\Omega^{-1}E\left[VV_M'\right]$. This shows that

$$E\left[d_1 d_1'\right] = P'\Omega^{-1}P - P_M'\Omega_M^{-1}P_M + O(n^{-1}) = -H_{11} + O(n^{-1}).$$

For iii) consider

$$\begin{aligned} E\left[d_2 d_2'\right] &= \frac{1}{n}\sum\nolimits_{t,s=1}^{n-m}\sum\nolimits_{j_1,..,j_4=1}^{M}\Big[\Gamma_{j_1}^{xy}(1-k\left(j_1/M\right))\vartheta_{j_1,j_2}^{M}(1-k\left(j_2/M\right)) \\ &\quad \times E(v_{t,j_2}v_{s,j_3}')(1-k\left(j_3/M\right))\vartheta_{j_3,j_4}^{M}(1-k\left(j_4/M\right))\Gamma_{j_4}^{yx}\Big] \\ &= M^{-4q}\sum_{j_1,..,j_4=1}^{M}\Bigg[\left|j_1\right|^q\Gamma_{j_1}^{xy}\frac{(1-k\left(j_1/M\right))}{\left|j_1\right|^q M^{-q}}\vartheta_{j_1,j_2}^{M}\left|j_2\right|^q\frac{(1-k\left(j_2/M\right))}{\left|j_2\right|^q M^{-q}} \\ &\quad \times\sum\nolimits_{l=-m+1}^{m-1}\tfrac{n-|l|}{n}\gamma_l^{\varepsilon}\Gamma_{l-j_2+j_3}^{yy}\left|j_3\right|^q\frac{(1-k\left(j_3/M\right))}{\left|j_3\right|^q M^{-q}}\vartheta_{j_3,j_4}^{M}\frac{(1-k\left(j_4/M\right))}{\left|j_4\right|^q M^{-q}}\left|j_4\right|^q\Gamma_{j_4}^{yx}\Bigg] + o(n^{-1}). \end{aligned}$$

Using the fact that $\left|\frac{(1-k(j/M))}{|j|^q M^{-q}}\right| < C$ for some $C < \infty$, $|1-k(j_2/M)| \leq 2$ and $\left\|\sum_{l=-m+1}^{m-1} \frac{n-|l|}{n} \gamma_l^{\varepsilon} \Gamma_{l-j_2+j_3}^{yy}\right\|$ is uniformly bounded in j_2 and j_3 leads to

$$\left\|E\left[d_2 d_2'\right]\right\| \leq CM^{-4q}\left(\sum_{j_1,j_2=1}^{M} |j_1|^q |j_2|^q \left\|\Gamma_{j_1}^{xy} \vartheta_{j_1,j_2}^M\right\|\right)^2$$

where $\sum_{j_1,j_2=1}^{M} |j_1|^q |j_2|^q \left\|\Gamma_{j_1}^{xy} \vartheta_{j_1,j_2}^M\right\| \leq C \sum_{j_1=1}^{M} |j_1|^{2q} \left\|\Gamma_{j_1}^{xy}\right\| = O(1)$ follows from Lemmas 2.6vii) and 2.7v). Thus $E\left[d_2 d_2'\right] = O(M^{-4q})$.

For iv) write $d_3 = d_{31} + d_{32}$ where

$$d_{31} = \frac{1}{\sqrt{n}} \sum_{t=1}^{n-m} \sum_{j_1,j_2=1}^{M} \Gamma_{j_1}^{xy} \left(1-k\left(j_1/M\right)\right) \vartheta_{j_1,j_2}^M k\left(j_2/M\right) v_{t,j_2} \tag{3.2}$$

and

$$d_{32} = \frac{1}{\sqrt{n}} \sum_{t=1}^{n-m} \sum_{j_1,j_2=1}^{M} \Gamma_{j_1}^{xy} k\left(j_1/M\right) \vartheta_{j_1,j_2}^M \left(1-k\left(j_2/M\right)\right) v_{t,j_2} \tag{3.3}$$

such that

$$M^q E\left\|d_{31}\right\| \leq \sum_{j_1,j_2=1}^{M} |j_1|^q \left\|\Gamma_{j_1}^{xy}\right\| \left\|\vartheta_{j_1,j_2}^M\right\| \left|\frac{1-k\left(j_1/M\right)}{|j_1|^q M^{-q}}\right| E\left\|\frac{1}{\sqrt{n}} \sum_{t=1}^{n-m} v_{t,j_2}\right\| = O(1).$$

where

$$E\left\|\frac{1}{\sqrt{n}} \sum_{t=1}^{n-m} v_{t,j_2}\right\| = O(1)$$

uniformly in j_2 because

$$E\left[\left\|n^{-1/2} \sum_{t=1}^{n-m} v_{t,j_2}\right\|^2\right] = \sum_{l=-m+1}^{m-1} \frac{n-m-|l|}{n} \gamma_l^{\varepsilon} \Gamma_l^{yy} = O(1) \tag{3.4}$$

uniformly in j_2 by Lemma 2.2iii). The result now follows from the Markov inequality. The same arguments, using Lemmas 2.6vii) and 2.7v) to bound $\sum_{j_1,j_2=1}^{M} |j_2|^q \left\|\Gamma_{j_1}^{xy}\right\| \left\|\vartheta_{j_1,j_2}^M\right\|$ apply to d_{32}.

For v) note that

$$\begin{aligned} \hat{\Gamma}_j^{xy} - \check{\Gamma}_j^{xy} &= \frac{n-\max\left(r-m,j\right)}{n}(\bar{x}-\mu_x)(\bar{y}-\mu_y)' \\ &\quad +(\bar{x}-\mu_x)n^{-1} \sum_{t=\max(r-m,j)+1}^{n-m} (y_{t+1-j}-\mu_y)' + n^{-1} \sum_{t=\max(r-m,j)+1}^{n-m} (x_{t+m}-\mu_x)(\bar{y}-\mu_y)' \end{aligned}$$

such that d_4 can be analyzed by considering

$$\begin{aligned} d_{41} &= (\bar{x}-\mu_x)(\bar{y}-\mu_y)' n^{-1/2} \sum_{j_1,j_2=1}^{M} \sum_{t=1}^{n-m} \frac{n-\max\left(r-m,j_1\right)}{n} k\left(j_1/M\right) \vartheta_{j_1,j_2}^M k\left(j_2/M\right) v_{t,j_2}, \\ d_{42} &= (\bar{x}-\mu_x)' n^{-3/2} \sum_{j_1,j_2=1}^{M} \sum_{t=\max(1,j_2)}^{n-m} (y_{t+1-j_1}-\mu_y) k\left(j_1/M\right) \vartheta_{j_1,j_2}^M k\left(j_2/M\right) \sum_{t=1}^{n-m} v_{t,j_2} \end{aligned}$$

and with the remaining terms collected in d_{43}. Then

$$\|d_{41}\| \leq \|\bar{x}-\mu_x\| \left\|\bar{y}-\mu_y\right\| \left\| n^{-1/2} \sum\nolimits_{j_1,j_2=1}^{M} \sum\nolimits_{t=1}^{n-m} k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) v_{t,j_2} \right\|$$

The third term in the previous display can be bounded in expectation by considering

$$\begin{aligned} & E\left\| n^{-1/2} \sum\nolimits_{j_1,j_2=1}^{M} \sum\nolimits_{t=1}^{n-m} k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) v_{t,j_2} \right\| \\ \leq & \sum\nolimits_{j_1,j_2=1}^{M} \left\|\vartheta_{j_1,j_2}^{M}\right\| E\left\| n^{-1/2} \sum\nolimits_{t=1}^{n-m} v_{t,j_2} \right\| = O\left(M\right) \end{aligned}$$

by Lemma 2.7iv) and using $\|\bar{x}-\mu_x\| \left\|\bar{y}-\mu_y\right\| = O_p(n^{-1})$ shows that $d_{41} = O_p(M/n)$. For d_{42} write

$$\begin{aligned} \|d_{42}\| \leq & \|\bar{x}-\mu_x\| \\ & \times \left\| n^{-3/2} \sum\nolimits_{j_1,j_2=1}^{M} \sum\nolimits_{t_1=1}^{n-m} \sum\nolimits_{t_2=\max(r-m,j_1)+1}^{n-m} (y_{t_2+1-j_1}-\mu_y) k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) v_{t_1,j_2} \right\|. \end{aligned}$$

Then the second term in the previous display is bounded in expectation by considering

$$\begin{aligned} & \sum\nolimits_{j_1,j_2=1}^{M} \left\|\vartheta_{j_1,j_2}^{M}\right\| \left(E\left\| n^{-1} \sum\nolimits_{t_2=\max(r-m,j_1)+1}^{n-m} (y_{t_2+1-j_1}-\mu_y) \right\|^2 E\left\| n^{-1/2} \sum\nolimits_{t_1=1}^{n-m} v_{t_1,j_2} \right\|^2 \right)^{1/2} \\ = & O(M/\sqrt{n}) \end{aligned}$$

where $\sum_{j_1,j_2=1}^{M} \left\|\vartheta_{j_1,j_2}^{M}\right\| = O\left(M\right)$ by Lemma 2.7iv) and by (3.4)

$$E\left\| n^{-1/2} \sum\nolimits_{t_1=1}^{n-m} v_{t_1,j_2} \right\|^2 = O(1)$$

uniformly in j_2. Further,

$$E\left\| n^{-1} \sum\nolimits_{t_2=\max(r-m,j_1)+1}^{n-m} (y_{t_2+1-j_1}-\mu_y) \right\|^2 = O(n^{-1})$$

uniformly in j_1. Form $\|\bar{x}-\mu_x\| = O_p\left(n^{-1/2}\right)$ it follows that $d_{42} = O_p\left(n^{-1/2}\right) O_p(M/n^{1/2}) = O_p\left(M/n\right).$ The result that $d_{43} = O_p\left(M/n\right)$ follows in the same way as the result for d_{42} by noting that

$$n^{-1} \sum\nolimits_{t=\max(r-m,j)+1}^{n-m} (x_{t+m}-\mu_x) = O_p\left(n^{-1/2}\right).$$

For the proof of vi) decompose

$$\begin{aligned} d_5 = & \sum_{j_1,j_2=1}^{M} n^{-1/2} \sum\nolimits_{t=1}^{n-m} \left(\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right) k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) v_{t,j_2} \\ = & n^{-3/2} \sum_{j_1,j_2=1}^{M} \sum\nolimits_{t=\max(r-m,j_1)+1}^{n-m} \check{w}_{t,j_1} k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) \sum\nolimits_{t=1}^{n-m} v_{t,j_2} \\ & \sum_{j_1,j_2=1}^{M} -\frac{m+\max\left(r-m,j_1\right)}{n} \Gamma_{j_1}^{xy} k\left(j_1/M\right) \vartheta_{j_1,j_2}^{M} k\left(j_2/M\right) n^{-1/2} \sum\nolimits_{t=1}^{n-m} v_{t,j_2} \\ \equiv & d_{51}+d_{52}. \end{aligned}$$

For d_{52} note that

$$E\left[\|d_{52}\|\right]=C\frac{M}{n}\sum_{j_1,j_2=1}^{M}\left\|\Gamma_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|E\left[\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_2}\right\|\right]=O\left(M/n\right)$$

by Lemmas 2.3iii) and 2.7iv) and (3.4) such that $d_{52}=O_p\left(M/n\right)$. Finally, $d_{51}=O_p\left(M/n^{-1/2}\right)$ follows directly from Lemma A.12x) in the main text.

For vii) define the terms

$$\begin{aligned} d_{61} &= \left(\hat{P}_M-\check{P}_M\right)' W_M\Omega_M^{-1}(\Omega_M-\hat{\Omega}_M)\Omega_M^{-1}W_MV_M \\ d_{62} &= \left(\check{P}_M-P_M\right)' W_M\Omega_M^{-1}(\Omega_M-\hat{\Omega}_M)\Omega_M^{-1}W_MV_M \end{aligned}$$

such that $d_6=d_{61}+d_{62}$. Note that $\hat{\Omega}_M$ can be replaced w.p.a 1 by $\hat{\Omega}_M^*$ by Lemma 2.10. For d_{61} note that

$$\begin{aligned} \|d_{61}\| &\le \sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{\Delta}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \\ &+\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \end{aligned} \tag{3.5}$$

where the first line in (3.5) can be bounded by

$$\sup_{j_1,j_2}\left\|\vartheta_{j_1,j_2}^{M}\right\|\sum_{j_1=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\sum_{j_2,..,j_4=1}^{M}\left\|\hat{\omega}_{j_2,j_3}^{\Delta}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|$$

By Lemma 2.8ii) the term $\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|$ is $O(n^{-1})$ in expectation independently of j_1 such that

$$\sum_{j_1=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|=O_p\left(M/n\right).$$

By Lemma 2.9vi) $E\left[\left\|\hat{\omega}_{j_2,j_3}^{\Delta}\right\|^2\right]=O\left(n^{-1}\right)$ while $E\left[\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|^2\right]=O\left(1\right)$ uniformly in j_4 by (3.4) such that by the Cauchy-Schwartz inequality and Lemma 2.7iv) it follows that

$$\sum_{j_2,..,j_4=1}^{M}\left\|\hat{\omega}_{j_2,j_3}^{\Delta}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|=O_p\left(M^2/n^{1/2}\right).$$

Then the first term in d_{61} is $O\left(M/n\right)O_p\left(M^2/n^{1/2}\right)=O_p\left(M^3/n^{3/2}\right)=o_p\left(M/n^{1/2}\right)$. For the second term in (3.5) use the decomposition $\hat{\omega}_{j_2,j_3}^{R}=\sum_{j=1}^{4}\hat{\omega}_{j_2,j_3}^{R_j}$ and consider each term separately. For R_1

note that

$$\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R_1}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \tag{3.6}$$
$$\leq C\frac{M}{n}\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|$$
$$= O\left(M/n\right)O_p\left(M^2/n\right)=o_p\left(M/n\right)$$

by Lemmas 2.8 and 2.9ii). For R_2 use

$$\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R_2}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \tag{3.7}$$
$$\leq \sup_{j_1,j_2}\left\|\vartheta_{j_1,j_2}^{M}\right\|\xi_n\sum_{j_1=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\sum_{j_2,..,j_4=1}^{M}\left\|\hat{\omega}_{j_2,j_3}^{\bar{R}_2}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|$$
$$= O_p\left(M/n\right)O_p\left(n^{-1/2}\right)O_p\left(M^2/n^{1/2}\right)=O_p\left(M^3/n^2\right)=o_p\left(M/n\right).$$

by Lemmas 2.8 and 2.9iii) and the Markov and Cauchy-Schwartz inequalities. Next,

$$\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R_3}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|=O_p\left(M/n^{3/2}\right)=o_p\left(M/n\right) \tag{3.8}$$

by Lemmas 2.8 and 2.9iv) and

$$\sum_{j_1,..,j_4=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R_4}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \tag{3.9}$$
$$\leq \sup_{j_1,j_2}\left\|\vartheta_{j_1,j_2}^{M}\right\|\sum_{j_1=1}^{M}\left\|\hat{\Gamma}_{j_1}^{xy}-\check{\Gamma}_{j_1}^{xy}\right\|\sum_{j_2,..,j_4=1}^{M}\left\|\hat{\omega}_{j_2,j_3}^{R_4}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|$$
$$= O_p\left(M/n\right)O_p\left(M^2/n\right)=o_p\left(M/n\right).$$

by Lemmas 2.8 and 2.9v). Then it follows form (3.6), (3.7), (3.8) and (3.9) that

$$\left\|d_{61}\right\|=O_p(M^3/n^{3/2})=o_p\left(M/n^{1/2}\right). \tag{3.10}$$

For d_{62} first consider

$$\sum_{j_1,..,j_4=1}^{M}\left\|\check{\Gamma}_{j_1}^{xy}-\Gamma_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R}\right\|\left\|\vartheta_{j_3,j_4}^{M}\right\|\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|=o_p\left(M/n^{1/2}\right) \tag{3.11}$$

by the same inequalities as (3.6), (3.7), (3.8) and (3.9) except that $\left\|\hat{\Gamma}_{j_1}^{xy} - \check{\Gamma}_{j_1}^{xy}\right\|$ is replaced by $\left\|\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right\| = O_p\left(n^{-1/2}\right)$. Next consider

$$\begin{aligned}(3.12)\qquad & E\left\|\sum_{j_1,..,j_4=1}^{M}\left(\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right)k\left(j_1/M\right)\vartheta_{j_1,j_2}^{M}\hat{\omega}_{j_2,j_3}^{\Delta}\vartheta_{j_3,j_4}^{M}k\left(j_4/M\right)n^{-1/2}\textstyle\sum_{t=1}^{n-m}v_{t,j_4}\right\| \\ \leq & \sum_{j_1=1}^{M}\left(E\left[\left\|\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right\|^2\right]E\left[\left\|\sum_{j_2,..,j_4=1}^{M}\vartheta_{j_1,j_2}^{M}\hat{\omega}_{j_2,j_3}^{\Delta}\vartheta_{j_3,j_4}^{M}n^{-1/2}\textstyle\sum_{t=1}^{n-m}v_{t,j_4}\right\|^2\right]\right)^{1/2} \\ = & O\left(M\right)O\left(n^{-1/2}\right)O\left(M/n^{1/2}\right) = O\left(M^2/n\right) = o\left(M/n^{1/2}\right)\end{aligned}$$

where $E\left[\left\|\check{\Gamma}_{j_1}^{xy} - \Gamma_{j_1}^{xy}\right\|^2\right] = O\left(n^{-1}\right)$ uniformly in j_1 by Lemma 2.8 and

$$(3.13)\qquad E\left[\left\|\sum_{j_2,..,j_4=1}^{M}\vartheta_{j_1,j_2}^{M}\hat{\omega}_{j_2,j_3}^{\Delta}\vartheta_{j_3,j_4}^{M}n^{-1/2}\textstyle\sum_{t=1}^{n-m}v_{t,j_4}\right\|^2\right] = O\left(M^2/n\right)$$

by the same argument as in (3.15) below and in Lemma A.12xi) in the main text because Lemma 2.7iv) can be used to uniformly bound $\sum_{j_2=1}^{M}\left\|\vartheta_{j_1,j_2}^{M}\right\|$. Combining (3.11) and (3.12) shows that $d_{62} = o_p\left(M/n^{1/2}\right)$. Together with (3.10) these results imply that $d_6 = o_p\left(M/n^{1/2}\right)$.

For viii) let $a_{j_2}^{M} = \sum_{j_1=1}^{M}\Gamma_{j_1}^{xy}k\left(j_1/M\right)\vartheta_{j_1,j_2}^{M}$. The term d_7 is bounded by

$$\begin{aligned}(3.14)\qquad \left\|d_7\right\| \leq & \left\|\textstyle\sum_{j_2,...,j_4=1}^{M}a_{j_2}^{M}\hat{\omega}_{j_2j_3}^{\Delta}\vartheta_{j_3,j_4}^{M}k\left(j_4/M\right)n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\| \\ & + \sum_{j_1,..,j_3=1}^{M}\left\|\Gamma_{j_1}^{xy}\right\|\left\|\vartheta_{j_1,j_2}^{M}\right\|\left\|\hat{\omega}_{j_2,j_3}^{R}\right\|\left\|\textstyle\sum_{j_4=1}^{M}\vartheta_{j_3,j_4}^{M}k\left(j_4/M\right)n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|\end{aligned}$$

where because $E\left\|n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|^2$ is bounded and by Lemma 2.9 and arguments similar to the proof of Lemma 3.2vii) it follows that the second term in (3.14) is $O_p\left(n^{-1/2}\right) + O_p\left(Mn^{-1}\right)$. Furthermore, for r_1 and r_2 defined in the proof of Lemma 2.8 one obtains

$$\begin{aligned}(3.15)\qquad & E\left\|\textstyle\sum_{j_2,...,j_4=1}^{M}a_{j_2}^{M}\hat{\omega}_{j_2j_3}^{\Delta}\vartheta_{j_3,j_4}^{M}k\left(j_4/M\right)n^{-1/2}\sum_{t=1}^{n-m}v_{t,j_4}\right\|^2 \\ = & \; n^{-3}\sum_{j_2,...,j_7=1}^{M}\sum_{t,s=j_4,j_5}^{n-m}\sum_{l_1,l_2=-m+1}^{m-1}\sum_{t_2,t_3=r_1}^{r_2}\left\{\gamma_{l_1}^{\varepsilon}\gamma_{l_2}^{\varepsilon}\right. \\ & \left.\times\operatorname{tr}E\left[a_{j_2}^{M}\breve{w}_{t_2-j_2,j_3+l_1-j_2}^{y}\vartheta_{j_3,j_4}^{M}k\left(j_4/M\right)v_{t,j_4}v_{s,j_5}'k\left(j_5/M\right)\vartheta_{j_5,j_6}^{M}\breve{w}_{t_3-j_7,j_6+l_2-j_7}^{y\prime}a_{j_7}^{M\prime}\right]\right\}.\end{aligned}$$

Using Lemma 2.1, (3.15) can be written as

$$
\begin{aligned}
& n^{-3}\sum_{j_2,\ldots,j_7=1}^{M}\sum_{t,s=j_4,j_5}^{n-m}\sum_{l_1,l_2=-m+1}^{m-1}\sum_{t_2,t_3=r_1}^{r_2}\gamma^{\varepsilon}_{l_1}\gamma^{\varepsilon}_{l_2}\left\{\left\{\left(\operatorname{vec}\vartheta^{M\prime}_{j_5,j_6}\right)' E\left[v_{s,j_5}\otimes \breve{w}^{y\prime}_{t_2-j_7,j_6+l_1-j_7}\right]\right.\right.\\
& \times\left(I\otimes a^{M\prime}_{j_7}a^{M}_{j_2}\right)E\left[v'_{t,j_4}\otimes\breve{w}^{y}_{t_3-j_3,j_3+l_1-j_2}\right]\operatorname{vec}\vartheta^{M}_{j_3,j_4}\Big\}\\
& +\gamma^{\varepsilon}_{l_1}\gamma^{\varepsilon}_{l_2}\operatorname{tr}\left[\left(\vartheta^{M}_{j_3,j_4}E\left[v_{t,j_4}v'_{s,j_5}\right]\vartheta^{M}_{j_5,j_6}\right)\left(E\left[\breve{w}^{y\prime}_{t_2-j_7,j_6+l_1-j_7}a^{M\prime}_{j_7}a^{M}_{j_2}\breve{w}^{y}_{t_3-j_2,j_3+l_1-j_2}\right]\right)\right]\\
& +\gamma^{\varepsilon}_{l_1}\gamma^{\varepsilon}_{l_2}\operatorname{tr}\left[\left(E\left[a^{M}_{j_7}\breve{w}^{y}_{t_3-j_7,j_6+l_2-j_7}\otimes\vartheta^{M}_{j_3,j_4}v_{t,j_4}\right]\right)\left(E\left[\operatorname{vec}(v'_{s,j_5}\vartheta^{M}_{j_5,j_6})\operatorname{vec}(\breve{w}^{y\prime}_{t_2-j_2,j_3+l_1-j_2}a^{M}_{j_2}{}')'\right]\right)\right]+\mathcal{K}_4\Big\}.
\end{aligned}
$$

From Lemma 2.2i) it follows that

$$
E\left[v_{t,j_4}\otimes\breve{w}^{y\prime}_{t_2-j_2,j_3+l_1-j_2}\right]=((\operatorname{vec}(\Gamma^{yy}_{t_2-t+j_4-j_3-l_1})\otimes\left(\Gamma^{\varepsilon y}_{t-t_2+j_2}\right)')+K_{pp}(\Gamma^{\varepsilon y}_{t-t_2+j_3+l_1}\otimes\Gamma^{yy}_{t-j_4-t_2+j_2})+\mathcal{K}^1_4).
$$

As in the proof of Lemma A.12xi) of the main paper, the term

$$
\begin{aligned}
(3.16)\quad & E\left[d_7\right]\\
= \; & n^{-3/2}\sum_{j_2,\ldots,j_4=1}^{M}\sum_{t=1,t_2=r_1}^{n,r_2}\left(I\otimes a^{M}_{j_2}\right)\left(\operatorname{vec}\left(\textstyle\sum_{l_1=-m+1}^{m-1}\gamma^{\varepsilon}_{l_1}\Gamma^{yy}_{t_2-t+j_4-j_3+l_1}\right)'\otimes\left(\Gamma^{\varepsilon y}_{t-t_2+j_2}\right)\right)\operatorname{vec}\vartheta^{M}_{j_3,j_4}\\
= \; & O(M/n^{1/2})
\end{aligned}
$$

is the largest term. This implies that $E\left[\|d_7\|\right]=O(M/n^{1/2})$ and that $E\left[\|d_7-E\left[d_7\right]\|^2\right]=o\left(M^2/n\right).$ Then

$$
\begin{aligned}
& \|E\left[\left(d_7-E\left[d_7\right]\right)\left(d_5-E\left[d_5\right]\right)\right]\|\\
\leq\; & \left(E\left[\|d_7-E\left[d_7\right]\|^2\right]E\left[\|d_5-E\left[d_5\right]\|^2\right]\right)^{1/2}=o\left(M^2/n\right).
\end{aligned}
$$

This shows that $E\left[d_5d_7'\right]=E\left[d_5\right]E[d_7]'+o\left(M^2/n\right).$

For ix) recall that $B=\hat{\Omega}^{-1}_M(\Omega_M-\hat{\Omega}_M)\Omega^{-1}_M(\Omega_M-\hat{\Omega}_M)\Omega^{-1}_M$ and write

$$
d_8=\left(\hat{P}_M-P_M\right)'W_MBW_MV_M+P'_MW_MBW_MV_M. \tag{3.17}
$$

Then, similarly to the proof of Lewis and Reinsel (1985, Theorem 2),

$$
\begin{aligned}
(3.18)\qquad \left\|P'_MW_MBW_MV_M\right\| \;&\leq\; \|P_M\|\left\|\hat{\Omega}^{-1}_M\right\|_2^2\left\|\Omega_M-\hat{\Omega}_M\right\|^2M^{1/2}\left\|\Omega^{-1}_M\right\|_2^2\left\|M^{-1/2}V_M\right\|\\
&=\; O_p\left(M^2/n\right)M^{1/2}O_p\left(1\right)=O_p\left(M/n^{1/2}\right)O_p\left(M^{3/2}/n^{1/2}\right)\\
&=\; o_p\left(M/n^{1/2}\right)
\end{aligned}
$$

because $M^3/n \to 0$ such that $M^{3/2}/n^{1/2} = O\left(\sqrt{M^3/n}\right) = o(1)$, $\|P_M\| = O(1)$, $\left\|\hat{\Omega}_M^{-1}\right\|_2^2 = O_p(1)$ by Lemma 2.11, $\left\|\Omega_M - \hat{\Omega}_M\right\|^2 = O_p\left(M^2/n\right)$ by Lemma 2.10 and

$$\begin{aligned} E\left\|M^{-1/2}V_M\right\|^2 &= M^{-1}E\left[n^{-1}\sum_{t,s=1}^{n}\varepsilon_{t+m}\varepsilon_{s+m}\operatorname{tr}\left(z_{t,M}'z_{s,M}\right)\right] \\ &= \sum_{l=-m+1}^{m-1}\gamma^{\varepsilon}(l)\operatorname{tr}\left(\Gamma_l^{yy}\right) + o(1) = O(1) \end{aligned}$$

such that $\left\|M^{-1/2}V_M\right\|^2 = O_p(1)$.

Similarly,

$$\begin{aligned} \left\|\left(\hat{P}_M - P_M\right)' W_M B W_M V_M\right\| &\leq \left\|\hat{P}_M - P_M\right\| \left\|\hat{\Omega}_M^{-1}\right\|_2^2 \left\|\Omega_M - \hat{\Omega}_M\right\|^2 \left\|\Omega_M^{-1}\right\|_2^2 \|V_M\| \\ &= O_p\left(M/n^{1/2}\right) O_p(1)\, O_p\left(M^2/n\right) O_p(M) = o_p\left(M/n^{1/2}\right) \end{aligned}$$

and that $d_8 = o_p\left(M/n^{1/2}\right)$.

For x) note that

$$\begin{aligned} d_9 &= \hat{P}_M' W_M \hat{\Omega}_M^{-1} W_M \left(\tilde{V}_M - V_M\right) \\ &= \hat{P}_M' W_M \hat{\Omega}_M^{-1} W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_t \left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right] \\ &\quad - \sum_{j_1,j_2=1}^{M} \hat{\Gamma}_{j_1}^{xy} \hat{\vartheta}_{j_1,j_2}^{M} k(j_1/M)\, k(j_2/M)\, n^{-1/2} \sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2} - \bar{y}\right) \\ &= d_{91} + d_{92}. \end{aligned}$$

Then, for d_{91}, one obtains

$$\begin{aligned} d_{91} &= \left(\hat{P}_M - P_M\right)' W_M \left(\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right) W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m}\left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right] \\ &\quad + \left(\hat{P}_M - P_M\right)' W_M \Omega_M^{-1} W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m}\left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right] \\ &\quad + P_M' W_M \left(\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right) W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m}\left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right] \\ &\quad + P_M' W_M \Omega_M^{-1} W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m}\left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right] \end{aligned} \tag{3.19}$$

where $\left\|\hat{P}_M - P_M\right\| = O_p\left(M/n^{1/2}\right)$ by Lemma 2.8iii) and vii), $\left\|\hat{\Omega}_M^{-1} - \Omega_M^{-1}\right\|_2 = O_p\left(M/n^{1/2}\right)$ by Lemma 2.11, and $\left\|n^{-1/2}\sum_{t=1}^{n-m}\varepsilon_{t+m}\left[\mathbf{1}_M \otimes \left(\bar{y} - \mu_y\right)\right]\right\| = O_p\left(M/n^{1/2}\right)$. Then the first three terms in 3.19 are $o_p\left(M/n^{1/2}\right)$ while

$$\begin{aligned}
&\left\| P_M' W_M \Omega_M^{-1} W_M n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m} \left[\mathbf{1}_M \otimes (\bar{y}-\mu_y)\right] \right\| \\
= &\left\| (\bar{y}-\mu_y) \sum_{j_1,j_2=1}^{M} \Gamma_{j_1}^{xy} \vartheta_{j_1,j_2}^{M} k(j_1/M)\, k(j_2/M)\, n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m} \right\| \\
\leq &\left\| (\bar{y}-\mu_y) \right\| \sum_{j_1,j_2=1}^{M} \left\| \Gamma_{j_1}^{xy} \right\| \left\| \vartheta_{j_1,j_2}^{M} \right\| \left\| n^{-1/2} \sum_{t=1}^{n-m} \varepsilon_{t+m} \right\| = O_p\left(n^{-1/2}\right).
\end{aligned}$$

This shows that $d_{91} = o_p\left(M/n^{-1/2}\right)$. For d_{92} replace $n^{-1/2}\sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2}-\bar{y}\right)$ with

$$n^{-1/2} \sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2}-\mu_y\right)$$

where the difference involving $n^{-1/2}\sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(\bar{y}-\mu_y\right)$ is of lower order by the same arguments as in d_{91}. Then,

$$\begin{aligned}
\|d_{92}\| &\leq \sum_{j_1,j_2=1}^{M} \left\| \hat{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\vartheta}_{j_1,j_2}^{M} \right\| \left\| n^{-1/2} \sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2}-\mu_y\right) \right\| \\
&\leq \left(\sum_{j_2=1}^{M} \left(\sum_{j_1=1}^{M} \left\| \hat{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\vartheta}_{j_1,j_2}^{M} \right\| \right)^2 \right)^{1/2} \left(\sum_{j_2=1}^{M} \left\| n^{-1/2} \sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2}-\mu_y\right) \right\|^2 \right)^{1/2}
\end{aligned}$$

by the Cauchy-Schwartz inequality. Then,

$$\left(\sum_{j_2=1}^{M} \left(\sum_{j_1=1}^{M} \left\| \hat{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\vartheta}_{j_1,j_2}^{M} \right\| \right)^2 \right)^{1/2} \leq \sum_{j_1,j_2=1}^{M} \left\| \hat{\Gamma}_{j_1}^{xy} \right\| \left\| \hat{\vartheta}_{j_1,j_2}^{M} \right\| = O_p(1)$$

by the same arguments as for d_{91}. Also,

$$\sum_{j_2=1}^{M} E\left[\left\| n^{-1/2} \sum_{t=1}^{j_2-1} \varepsilon_{t+m}\left(y_{t+1-j_2}-\mu_y\right) \right\|^2 \right] = \sum_{j_2=1}^{M} n^{-1} \sum_{l=-\min(m,j_2)+1}^{\min(m,j_2)-1} \gamma_l^{\varepsilon} \Gamma_l^{yy} = O(M/n)$$

such that by the Markov inequality $d_{92} = O_p\left(\sqrt{M/n}\right) = o_p\left(M/n^{1/2}\right)$. ■

4. Auxiliary Lemmas for Theorem 4.1

Lemma 4.1. *Let Assumptions B and C be satisfied. Let $\hat{\Sigma}_h$ be as defined (7.5) and let $\Sigma = E\left[u_t u_t'\right]$. Then,*

$$\left\| \hat{\Sigma}_{\hat{h}} - \Sigma \right\| = O_p(n^{-1/2}).$$

Proof. Recall H_n from Definition 1.2. Consider $\left\|\hat{\Sigma}_{\hat{h}}-\Sigma\right\|$ where

$$P\left(\left\|\hat{\Sigma}_{\hat{h}}-\Sigma\right\|>\epsilon\right)\leq P\left(\sup_{h\in H_n}\left\|\hat{\Sigma}_h-\Sigma\right\|>\epsilon\right)+o(1)$$

by Lemma 2.2 of Kuersteiner (2005) such that we focus on $\sup_{h\in H_n}\left\|\hat{\Sigma}_h-\Sigma\right\|$. By Hannan and Deistler (1988, Theorem 7.4.6) it follows that $\sup_{h\in H_n}\left\|\hat{\Sigma}_h-\Sigma\right\| = \sup_{h\in H_n}\left\|\Sigma_h-\Sigma\right\|(1+o_p(1))+O_p(n^{-2/3}\log n)$ where by the arguments in the proof of Lemma 2.7,

$$\sup_{h\in H_n}\left\|\Sigma_h-\Sigma\right\| = O(\sum_{j=h_{\min}}^{\infty}\left\|\pi_j\right\|)=o(n^{-1/2}) \tag{4.1}$$

by Hannan and Deistler (1988, Theorem 6.6.12). Thus, $\left\|\hat{\Sigma}_{\hat{h}}-\Sigma\right\|=O_p(n^{-1/2})$. ■

Lemma 4.2. *Let Assumptions B and C hold. Then,* $\sup_j\left\|\hat{\pi}_{j,\hat{h}}-\pi_j\right\|=O_p\left((\log n/n)^{1/2}\right)$.

Proof. See Kuersteiner (2005, Theorem 2.4). ■

Lemma 4.3. *Let Assumptions B, C and D be satisfied. Let* $\hat{\Gamma}^{yy}_{j,\hat{h}}=\Gamma\left(j,\hat{\pi}_{\hat{h}}\left(z\right)\right)$ *be as defined in Assumption C. Let* $\hat{\pi}_{\hat{h}}\left(\hat{h}\right)$ *be defined as in (7.4) and Step 10. Let* ν *be as defined in Assumption C. Then, for some* ν_* *such that* $\nu<\nu_*<1$ *it follows that*

$$\left\|\hat{\Gamma}^{yy}_{j,\hat{h}}-\Gamma^{yy}_j\right\|\leq j\nu^jO_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)+j^3\nu_*^jO_p\left(h^3_{\max}\left(\log n/n\right)^{3/2}\right)$$

uniformly in j *where the* $O_p\left(.\right)$ *does not depend on* j.

Proof. Note that

$$\sup_{|z|\leq 1}\left\|\hat{\pi}_{\hat{h}}(z)-\pi\left(z\right)\right\|\leq\sum_{j=1}^{\hat{h}}\left\|\hat{\pi}_{j,\hat{h}}-\pi_j\right\|+\sum_{j=h_{\min}}^{\infty}\left\|\pi_j\right\|=O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)$$

The first term is $O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)$ by Lemma 4.2 while the last term is $o\left(n^{-1/2}\right)$. By Assumption D it follows that

$$\begin{aligned}\left\|\hat{\Gamma}^{yy}_{j,\hat{h}}-\Gamma^{yy}_j\right\| &= \left\|\Gamma\left(j,\hat{\pi}_{\hat{h}}\left(z\right)\right)-\Gamma\left(j,\pi\left(z\right)\right)\right\|\\ &= j\nu^jO_p\left(\sup_{|z|\leq 1}\left\|\hat{\pi}_{\hat{h}}\left(z\right)-\pi\left(z\right)\right\|\right)+j^3\nu_*^jO_p\left(\sup_{|z|\leq 1}\left\|\hat{\pi}_{\hat{h}}\left(z\right)-\pi\left(z\right)\right\|^3\right)\\ &= j\nu^jO_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)+j^3\nu_*^jO_p\left(h^3_{\max}\left(\log n/n\right)^{3/2}\right)\end{aligned}$$

which establishes the result. ■

For the next lemma the following definitions are introduced. As in (7.7) a typical block of the matrix $\hat{\Omega}^*_{M,\hat{h}}$ is given by

$$\widehat{\tilde{\Gamma}}^{yy}_{j,\hat{h}} = \sum_{l=-m+1}^{m-1} \hat{\gamma}^{\varepsilon}_{\hat{\theta}}(l)\,\hat{\Gamma}^{yy}_{j-l,\hat{h}}$$

where $\hat{\Gamma}^{yy}_{j,\hat{h}}$ is the j-th autocovariance matrix computed from the $\mathrm{VAR}\left(\hat{h}\right)$ approximation of y_t and

$$\hat{\gamma}^{\varepsilon}_{\hat{\theta}}(l) = (2\pi)^{-1} \int_{-\pi}^{\pi} \hat{\sigma}^2_{\varepsilon} \left|\hat{\theta}\left(e^{i\lambda}\right)\right|^2 e^{i\lambda l} d\lambda.$$

Note that $\widehat{\tilde{\Gamma}}^{yy}_{j,\hat{h}}$ can be computed for j such that$|j| \leq k_{\max}$. Similarly define the population analog $\tilde{\Gamma}^{yy}_{j} = \sum_{l=-m+1}^{m-1} \gamma^{\varepsilon}_l \Gamma^{yy}_{j-l}$. For $h \leq k_{\max}$ let $\widehat{\tilde{\Gamma}}_{h,\hat{h}}$ the $hp \times hp$ matrix whose (m,n)th block is $\widehat{\tilde{\Gamma}}^{yy}_{n-m,\hat{h}}$ and

$$\widehat{\tilde{\Gamma}}'_{1,h,\hat{h}} = \left[\widehat{\tilde{\Gamma}}^{yy}_{1,\hat{h}}, ..., \widehat{\tilde{\Gamma}}^{yy}_{h,\hat{h}}\right].$$

Now, consider the following pseudo-regression coefficients

$$\widehat{\tilde{\pi}}_{h,\hat{h}}(h) = \left(\widehat{\tilde{\pi}}_{1,h}, ..., \widehat{\tilde{\pi}}_{h,h}\right) = \widehat{\tilde{\Gamma}}'_{1,h,\hat{h}} \widehat{\tilde{\Gamma}}^{-1}_{h,\hat{h}}$$

for $h \leq k_{\max}$. Also define $\widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{i\lambda}\right) = I - \sum_{j=1}^{h} \widehat{\tilde{\pi}}_{j,h} e^{i\lambda j}$ and $\tilde{\pi}_h\left(e^{i\lambda}\right) = I - \sum_{j=1}^{h} \tilde{\pi}_{j,h} e^{i\lambda j}$ where $\tilde{\pi}_{j,h}$ is defined in (2.6). Let $K_{\hat{h}}\left(e^{i\lambda}\right)$ be the spectral density with Fourier coefficients $\widehat{\tilde{\Gamma}}^{yy}_{j,\hat{h}} = (2\pi)^{-1} \int_{-\pi}^{\pi} K_{\hat{h}}\left(e^{i\lambda}\right) e^{i\lambda j} d\lambda$ and let the population analog be

$$K(e^{i\lambda}) = \sigma^2_{\varepsilon} \left|\theta\left(e^{i\lambda}\right)\right|^2 \pi\left(e^{i\lambda}\right)^{-1} \Sigma \pi'\left(e^{-i\lambda}\right)^{-1}.$$

Following Hannan and Deistler (1988, p. 258) define $\hat{\Sigma}_{h,\hat{h}} = (2\pi)^{-1} \int_{-\pi}^{\pi} \widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{-i\lambda}\right) K_{\hat{h}}\left(e^{i\lambda}\right) d\lambda$ and note that in the same way one can write $\Sigma_h = (2\pi)^{-1} \int_{-\pi}^{\pi} \tilde{\pi}_h\left(e^{-i\lambda}\right) K\left(e^{i\lambda}\right) d\lambda$ where Σ_h was defined as $\Sigma_h = E\left[u_{t,h} u'_{t,h}\right]$.

The next lemma establishes rates of convergence for components that enter into the difference between $\hat{\Omega}_{M,\hat{h}}$, $\hat{\Omega}_{k_{\max},\hat{h}}$ and Ω.

Lemma 4.4. *Using the definitions introduced above, and for $h \leq k_{\max}$ it holds uniformly in $j \leq h$ that*

i) $\left\|\widehat{\tilde{\Gamma}}^{yy}_{j,\hat{h}} - \tilde{\Gamma}^{yy}_{j}\right\| = |j|\,\nu^{|j|} O_p\left(h_{\max}(\log n/n)^{1/2}\right) + |j|^3\,\nu_*^{|j|} O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)$,

ii) $\left\|\widehat{\tilde{\pi}}_{j,h} - \tilde{\pi}_{j,h}\right\| = j^2 \nu^j O_p\left(h_{\max}(\log n/n)^{1/2}\right) + j^4 \nu_*^j O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)$,

iii) $\left\|\hat{\Sigma}^{-1}_{h,\hat{h}} - \hat{\Sigma}^{-1}_{k_{\max},\hat{h}}\right\| = h^2 \nu^h O_p\left(1 + h_{\max}(\log n/n)^{1/2}\right) + h^4 \nu_*^h O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)$,

iv) $\left\|\hat{\Sigma}^{-1}_{h,\hat{h}} - \Sigma^{-1}_{k_{\max},\hat{h}} - \Sigma^{-1}_h + \Sigma^{-1}\right\| = h^2 \nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right) + h^5 \nu_*^h O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)$,

v) $\sum_{s=1}^{h} \left\|\hat{\Sigma}^{-1}_{h,\hat{h}} \widehat{\tilde{\pi}}_{s,h} - \Sigma^{-1}_{k_{\max},\hat{h}} \widehat{\tilde{\pi}}_{s,k_{\max}} - \Sigma^{-1}_h \tilde{\pi}_{s,h} + \Sigma^{-1} \tilde{\pi}_s\right\|$

$= h^2\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right) + h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right),$

vi) $\left\|\hat{\Sigma}^{-1}_{k_{\max},\hat{h}} - \Sigma^{-1}\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right),$

vii) $\sum_{j=0}^{h}\left\|\hat{\tilde{\pi}}_{j,h} - \hat{\tilde{\pi}}_{j,h_{\max}}\right\| = h^2\nu^h O_p\left(1 + h_{\max}(\log n/n)^{1/2}\right) + h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right),$

viii) $\sum_{s=1}^{h}\left\|\hat{\tilde{\pi}}_{s,h} - \hat{\tilde{\pi}}_{s,k_{\max}} - \tilde{\pi}_{s,h} + \tilde{\pi}_s\right\| = h^2\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right) + h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right).$

Proof. For i) use the bound

$$\begin{aligned}
\left\|\hat{\tilde{\Gamma}}^{yy}_{j,\hat{h}} - \tilde{\Gamma}^{yy}_j\right\| &\le \left\|\sum_{l=-m+1}^{m-1}\hat{\gamma}^{\varepsilon}(l)\left(\hat{\Gamma}^{yy}_{j-l,\hat{h}} - \Gamma^{yy}_{j-l}\right)\right\| + \left\|\sum_{l=-m+1}^{m-1}\left(\hat{\gamma}^{\varepsilon}(l) - \gamma^{\varepsilon}(l)\right)\Gamma^{yy}_{j-l}\right\| \\
&\le \sum_{l=-m+1}^{m-1}\left|\hat{\gamma}^{\varepsilon}(l)\right|\left(|j-l|\,\nu^{|j-l|}O_p\left(h_{\max}(\log n/n)^{1/2}\right) + |j-l|^3\,\nu_*^{|j-l|}O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\right) \\
&\quad + O_p\left(n^{-1/2}\right)\sum_{l=-m+1}^{m-1}\left\|\Gamma^{yy}_{j-l}\right\| \\
&\le |j|\,\nu^{|j|}O_p\left(h_{\max}(\log n/n)^{1/2} + n^{-1/2}\right) + |j|^3\,\nu_*^{|j|}O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)
\end{aligned}$$

where Lemma 4.3 was used.

For ii) let $\tilde{\Gamma}_h$ be the $hp \times hp$ matrix whose (m,n)th block is $\tilde{\Gamma}^{yy}_{n-m}$. Then

$$\begin{aligned}
&\left\|\hat{\tilde{\Gamma}}_{h,\hat{h}} - \tilde{\Gamma}_h\right\|^2 \\
&= \sum_{j=-h+1}^{h-1}|h-j|\left\|\hat{\tilde{\Gamma}}_{j,\hat{h}} - \tilde{\Gamma}^{yy}_j\right\|^2 \\
&\le \sum_{j=-k_{\max}+1}^{k_{\max}-1}|k_{\max}-j|\left(|j|\,\nu^{|j|}O_p\left(h_{\max}(\log n/n)^{1/2}\right) + |j|^3\,\nu_*^{|j|}O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\right)^2 \\
&= O_p\left(h_{\max}^2(\log n/n)^{1/2}\right)
\end{aligned} \tag{4.2}$$

where the inequality is based on Lemma 4.3. Define

$$\nu_n(k) = k^2\nu^k h_{\max}(\log n/n)^{1/2} + k^4\nu_*^k h_{\max}^3(\log n/n)^{3/2},$$

$$\nu_h = \operatorname{diag}\left(\nu_n(1)^{-1}, \ldots, \nu_n(k)^{-1}, \ldots, \nu_n(h)^{-1}\right)'$$

and $V_h = (\nu_h \otimes I_p)$. Then consider

$$\begin{aligned}
\left\|V_h\left(\hat{\tilde{\pi}}_{h,\hat{h}}(h) - \tilde{\pi}_h(h)\right)\right\|_2 &= \left\|\left(V_h\hat{\tilde{\Gamma}}_{h,\hat{h}}V_h^{-1}\right)^{-1}\left(V_h\hat{\tilde{\Gamma}}_{h,\hat{h}}V_h^{-1}\right)V_h\left(\hat{\tilde{\pi}}_{h,\hat{h}}(h) - \tilde{\pi}_h(h)\right)\right\|_2 \\
&\le \left\|V_h\hat{\tilde{\Gamma}}^{-1}_{h,\hat{h}}V_h^{-1}\right\|_2\left\|V_h\hat{\tilde{\Gamma}}_{h,\hat{h}}\left(\hat{\tilde{\pi}}_{h,\hat{h}}(h) - \tilde{\pi}_h(h)\right)\right\|_2.
\end{aligned} \tag{4.3}$$

Now note that for any matrix A, $Ax = \lambda x$ where x is an eigenvector and λ is the corresponding eigenvalue. It follows that $x'A'Ax = \lambda^2 x'x$ such that $\|A\|_2 = \lambda$ where λ is the largest eigenvalue of A.

Also note that for any pair x, λ such that $V_h \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} V_h^{-1} x = \lambda x$ implies that $\widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} V_h^{-1} x = \lambda V_h^{-1} x$. In other words, the eigenvalues of $V_h \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} V_h^{-1}$ and $\widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1}$ are the same. Then,

$$\left\| V_h \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} V_h^{-1} \right\|_2 = \left\| \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} \right\|_2 \leq \left\| \tilde{\Gamma}_h^{-1} \right\|_2 + \left\| \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} - \tilde{\Gamma}_h^{-1} \right\|_2$$

where $\left\| \tilde{\Gamma}_h^{-1} \right\|_2$ is bounded by Lewis and Reinsel (1985, p. 397) and $\left\| \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} - \tilde{\Gamma}_h^{-1} \right\|_2 = o_p(1)$ by (4.2) and the same argument as in Lewis and Reinsel (1985, p. 397). Thus, for some $C < \infty$,

$$\left\| V_h \widehat{\tilde{\Gamma}}_{h,\hat{h}}^{-1} V_h^{-1} \right\|_2 \leq C \text{ w.p.a.1.} \tag{4.4}$$

such that the first term in the second line of (4.3) is bounded.

Next, for the second term in (4.3), consider

$$\left\| \left[V_h \widehat{\tilde{\Gamma}}_{h,\hat{h}} \left(\widehat{\tilde{\pi}}_{h,\hat{h}}(h) - \tilde{\pi}_h(h) \right) \right]_k \right\|^2 = \nu_n(k)^{-2} \left\| \sum_{j=1}^h \widehat{\tilde{\Gamma}}_{k-j,\hat{h}}^{yy} \left(\widehat{\tilde{\pi}}_{j,h} - \tilde{\pi}_{j,h} \right)' \right\|^2 \tag{4.5}$$

where from Hannan and Kavalieris (1986, p.39, Equation 4.3) it follows that for $k = 1, ..., h$,

$$\sum_{j=1}^h \left(\widehat{\tilde{\pi}}_{j,h} - \tilde{\pi}_{j,h} \right) \widehat{\tilde{\Gamma}}_{j-k,\hat{h}} = -\sum_{j=0}^h \tilde{\pi}_j \left(\widehat{\tilde{\Gamma}}_{j-k,\hat{h}} - \tilde{\Gamma}_{j-k}^{yy} \right) + \sum_{j=1}^h \left(\tilde{\pi}_j - \tilde{\pi}_{j,h} \right) \left(\widehat{\tilde{\Gamma}}_{j-k,\hat{h}} - \tilde{\Gamma}_{j-k}^{yy} \right). \tag{4.6}$$

Then, using (4.6) one obtains the bound

$$\begin{aligned} (4.7) \qquad & \nu_n(k)^{-1} \left\| \sum_{j=1}^h \left(\widehat{\tilde{\pi}}_{j,h} - \tilde{\pi}_{j,h} \right) \widehat{\tilde{\Gamma}}_{j-k,\hat{h}}^{yy} \right\| \\ \leq\ & \sum_{j=0}^h \nu_n(k)^{-1} \left\| \tilde{\pi}_j \right\| \left\| \widehat{\tilde{\Gamma}}_{j-k,\hat{h}}^{yy} - \tilde{\Gamma}_{j-k}^{yy} \right\| + \sum_{j=1}^h \nu_n(k)^{-1} \left\| \tilde{\pi}_j - \tilde{\pi}_{j,h} \right\| \left\| \widehat{\tilde{\Gamma}}_{j-k,\hat{h}}^{yy} - \tilde{\Gamma}_{j-k}^{yy} \right\| \\ \leq\ & C\nu_n(k)^{-1} \sum_{j=0}^h \nu^j \left(|k-j| \, \nu^{|k-j|} O_p\left(h_{\max} (\log n/n)^{1/2} \right) \right) \\ & + C\nu_n(k)^{-1} \sum_{j=0}^h \nu^j |k-j|^3 \nu_*^{|k-j|} O_p\left(h_{\max}^3 (\log n/n)^{3/2} \right) \\ & + O_p\left(h_{\max} (\log n/n)^{1/2} \right) \nu_n(k)^{-1} \sum_{j=h}^{\infty} \left\| \pi_j \right\| \\ =\ & O_p(1) + O_p(1) + O_p(1) \end{aligned}$$

where the second inequality uses

$$\begin{aligned} \sum_{j=1}^h \nu_n(k)^{-1} \left\| \tilde{\pi}_j - \tilde{\pi}_{j,h} \right\| \left\| \widehat{\tilde{\Gamma}}_{j-k,\hat{h}}^{yy} - \tilde{\Gamma}_{j-k}^{yy} \right\| &\leq \sup_{|j| \leq h} \left\| \widehat{\tilde{\Gamma}}_{j,\hat{h}}^{yy} - \tilde{\Gamma}_j^{yy} \right\| \nu_n(k)^{-1} \sum_{j=1}^h \left\| \tilde{\pi}_j - \tilde{\pi}_{j,h} \right\| \\ &\leq O_p\left(h_{\max} (\log n/n)^{1/2} \right) \nu_n(k)^{-1} \sum_{j=h}^{\infty} \left\| \pi_j \right\| \end{aligned}$$

by Lemma 4.4i) and Hannan and Deistler (1988, Theorem 6.6.12). The last line in (4.7) follows because

$$\nu_n(k) \geq k^2 \nu^k h_{\max} (\log n/n)^{1/2}$$

such that

$$\begin{aligned}
\nu_n(k)^{-1}\sum_{j=0}^{h}\nu^j|k-j|\nu^{|k-j|} &\le \nu_n(k)^{-1}\sum_{j=0}^{k-1}\nu^j(k-j)\nu^{k-j}+\nu_n(k)^{-1}\sum_{j=k}^{\infty}\nu^j(j-k)\nu^{j-k}\\
&\le \frac{k^2\nu^k}{k^2\nu^k h_{\max}(\log n/n)^{1/2}}+\frac{\nu^k\nu^2/\left(1-\nu^2\right)^2}{k^2\nu^k h_{\max}(\log n/n)^{1/2}}\\
&= O\left(\frac{n^{1/2}}{h_{\max}(\log n)^{1/2}}\right)
\end{aligned}$$

and

$$\nu_n(k)\ge k^4\nu_*^k h_{\max}^3(\log n/n)^{3/2}$$

$$\begin{aligned}
\nu_n(k)^{-1}\sum_{j=0}^{h}\nu^j|k-j|^3\nu_*^{|k-j|} &\le \sum_{j=0}^{h}\nu_*^j|k-j|^3\nu_*^{|k-j|}\\
&\le \frac{k^3\sum_{j=0}^{k-1}\nu_*^j\nu_*^{k-j}}{k^4\nu_*^k h_{\max}^3(\log n/n)^{3/2}}+\frac{\nu_*^{-k}\sum_{j=k}^{\infty}j^3\nu_*^{2j}}{k^4\nu_*^k h_{\max}^3(\log n/n)^{3/2}}\\
&= O\left(\frac{\nu_*^k k^4}{k^4\nu_*^k h_{\max}^3(\log n/n)^{3/2}}\right)=O\left(\frac{n^{3/2}}{h_{\max}^3(\log n)^{3/2}}\right)
\end{aligned}$$

as well as

$$\begin{aligned}
\nu_n(k)^{-1}\sum_{j=h}^{\infty}\|\pi_j\| &\le \frac{C\nu^h n^{1/2}}{k^2\nu^k h_{\max}(\log n)^{1/2}}\\
&\le \frac{Cn^{1/2}}{h_{\max}(\log n)^{1/2}}=O\left(\frac{Cn^{1/2}}{h_{\max}(\log n)^{1/2}}\right).
\end{aligned}$$

The last inequality follows from $h\ge k$ and $\nu^{h-k}\le 1$ as well as $k\ge 1$. Then, (4.4), (4.5), (4.6) and (4.7) together wiht (4.3) imply that

$$\left\|V_h\left(\hat{\tilde{\pi}}_{h,\hat{h}}(h)-\tilde{\pi}_h(h)\right)\right\|_2=O_p(1)$$

which establishes ii).

For iii) note that

$$\begin{aligned}
\left\|\hat{\tilde{\pi}}_{j,h}-\tilde{\pi}_j\right\| &\le \left\|\hat{\tilde{\pi}}_{j,h}-\tilde{\pi}_{j,h}\right\|+\|\tilde{\pi}_{j,h}-\tilde{\pi}_j\|\\
&\le \left\|\hat{\tilde{\pi}}_{j,h}-\tilde{\pi}_{j,h}\right\|+\sum_{j=h}^{\infty}\|\tilde{\pi}_j\|\\
&= \left\|\hat{\tilde{\pi}}_{j,h}-\tilde{\pi}_{j,h}\right\|+O\left(h\nu^h\right).
\end{aligned} \tag{4.8}$$

Then use Hannan and Deistler (1988, p. 269), and note that the orthogonality conditions defining $\widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{-i\lambda}\right)$ can be written as

$$(4.9) \qquad (2\pi)^{-1}\int_{-\pi}^{\pi}\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{-i\lambda}\right)K_{\hat{h}}\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda = I\left\{k=0\right\};\text{ for } k=0,...,h$$

$$(4.10) \qquad (2\pi)^{-1}\int_{-\pi}^{\pi}\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{k_{\max},\hat{h}}\left(e^{-i\lambda}\right)K_{\hat{h}}\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda = I\left\{k=0\right\};\text{ for } k=0,...,k_{\max}$$

as well as for the population regression parameters

$$(4.11) \qquad (2\pi)^{-1}\int_{-\pi}^{\pi}\Sigma_{h}^{-1}\tilde{\pi}_{h}\left(e^{-i\lambda}\right)K\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda = I\left\{k=0\right\};\text{ for } k=0,...,h$$

$$(4.12) \qquad (2\pi)^{-1}\int_{-\pi}^{\pi}\Sigma^{-1}\tilde{\pi}\left(e^{-i\lambda}\right)K\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda = I\left\{k=0\right\};\text{ for } k=0,...$$

Now define parameters that measure the difference in parameter values when h lags are used for the pseudo-regression parameters $\widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{i\lambda}\right)$ rather than $k_{\max}$ lags

$$\widehat{\tilde{\pi}}_{h,\hat{h}}^{\Delta}\left(e^{i\lambda}\right)=\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{h,\hat{h}}\left(e^{i\lambda}\right)-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\left(I-\sum_{j=1}^{h}\widehat{\tilde{\pi}}_{j,k_{\max}}e^{i\lambda j}\right),$$

and a similar measure for the corresponding population parameters

$$\tilde{\pi}_{h}^{\Delta}\left(e^{i\lambda}\right)=\Sigma_{h}^{-1}-\Sigma^{-1}-\sum_{j=1}^{h}\left(\Sigma_{h}^{-1}\tilde{\pi}_{j,h}-\Sigma^{-1}\tilde{\pi}_{j}e^{i\lambda j}\right).$$

First note that from (4.9) and (4.10) and for $0\leq k\leq h$

$$(2\pi)^{-1}\int_{-\pi}^{\pi}\widehat{\tilde{\pi}}_{h,\hat{h}}^{\Delta}\left(e^{i\lambda}\right)K_{\hat{h}}\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda=-(2\pi)^{-1}\int_{-\pi}^{\pi}\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\sum_{j=h+1}^{k_{\max}}\widehat{\tilde{\pi}}_{j,k_{\max}}e^{i\lambda j}K_{\hat{h}}\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda.$$

It now follows by the discussion in Hannan and Deistler (1988, p.270) and because $K_{\hat{h}}\left(e^{i\lambda}\right)$ can be factorized

$$K_{\hat{h}}\left(e^{i\lambda}\right)=\hat{\sigma}_{\varepsilon}^{2}\left|\hat{\theta}\left(e^{i\lambda}\right)\right|^{2}\tilde{K}_{\hat{h}}^{y}\left(e^{i\lambda}\right)\tilde{K}_{\hat{h}}^{y}\left(e^{-i\lambda}\right)'$$

by Assumption D where $K_{\hat{h}}\left(e^{i\lambda}\right)$ has absolutely summable Fourier coefficients w.p.a.1 that the inequality of Baxter (1963) is applicable[2]. It then follows similarly to the inequalities in Hannan and Deistler (1988, p. 270) that

$$(4.13) \qquad \begin{aligned} &\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\|+\sum_{s=1}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,h}-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,k_{\max}}\right\| \\ \leq\; &\left\|\hat{\Sigma}_{\hat{h}}^{-1}\right\|\sum_{s=0}^{h}\sum_{j=h+1}^{k_{\max}}\left\|\widehat{\tilde{\pi}}_{j,k_{\max}}\right\|\left\|\widehat{\Gamma}_{s-j,\hat{h}}^{yy}\right\| \\ \leq\; &C\sum_{j=h+1}^{k_{\max}}\left\|\widehat{\tilde{\pi}}_{j,k_{\max}}\right\| \text{ w.p.a.1} \end{aligned}$$

[2]Baxter's (1963, Theorem 1.1) is formulated in terms of conditions on $\log f(\theta)$ (Baxter's notation). However, inspection of the proof shows that summability conditions on the Fourier coefficients of $f(\theta)$ and the fact that $f(\theta)$ can be factorized are at the heart of the proof. Both of these properties are satisfied for the application at hand. See also Hannan and Deistler (1988, p. 270) for a related discussion.

because $\left\|\hat{\Sigma}_{\hat{h}}^{-1}\right\| \sum_{s=-k_{\max}}^{k_{\max}} \left\|\widehat{\widetilde{\Gamma}}_{s,\hat{h}}^{yy}\right\| \leq C$ for some C w.p.a.1 by Lemma 4.4i). Furthermore,

$$\begin{aligned}
\sum_{j=h+1}^{k_{\max}} \left\|\widehat{\tilde{\pi}}_{j,k_{\max}}\right\| &\leq \sum_{j=h+1}^{k_{\max}} \left(\left\|\tilde{\pi}_j\right\| + \left\|\widehat{\tilde{\pi}}_{j,k_{\max}} - \tilde{\pi}_j\right\|\right) \\
&= h\nu^h + h^2\nu^h O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right) + O\left(k_{\max}^2 \nu^{k_{\max}}\right) \\
&\quad + h^4 \nu_*^h O_p\left(h_{\max}^3 \left(\log n/n\right)^{3/2}\right) \\
&= h^2\nu^h O_p\left(1 + h_{\max}\left(\log n/n\right)^{1/2}\right) + h^4\nu_*^h O_p\left(h_{\max}^3\left(\log n/n\right)^{3/2}\right)
\end{aligned}$$

by Lemma 4.4ii), (4.8) and $O\left(k_{\max}^2 \nu^{k_{\max}}\right) = o\left(h^2\nu^h\right)$. This implies that

$$\begin{aligned}
\left\|\hat{\Sigma}_{h,\hat{h}}^{-1} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\| &= h^2\nu^h\left(1 + O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\right) \\
&\quad + h^4\nu_*^h O_p\left(h_{\max}^3\left(\log n/n\right)^{3/2}\right)
\end{aligned} \tag{4.14}$$

as well as

$$\begin{aligned}
\sum_{s=0}^{h} \left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,k_{\max}}\right\| &= h^2\nu^h\left(1 + O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\right) \\
&\quad + h^4\nu_*^h O_p\left(h_{\max}^3\left(\log n/n\right)^{3/2}\right).
\end{aligned} \tag{4.15}$$

To show iv) and v) note that (4.9)-(4.12) imply

$$(2\pi)^{-1}\int_{-\pi}^{\pi}\left(\widehat{\tilde{\pi}}_{h,\hat{h}}^{\Delta}\left(e^{-i\lambda}\right) - \tilde{\pi}_h^{\Delta}\left(e^{-i\lambda}\right)\right) K\left(e^{i\lambda}\right) e^{-i\lambda k} d\lambda = (2\pi)^{-1}\int_{-\pi}^{\pi} \hat{H}_{\hat{h}}(e^{i\lambda}) e^{i\lambda k} d\lambda$$

for $k = 0, 1, ..., h$ where

$$\begin{aligned}
\hat{H}_{\hat{h}}(e^{i\lambda}) &= \widehat{\tilde{\pi}}_{h,\hat{h}}^{\Delta}\left(e^{-i\lambda}\right)\left(K_{\hat{h}}\left(e^{i\lambda}\right) - K\left(e^{i\lambda}\right)\right) \\
&\quad + \textstyle\sum_{j=h+1}^{k_{\max}}\left(\Sigma^{-1}\tilde{\pi}_j - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right) e^{i\lambda j} K_{\hat{h}}\left(e^{i\lambda}\right) \\
&\quad - \textstyle\sum_{j=h+1}^{k_{\max}} \Sigma^{-1}\tilde{\pi}_j e^{i\lambda s}\left(K_{\hat{h}}\left(e^{i\lambda}\right) - K\left(e^{i\lambda}\right)\right) \\
&\quad + \textstyle\sum_{j=k_{\max}+1}^{\infty} \Sigma^{-1}\tilde{\pi}_j e^{i\lambda s} K\left(e^{i\lambda}\right).
\end{aligned}$$

Use the notation $\hat{H}_{j,h}$ for the Fourier coefficients of $\hat{H}_{\hat{h}}(e^{i\lambda})$. By Baxter (1963) and Hannan and Deistler (1988, p. 269) it follows that

$$\begin{aligned}
&\left\|\hat{\Sigma}_{h,\hat{h}}^{-1} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \Sigma_h^{-1} + \Sigma^{-1}\right\| + \sum_{s=1}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,k_{\max}} - \Sigma_h^{-1}\tilde{\pi}_{s,h} + \Sigma^{-1}\tilde{\pi}_s\right\| \\
&\leq \sum_{s=0}^{h}\left\|\hat{H}_{s,h}\right\|.
\end{aligned} \tag{4.16}$$

Note that $\widehat{\tilde{\Gamma}}_{j,\hat{h}}^{yy}$ and $\tilde{\Gamma}_j^{yy}$ are the Fourier coefficients of $K_{\hat{h}}\left(e^{i\lambda}\right)$ and $K\left(e^{i\lambda}\right)$ respectively. It follows that

$$\begin{aligned}
\hat{H}_{s,h} &= \int_{-\pi}^{\pi} \hat{H}_{\hat{h}}(e^{i\lambda})e^{i\lambda s} = \sum_{j=0}^{h}\left(\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right)\left(\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy} - \tilde{\Gamma}_{s-j}^{yy}\right) \\
&+ \textstyle\sum_{j=h+1}^{k_{\max}}\left(\Sigma^{-1}\tilde{\pi}_j - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right)\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy} \\
&- \textstyle\sum_{j=h+1}^{k_{\max}}\Sigma^{-1}\tilde{\pi}_j\left(\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy} - \tilde{\Gamma}_{s-j}^{yy}\right) \\
&+ \textstyle\sum_{j=k_{\max}+1}^{\infty}\Sigma^{-1}\tilde{\pi}_j\tilde{\Gamma}_{s-j}^{yy}
\end{aligned}$$

such that

$$\begin{aligned}
\sum_{s=0}^{h}\left\|\hat{H}_{s,h}\right\| &\leq \sum_{j=0}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right\|\sum_{s=-h}^{h}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy} - \tilde{\Gamma}_s^{yy}\right\| \\
&+ \textstyle\sum_{j=h+1}^{k_{\max}}\left\|\Sigma^{-1}\tilde{\pi}_j - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right\|\displaystyle\sum_{s=0}^{h}\left\|\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy}\right\| \\
&+ \textstyle\sum_{j=h+1}^{k_{\max}}\left\|\Sigma^{-1}\right\|\left\|\tilde{\pi}_j\right\|\displaystyle\sum_{s=-h}^{h}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy} - \tilde{\Gamma}_s^{yy}\right\| \\
&+ \textstyle\sum_{j=k_{\max}+1}^{\infty}\left\|\Sigma^{-1}\right\|\left\|\tilde{\pi}_j\right\|\displaystyle\sum_{s=0}^{h}\left\|\tilde{\Gamma}_{s-j}^{yy}\right\|.
\end{aligned}$$

By Lemma 4.4i) $\sum_{s=-h}^{h}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy}\right\|$ is bounded uniformly in h such that $\sum_{s=-k_{\max}}^{k_{\max}}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy}\right\| = O_p(1)$. Now,

$$\sum_{s=-h}^{h}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy} - \tilde{\Gamma}_s^{yy}\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right) \tag{4.17}$$

by Lemma 4.4i) such that togehter with (4.15)

$$\begin{aligned}
\sum_{j=0}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right\|\sum_{s=-h}^{h}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy} - \tilde{\Gamma}_s^{yy}\right\| &= h^2\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right) \\
&\quad + h^4\nu_*^h O_p\left(h_{\max}^4(\log n/n)^2\right),
\end{aligned} \tag{4.18}$$

while by 4.4i) and (4.17)

$$\sum_{j=h+1}^{k_{\max}}\sum_{s=0}^{h}\left\|\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy}\right\| = O_p(1)$$

and using the bound

$$\left\|\Sigma^{-1}\tilde{\pi}_j - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,k_{\max}}\right\| \leq \left\|\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\|\left\|\widehat{\tilde{\pi}}_{j,k_{\max}} - \tilde{\pi}_j\right\| + \left\|\Sigma^{-1} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\|\left\|\tilde{\pi}_j\right\|$$

as well as (4.8) and Lemmas 4.4ii) and vi)

$$\begin{aligned}\textstyle\sum_{j=h+1}^{k_{\max}}\left\|\Sigma^{-1}\tilde{\pi}_j-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\hat{\tilde{\pi}}_{j,k_{\max}}\right\| &\leq \left\|\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\|\textstyle\sum_{j=h+1}^{k_{\max}}\left\|\hat{\tilde{\pi}}_{j,k_{\max}}-\tilde{\pi}_j\right\| \\ &\quad+\left\|\Sigma^{-1}-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\right\|\textstyle\sum_{j=h+1}^{k_{\max}}\|\tilde{\pi}_j\| \\ &= O_p\left(h_{\max}(\log n/n)^{1/2}\right)\left(\textstyle\sum_{j=h+1}^{k_{\max}}j\nu^j+k_{\max}^2\nu^{k_{\max}}+h\nu^h\right) \\ &\quad+O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\textstyle\sum_{j=h+1}^{k_{\max}}j^5\nu_*^j \\ &= h\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right)+h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\end{aligned}$$

where the last line uses the fact that $k_{\max}^2\nu^{k_{\max}}=O\left(h\nu^h\right)$ .It then follows that

$$\begin{aligned}\text{(4.19)}\qquad \textstyle\sum_{j=h+1}^{k_{\max}}\left\|\Sigma^{-1}\tilde{\pi}_j-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\hat{\tilde{\pi}}_{j,k_{\max}}\right\|\sum_{s=0}^{h}\left\|\hat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy}\right\| &= h^2\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right) \\ &\quad+h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\end{aligned}$$

and

$$\text{(4.20)}\qquad \textstyle\sum_{j=h+1}^{k_{\max}}\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_j\|\sum_{s=-h}^{h}\left\|\hat{\tilde{\Gamma}}_{s,\hat{h}}^{yy}-\tilde{\Gamma}_s^{yy}\right\|=h\nu^h O_p\left(h_{\max}(\log n/n)^{1/2}\right).$$

Finally,

$$\text{(4.21)}\qquad \textstyle\sum_{j=k_{\max}+1}^{\infty}\left\|\Sigma^{-1}\right\|\|\tilde{\pi}_j\|\sum_{s=0}^{h}\left\|\tilde{\Gamma}_{s-j}^{yy}\right\|=k_{\max}\nu^{k_{\max}}O_p(1)$$

where $k_{\max}=\sqrt{n/\log n}$ implies that $k_{\max}\nu^{k_{\max}}O_p(1)=o_p\left(n^{-1/2}\right)h_{\max}\nu^{h_{\max}}$. It thus follows from (4.18), (4.19), (4.20) and (4.21) that

$$\sum_{s=0}^{h}\left\|\hat{H}_{s,h}\right\|=h^2\nu^h O_p\left(h_{\max}\sqrt{\log n/n}\right)+h^5\nu_*^h O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)$$

which establishes iv) and v).

For vi) consider

$$(2\pi)^{-1}\left(\int_{-\pi}^{\pi}\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\hat{\tilde{\pi}}_{k_{\max},\hat{h}}\left(e^{-i\lambda}\right)-\textstyle\sum_{j=0}^{k_{\max}}\Sigma^{-1}\tilde{\pi}_j e^{i\lambda j}\right)K_{\hat{h}}\left(e^{i\lambda}\right)e^{-i\lambda k}d\lambda=(2\pi)^{-1}\int_{-\pi}^{\pi}\hat{H}_{\hat{h}}(e^{i\lambda})e^{i\lambda k}$$

for $k=0,1,...,k_{\max}$ where

$$\begin{aligned}\hat{H}_{\hat{h}}(e^{i\lambda}) &= \textstyle\sum_{j=0}^{k_{\max}}\Sigma^{-1}\tilde{\pi}_j e^{i\lambda j}\left(K_{\hat{h}}\left(e^{i\lambda}\right)-K\left(e^{i\lambda}\right)\right) \\ &\quad+\textstyle\sum_{j=k_{\max}+1}^{\infty}\Sigma^{-1}\tilde{\pi}_j e^{i\lambda j}K\left(e^{i\lambda}\right)\end{aligned}$$

where by Baxter's inequality, as in (4.16),

$$\left\|\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}-\Sigma^{-1}\right\|+\sum_{s=1}^{k_{\max}}\left\|\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,k_{\max}}-\Sigma^{-1}\tilde{\pi}_s\right\|\leq\sum_{s=0}^{k_{\max}}\left\|\hat{H}_{s,h}\right\| \tag{4.22}$$

and

$$\hat{H}_{s,h}=\int_{-\pi}^{\pi}\hat{H}_{\hat{h}}(e^{i\lambda})e^{i\lambda s}=\sum_{j=0}^{k_{\max}}\Sigma^{-1}\tilde{\pi}_j\left(\widehat{\tilde{\Gamma}}_{s-j,\hat{h}}^{yy}-\tilde{\Gamma}_{s-j}^{yy}\right)+\sum_{j=k_{\max}+1}^{\infty}\Sigma^{-1}\tilde{\pi}_j\tilde{\Gamma}_{s-j}^{yy} \tag{4.23}$$

Then,

$$\begin{aligned}\sum_{s=0}^{k_{\max}}\left\|\hat{H}_{s,h}\right\| &\leq \left\|\Sigma^{-1}\right\|\sum_{j=0}^{k_{\max}}\left\|\tilde{\pi}_j\right\|\sum_{s=-k_{\max}}^{k_{\max}}\left\|\widehat{\tilde{\Gamma}}_{s,\hat{h}}^{yy}-\tilde{\Gamma}_s^{yy}\right\|+\left\|\Sigma^{-1}\right\|\sum_{j=k_{\max}+1}^{\infty}\left\|\tilde{\pi}_j\right\|\sum_{s=-\infty}^{\infty}\left\|\tilde{\Gamma}_s^{yy}\right\| \\ &= O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\end{aligned} \tag{4.24}$$

from (4.17), the fact that $\sum_{j=0}^{h}\left\|\tilde{\pi}_j\right\|=O(1)$ uniformly in h and

$$\sum_{j=k_{\max}+1}^{\infty}\Sigma^{-1}\left\|\tilde{\pi}_j\right\|=O\left(k_{\max}\nu^{k_{\max}}\right)=o\left(n^{-1/2}\right).$$

Then, (4.22) and (4.24) imply that $\left\|\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}-\Sigma^{-1}\right\|=O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)$.

For vii) one obtains from (4.14) and (4.15) that

$$\begin{aligned}\sum_{j=0}^{h}\left\|\widehat{\tilde{\pi}}_{j,h}-\widehat{\tilde{\pi}}_{j,h_{\max}}\right\| &\leq \left\|\hat{\Sigma}_{h,\hat{h}}\right\|\left(\sum_{j=0}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,h}-\hat{\Sigma}_{h_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{j,h_{\max}}\right\|+\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}-\hat{\Sigma}_{h_{\max},\hat{h}}^{-1}\right\|\sum_{j=0}^{h}\left\|\widehat{\tilde{\pi}}_{j,h_{\max}}\right\|\right) \\ &= h^2\nu^h\left(1+O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\right)+h^5\nu_*^hO_p\left(h_{\max}^3\left(\log n/n\right)^{3/2}\right)\end{aligned}$$

because $\sum_{j=0}^{h}\left\|\widehat{\tilde{\pi}}_{j,h_{\max}}\right\|=O_p(1)$ by Lemma 4.4ii).

For viii), similarly to vii)

$$\sum_{s=1}^{h}\left\|\widehat{\tilde{\pi}}_{s,h}-\widehat{\tilde{\pi}}_{s,k_{\max}}-\tilde{\pi}_{s,h}+\tilde{\pi}_s\right\|$$

$$\leq \left\|\hat{\Sigma}_{h,\hat{h}}\right\|\sum_{s=1}^{h}\left\|\hat{\Sigma}_{h,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,h}-\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s,k_{\max}}-\Sigma_h^{-1}\tilde{\pi}_{s,h}+\Sigma^{-1}\tilde{\pi}_s\right\| \tag{4.25}$$

$$+\left\|\hat{\Sigma}_{h,\hat{h}}\right\|\sum_{s=1}^{h}\left\|\left(\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}-\hat{\Sigma}_{h,\hat{h}}^{-1}\right)\left(\widehat{\tilde{\pi}}_{s,k_{\max}}-\tilde{\pi}_s\right)\right\| \tag{4.26}$$

$$+\left\|\hat{\Sigma}_{h,\hat{h}}\right\|\sum_{s=1}^{h}\left\|\left(\hat{\Sigma}_{k_{\max},\hat{h}}^{-1}-\Sigma^{-1}\right)\left(\tilde{\pi}_s-\tilde{\pi}_{s,h}\right)-\left(\hat{\Sigma}_{h,\hat{h}}^{-1}-\hat{\Sigma}_{h_{\max},\hat{h}}^{-1}-\Sigma_h^{-1}+\Sigma^{-1}\right)\tilde{\pi}_{s,h}\right\| \tag{4.27}$$

where each term is considered separately below. First, (4.25) is

$$\begin{aligned}
(4.28) \qquad & \sum_{s=1}^{h} \left\| \hat{\Sigma}_{h,\hat{h}}^{-1} \widehat{\tilde{\pi}}_{s,h} - \hat{\Sigma}_{k_{\max},\hat{h}}^{-1} \widehat{\tilde{\pi}}_{s,k_{\max}} - \Sigma_{h}^{-1} \tilde{\pi}_{s,h} + \Sigma^{-1} \tilde{\pi}_{s} \right\| \\
&= h^{2} \nu^{h} O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right) + h^{5} \nu_{*}^{h} O_{p}\left(h_{\max}^{3} \left(\log n/n\right)^{3/2} \right)
\end{aligned}$$

by Lemma 4.4v), (4.26) is bounded by

$$\begin{aligned}
(4.29) \qquad & \sum_{s=1}^{h} \left\| \left(\hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \hat{\Sigma}_{h,\hat{h}}^{-1} \right) \left(\widehat{\tilde{\pi}}_{s,k_{\max}} - \tilde{\pi}_{s} \right) \right\| \\
&\leq \left\| \hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \hat{\Sigma}_{h,\hat{h}}^{-1} \right\| \sum_{s=1}^{h} \left\| \widehat{\tilde{\pi}}_{s,h_{\max}} - \tilde{\pi}_{s} \right\| \\
&= h^{2} \lambda^{h} O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right) + h^{5} \nu_{*}^{h} O_{p}\left(h_{\max}^{4} \left(\log n/n\right)^{2} \right)
\end{aligned}$$

where $\left\| \hat{\Sigma}_{h_{\max},\hat{h}}^{-1} - \hat{\Sigma}_{h,\hat{h}}^{-1} \right\| = h^{2} \lambda^{h} O_{p}\left(1 + h_{\max} \left(\log n/n\right)^{1/2} \right) + h^{5} \nu_{*}^{h} O_{p}\left(h_{\max}^{3} \left(\log n/n\right)^{3/2} \right)$ by Lemma 4.4iii) and

$$\sum_{s=1}^{h} \left\| \widehat{\tilde{\pi}}_{s,k_{\max}} - \tilde{\pi}_{s} \right\| = O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right) + O\left(k_{\max} \nu^{k_{\max}} \right) = O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right)$$

by Lemma 4.4ii), (4.8) and $k_{\max} \nu^{k_{\max}} = o\left(n^{-1/2}\right)$. For (4.27) first consider

$$\begin{aligned}
(4.30) \qquad & \sum_{s=1}^{h} \left\| \left(\hat{\Sigma}_{h,\hat{h}}^{-1} - \hat{\Sigma}_{h_{\max},\hat{h}}^{-1} - \Sigma_{h}^{-1} + \Sigma^{-1} \right) \tilde{\pi}_{s,h} \right\| \\
&\leq \left\| \hat{\Sigma}_{h,\hat{h}}^{-1} - \hat{\Sigma}_{h_{\max},\hat{h}}^{-1} - \Sigma_{h}^{-1} + \Sigma^{-1} \right\| \sum_{s=1}^{h} \left\| \tilde{\pi}_{s,h} \right\| \\
&= h^{2} \nu^{h} O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right) + h^{5} \nu_{*}^{h} O_{p}\left(h_{\max}^{3} \left(\log n/n\right)^{3/2} \right)
\end{aligned}$$

by Lemma 4.4iv) and the fact that $\sum_{s=1}^{h} \left\| \tilde{\pi}_{s,h} \right\| = O(1)$ uniformly in h. For the remaining term in (4.27) consider

$$\begin{aligned}
(4.31) \qquad \sum_{s=1}^{h} \left\| \left(\hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \Sigma^{-1} \right) \left(\tilde{\pi}_{s} - \tilde{\pi}_{s,h} \right) \right\| &\leq \left\| \hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \Sigma^{-1} \right\| \sum_{s=1}^{h} \left\| \tilde{\pi}_{s} - \tilde{\pi}_{s,h} \right\| \\
&\leq \left\| \hat{\Sigma}_{k_{\max},\hat{h}}^{-1} - \Sigma^{-1} \right\| \sum_{s=h+1}^{\infty} \left\| \tilde{\pi}_{s} \right\| \\
&= O\left(h \nu^{h} \right) O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right)
\end{aligned}$$

by Lemma 4.4vi). It now follows from (4.25), (4.26), (4.27), (4.28), (4.29), (4.30) and (4.31) that

$$\sum_{s=1}^{h} \left\| \widehat{\tilde{\pi}}_{s,h} - \widehat{\tilde{\pi}}_{s,k_{\max}} - \tilde{\pi}_{s,h} + \tilde{\pi}_{s} \right\| = h^{2} \nu^{h} O_{p}\left(h_{\max} \left(\log n/n\right)^{1/2} \right) + h^{5} \nu_{*}^{h} O_{p}\left(h_{\max}^{3} \left(\log n/n\right)^{3/2} \right).$$

■

Lemma 4.5. *Let $\hat{\Omega}_{M,\hat{h}}$ be defined in (7.7). Then*
i)

$$\begin{aligned}\left\|\hat{\vartheta}^{M}_{j_1,j_2,\hat{h}}-\hat{\vartheta}^{k_{\max}}_{j_1,j_2,\hat{h}}-\vartheta^{M}_{j_1,j_2}+\vartheta_{j_1,j_2}\right\| &= (M-j_1+1)^2\,\nu^{M-j_1}O_p\left(h_{\max}(\log n/n)^{1/2}\right)\\ &\quad+(M-j_1+1)^5\,\nu_*^{M-j_1}O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)\end{aligned}$$

uniformly in M and $j_1,j_2\le M$,
ii) $\left\|\hat{\vartheta}^{k_{\max}}_{j_1,j_2,\hat{h}}-\vartheta_{j_1,j_2}\right\|=O_p\left(h_{\max}(\log n/n)^{1/2}\right)$ *uniformly in j_1 and j_2.*

Proof. Without loss of generality assume $j_2\ge j_1$. The reverse case follows by symmetry. For i) note that w.p.a.1, $\hat{\vartheta}^{M}_{j_1,j_2,\hat{h}}$ is the j_1,j_2-th block of the inverse of $\hat{\Omega}^{*}_{M,\hat{h}}=\sum_{l=-m+1}^{m-1}\hat{\gamma}^{\varepsilon}_{\hat{\theta}}(l)\,\hat{\Omega}_{\hat{h}}(l)$. Then, by the same argument as in Lemma 2.4, the j,k-th block of $\hat{\Omega}^{*}_{M,\hat{h}}$ can be represented as

$$\textstyle\sum_{l=-m+1}^{m-1}\hat{\gamma}^{\varepsilon}_{\hat{\theta}}(l)\,\hat{\Gamma}^{yy}_{k-j-l,\hat{h}}=\int_{-\pi}^{\pi}K_{\hat{h}}\left(e^{i\lambda}\right)e^{i\lambda(k-j)}d\lambda$$

where $K_{\hat{h}}\left(e^{i\lambda}\right)$ is defined in Lemma 4.4. By the same argument as in Lemma 2.7 it then follows that $\hat{\vartheta}^{M}_{j_1,j_2,\hat{h}}$ is based on the finite dimensional inverse of the variance covariance matrix of a process with spectrum $K_{\hat{h}}\left(e^{i\lambda}\right)$. This implies that $\hat{\vartheta}^{M}_{j_1,j_2,\hat{h}}=\sum_{s=0}^{j_1-1}\hat{\tilde{\pi}}'_{s,s+M-j_1}\hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}}\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}$ where $\hat{\tilde{\pi}}'_{s,s+M-j_1}$ and $\hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}}$ are defined in Lemma 4.4. Note that

$$\begin{aligned}\hat{\vartheta}^{M}_{j_1,j_2,\hat{h}}-\hat{\vartheta}^{k_{\max}}_{j_1,j_2,\hat{h}}-\left(\vartheta^{M}_{j_1,j_2}-\vartheta_{j_1,j_2}\right) &= \textstyle\sum_{s=0}^{j_1-1}\Big(\hat{\tilde{\pi}}'_{s,s+M-j_1}\hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}}\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}\\ &\quad-\hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1}\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}}\hat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1}\\ &\quad-\tilde{\pi}'_{s,s+M-j_1}\Sigma^{-1}_{s+M-j_1}\tilde{\pi}_{s+j_2-j_1,s+M-j_1}+\tilde{\pi}'_s\Sigma^{-1}\tilde{\pi}_{s+j_2-j_1}\Big)\end{aligned}$$

such that one can represent

$$
\begin{aligned}
(4.32)\quad & \hat{\vartheta}^{M}_{j_1,j_2,\hat{h}} - \hat{\vartheta}^{k_{\max}}_{j_1,j_2,\hat{h}} - \left(\vartheta^{M}_{j_1,j_2} - \vartheta_{j_1,j_2}\right) \\
= \quad & \textstyle\sum_{s=0}^{j_1-1} \left(\hat{\tilde{\pi}}'_{s,s+M-j_1} \hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}} - \hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \tilde{\pi}'_{s,s+M-j_1} \Sigma^{-1}_{s+M-j_1} + \tilde{\pi}'_s \Sigma^{-1} \right) \\
& \times \hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} \\
& + \textstyle\sum_{s=0}^{j_1-1} \hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} \\
& \times \left(\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} - \hat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1} - \tilde{\pi}_{s+j_2-j_1,s+M-j_1} + \tilde{\pi}_{s+j_2-j_1} \right) \\
& + \textstyle\sum_{s=0}^{j_1-1} \left(\hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} - \tilde{\pi}_s \right)' \left(\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \Sigma^{-1} \right) \left(\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1} \right) \\
& + \textstyle\sum_{s=0}^{j_1-1} \tilde{\pi}'_s \left(\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \Sigma^{-1} \right) \left(\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1} \right) \\
& - \textstyle\sum_{s=0}^{j_1-1} \tilde{\pi}'_{s,s+M-j_1} \left(\Sigma^{-1} - \Sigma^{-1}_{s+M-j_1} \right) \left(\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1,s+M-j_1} \right) \\
& + \textstyle\sum_{s=0}^{j_1-1} \left(\hat{\tilde{\pi}}_{s,s+k_{\max}-j_1} - \tilde{\pi}_s \right)' \Sigma^{-1} \left(\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1} \right) \\
& + \textstyle\sum_{s=0}^{j_1-1} \left(\tilde{\pi}_s - \tilde{\pi}_{s,s+M-j_1} \right)' \Sigma^{-1} \left(\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} \right) \\
\equiv \quad & \textstyle\sum_{j=1}^{7} \psi_{j,n}
\end{aligned}
$$

where $\psi_{j,n}$ is the j-th term in the above decomposition. For $\psi_{1,n}$ note that by Lemma 4.4v)

$$
\begin{aligned}
& \left\| \hat{\tilde{\pi}}'_{s,s+M-j_1} \hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}} - \hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \tilde{\pi}'_{s,s+M-j_1} \Sigma^{-1}_{s+M-j_1} + \tilde{\pi}'_s \Sigma^{-1} \right\| \\
\leq \quad & \sum_{u=1}^{s+M-j_1} \left\| \hat{\tilde{\pi}}'_{u,s+M-j_1} \hat{\Sigma}^{-1}_{s+M-j_1,\hat{h}} - \hat{\tilde{\pi}}'_{u,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \tilde{\pi}'_{u,s+M-j_1} \Sigma^{-1}_{s+M-j_1} + \tilde{\pi}'_u \Sigma^{-1} \right\| \\
= \quad & (s+M-j_1)^2 \nu^{s+M-j_1} O_p\left(h_{\max} (\log n/n)^{1/2} \right) + (s+M-j_1)^5 \nu_*^{s+M-j_1} O_p\left(h^3_{\max} (\log n/n)^{3/2} \right).
\end{aligned}
$$

Next note that

$$
\left\| \hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} \right\| \leq \left\| \hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1,s+M-j_1} \right\| + \left\| \tilde{\pi}_{s+j_2-j_1} \right\|
$$

where

$$
(4.33)\qquad \textstyle\sum_{s=0}^{j_1-1} \left\| \tilde{\pi}_{s+j_2-j_1} \right\| = O(1)
$$

uniformly in j_1 and by Lemma 4.4ii), using $j_2 - j_1 \geq 0$,

$$
\begin{aligned}
(4.34)\quad \textstyle\sum_{s=0}^{j_1-1} \left\| \hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1} \right\| \quad = \quad & |j_2-j_1|^2 \nu^{|j_2-j_1|} O_p\left(h_{\max} (\log n/n)^{1/2} \right) \\
& + |j_2-j_1|^5 \nu_*^{|j_2-j_1|} O_p\left(h^3_{\max} (\log n/n)^{3/2} \right).
\end{aligned}
$$

Together, (4.33) and (4.34) imply that $\sup_{j_1,j_2}\left\|\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}\right\| = O_p(1)$. It then follows that

$$\begin{aligned}\left\|\psi_{1,n}\right\| &\leq (M-j_1+1)^2\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)^{1/2}\right) \\ &\quad \times \textstyle\sum_{s=0}^{j_1-1}(s+1)^2\,\nu^s\left\|\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}\right\| \\ &\quad + (M-j_1+1)^5\,\nu_*^{M-j_1} O_p\left(h_{\max}^3(\log n/n)^{3/2}\right) \\ &\quad \times \textstyle\sum_{s=0}^{j_1-1}(s+1)^5\,\nu_*^s\left\|\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}\right\| \\ &= (M-j_1+1)\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)^{1/2}\right) \\ &\quad + (M-j_1+1)^5\,\nu_*^{M-j_1} O_p\left(h_{\max}^3(\log n/n)^{3/2}\right).\end{aligned} \tag{4.35}$$

For

$$\psi_{2,n} = \textstyle\sum_{s=0}^{j_1-1}\hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1}\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}}\left(\hat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1} - \hat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1} - \tilde{\pi}_{s+j_2-j_1,s+M-j_1} + \tilde{\pi}_{s+j_2-j_1}\right)$$

and by Lemma 4.4viii) and the same argument as for $\psi_{1,n}$ it follows that

$$\psi_{2,n} = (M-j_1+1)^2\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)^{1/2}\right) + (M-j_1+1)^5\,\nu_*^{M-j_1} O_p\left(h_{\max}^3(\log n/n)^{3/2}\right). \tag{4.36}$$

By Lemma 4.4ii), (4.8), $j_1 \leq M_{\max}$ and $\nu^{k_{\max}-M_{\max}} = o(n^{-1})$ it follows that

$$\textstyle\sum_{s=0}^{j_1-1}\left\|\hat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} - \tilde{\pi}_s\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right) \tag{4.37}$$

and noting that the proof of Lemma 4.4vi) remains unchanged upon replacing $k_{\max}$ with $k_{\max}-M_{\max}$ one obtains

$$\left\|\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \Sigma^{-1}\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right). \tag{4.38}$$

Also,

$$\left\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1}\right\| \leq \sum_{u=M-j_1}^{\infty}\left\|\tilde{\pi}_u\right\| = O\left((M-j_1)\,\nu^{M-j_1}\right) \tag{4.39}$$

uniformly in $s < j_1$ by Kuersteiner (2005, p.99). Together, (4.37), (4.38) and (4.39) imply that

$$\begin{aligned}\left\|\psi_{3,n}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\left\|\hat{\tilde{\pi}}_{s,s+k_{\max}-j_1} - \tilde{\pi}_s\right\|\left\|\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \Sigma^{-1}\right\|\left\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1}\right\| \\ &= (M-j_1)\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)\right) = (M-j_1)\,\nu^{M-j_1} o_p\left((\log n/n)^{1/2}\right).\end{aligned} \tag{4.40}$$

For $\psi_{4,n}$ one obtains similarly that

$$\begin{aligned}\left\|\psi_{4,n}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\left\|\tilde{\pi}'_s\right\|\left\|\hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \Sigma^{-1}\right\|\left\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1} - \tilde{\pi}_{s+j_2-j_1}\right\| \\ &= (M-j_1)\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)^{1/2}\right)\textstyle\sum_{s=0}^{\infty}\left\|\tilde{\pi}'_s\right\| \\ &= (M-j_1)\,\nu^{M-j_1} O_p\left(h_{\max}(\log n/n)^{1/2}\right).\end{aligned} \tag{4.41}$$

By (4.1) Hannan and Deistler (1988, Theorem 6.6.12) it follows that

$$\left\|\Sigma^{-1}-\Sigma_{s+M-j_1}^{-1}\right\| = O\left((M-j_1+s)\,\nu^{s+M-j_1}\right).$$

By Lemma 4.4ii)

$$\begin{aligned}\left\|\widehat{\tilde{\pi}}_{j,s+M-j_1}-\tilde{\pi}_{j,s+M-j_1}\right\| &= j^2\nu^j O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)+j^5\nu_*^j O_p\left(h_{\max}^3\left(\log n/n\right)^{3/2}\right)\\ &= O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\end{aligned}$$

uniformly in j. This shows that

$$(4.42)\quad \begin{aligned}\left\|\psi_{5,n}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\left\|\tilde{\pi}_{s,s+M-j_1}\right\|\left\|\Sigma^{-1}-\Sigma_{s+M-j_1}^{-1}\right\|\left\|\widehat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}-\tilde{\pi}_{s+j_2-j_1,s+M-j_1}\right\|\\ &= (M-j_1)\,\nu^{M-j_1}O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right).\end{aligned}$$

By arguments similar to the ones used for $\psi_{3,n}$, in particular using (4.37) and (4.39), it follows that

$$(4.43)\quad \begin{aligned}\left\|\psi_{6,n}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\left\|\widehat{\tilde{\pi}}_{s,s+k_{\max}-j_1}-\tilde{\pi}_s\right\|\left\|\Sigma^{-1}\right\|\left\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1}-\tilde{\pi}_{s+j_2-j_1}\right\|\\ &\leq (M-j_1+1)^2\,\nu^{M-j_1}O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\textstyle\sum_{s=0}^{j_1-1}(s+1)\,\nu^{2s}\\ &= (M-j_1+1)^2\,\nu^{M-j_1}O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\end{aligned}$$

because $\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1}-\tilde{\pi}_{s+j_2-j_1}\| \leq \sum_{j=M-j_1}^{\infty}\|\tilde{\pi}_j\| \leq C\,(M-j_1)\,\nu^{M-j_1}$. As before

$$\begin{aligned}\sum_{s=1}^{j_1-1}\left\|\tilde{\pi}_s-\tilde{\pi}_{s,s+M-j_1}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\sum_{u=s}^{\infty}\left\|\tilde{\pi}_{u+M-j_1}\right\| \leq c\sum_{s=0}^{j_1-1}\sum_{u=s}^{\infty}(u+M-j_1)\,\nu^{u+M-j_1}\\ &\leq c\,(M-j_1+1)\,\nu^{M-j_1}\textstyle\sum_{s=0}^{j_1-1}\sum_{u=s}^{\infty}(u+1)\,\nu^u = (M-j_1+1)\,\nu^{M-j_1}O(1)\end{aligned}$$

such that

$$(4.44)\quad \begin{aligned}\left\|\psi_{7,n}\right\| &\leq \textstyle\sum_{s=0}^{j_1-1}\left\|\tilde{\pi}_s-\tilde{\pi}'_{s,s+M-j_1}\right\|\left\|\Sigma^{-1}\right\|\left\|\tilde{\pi}_{s+j_2-j_1,s+M-j_1}-\widehat{\tilde{\pi}}_{s+j_2-j_1,s+M-j_1}\right\|\\ &= (M-j_1+1)\,\nu^{M-j_1}O_p(h_{\max}\left(\log n/n\right)^{1/2}).\end{aligned}$$

The result then follows from (4.32), (4.35), (4.36), (4.40), (4.41), (4.42), (4.43) and (4.44).

For ii) consider

$$\begin{aligned}\hat{\vartheta}_{j_1,j_2,\hat{h}}^{k_{\max}}-\vartheta_{j_1,j_2} &= \textstyle\sum_{s=0}^{j_1-1}\left(\widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1}\hat{\Sigma}_{s+k_{\max}-j_1,\hat{h}}^{-1}\widehat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1}-\tilde{\pi}'_s\Sigma^{-1}\tilde{\pi}_{s+j_2-j_1}\right)\\ &= \textstyle\sum_{s=0}^{j_1-1}\widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1}\hat{\Sigma}_{s+k_{\max}-j_1,\hat{h}}^{-1}\left(\widehat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1}-\tilde{\pi}_{s+j_2-j_1}\right)\\ &\quad+\textstyle\sum_{s=0}^{j_1-1}\left(\widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1}\hat{\Sigma}_{s+k_{\max}-j_1,\hat{h}}^{-1}-\tilde{\pi}'_s\Sigma^{-1}\right)\tilde{\pi}_{s+j_2-j_1}\end{aligned}$$

where

$$
\begin{aligned}
& \left\| \sum_{s=0}^{j_1-1} \widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} \left(\widehat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1} - \tilde{\pi}_{s+j_2-j_1} \right) \right\| \\
\leq \; & \sum_{s=0}^{j_1-1} \left\| \widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \right\| \left\| \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} \left(\widehat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1} - \tilde{\pi}_{s+j_2-j_1} \right) \right\| \\
\leq \; & \sum_{s=0}^{j_1-1} \left\| \widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \right\| \left\| \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} \widehat{\tilde{\pi}}_{s+j_2-j_1,s+k_{\max}-j_1} - \Sigma^{-1} \tilde{\pi}_{s+j_2-j_1} \right\| \\
& + \sum_{s=0}^{j_1-1} \left\| \widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \right\| \left\| \Sigma^{-1} - \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} \right\| \left\| \tilde{\pi}_{s+j_2-j_1} \right\| \\
= \; & O_p \left(h_{\max} (\log n/n)^{1/2} \right)
\end{aligned}
$$

by Lemma 4.4vi), (4.22), (4.23) and (4.24). Similarly

$$
\begin{aligned}
& \left\| \sum_{s=0}^{j_1-1} \left(\widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \tilde{\pi}'_s \Sigma^{-1} \right) \tilde{\pi}_{s+j_2-j_1} \right\| \\
\leq \; & \sum_{s=0}^{j_1-1} \left\| \widehat{\tilde{\pi}}'_{s,s+k_{\max}-j_1} \hat{\Sigma}^{-1}_{s+k_{\max}-j_1,\hat{h}} - \tilde{\pi}'_s \Sigma^{-1} \right\| \left\| \tilde{\pi}_{s+j_2-j_1} \right\| \\
= \; & O_p \left(h_{\max} (\log n/n)^{1/2} \right)
\end{aligned}
$$

such that the result follows. ■

Lemma 4.6. *Let Assumptions B, C and D be satisfied. Let $\hat{D}_{M,\hat{h}}$ be as defined in (7.8). Define $k_{\max}$ as $k_{\max} = \sqrt{n/\log n}$. Let $\hat{D}_{k_{\max}\hat{h}} = \sum_{k_1 k_2=1}^{k_{\max}} \hat{\Gamma}^{xy}_{k_1,\hat{h}} \hat{\vartheta}^{k_{\max}}_{k_1 k_2,\hat{h}} \hat{\Gamma}^{yx}_{-k_2,\hat{h}}$. Then,*
i) $\hat{P}'_{M,\hat{h}} \hat{\Omega}^{-1}_{M,\hat{h}} \hat{P}_{M,\hat{h}} - \hat{D}_{k_{\max},\hat{h}} - P'_M \Omega^{-1}_M P_M + D = M^3 \nu^M O_p \left(h_{\max} (\log n/n)^{1/2} \right) + o_p \left(n^{-1} \right)$ uniformly for $M \leq M_{\max}$
ii) $\varphi_n (M)^{-1} \left(\hat{P}'_{M,\hat{h}} \hat{\Omega}^{-1}_{M,\hat{h}} \hat{P}_{M,\hat{h}} - \hat{D}_{k_{\max},\hat{h}} - P'_M \Omega^{-1}_M P_M + D \right) = M^3 O_p \left(h_{\max} (\log n/n)^{1/2} \right) + o_p \left(M^{-2} \right)$.

Proof. For i) let $\hat{H}_{11,\hat{h}} = \hat{P}'_{M,\hat{h}} \hat{\Omega}^{-1}_{M,\hat{h}} \hat{P}_{M,\hat{h}} - \hat{D}_{k_{\max},\hat{h}}$ and $H_{11} = P'_M \Omega^{-1}_M P_M - D$. Then it follows that

$$
\begin{aligned}
(4.45) \quad \hat{H}_{11,\hat{h}} - H_{11} = \; & \sum_{j_1,j_2}^{M} \left[\hat{\Gamma}^{xy}_{j_1,\hat{h}} \left(\hat{\vartheta}^{M}_{j_1 j_2,\hat{h}} - \hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}} \right) \hat{\Gamma}^{yx}_{-j_2,\hat{h}} - \Gamma^{xy}_{j_1} \left(\vartheta^{M}_{j_1 j_2} - \vartheta_{j_1 j_2} \right) \Gamma^{yx}_{-j_2} \right] \\
& - \sum_{j_1=1}^{M} \sum_{j_2=1}^{k_{\max}-M} \left(\hat{\Gamma}^{xy}_{j_1,\hat{h}} \hat{\vartheta}^{k_{\max}}_{j_1,j_2+M,\hat{h}} \hat{\Gamma}^{yx}_{-j_2-M,\hat{h}} - \Gamma^{xy}_{j_1} \vartheta_{j_1,j_2+M} \Gamma^{yx}_{-j_2-M} \right) \\
& - \sum_{j_1=1}^{k_{\max}-M} \sum_{j_2=1}^{M} \left(\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} \hat{\vartheta}^{k_{\max}}_{j_1+M,j_2,\hat{h}} \hat{\Gamma}^{yx}_{-j_2,\hat{h}} - \Gamma^{xy}_{j_1+M} \vartheta_{j_1+M,j_2} \Gamma^{yx}_{-j_2} \right) \\
& - \sum_{j_1,j_2=1}^{k_{\max}-M} \hat{\Gamma}^{xy}_{j_1+M,\hat{h}} \hat{\vartheta}^{k_{\max}}_{j_1+M,j_2+M,\hat{h}} \hat{\Gamma}^{yx}_{-j_2-M,\hat{h}} - \Gamma^{xy}_{j_1+M} \vartheta_{j_1+M,j_2+M} \Gamma^{yx}_{-j_2-M} \\
& + \sum_{j_1=1}^{k_{\max}} \sum_{j_2=1}^{\infty} \Gamma^{xy}_{j_1} \vartheta_{j_1,j_2+k_{\max}} \Gamma^{yx}_{-j_2-k_{\max}} \\
& + \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{k_{\max}} \Gamma^{xy}_{j_1+k_{\max}} \vartheta_{j_1+k_{\max},j_2} \Gamma^{yx}_{-j_2} \\
& + \sum_{j_1,j_2=1}^{\infty} \Gamma^{xy}_{j_1+k_{\max}} \vartheta_{j_1+k_{\max},j_2+k_{\max}} \Gamma^{yx}_{-j_2-k_{\max}}
\end{aligned}
$$

such that $\hat{H}_{11,\hat{h}} - H_{11} = \sum_{j=1}^{7} \psi_{jn}$ where the definitions of ψ_{jn} correspond to the previous display. Without loss of generality replace Γ_j^{xy} by Γ_j^{yy}. The non-stochastic remainder terms ψ_{5n}, ψ_{6n} and $\psi_{7n}$of $\hat{H}_{11,\hat{h}} - H_{11}$ are $O(k_{\max}\nu^{k_{\max}})$ by Lemma 3.1i) where $O(k_{\max}\nu^{k_{\max}}) = o\left(n^{-1}\right)$. For ψ_{4n} consider

$$
\begin{aligned}
(4.46) \qquad \|\psi_{4n}\| \leq\ & \sum_{j_1,j_2=1}^{k_{\max}-M} \left\|\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} - \Gamma^{xy}_{j_1+M}\right\| \left\|\hat{\vartheta}^{k_{\max}}_{j_1+M j_2+M,\hat{h}}\right\| \left\|\hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| \\
&+ \sum_{j_1,j_2=1}^{k_{\max}-M} \left\|\Gamma^{xy}_{j_1+M}\right\| \left\|\hat{\vartheta}^{k_{\max}}_{j_1+M,j_2+M,\hat{h}} - \vartheta_{j_1+M,j_2+M}\right\| \left\|\hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| \\
&+ \sum_{j_1,j_2=1}^{k_{\max}-M} \left\|\Gamma^{xy}_{j_1+M}\right\| \left\|\vartheta_{j_1+M,j_2+M}\right\| \left\|\Gamma^{yx}_{-j_2-M} - \hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\|.
\end{aligned}
$$

For the first term in (4.46) use Lemmas 4.3, 4.5ii) and the fact that $\|\vartheta_{j_1,j_2}\|$ is bounded uniformly in j_1 and j_2 by (2.2) to obtain the bound

$$
\begin{aligned}
(4.47) \qquad & \sum_{j_1,j_2=1}^{k_{\max}-M} \left\|\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} - \Gamma^{xy}_{j_1+M}\right\| \left\|\hat{\vartheta}^{k_{\max}}_{j_1+M j_2+M,\hat{h}}\right\| \left\|\hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| \\
\leq\ & (M+1)^2 \nu^{2M} O_p\left(h_{\max} (\log n/n)^{1/2}\right) O_p\left(1 + h_{\max} (\log n/n)^{1/2}\right) \sum_{j_1,j_2=1}^{k_{\max}-M} j_1 j_2 \nu^{j_1+j_2} \\
& + (M+1)^4 \nu^M \nu_*^M O_p\left(h_{\max} (\log n/n)^{1/2}\right) \\
& \times O_p\left(h^3_{\max} (\log n/n)^{3/2}\right) \sum_{j_1,j_2=1}^{k_{\max}-M} (j_2+1)^3 (j_1+1) \nu_*^{j_2} \nu^{j_1} \\
& + (M+1)^4 \nu^M \nu_*^M O_p\left(1 + h_{\max} (\log n/n)^{1/2}\right) \\
& \times O_p\left(h^3_{\max} (\log n/n)^{3/2}\right) \sum_{j_1,j_2=1}^{k_{\max}-M} (j_1+1)^3 (j_2+1) \nu^{j_2} \nu_*^{j_1} \\
& + (M+1)^6 \nu_*^{2M} O_p\left(h^6_{\max} (\log n/n)^3\right) \sum_{j_1,j_2=1}^{k_{\max}-M} (j_2+1)^3 (j_1+1)^3 \nu_*^{j_2} \nu_*^{j_1} \\
=\ & \nu^M O_p\left(h_{\max} (\log n/n)^{1/2}\right) + o\left(n^{-1}\right)
\end{aligned}
$$

because $(M+1)^6 \nu_*^M$ is bounded uniformly in M and $h^3_{\max} (\log n/n)^{3/2} = o\left(n^{-1}\right)$. The second term in (4.46) is bounded by Lemmas 4.3 and 4.5ii) such that

$$
\begin{aligned}
(4.48) \qquad & \sum_{j_1,j_2=1}^{k_{\max}-M} \left\|\Gamma^{xy}_{j_1+M}\right\| \left\|\hat{\vartheta}^{k_{\max}}_{j_1+M,j_2+M,\hat{h}} - \vartheta_{j_1+M,j_2+M}\right\| \left\|\hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| \\
\leq\ & (M+1) \nu^{2M} O_p\left(h_{\max} (\log n/n)^{1/2}\right) \sum_{j_1,j_2=1}^{k_{\max}-M} j_2 \nu^{j_1} \nu^{j_2} \left(O_p\left(1 + h_{\max} (\log n/n)^{1/2}\right)\right) \\
& + (M+1)^3 \nu^M \nu_*^M O_p\left(h_{\max} (\log n/n)^{1/2}\right) \\
& \times \sum_{j_1,j_2=1}^{k_{\max}-M} \nu^{j_1} (j_2+1)^3 \nu_*^{j_2} O_p\left(h^3_{\max} (\log n/n)^{3/2}\right) \\
=\ & \nu^M O_p\left(h_{\max} (\log n/n)^{1/2}\right) + o_p\left(n^{-1}\right)
\end{aligned}
$$

where Lemma 4.5ii) was used for a uniform bound on $\left\|\hat{\vartheta}^{k_{\max}}_{j_1+M,j_2+M,\hat{h}} - \vartheta_{j_1+M,j_2+M}\right\|$. The third term in (4.46) is bounded by using Lemma 4.3 for $\left\|\Gamma^{yx}_{-j_2-M} - \hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right)$ such that

$$
\begin{aligned}
(4.49)\qquad & \textstyle\sum_{j_1,j_2=1}^{k_{\max}-M}\left\|\Gamma^{xy}_{j_1+M}\right\|\left\|\vartheta_{j_1+M,j_2+M}\right\|\left\|\Gamma^{yx}_{-j_2-M} - \hat{\Gamma}^{yx}_{-j_2-M,\hat{h}}\right\| \\
\leq\ & \nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right)\textstyle\sum_{j_1,j_2=1}^{k_{\max}-M}\nu^{j_1}\left\|\vartheta_{j_1+M,j_2+M}\right\| \\
=\ & \nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right),
\end{aligned}
$$

and form (4.47), (4.48) and (4.49) it follows that

$$
\left\|\psi_{4n}\right\| = \nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right) + o_p\left(n^{-1}\right).
$$

In the same way $\left\|\psi_{3n}\right\|$ is analyzed as

$$
\begin{aligned}
(4.50)\qquad \left\|\psi_{3n}\right\| \leq\ & \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} - \Gamma^{xy}_{j_1+M}\right\|\left\|\hat{\vartheta}^{k_{\max}}_{j_1+Mj_2,\hat{h}}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\Gamma^{xy}_{j_1+M}\right\|\left\|\hat{\vartheta}^{k_{\max}}_{j_1+M,j_2,\hat{h}} - \vartheta_{j_1+M,j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\Gamma^{xy}_{j_1+M}\right\|\left\|\vartheta_{j_1+M,j_2}\right\|\left\|\Gamma^{yx}_{-j_2} - \hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\|
\end{aligned}
$$

where the first term in (4.50) is bounded by

$$
\begin{aligned}
(4.51)\qquad & \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} - \Gamma^{xy}_{j_1+M}\right\|\left\|\hat{\vartheta}^{k_{\max}}_{j_1+Mj_2,\hat{h}} - \vartheta_{j_1+M,j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{xy}_{j_1+M,\hat{h}} - \Gamma^{xy}_{j_1+M}\right\|\left\|\vartheta_{j_1+M,j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
\leq\ & (M+1)\,\nu^M O_p\left(h^2_{\max}\log n/n\right)\textstyle\sum_{j_1=1}^{k_{\max}-M} j_1\nu^{j_1}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + (M+1)^3\,\nu^M_* O_p\left(h^4_{\max}(\log n/n)^2\right)\textstyle\sum_{j_1=1}^{k_{\max}-M} j_1^3\nu_*^{j_1}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + (M+1)\,\nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right)\textstyle\sum_{j_1=1}^{k_{\max}-M} j_1\nu^{j_1}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
& + \nu^M (M+1)^3\,\nu^M_* O_p\left(h^3_{\max}(\log n/n)^{3/2}\right)\textstyle\sum_{j_1=1}^{k_{\max}-M} j_1^3\nu_*^{j_1}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
=\ & (M+1)\,\nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right) + o_p\left(n^{-1}\right)
\end{aligned}
$$

where $\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| = O_p(1)$ by Lemma 4.3, $\left\|\vartheta_{j_1+M,j_2}\right\| \leq CM\nu^M$ by Lemma 2.5ii), $M\nu^M$ is uniformly bounded in M for a fixed $\nu \in (0,1)$,and $h^3_{\max}(\log n/n)^{3/2} = o\left(n^{-1}\right)$.

The second term in (4.50) is bounded by

$$
\begin{aligned}
(4.52)\qquad & \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\Gamma^{xy}_{j_1+M}\right\|\left\|\hat{\vartheta}^{k_{\max}}_{j_1+M,j_2,\hat{h}} - \vartheta_{j_1+M,j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
\leq\ & C(M+1)\,\nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)\textstyle\sum_{j_1=1}^{k_{\max}-M} j_1\nu^{j_1}\sum_{j_2=1}^{M}\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
=\ & (M+1)\,\nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)
\end{aligned}
$$

by Lemmas 4.3 and 4.5ii). Finally, the third term in (4.50) is bounded by

$$\begin{aligned}
(4.53)\qquad & \textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\left\|\Gamma^{xy}_{j_1+M}\right\|\left\|\vartheta_{j_1+M,j_2}\right\|\left\|\Gamma^{yx}_{-j_2}-\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
\leq\ & CM\nu^{2M}\textstyle\sum_{j_1=1}^{k_{\max}-M}\sum_{j_2=1}^{M}\nu^{j_1} \\
& \times\left(j_2\nu^{j_2}O_p\left(h_{\max}(\log n/n)^{1/2}\right)+j_2^3\nu_*^{j_2}O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\right) \\
=\ & M\nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right)
\end{aligned}$$

such that from (4.51), (4.52) and (4.53) it follows that

$$\|\psi_{3n}\| = M\nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right)+o_p\left(n^{-1}\right).$$

By symmetry, this implies that

$$\|\psi_{2n}\| = M\nu^M O_p\left(h_{\max}(\log n/n)^{1/2}\right)+o_p\left(n^{-1}\right)$$

as well.

For ψ_{1n} consider

$$\begin{aligned}
(4.54)\qquad \psi_{1n} =\ & \textstyle\sum_{j_1,j_2}^{M}\hat{\Gamma}^{xy}_{j_1,\hat{h}}\left(\hat{\vartheta}^{M}_{j_1j_2,\hat{h}}-\hat{\vartheta}^{k_{\max}}_{j_1j_2,\hat{h}}-\left(\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right)\right)\hat{\Gamma}^{yx}_{-j_2,\hat{h}} \\
& +\textstyle\sum_{j_1,j_2}^{M}\left(\hat{\Gamma}^{xy}_{j_1,\hat{h}}-\Gamma^{xy}_{j_1}\right)\left(\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right)\Gamma^{yx}_{-j_2} \\
& +\textstyle\sum_{j_1,j_2}^{M}\Gamma^{xy}_{j_1}\left(\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right)\left(\hat{\Gamma}^{yx}_{-j_2,\hat{h}}-\Gamma^{yx}_{-j_2}\right) \\
& -\textstyle\sum_{j_1,j_2}^{M}\left(\hat{\Gamma}^{xy}_{j_1,\hat{h}}-\Gamma^{xy}_{j_1}\right)\left(\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right)\left(\hat{\Gamma}^{yx}_{-j_2,\hat{h}}-\Gamma^{yx}_{-j_2}\right)
\end{aligned}$$

where the first term in (4.54) is bounded by

$$\begin{aligned}
& \textstyle\sum_{j_1,j_2}^{M}\left\|\hat{\Gamma}^{xy}_{j_1,\hat{h}}\right\|\left\|\hat{\vartheta}^{M}_{j_1j_2,\hat{h}}-\hat{\vartheta}^{k_{\max}}_{j_1j_2,\hat{h}}-\left(\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right)\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\
\leq\ & \textstyle\sum_{j_1,j_2}^{M}\nu^{j_1}(M-j_1+1)^2\nu^{M-j_1}\nu^{j_2}O_p\left(h_{\max}\sqrt{\log n/n}\right) \\
& +\textstyle\sum_{j_1,j_2}^{M}\nu^{j_1}(M-j_1+1)^5\nu_*^{M-j_1}\nu^{j_2}O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)+o_p\left(M^2\nu^M h_{\max}\sqrt{\log n/n}\right) \\
\leq\ & M^3\nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)+M^6\nu_*^M O_p\left(h_{\max}^3(\log n/n)^{3/2}\right) \\
=\ & M^3\nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)+o_p\left(n^{-1}\right)
\end{aligned}$$

by Lemmas 2.3, 4.3 and 4.5i). The second term is bounded by

$$\begin{aligned}
& \textstyle\sum_{j_1,j_2}^{M}\left\|\hat{\Gamma}^{xy}_{j_1,\hat{h}}-\Gamma^{xy}_{j_1}\right\|\left\|\vartheta^{M}_{j_1j_2}-\vartheta_{j_1j_2}\right\|\left\|\Gamma^{yx}_{-j_2}\right\| \\
\leq\ & \nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)\textstyle\sum_{j_1,j_2}^{M}j_1\nu^{j_1}(M-j_2) \\
& +\nu^M O_p\left(h_{\max}^3(\log n/n)^{3/2}\right)\textstyle\sum_{j_1,j_2}^{M}j_1^3\nu_*^{j_1}(M-j_2) \\
=\ & M^2\nu^M O_p\left(h_{\max}\sqrt{\log n/n}\right)+o_p(1)
\end{aligned}$$

by Lemmas 4.3 and 2.7ii). The third term is the same by symmetry. The last term in (4.54) is of smaller order.

For ii) note that $\varphi_n(M) \geq (Mp)^2 n^{-1}\mathcal{A}$ and $\varphi_n(M) \geq \ell' D^{-1}\sigma_{1M} D^{-1}\ell \geq \nu^M \varepsilon$ by the assumptions in Theorem 4.1 such that

$$\begin{aligned} & \varphi_n(M)^{-1}\ell' D^{-1}\left(\hat{P}'_{M,\hat{h}}\hat{\Omega}^{-1}_{M,\hat{h}}\hat{P}_{M,\hat{h}} - \hat{D}_{k_{\max},\hat{h}} - P'_M \Omega_M^{-1} P_M + D\right) D^{-1}\ell \\ \leq\ & \frac{\nu^M O_p\left(M^3 h_{\max}(\log n/n)^{1/2}\right)}{\nu^M \varepsilon} + \frac{o_p\left(n^{-1}\right)}{(Mp)^2 n^{-1}\mathcal{A}} \\ =\ & O_p\left(M^3 h_{\max}(\log n/n)^{1/2}\right) + o_p\left(M^{-2}\right) \end{aligned}$$

where the last equality follows from Lemma 4.6i) above. ■

Lemma 4.7. *Let Assumptions B, C and D be satisfied. Let* $\hat{D}_{k_{\max},\hat{h}}$ *be defined as in Step 13. Then,*

$$\hat{D}_{k_{\max},\hat{h}} - D = O_p\left(h_{\max}\sqrt{\log n/n}\right).$$

Proof. Write

$$\begin{aligned} \hat{D}_{k_{\max},\hat{h}} - D \ =\ & \textstyle\sum_{j_1,j_2}^{k_{\max}}\left(\hat{\Gamma}^{xy}_{j_1,\hat{h}}\hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}}\hat{\Gamma}^{yx}_{-j_2,\hat{h}} - \Gamma^{xy}_{j_1}\vartheta_{j_1 j_2}\Gamma^{yx}_{-j_2}\right) \\ & + \textstyle\sum_{j_1=1}^{k_{\max}}\sum_{j_2=1}^{\infty}\Gamma^{xy}_{j_1}\vartheta_{j_1,j_2+k_{\max}}\Gamma^{yx}_{-j_2-k_{\max}} \\ & + \textstyle\sum_{j_1=1}^{\infty}\sum_{j_2=1}^{k_{\max}}\Gamma^{xy}_{j_1+k_{\max}}\vartheta_{j_1+k_{\max},j_2}\Gamma^{yx}_{-j_2} \\ & + \textstyle\sum_{j_1,j_2=1}^{\infty}\Gamma^{xy}_{j_1+k_{\max}}\vartheta_{j_1+k_{\max},j_2+k_{\max}}\Gamma^{yx}_{-j_2-k_{\max}}. \end{aligned}$$

where the last three terms are $O(k_{\max}\nu^{k_{\max}}) = o(\sqrt{\log n/n})$ by the same arguments as in the proof of Lemma 3.1i). For the first term consider

$$\begin{aligned} \left\|\textstyle\sum_{j_1,j_2}^{k_{\max}}\left(\hat{\Gamma}^{xy}_{j_1,\hat{h}}\hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}}\hat{\Gamma}^{yx}_{-j_2,\hat{h}} - \Gamma^{xy}_{j_1}\vartheta_{j_1 j_2}\Gamma^{yx}_{-j_2}\right)\right\| \ \leq\ & \textstyle\sum_{j_1,j_2}^{k_{\max}}\left\|\hat{\Gamma}^{xy}_{j_1,\hat{h}} - \Gamma^{xy}_{j_1}\right\|\left\|\hat{\vartheta}^{h_{\max}}_{j_1 j_2,\hat{h}}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\ & + \textstyle\sum_{j_1,j_2}^{k_{\max}}\left\|\Gamma^{xy}_{j_1}\right\|\left\|\hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}} - \vartheta_{j_1 j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\ & + \textstyle\sum_{j_1,j_2}^{k_{\max}}\left\|\Gamma^{xy}_{j_1}\right\|\left\|\vartheta_{j_1 j_2}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}} - \Gamma^{yx}_{-j_2}\right\| \end{aligned}$$

where by Lemma 4.3 it follows that $\left\|\hat{\Gamma}^{yx}_{j,\hat{h}} - \Gamma^{yx}_j\right\| = j\nu^j O_p(h_{\max}\sqrt{\log n/n}) + j^3\nu_*^j O_p(h^3_{\max}(\log n/n)^{3/2})$ uniformly in $j \leq k_{\max}$. By Lemma 4.5 $\left\|\hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}} - \vartheta_{j_1 j_2}\right\| = O_p\left(h_{\max}(\log n/n)^{1/2}\right)$. Then

$$\begin{aligned} & \textstyle\sum_{j_1,j_2}^{k_{\max}}\left\|\hat{\Gamma}^{xy}_{j_1,\hat{h}} - \Gamma^{xy}_{j_1}\right\|\left\|\hat{\vartheta}^{h_{\max}}_{j_1 j_2,\hat{h}}\right\|\left\|\hat{\Gamma}^{yx}_{-j_2,\hat{h}}\right\| \\ =\ & \textstyle\sum_{j_1=1}^{k_{\max}}\left(j_1\nu^{j_1}O_p(h_{\max}\sqrt{\log n/n}) + j_1^3\nu_*^{j_1}O_p(h^3_{\max}(\log n/n)^{3/2})\right) \\ & \times\textstyle\sum_{j_2=1}^{k_{\max}}\left(j_2\nu^{j_2}O_p(1 + h_{\max}\sqrt{\log n/n}) + j_2^3\nu_*^{j_2}O_p(h^3_{\max}(\log n/n)^{3/2})\right) \\ =\ & O_p\left(h_{\max}\sqrt{\log n/n}\right). \end{aligned}$$

The other terms can be bounded in a similar way. This establishes the Lemma. ■

Lemma 4.8. *Assume that Conditions B and C hold. Let* $\widehat{\mathcal{A}}_1$ *be defined in 7.9. Then,* $\sqrt{n}\left(\widehat{\mathcal{A}}_1 - \mathcal{A}_1\right) = O_p(1)$.

Proof. To show consistency of

$$\sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{\varepsilon x} = \sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_{m-j}^{yx} + \tilde{\beta}' \sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{xx}$$

consider without loss of generality $\sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{yy}$ since x_t is composed of elements of $y_t, y_{t-1}, ..., y_{t-r}$. Next show that

$$\sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{yy} - \sum_{j=-(n-1)/2}^{(n-1)/2} \zeta_j \Gamma_j^{yy} = O_p(n^{-1/2}).$$

Write

$$\begin{aligned} & \sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{yy} - \sum_{j=-(n-1)/2}^{(n-1)/2} \zeta_j \Gamma_j^{yy} \\ = \ & \sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \left(\hat{\Gamma}_j^{yy} - \Gamma_j^{yy}\right) - \sum_{j=-(n-1)/2}^{(n-1)/2} \left(\hat{\zeta}_j - \zeta_j\right) \Gamma_j^{yy}. \end{aligned}$$

First consider

$$n^{1/2} \sum_{j=-(n-1)/2}^{(n-1)/2} \left\|\hat{\zeta}_j - \zeta_j\right\| \left\|\Gamma_j^{yy}\right\| \leq n^{1/2} \sup_j \left\|\hat{\zeta}_j - \zeta_j\right\| \sum_{j=-(n-1)/2}^{(n-1)/2} \left\|\Gamma_j^{yy}\right\|$$

where $P(\sup_j \left\|\hat{\zeta}_j - \zeta_j\right\| > Cn^{-1/2})$ goes to zero for some C large enough because $\hat{\theta} - \theta = O_p\left(n^{-1/2}\right)$ by assumption.

There exists $\delta > 0$ such that $|\theta - \theta_0| < \delta$ implies $\theta(z)$ has no zeros on or inside the unit circle. Consider

$$\begin{aligned} P\left(n^{1/2} \sum_{j=-(n-1)/2}^{(n-1)/2} \left|\hat{\zeta}_j\right| \left\|\hat{\Gamma}_j^{yy} - \Gamma_j^{yy}\right\| > \eta\right) \ \leq \ & P\left(n^{1/2} \sup_{|\theta-\theta_0|<\delta} \sum_{j=-(n-1)/2}^{(n-1)/2} \left|\zeta_j(\theta)\right| \left\|\hat{\Gamma}_j^{yy} - \Gamma_j^{yy}\right\| > \eta\right) \\ & + P\left(\left|\hat{\theta} - \theta_0\right| \geq \delta\right) \end{aligned}$$

Use the triangular inequality $\left\|\hat{\Gamma}_j^{yy} - \Gamma_j^{yy}\right\| \leq \left\|\hat{\Gamma}_j^{yy} - \check{\Gamma}_j^{yy}\right\| + \left\|\check{\Gamma}_j^{yy} - \Gamma_j^{yy}\right\|$ such that

$$\begin{aligned} & n^{1/2} \sup_{|\theta-\theta_0|<\delta} \sum_{j=-(n-1)/2}^{(n-1)/2} \left|\zeta_j(\theta)\right| \left\|\hat{\Gamma}_j^{yy} - \check{\Gamma}_j^{yy}\right\| \\ \leq \ & \left(\sup_{|\theta-\theta_0|<\delta} \sum_{j=-(n-1)/2}^{(n-1)/2} |j| \left|\zeta_j(\theta)\right|\right) \left(n \sum_{j=-(n-1)/2}^{(n-1)/2} |j|^{-2} \left\|\hat{\Gamma}_j^{yy} - \check{\Gamma}_j^{yy}\right\|^2\right)^{1/2} \\ = \ & O_p(1) \end{aligned}$$

where the inequality follows from the Cauchy-Schwartz inequality and the last line is obtained by Lemma 2.8i) which shows that $n^{-1}E\left\|\hat{\Gamma}_j^{yy}-\check{\Gamma}_j^{yy}\right\|^2$ is uniformly bounded in j and the fact that

$$\sup_{|\theta-\theta_0|<\delta}\sum\nolimits_{j=-(n-1)/2}^{(n-1)/2}|j|\left|\zeta_j(\theta)\right|=O(1)$$

uniformly in n. In the same way it follows that

$$n^{1/2}\sup_{|\theta-\theta_0|<\delta}\sum\nolimits_{j=-(n-1)/2}^{(n-1)/2}\left|\zeta_j(\theta)\right|\left\|\check{\Gamma}_j^{yy}-\Gamma_j^{yy}\right\|=O_p(1)$$

because by Lemma 2.8v), $E\left[\left\|\check{\Gamma}_j^{yy}-\Gamma_j^{yy}\right\|^2\right]=O\left(n^{-1}\right)$ uniformly in j. This establishes that $\widehat{\mathcal{A}}_1-\mathcal{A}_1=O_p(n^{-1/2})$. ■

Lemma 4.9. *Assume that Conditions B, C and D hold. Let $\widehat{\mathcal{A}}_2$ be defined in 7.10. Then,* $\left(\widehat{\mathcal{A}}_2-\mathcal{A}_2\right)=O_p\left(h_{\max}(\log n/n)^{1/2}\right)$.

Proof. For $\widehat{\mathcal{A}}_2$ note that$\sum_{j_1,j_2=1}^{k_{\max}}\hat{\Gamma}_{j_1,\hat{h}}^{xy}\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\hat{\Gamma}_{-j_2}^{\varepsilon y}$

$$\begin{aligned}\left\|\widehat{\mathcal{A}}_2-\mathcal{A}_2\right\| &\leq \sum\nolimits_{j_2,j_1=1}^{k_{\max}}\left\|\hat{\Gamma}_{j_1,\hat{h}}^{xy}-\Gamma_{j_1}^{xy}\right\|\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}\right\| \\ &\quad+\sum\nolimits_{j_2,j_1=1}^{k_{\max}}\left\|\Gamma_{j_1}^{xy}\right\|\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}-\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}\right\| \\ &\quad+\sum\nolimits_{j_2=1}^{k_{\max}}\left\|\sum\nolimits_{j_1=1}^{k_{\max}}\Gamma_{j_1}^{xy}\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\|.\end{aligned}\tag{4.55}$$

By Lemma 4.3

$$\left\|\hat{\Gamma}_{j,\hat{h}}^{xy}-\Gamma_j^{xy}\right\|=j\nu^jO_p(h_{\max}(\log n/n)^{1/2})+j^3\nu^jO_p(h_{\max}^3(\log n/n)^{3/2})$$

where the $O_p(.)$- term is uniform in j. It then follows that

$$\begin{aligned}\sum\nolimits_{j_2,j_1=1}^{k_{\max}}\left\|\hat{\Gamma}_{j_1,\hat{h}}^{xy}-\Gamma_{j_1}^{xy}\right\|\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}\right\| &\leq O_p(h_{\max}(\log n/n)^{1/2})\sum\nolimits_{j_2,j_1=1}^{k_{\max}}j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\| \\ &\quad+O_p(h_{\max}(\log n/n)^{1/2})\sum\nolimits_{j_2,j_1=1}^{k_{\max}}j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\Gamma_{-j_2}^{\varepsilon y}\right\|\end{aligned}$$

where $\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|$ is uniformly bounded by Lemma 4.5ii) and (2.2). Then

$$\begin{aligned}\sum\nolimits_{j_2,j_1=1}^{k_{\max}}j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\Gamma_{-j_2}^{\varepsilon y}\right\| &\leq O_p\left(h_{\max}(\log n/n)^{1/2}\right)\sum\nolimits_{j_2,j_1=1}^{k_{\max}}j_1\nu^{j_1}\left\|\Gamma_{-j_2}^{\varepsilon y}\right\| \\ &\quad+\sum\nolimits_{j_2,j_1=1}^{k_{\max}}j_1\nu^{j_1}\left\|\vartheta_{j_1j_2}\right\|\left\|\Gamma_{-j_2}^{\varepsilon y}\right\| \\ &= O_p(1).\end{aligned}$$

by Lemma 4.5ii). Furthermore,

$$\begin{aligned}\sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\| &\leq \sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}-\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\| \\ &\quad+\sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\|\end{aligned}$$

where $\sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\vartheta_{j_1j_2}\right\|\left(E\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\|^2\right)^{1/2}=O(k_{\max}n^{-1/2})=o(1)$ and

$$\begin{aligned}\sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}-\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\| &\leq O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)\sum_{j_2,j_1=1}^{k_{\max}} j_1\nu^{j_1}\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\| \\ &= O_p(h_{\max}\left(\log n/n\right)^{1/2}k_{\max}n^{-1/2})=o_p\left(1\right)\end{aligned}$$

such that $\sum_{j_2,j_1=1}^{k_{\max}}\left\|\hat{\Gamma}_{j_1,\hat{h}}^{xy}-\Gamma_{j_1}^{xy}\right\|\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}\right\|=O_p\left(h_{\max}\left(\log n/n\right)^{1/2}\right)$. Similar arguments show that

$$\sum_{j_2,j_1=1}^{k_{\max}}\left\|\Gamma_{j_1}^{xy}\right\|\left\|\hat{\vartheta}_{j_1j_2,\hat{h}}^{k_{\max}}-\vartheta_{j_1j_2}\right\|\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}\right\|=O_p(h_{\max}\left(\log n/n\right)^{1/2}).$$

For the last term in (4.55) note that

$$\sum_{j_2=1}^{k_{\max}}\left\|\sum_{j_1=1}^{k_{\max}}\Gamma_{j_1}^{xy}\vartheta_{j_1j_2}\right\|\left(E\left\|\hat{\Gamma}_{-j_2}^{\varepsilon y}-\Gamma_{-j_2}^{\varepsilon y}\right\|^2\right)^{1/2}=O\left(n^{-1/2}\right).$$

This establishes that $\left\|\widehat{\mathcal{A}}_2-\mathcal{A}_2\right\|=O_p((\log n/n)^{1/2})$. ■

5. Bias and MSE Calculations

In this section the bias approximations derived in the main text are obtained for the model used in the Monte Carlo simulations. Recall the model

$$
\begin{aligned}
y_{t,1} &= \beta y_{t,2} + u_{t,1} - \theta u_{t-1,1} \\
y_{t,2} &= \phi y_{t-1,2} + u_{t,2}
\end{aligned}
\tag{5.1}
$$

and let $\rho = cov\left(u_{t,1}, u_{t,2}\right)$ for the following calculations. The constants $\mathcal{A}_1 = (4\pi)^{-1} \int_{-\pi}^{\pi} f_{\varepsilon x}(\lambda) f_{\varepsilon}^{-1}(\lambda) d\lambda$, $\mathcal{A}_2 = 2^{-1} \int_{-\pi}^{\pi} f^a\left(\lambda\right) f_{\varepsilon y}(\lambda) d\lambda$ and D are computed as follows. First,

$$f_{\varepsilon}(\lambda) = \frac{1}{2\pi}\left(1 - \theta e^{-i\lambda}\right)\left(1 - \theta e^{i\lambda}\right)$$

such that

$$f_{\varepsilon}(\lambda)^{-1} = 2\pi \frac{1}{\left(1 - \theta e^{-i\lambda}\right)\left(1 - \theta e^{i\lambda}\right)} = 2\pi \sum_{j=-\infty}^{\infty} \frac{\theta^{|j|}}{1-\theta^2} e^{-i\lambda j}.$$

Let $y_t = \left(y_{t,1}, y_{t,2}\right)'$. Then, using $\theta\left(L\right) = \left(1 - \theta L\right)$ and $\phi\left(L\right) = \left(1 - \phi L\right)$

$$y_t = \begin{bmatrix} \theta\left(L\right) & \phi\left(L\right)^{-1} \\ 0 & \phi\left(L\right)^{-1} \end{bmatrix} u_t$$

and

$$\pi\left(L\right) y_t := \begin{bmatrix} \theta\left(L\right)^{-1} & -\theta\left(L\right)^{-1} \\ 0 & \phi\left(L\right) \end{bmatrix} y_t = u_t$$

where $\pi\left(L\right) = \sum_{j=0}^{\infty} \pi_j L^j$ has coefficient matrices

$$\pi_0 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \pi_1 = \begin{bmatrix} -\theta & \theta \\ 0 & -\phi \end{bmatrix}, \pi_j = \begin{bmatrix} -\theta^j & \theta^j \\ 0 & 0 \end{bmatrix}.$$

Next, compute

$$\tilde{\pi}\left(L\right) = \theta\left(L\right)^{-1} \pi\left(L\right) = \begin{bmatrix} \theta\left(L\right)^{-2} & -\theta\left(L\right)^{-2} \\ 0 & \theta\left(L\right)^{-1} \phi\left(L\right) \end{bmatrix}$$

with coefficient matrices

$$\tilde{\pi}_0 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \tilde{\pi}_1 = \begin{bmatrix} -2\theta & 2\theta \\ 0 & \theta - \phi \end{bmatrix}, \tilde{\pi}_j = \begin{bmatrix} -\left(j+1\right)\theta^j & \left(j+1\right)\theta^j \\ 0 & \theta^{j-1}(\theta - \phi) \end{bmatrix}.$$

We have

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \Sigma^{-1} = \begin{bmatrix} 1/\left(1-\rho^2\right) & -\rho/\left(1-\rho^2\right) \\ -\rho/\left(1-\rho^2\right) & 1/\left(1-\rho^2\right) \end{bmatrix}$$

and for $j \leq k$,

$$\vartheta_{j,k} = \sum_{l=1}^{j-1} \begin{bmatrix} -(l+1)\,\theta^{l} & (l+1)\,\theta^{l} \\ 0 & \theta^{l-1}(\theta-\phi) \end{bmatrix}' \Sigma^{-1} \begin{bmatrix} -(l+k-j+1)\,\theta^{l+k-j} & (l+k-j+1)\,\theta^{l+k-j} \\ 0 & \theta^{l+k-j-1}(\theta-\phi) \end{bmatrix}$$
$$+ \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}' \Sigma^{-1} \begin{bmatrix} -(k-j+1)\,\theta^{l+k-j} & (k-j+1)\,\theta^{k-j} \\ 0 & \theta^{k-j-1}(\theta-\phi) \end{bmatrix}.$$

Next,

$$\begin{aligned} \Gamma_{t-s}^{\varepsilon x} &= E\left[\varepsilon_t y_{s,2}\right] \\ &= E\left[(u_{t,1}-\theta u_{t-1,1})\sum_{j=0}^{\infty}\phi^{j}u_{s-j,2}\right] = \begin{cases} 0 & s<t-1 \\ -\theta\rho & s=t-1 \\ \phi^{s-t}\rho\,(1-\theta\phi) & s>t-1 \end{cases}, \end{aligned}$$

$$\begin{aligned} \Gamma_{t-s}^{\varepsilon y} &= E\left[\varepsilon_{t+2} y_{s}\right] \\ &= E\left[(u_{t+2,1}-\theta u_{t+1,1})\begin{pmatrix} (u_{s,1}-\theta u_{s-1,1})+\sum_{j=0}^{\infty}\phi^{j}u_{s-j,2} \\ \sum_{j=0}^{\infty}\phi^{j}u_{s-j,2} \end{pmatrix}\right] \\ &= \begin{cases} 0 & s<t+1 \\ \begin{bmatrix} -\theta\,(1+\rho) \\ -\theta\rho \end{bmatrix} & s=t+1 \\ \begin{bmatrix} \left(1+\theta^{2}\right)+\rho\,(1-\theta\phi) \\ \rho\,(1-\theta\phi) \end{bmatrix} & s=t+2 \\ \begin{bmatrix} -\theta+\phi\rho\,(1-\theta\phi) \\ \phi\rho\,(1-\theta\phi) \end{bmatrix} & s=t+3 \\ \begin{bmatrix} \phi^{s-t-2}\rho\,(1-\theta\phi) \\ \phi^{s-t-2}\rho\,(1-\theta\phi) \end{bmatrix} & s>t+3 \end{cases} \end{aligned}$$

and

$$\begin{aligned} \Gamma_{j}^{xy} &= E\left[y_{t+m,2}y_{t-j+1}\right] \\ &= E\left[\sum_{k=0}^{\infty}\phi^{k}u_{t+2-k,2}\begin{pmatrix} u_{t-j+1,1}-\theta u_{t-j,1}+\sum_{k=0}^{\infty}\phi^{k}u_{t-j+1-k,2} \\ \sum_{k=0}^{\infty}\phi^{k}u_{t-j+1-k,2} \end{pmatrix}\right] \\ &= \begin{bmatrix} \phi^{1+j}\,(1-\phi\theta)\,\rho+\phi^{1+j}/\left(1-\phi^{2}\right) \\ \phi^{1+j}/\left(1-\phi^{2}\right) \end{bmatrix} \end{aligned}$$

After substitution one obtains[3]

$$\begin{aligned}
\mathcal{A}_1 &= (4\pi)^{-1}\int_{-\pi}^{\pi}\frac{1}{2\pi}\sum_{k=-\infty}^{1}\Gamma_k^{\varepsilon x}e^{-i\lambda k}2\pi\sum_{j=-\infty}^{\infty}\frac{\theta^{|j|}}{1-\theta^2}e^{-i\lambda j}d\lambda = \frac{1}{2\left(1-\theta^2\right)}\sum_{k=-\infty}^{1}\Gamma_k^{\varepsilon x}\theta^{|k|} \\
&= \Gamma_1^{\varepsilon x}\theta+\Gamma_0^{\varepsilon x}+\sum_{k=-\infty}^{-1}\Gamma_k^{\varepsilon x}\theta^{|k|} \\
&= \frac{\rho}{2}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{A}_2 &= 2^{-1}\int_{-\pi}^{\pi} f^a\left(\lambda\right)f_{\varepsilon y}(\lambda)d\lambda \\
&= 2^{-1}\int_{-\pi}^{\pi}\sum_{j_1,j_2=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2}e^{-i\lambda j_2}\sum_{k=-\infty}^{-1}\frac{1}{2\pi}\Gamma_k^{\varepsilon y}e^{-i\lambda k} \\
&= \sum_{j_1,j_2=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2}\Gamma_{-j_2}^{\varepsilon y}
\end{aligned}$$

such that

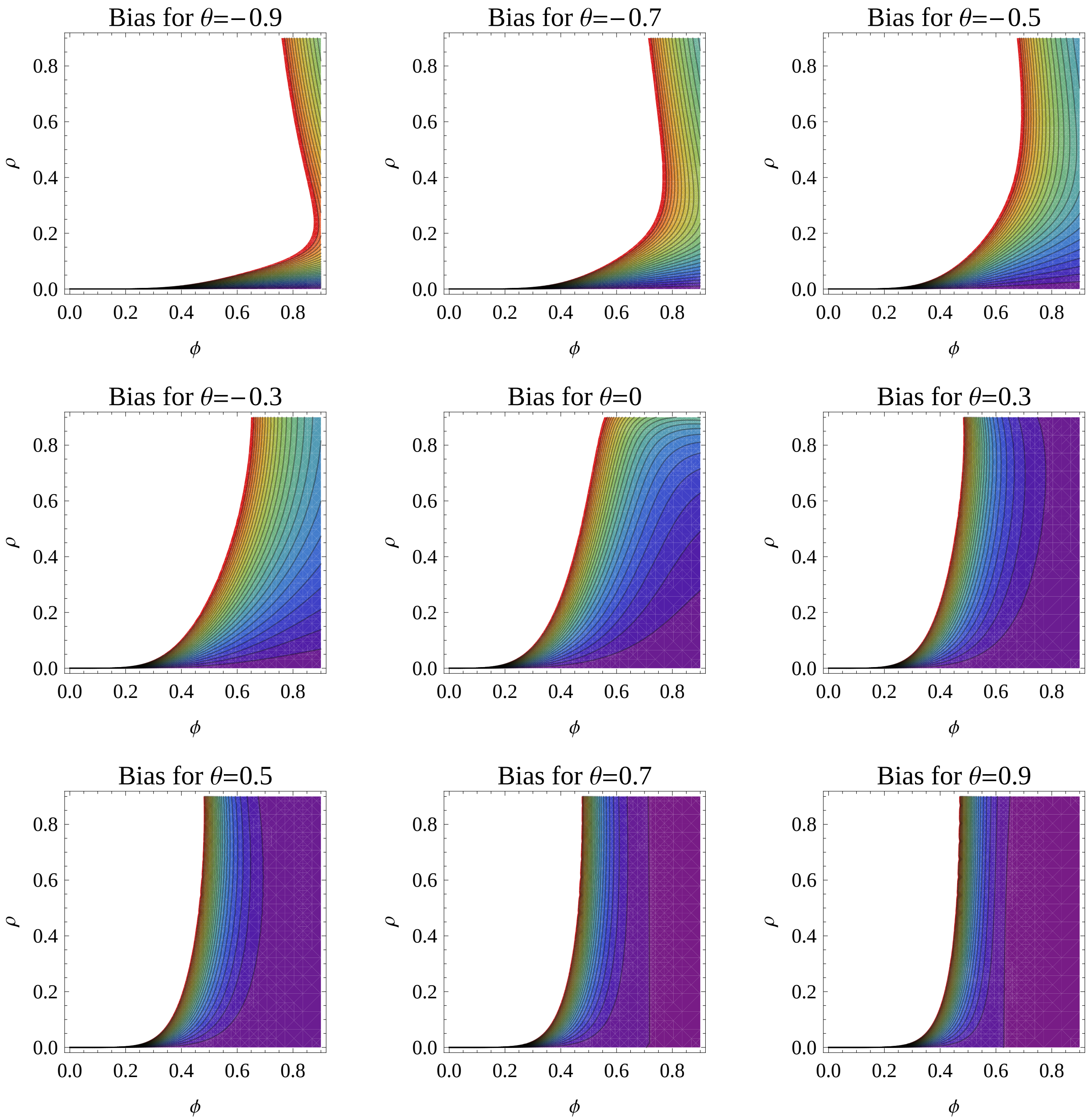
$$\begin{aligned}
\mathcal{A}_2 = \; & \frac{\rho}{2\left(\rho^2-1\right)\phi^2\left(\phi^2-1\right)\left(\theta\phi-1\right)}(\theta^6\phi^4\left(\phi^2-1\right)\left(\rho^2\left(3\phi^6+2\right)+9\phi^4\right) \\
& +\theta^5\phi^3\left(-\rho^2(\phi-1)(\phi+1)\left(\left((3\phi(\phi+1)+8)\phi^4+2\right)\phi^2+4\right)+(\phi((\phi-17)\phi-1)+21)\phi^4+1\right) \\
& +\theta^4\phi^2\left(11\rho^2\phi^{10}+7\rho^2\phi^9-\left(5\rho^2+1\right)\phi^8-\left(7\rho^2+3\right)\phi^7\right) \\
& +\theta^4\phi^2\left(\left(10-6\rho^2\right)\phi^6+6\left(\rho^2-3\right)\phi^4-2\left(3\rho^2+1\right)\phi^2+2\phi^5-2\right) \\
& +\theta^3\phi\left(-\rho^2(\phi-1)(\phi+1)\left(\phi^2\left((3\phi(5\phi+1)-1)\phi^4+6\right)-3\right)\right) \\
& +\theta^3\phi\left(3\phi^2+\left(\phi\left(\phi\left(\phi\left(\phi^3-\phi^2+\phi+4\right)-7\right)-1\right)+13\right)\phi^4+2\right) \\
& +\theta^2\left(\rho^2(\phi-1)(\phi+1)\left(9\phi^8-3\phi^7-4\phi^6+2\phi^2-1\right)\right) \\
& -\theta^2\left(\phi^2+\left(\left((\phi-1)\phi\left(3\phi^2-4\right)+5\right)\phi^2+2\right)\phi^4+1\right) \\
& +\theta\phi^3\left(\phi^2\left(-2\rho^2(\phi-2)(\phi-1)(\phi+1)^2+\phi(\phi(\phi(3\phi-2)-4)+2)+4\right)+1\right) \\
& +\phi^4\left(-\rho^2\left(\phi^2-1\right)-\phi^4+\phi^2-1\right))
\end{aligned}$$

while

$$\begin{aligned}
D &= \sum_{j_1,j_2=1}^{\infty}\Gamma_{j_1}^{xy}\vartheta_{j_1,j_2}\Gamma_{-j_2}^{xy\prime} \\
&= \frac{\phi^4\left(1+\rho^2\left(4\theta^2\phi^2(-2+\theta\phi)^2\right)-1\right)}{\left(\rho^2-1\right)\left(\theta\phi-1\right)^2\left(\phi^2-1\right)}
\end{aligned}$$

[3]Mathematica code used to carry out the calculations below can be downloaded from the author's web-page.

It can be noted that for $\theta = 0$, $D = \frac{\phi^4}{1-\phi^2}$. It follows that D^{-1} is then equal to the asymptotic variance of the 2SLS estimator using $y_{t-2,2}$ as the only instrument. The approximate bias for the truncated kernel is a complicated function of the underlying parameters because of the complexity of $\mathcal{A}_2$. Contour plots that depict the bias as a function of ϕ and ρ are reported for various values of θ. The magnitude of the bias increases from purple to red. White areas in the graphs indicate that the bias exceeds the set upperbound used to produce the graphs. These graphs confirm findings in the Monte Carlo simulations, namely that the bias generally increases with ρ, decreases with ϕ and decreases in θ. In particular, one finds that the bias is largest for $\theta = -.9$ and gradually decreases as θ moves to .9. The contour plots reflect this by the decreasing white area as θ increases.



For $M = 1$ the GMM variance can be obtained in tractable closed form as

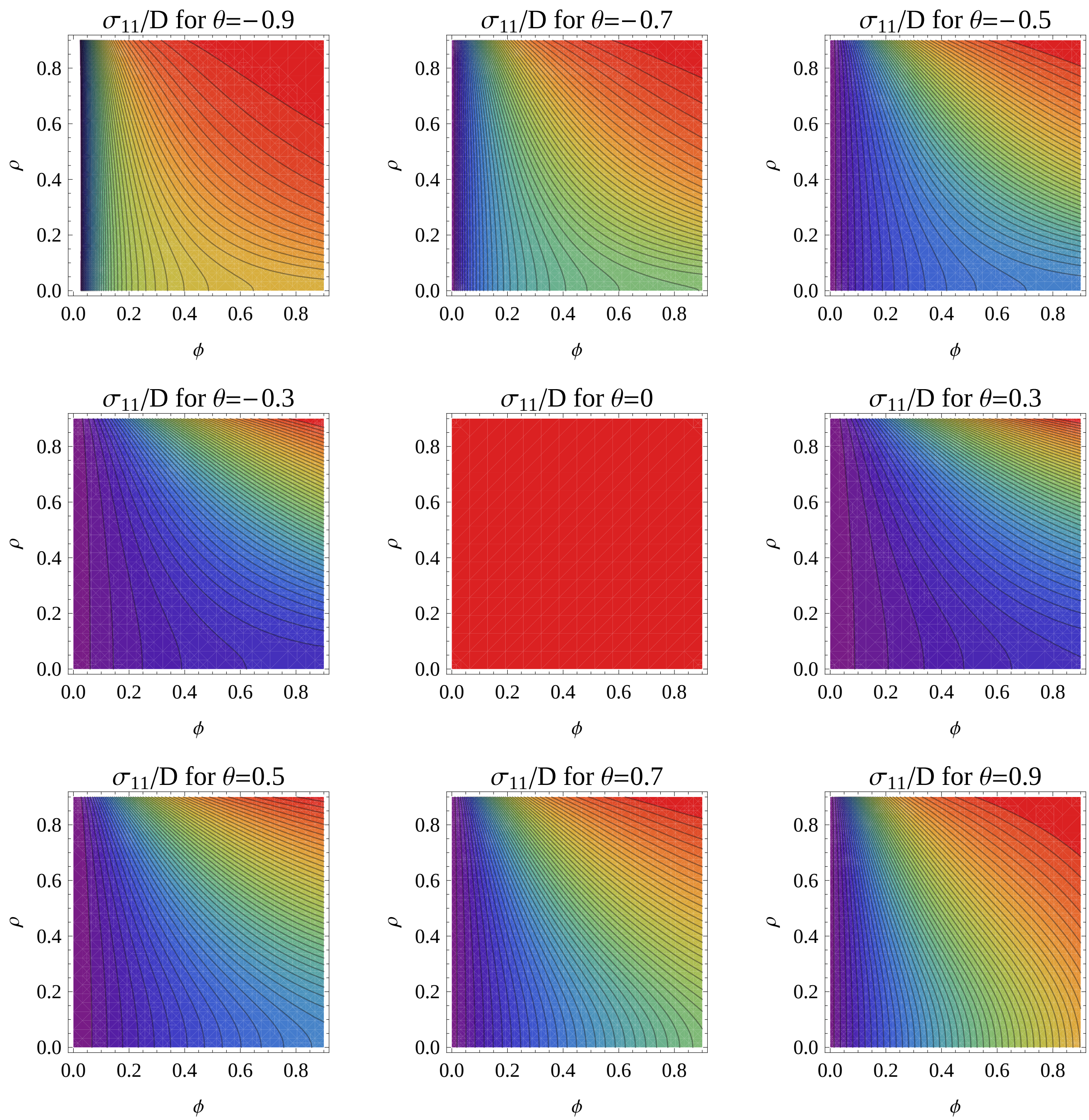
$$D_1 = \frac{\phi^4}{\left(1-\phi^2\right)\left(1-\theta(-1+2\phi)\right)}.$$

It follows that

$$\sigma_{11}/D = \left(D - D_1\right)/D = \frac{\theta\left(-1+\theta\phi^2+\rho^2\left(1+\theta\phi^2\left(-17+4\theta\left(-4+\phi\left(12+\theta\left(4-(9+\theta)\phi+2\theta\phi^2\right)\right)\right)\right)\right)\right)}{\left(-1+\theta(-1+2\phi)\right)\left(1+\rho^2\left(-1+4\theta^2\phi^2(-2+\theta\phi)^2\right)\right)}.$$

Also note that D_1 reduces to $\frac{\phi^4}{1-\phi^2}$ when $\theta = 0$ which implies that $\sigma_{11}/D = 0$ in that case.

Contour plots of σ_{11}/D show that the discrepancy between the efficiency bound and the asymptotic variance for $M = 1$ generally increases in ρ and ϕ as well as in $|\theta|$.



6. Additional Monte Carlo Results

A small Monte Carlo experiment is conducted to assess the performance of the proposed moment selection methods. The appendix presents results for an extended set of estimators and some additional simulations designs. For the simulations the following data generating process is used

$$
\begin{aligned}
y_{t,1} &= \beta y_{t,2} + \varepsilon_t - \theta \varepsilon_{t-1} \\
y_{t,2} &= \phi y_{t-1,2} + u_{t,2} - \gamma u_{t-1,2}. \\
\varepsilon_t &= u_{t,1} h_t^{1/2} \\
h_t &= .1 + (\delta - .1)\varepsilon_{t-1}^2 + (1-\delta)\, h_{t-1}
\end{aligned} \tag{6.1}
$$

with $u_t = (u_{t,1}, u_{t,2})' \sim N(0, \Sigma)$ where Σ has elements $\sigma_1^2 = \sigma_2^2 = 1$ and σ_{12}. The parameter β is the parameter to be estimated and is set to $\beta = 1$ in all simulations. All remaining parameters are nuisance parameters not explicitly estimated. The parameter σ_{12} is one of the determinants of the small sample bias of both Ordinary Least Squares (OLS) and GMM estimators and is varied over $\sigma_{12} = \{.1, .5, .9\}$. The parameters ϕ and γ control the quality of lagged instruments. The parameter ϕ is chosen from $\{.1, .3, .5\}$. The parameter γ is set to zero except on Tables 4-6 where it is set to .5. Low values of ϕ imply that the model is poorly identified. The parameter θ is varied over the set $\{-.9, -.5, 0, .5, .9\}$. Except on Tables 7-9 where $\delta = .2$ the errors are homoskedastic. In those cases $h_t = 1$ for all t.

Samples of size $n = \{128, 512\}$ from Model (6.1) are generated. Starting values are $y_0 = 0$ and $u_0 = 0$. In each sample the first 1,000 observations are discarded to eliminate dependence on initial conditions. The simulations are based on 1,000 replications.

A simple bias corrected GMM estimator can be constructed from the estimated bias by subtracting the approximate bias term from the estimator. The feasible bias corrected estimator is defined as

$$
\hat{\beta}_{n,M}^{BC} = \hat{\beta}_{n,\hat{M}^*} - \frac{\hat{M}^* p}{n} \hat{D}_{k_{\max},\hat{h}}^{-1} \left(\widehat{\mathcal{A}}_1 \int_{-1}^{1} k^2(x)dx + \widehat{\mathcal{A}}_2 \int_{-1}^{1} k(x)dx \right) \tag{6.2}
$$

where $\hat{M}^*$ is the optimal data-dependent bandwidth selected for the kernel $k(.)$ used to compute $\hat{\beta}_{n,\hat{M}^*}$. The performance of this bias corrected estimator in finite samples is reported in this Appendix.

The description of table columns can be found in the main paper, as well as the description of most estimators used. The four additional estimators reported here but not in the main paper are GMM-BRM which is kernel weighted GMM based on the bias minimal kernel given in (3.16) in the main paper, GMM-Trunc-BC the bias corrected GMM estimator defined in (6.2) above, using the truncated kernel. GMM-Bartlett is the kernel weighted estimator where instead of the Tukey-Hanning kernel the Bartlett kernel was used. WWA is the estimator suggested by West, Wong and Anatolyev (2009). It estimates a heteroskedasticity robust weight matrix which could add noise to the estimators. To asses to what extent that is the case, and to facilitate comparison with estimators that do not use robust weight matrices, a modified version of the estimator that imposes homoskedasticity, is reported and denoted by WWA-Hom.

The results indicate that GMM-Trunc-BC performs poorly when identification is weak. In those instances it is not successful at reducing bias but instead increases variability, in some cases dramatically. When identification is strong, ie. $\phi = .5$ it can lead to significant reductions in bias relative to GMM-Trunc, but the associated increase in variance often leads to an overall less favorable measure for MAE. Also, size distortions with GMM-Trunc-BC are generally worse than with GMM-Trunc. GMM-BRM only slightly improves the bias of GMM-BR in some cases but also suffers from inflated variability in many instances. GMM-Bartlett performs almost as well as GMM-Tuk-Han and when identification is strong it is sometimes slightly better. Overall, these two procedures are quite similar. The difference between WWA and WWA-Hom is generally negligible in these experiments. The assessment of WWA in Section 5 of the paper thus also applies for WWA-Hom.

7. Step by Step Procedure to Compute the Criterion $\hat{\varphi}_n$[4]

1. Obtain a first stage consistent estimate of β by

$$\tilde{\beta}_{n,\tilde{M}} = \left(\hat{P}'_{\tilde{M}}\hat{P}_{\tilde{M}}\right)^{-1}\hat{P}'_{\tilde{M}}\hat{P}^y_{\tilde{M}} \tag{7.1}$$

   by choosing $\tilde{M}$ such that $\tilde{M}p > d$ and $\tilde{M}$ is the lag-length selected by the Ng and Perron (1995) criterion of a VAR fitted to y_t (see Kuersteiner, 2005 and Step 7 below).

2. Obtain estimated residuals

$$\tilde{\varepsilon}_t = y_t - \bar{y} - \tilde{\beta}_{n,\tilde{M}}\left(x_t - \bar{x}\right).$$

3. Let $\hat{\gamma}^{\varepsilon}(j) = n^{-1}\sum_{t=r+j+1}^{n}\tilde{\varepsilon}_t\tilde{\varepsilon}_{t-j}$ for $j = 0, .., m-1$. Let $\hat{\Omega}_{\tilde{M}}(l) = \frac{1}{n}\sum_{t=1}^{n} z_{t,\tilde{M}}z'_{t-l,\tilde{M}}$ for $l \geq 0$, $\hat{\Omega}_{\tilde{M}}(l) = \hat{\Omega}_{\tilde{M}}(-l)'$ for $l < 0$ and $\hat{\Omega}^*_{\tilde{M}} = \sum_{l=-m+1}^{m-1}\hat{\gamma}^{\varepsilon}(l)\hat{\Omega}_{\tilde{M}}(l)$,

$$\hat{\Omega}_{\tilde{M}} = \hat{\Omega}^*_{\tilde{M}}\mathbf{1}\left\{\min\hat{\xi}_{\Omega} \geq 0\right\} + \left(1 - \mathbf{1}\left\{\min\hat{\xi}_{\Omega} \geq 0\right\}\right)\sum_{l=-m+1}^{m-1}\left(1 - \frac{|l|}{m}\right)\hat{\gamma}^{\varepsilon}(l)\hat{\Omega}_{\tilde{M}}(l) \tag{7.2}$$

   where $\hat{\xi}_{\Omega}$ is the smallest eigenvalue of $\hat{\Omega}^*_{\tilde{M}}$.

4. Re-estimate

$$\breve{\beta}_{n,\tilde{M}} = \left(\hat{P}'_{\tilde{M}}\hat{\Omega}^{-1}_{\tilde{M}}\hat{P}_{\tilde{M}}\right)^{-1}\hat{P}'_{\tilde{M}}\hat{\Omega}^{-1}_{\tilde{M}}\hat{P}^y_{\tilde{M}}$$

[4] Matlabe code implementing this procedure can be downloaded from the author's web-page.

and

$$\hat{\varepsilon}_t = y_t - \bar{y} - \breve{\beta}_{n,\tilde{M}} (x_t - \bar{x}).$$

5. Fit an MA(m-1) model to $\hat{\varepsilon}_t$ to obtain an estimator $\hat{\theta}$ for the MA parameters $\theta(L) = 1 - \theta_1 L - ... - \theta_{m-1} L^{m-1}$. Let $\hat{\sigma}^2$ be an estimate of the innovation variance of the MA model. In the Monte Carlo simulations the Matlab routine armax is used to estimate these parameters.

6. Let $\hat{f}_\varepsilon(\lambda) = \hat{\sigma}^2 \left|\hat{\theta}\left(e^{-i\lambda}\right)\right|^2$ where $\hat{\theta}(L) = 1 - \hat{\theta}_1 L - ... - \hat{\theta}_{m-1} L^{m-1}$. Obtain

$$\hat{\zeta}_j = (2\pi)^{-1} \int_{-\pi}^{\pi} \hat{f}_\varepsilon^{-1}(\lambda) e^{i\lambda j} d\lambda. \tag{7.3}$$

Note that $\hat{\zeta}_j$ is the autocovariance function of an AR(m-1) process with parameters $\hat{\theta}$ and innovation variance $1/\hat{\sigma}^2$.

7. In order to estimate the parameters of the approximating VAR the following definitions are needed. Let $Y_{t,h} = \left(y_t' - \bar{y}', ..., y_{t-h+1}' - \bar{y}'\right)'$ and $M_h = \sum_{t=h+1}^n Y_{t-1,h} Y_{t-1,h}'$ and define $M_h^{-1}(1)$ to be the lower-right $p \times p$ block of M_h^{-1}. Let $\hat{\Gamma}_{1,h} = (n-h)^{-1} \sum_{t=h}^{n-1} Y_{t,h} \left(y_{t+1} - \bar{y}\right)'$ and

$$\hat{\Gamma}_h = (n-h)^{-1} \sum_{t=h}^{n-1} Y_{t,h} Y_{t,h}'.$$

Define $\pi(h) = (\pi_1', ..., \pi_h')'$ and set

$$\hat{\pi}_h(h) = \left(\hat{\pi}_{1,h}', \hat{\pi}_{2,h}', ..., \hat{\pi}_{h,h}'\right) = \hat{\Gamma}_{1,h}' \hat{\Gamma}_h^{-1}. \tag{7.4}$$

8. The estimated error covariance matrix of the approximating VAR is

$$\hat{\Sigma}_h = n^{-1} \sum_{t=h+1}^n \hat{u}_{t,h} \hat{u}_{t,h}' \tag{7.5}$$

where $\hat{u}_{t,h} = y_t - \hat{\pi}_{1,h} y_{t-1} - ... - \hat{\pi}_{h,h} y_{t-h}$.

9. The order of the approximating VAR is selected in a data-dependent way using the procedure of Ng and Perron (1995). Define the Wald statistic

$$J(h,h) = n\,(\text{vec}\,\hat{\pi}_{h,h})' \left(\hat{\Sigma}_{v,h} \otimes M_h^{-1}(1)\right)^{-1} (\text{vec}\,\hat{\pi}_{h,h}).$$

10. Choose $h_{\max} = (\log n)^2$ and $h_{\min} = ((\log \log n) \log n)/10$. Choose i) $\hat{h} = h$ if, at significance level α, $J(h,h)$ is the first statistic in the sequence $J(i,i), \{i = h_{\max}, ..., h_{\min}\}$, which is significantly different from zero or ii) $\hat{h} = h_{\min}$ if $J(i,i)$ is not significantly different from zero for all $i = h_{\max}, ..., h_{\min}$.

11. Using the data-dependent lag length $\hat{h}$, the approximating VAR is now estimated and implied auto-covariances are computed. Define $\hat{\pi}_{\hat{h}}(L) = I - \hat{\pi}_{1,\hat{h}} L - ... - \hat{\pi}_{\hat{h},\hat{h}} L^{\hat{h}}$. In order to estimate the autocovariance function of y_t, define the companion matrix

$$\hat{H}_{\hat{h}} = \begin{bmatrix} \hat{\pi}_{1,\hat{h}} & \hat{\pi}_{2,\hat{h}} & \cdots & \hat{\pi}_{\hat{h},\hat{h}} \\ I_p & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & I_p & 0 \end{bmatrix},$$

where $H_{\hat{h}}$ is of dimension $p\hat{h} \times p\hat{h}$ and let $E_{\hat{h}} = (I_p, 0, ..., 0)'$ be a $p\hat{h} \times p$ matrix. Then an approximation to the autocovariance function Γ_j^{yy} is obtained by first computing

$$\operatorname{vec} \hat{G}_{0,\hat{h}}^{yy} = \left(I_{p\hat{h}} - \hat{H}_{\hat{h}} \otimes \hat{H}_{\hat{h}}\right)^{-1} \operatorname{vec} \begin{bmatrix} \hat{\Sigma}_{\hat{h}} & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$\hat{\Gamma}_{j,\hat{h}}^{yy} = E_{\hat{h}}' \hat{G}_{0,\hat{h}}^{yy} \hat{H}_{\hat{h}}^{j} E_{\hat{h}} \tag{7.6}$$

for all $j \leq k_{\max} = O\left(\sqrt{n/\log n}\right)$ and $\hat{\Gamma}_{j,\hat{h}}^{yy} = \hat{\Gamma}_{-j,\hat{h}}^{yy\prime}$ for $-k_{\max} \leq j < 0$. The autocovariance matrices Γ_j^{xy} can be estimated by selecting the appropriate elements from $\hat{\Gamma}_{s,\hat{h}}^{yy}$. From these estimates construct the matrix $\hat{P}_{M,\hat{h}}'$.

12. Let $\hat{\gamma}_{\hat{\theta}}^{\varepsilon}(j) = (2\pi)^{-1} \int_{-\pi}^{\pi} \hat{f}_{\varepsilon}(\lambda) e^{i\lambda j}$ for $|j| = 0, .., m-1$. Define $\hat{\Omega}_{\hat{h}}(l)$ with typical k,j-th block $\hat{\Gamma}_{k-j-l,\hat{h}}^{yy}$, such that $\hat{\Omega}_{M,\hat{h}}^{*}$ is estimated as $\hat{\Omega}_{M,\hat{h}}^{*} = \sum_{l=-m+1}^{m-1} \hat{\gamma}_{\hat{\theta}}^{\varepsilon}(l) \hat{\Omega}_{\hat{h}}(l)$. Then

$$\hat{\Omega}_{M,\hat{h}} = \hat{\Omega}_{M,\hat{h}}^{*} \mathbf{1}\left\{\min \hat{\xi}_{\hat{h}} \geq 0\right\} + \sum_{l=-m+1}^{m-1} \left(1 - \frac{|l|}{m}\right) \hat{\gamma}_{\hat{\theta}}^{\varepsilon}(l) \hat{\Omega}_{\hat{h}}(l) \tag{7.7}$$

where $\min \hat{\xi}_{\hat{h}}$ is the smallest eigenvalue of $\hat{\Omega}_{M,\hat{h}}^{*}$. Form

$$\hat{D}_{M,\hat{h}} = \hat{P}_{M,\hat{h}}' \hat{\Omega}_{M,\hat{h}}^{-1} \hat{P}_{M,\hat{h}}. \tag{7.8}$$

13. Estimate D by $\hat{D}_{k_{\max},\hat{h}} = \hat{P}_{k_{\max},\hat{h}}' \hat{\Omega}_{k_{\max},\hat{h}}^{-1} \hat{P}_{k_{\max},\hat{h}}$ where $\hat{\Omega}_{k_{\max},\hat{h}}^{-1}$ is defined in (7.7).

14. Form $\hat{\Gamma}_j^{\varepsilon x} = n^{-1} \sum_{t=\max(j,r)+1}^{\min(n,n+j)} \hat{\varepsilon}_t x_{t-j}$ and compute the first component of the bias as

$$\widehat{\mathcal{A}}_1 = \frac{1}{2} \sum_{j=-(n-1)/2}^{(n-1)/2} \hat{\zeta}_j \hat{\Gamma}_j^{\varepsilon x}. \tag{7.9}$$

15. The second component of the bias is estimated as follows. Use $\hat{\Gamma}^{xy}_{k,\hat{h}}$ as estimated in Step 11 and $\hat{\vartheta}^{k_{\max}}_{kj,\hat{h}}$ is the k, j-th block of $\hat{\Omega}_{k_{\max},\hat{h}}$ as estimated in Step 13. Estimate

$$\hat{\Gamma}^{\varepsilon y}_{j} = n^{-1} \sum_{t=\max(j,r-m)}^{\min(n-m,n+j)} \hat{\varepsilon}_{t+m} y_{t+1-j}.$$

Then,

$$\widehat{\mathcal{A}}_2 = \frac{1}{2} \sum_{j_1,j_2=1}^{k_{\max}} \hat{\Gamma}^{xy}_{j_1,\hat{h}} \hat{\vartheta}^{k_{\max}}_{j_1 j_2,\hat{h}} \hat{\Gamma}^{\varepsilon y}_{-j_2} \tag{7.10}$$

16. The final step now consists in combining the different components to form an estimate of the bias term. For the truncated kernel let $\widehat{\mathcal{A}} = 4\ell' \hat{D}^{-1}_{k_{\max},\hat{h}} \left(\widehat{\mathcal{A}}_1 + \widehat{\mathcal{A}}_2\right) \left(\widehat{\mathcal{A}}_1 + \widehat{\mathcal{A}}_2\right)' \hat{D}^{-1}_{k_{\max},\hat{h}} \ell$ where $\widehat{\mathcal{A}}_1$ is defined in (7.9) and $\widehat{\mathcal{A}}_2$ is defined in (7.10). For general kernels form $\widehat{\mathcal{A}}_0 = \widehat{\mathcal{A}}_1 \int_{-1}^{1} k(x)^2 \, dx + \widehat{\mathcal{A}}_2 \int_{-1}^{1} k(x) \, dx$ and let $\widehat{\mathcal{A}} = \ell' \hat{D}^{-1}_{k_{\max},\hat{h}} \widehat{\mathcal{A}}_0 \widehat{\mathcal{A}}_0' \hat{D}^{-1}_{k_{\max},\hat{h}} \ell$

17. Form the criterion

$$\hat{\varphi}_n(M, \ell, k_{TR}(.)) = \frac{(Mp)^2}{n} \widehat{\mathcal{A}} + \ell' \hat{D}^{-1}_{k_{\max},\hat{h}} \left(\hat{D}_{k_{\max},\hat{h}} - \hat{D}_{M,\hat{h}}\right) \hat{D}^{-1}_{k_{\max},\hat{h}} \ell$$

and define $\hat{M} = \arg\min_{M \in I} \in \hat{\varphi}_n(M, \ell, k_{TR}(.))$.

18. Compute $\hat{\Omega}_{\hat{M}}$ as in Step 3 but using residuals $\hat{\varepsilon}_t$ in Step 4. Compute the estimator

$$\hat{\beta}_{n,\hat{M}} = \left(\hat{P}'_{\hat{M}} \hat{\Omega}^{-1}_{\hat{M}} \hat{P}_{\hat{M}}\right)^{-1} \hat{P}'_{\hat{M}} \hat{\Omega}^{-1}_{\hat{M}} \hat{P}^{y}_{\hat{M}}.$$

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Table 1a - homoskedastic DGP, ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.11	0.32	0.03	0.13	0.16	0.0	0.11	0.16	0.02	0.11	0.46	0.0
0.1	-0.9	GMM-1	0.15	3.93	10.79	1.48	0.05	1.0	0.07	3.65	4.88	1.31	0.05	1.0
0.1	-0.9	GMM-25	0.11	0.51	0.05	0.18	0.08	25.0	0.10	0.53	0.05	0.18	0.07	25.0
0.1	-0.9	GMM-Bartlett	0.14	1.86	0.83	0.62	0.03	3.7	0.13	1.46	0.42	0.48	0.01	5.5
0.1	-0.9	GMM-Tuk-Han	0.11	1.38	0.41	0.46	0.03	7.5	0.14	1.09	0.21	0.36	0.02	9.9
0.1	-0.9	GMM-BR	0.12	1.87	0.79	0.62	0.06	3.9	0.13	1.51	0.48	0.51	0.04	5.2
0.1	-0.9	GMM-BRM	0.10	2.31	1.63	0.82	0.36	1.8	0.12	2.00	1.00	0.70	0.38	2.3
0.1	-0.9	GMM-Trunc	0.23	2.50	2.69	0.92	0.07	1.0	0.26	2.01	2.06	0.80	0.08	2.0
0.1	-0.9	GMM-Trunc-BC	0.14	4.62	4.26E+11	28412.00	0.38	1.0	0.28	3.06	2.72	1.10	0.32	2.0
0.1	-0.9	CUE-1	0.09	9.42	49.69	3.74	0.01	1.0	-0.01	9.29	46.08	3.58	0.00	1.0
0.1	-0.9	CUE-25	0.02	9.82	45.67	3.56	0.82	25.0	0.01	8.88	42.35	3.41	0.83	25.0
0.1	-0.9	WWA-Hom	0.03	2.12	71.68	1.25	0.12	0.0	0.02	2.04	9.04	0.97	0.13	0.0
0.1	-0.9	WWA	0.03	2.12	109.83	1.33	0.12	0.0	0.02	2.04	9.71	0.98	0.13	0.0
0.1	-0.5	OLS	0.10	0.26	0.02	0.12	0.20	0.0	0.11	0.13	0.01	0.11	0.57	0.0
0.1	-0.5	GMM-1	0.14	3.06	24.65	1.33	0.04	1.0	0.06	3.07	4.73	1.12	0.03	1.0
0.1	-0.5	GMM-25	0.10	0.42	0.04	0.15	0.07	25.0	0.10	0.42	0.04	0.16	0.07	25.0
0.1	-0.5	GMM-Bartlett	0.11	1.80	1.89	0.65	0.02	2.5	0.09	1.48	0.77	0.53	0.01	3.1
0.1	-0.5	GMM-Tuk-Han	0.11	1.44	0.65	0.51	0.02	4.5	0.10	1.26	0.47	0.44	0.01	5.0
0.1	-0.5	GMM-BR	0.10	1.91	1.87	0.68	0.03	2.4	0.10	1.64	0.84	0.58	0.01	2.6
0.1	-0.5	GMM-BRM	0.11	2.13	2.66	0.81	0.23	1.1	0.07	2.27	2.62	0.83	0.24	1.2
0.1	-0.5	GMM-Trunc	0.16	2.13	17.58	0.89	0.04	1.0	0.14	2.07	2.27	0.72	0.04	2.0
0.1	-0.5	GMM-Trunc-BC	0.15	12.32	11585000.00	149.18	0.45	1.0	0.14	4.06	165.95	2.11	0.28	2.0
0.1	-0.5	CUE-1	0.10	7.44	35.61	2.97	0.01	1.0	0.02	7.64	37.63	3.09	0.00	1.0
0.1	-0.5	CUE-25	0.04	7.43	38.80	3.10	0.81	25.0	0.07	6.81	35.27	2.91	0.80	25.0
0.1	-0.5	WWA-Hom	0.06	2.11	3.60	0.80	0.10	0.0	0.09	1.91	279.31	1.29	0.09	0.0
0.1	-0.5	WWA	0.06	2.11	3.57	0.80	0.10	0.0	0.09	1.91	326.76	1.34	0.09	0.0
0.1	0	OLS	0.10	0.23	0.02	0.11	0.19	0.0	0.10	0.11	0.01	0.10	0.62	0.0
0.1	0	GMM-1	0.14	2.54	2.64	0.95	0.02	1.0	0.05	2.48	15.19	1.06	0.01	1.0
0.1	0	GMM-25	0.09	0.36	0.03	0.13	0.06	25.0	0.10	0.37	0.03	0.14	0.06	25.0
0.1	0	GMM-Bartlett	0.09	1.75	1.63	0.66	0.02	1.0	0.09	1.86	13.30	0.78	0.00	0.6
0.1	0	GMM-Tuk-Han	0.09	1.71	1.57	0.64	0.03	2.1	0.09	1.88	13.23	0.78	0.01	1.6
0.1	0	GMM-BR	0.10	1.80	1.64	0.67	0.02	1.1	0.09	1.85	13.31	0.79	0.01	0.8
0.1	0	GMM-BRM	0.11	1.94	1.71	0.71	0.18	0.5	0.08	2.07	13.73	0.87	0.14	0.4
0.1	0	GMM-Trunc	0.11	1.99	1.53	0.69	0.03	1.0	0.12	2.02	1.69	0.71	0.02	1.0
0.1	0	GMM-Trunc-BC	0.08	167.35	6.67E+09	3383.00	0.59	1.0	0.12	109.30	53286000.00	623.86	0.60	1.0
0.1	0	CUE-1	0.15	6.16	33.15	2.77	0.00	1.0	0.00	6.76	31.22	2.70	0.00	1.0
0.1	0	CUE-25	0.05	6.23	31.32	2.70	0.82	25.0	0.07	5.97	29.29	2.62	0.82	25.0
0.1	0	WWA-Hom	0.04	2.03	1.38	0.69	0.08	0.0	0.09	1.79	10.09	0.76	0.05	0.0
0.1	0	WWA	0.04	2.03	1.38	0.69	0.08	0.0	0.09	1.79	10.09	0.76	0.05	0.0
0.1	0.5	OLS	0.09	0.24	0.02	0.11	0.15	0.0	0.09	0.12	0.01	0.10	0.49	0.0
0.1	0.5	GMM-1	0.13	3.37	7.07	1.25	0.03	1.0	0.06	3.29	3.54	1.11	0.03	1.0
0.1	0.5	GMM-25	0.09	0.37	0.03	0.14	0.04	25.0	0.09	0.40	0.03	0.15	0.06	25.0
0.1	0.5	GMM-Bartlett	0.11	1.87	3.60	0.70	0.02	2.9	0.10	1.54	0.76	0.52	0.01	3.5
0.1	0.5	GMM-Tuk-Han	0.10	1.50	0.60	0.51	0.03	4.8	0.10	1.24	0.36	0.41	0.01	5.4
0.1	0.5	GMM-BR	0.10	1.97	3.76	0.73	0.03	2.5	0.09	1.62	1.13	0.58	0.02	2.8
0.1	0.5	GMM-BRM	0.08	2.30	4.44	0.88	0.23	1.1	0.09	2.23	1.79	0.76	0.22	1.3
0.1	0.5	GMM-Trunc	0.10	2.06	1.19	0.68	0.04	1.0	0.13	1.74	1.53	0.65	0.03	2.0
0.1	0.5	GMM-Trunc-BC	0.06	14.90	104350.00	43.47	0.44	1.0	0.10	3.76	711.28	2.56	0.23	2.0
0.1	0.5	CUE-1	0.11	8.15	36.92	3.20	0.01	1.0	0.01	7.63	38.53	3.18	0.00	1.0
0.1	0.5	CUE-25	0.03	7.25	31.00	2.81	0.80	25.0	0.00	7.27	36.55	3.01	0.83	25.0
0.1	0.5	WWA-Hom	0.04	3.18	1750800.00	47.30	0.20	0.0	0.07	1.91	76.36	1.48	0.12	0.0
0.1	0.5	WWA	0.04	3.19	8038000.00	95.11	0.20	0.0	0.07	1.91	76.40	1.48	0.12	0.0
0.1	0.9	OLS	0.08	0.29	0.02	0.11	0.11	0.0	0.09	0.15	0.01	0.09	0.33	0.0
0.1	0.9	GMM-1	0.15	4.08	10.14	1.58	0.06	1.0	0.11	3.93	5.11	1.37	0.05	1.0
0.1	0.9	GMM-25	0.10	0.45	0.04	0.16	0.04	25.0	0.10	0.47	0.04	0.17	0.05	25.0
0.1	0.9	GMM-Bartlett	0.14	1.72	0.83	0.60	0.03	4.4	0.11	1.21	0.33	0.41	0.02	6.6
0.1	0.9	GMM-Tuk-Han	0.11	1.24	0.40	0.42	0.03	8.3	0.12	0.95	0.17	0.32	0.02	11.2
0.1	0.9	GMM-BR	0.15	1.78	0.83	0.61	0.06	4.3	0.11	1.30	0.37	0.44	0.04	5.8
0.1	0.9	GMM-BRM	0.10	2.31	2.04	0.83	0.33	1.9	0.12	1.85	0.82	0.63	0.33	2.6
0.1	0.9	GMM-Trunc	0.19	2.22	2.84	0.86	0.06	2.0	0.21	1.68	1.43	0.67	0.06	4.0
0.1	0.9	GMM-Trunc-BC	0.15	4.01	2.19E+13	211810.00	0.37	2.0	0.19	2.56	1.91	0.92	0.27	4.0
0.1	0.9	CUE-1	0.12	11.10	52.35	4.03	0.01	1.0	-0.01	9.87	52.45	3.91	0.00	1.0
0.1	0.9	CUE-25	0.08	9.45	45.79	3.59	0.80	25.0	0.05	9.41	48.00	3.65	0.83	25.0
0.1	0.9	WWA-Hom	0.01	46.29	923790.00	65.10	0.53	0.0	0.01	18.52	10341.00	21.09	0.47	0.0
0.1	0.9	WWA	0.02	46.82	11221000.00	141.13	0.53	0.0	0.01	18.57	10325.00	21.12	0.47	0.0

Table 1b - homoskedastic DGP, ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.54	0.29	0.30	0.54	1.00	0.0	0.54	0.14	0.30	0.54	1.00	0.0
0.5	-0.9	GMM-1	0.66	3.39	9.90	1.54	0.11	1.0	0.62	3.38	5.67	1.35	0.09	1.0
0.5	-0.9	GMM-25	0.54	0.47	0.32	0.54	0.80	25.0	0.54	0.46	0.33	0.54	0.82	25.0
0.5	-0.9	GMM-Bartlett	0.61	1.46	0.96	0.75	0.22	5.3	0.61	1.13	0.56	0.65	0.26	8.0
0.5	-0.9	GMM-Tuk-Han	0.55	1.19	0.52	0.61	0.31	9.2	0.57	0.86	0.43	0.58	0.38	12.7
0.5	-0.9	GMM-BR	0.62	1.48	1.10	0.75	0.28	4.8	0.58	1.18	0.55	0.64	0.30	6.6
0.5	-0.9	GMM-BRM	0.61	2.05	3.35	0.93	0.55	2.2	0.62	1.65	0.89	0.77	0.61	3.0
0.5	-0.9	GMM-Trunc	0.64	1.78	4.82	1.01	0.38	3.0	0.66	1.49	1.40	0.89	0.47	6.0
0.5	-0.9	GMM-Trunc-BC	0.43	3.53	1.38E+10	4258.10	0.42	3.0	0.46	2.42	1.70	0.93	0.39	6.0
0.5	-0.9	CUE-1	0.65	9.03	43.07	3.53	0.05	1.0	0.61	7.73	33.73	3.11	0.05	1.0
0.5	-0.9	CUE-25	0.52	8.70	38.64	3.33	0.82	25.0	0.44	7.99	40.02	3.28	0.83	25.0
0.5	-0.9	WWA-Hom	0.33	2.02	33.54	1.09	0.12	0.0	0.30	1.97	6.27	0.90	0.12	0.0
0.5	-0.9	WWA	0.33	2.02	7.88	0.94	0.12	0.0	0.30	1.97	6.34	0.90	0.12	0.0
0.5	-0.5	OLS	0.52	0.23	0.28	0.52	1.00	0.0	0.52	0.11	0.27	0.52	1.00	0.0
0.5	-0.5	GMM-1	0.58	2.68	6.32	1.20	0.10	1.0	0.54	2.56	2.78	1.08	0.09	1.0
0.5	-0.5	GMM-25	0.52	0.37	0.29	0.51	0.82	25.0	0.52	0.37	0.29	0.52	0.87	25.0
0.5	-0.5	GMM-Bartlett	0.53	1.55	1.21	0.76	0.19	3.1	0.54	1.20	0.68	0.64	0.18	4.0
0.5	-0.5	GMM-Tuk-Han	0.52	1.20	0.65	0.63	0.28	5.2	0.52	1.05	0.50	0.58	0.26	6.2
0.5	-0.5	GMM-BR	0.56	1.57	1.35	0.78	0.22	2.7	0.55	1.29	0.81	0.69	0.20	3.2
0.5	-0.5	GMM-BRM	0.57	1.87	5.22	0.96	0.44	1.2	0.56	1.77	1.99	0.87	0.47	1.5
0.5	-0.5	GMM-Trunc	0.58	1.76	2.64	0.87	0.28	2.0	0.60	1.78	1.44	0.85	0.30	2.0
0.5	-0.5	GMM-Trunc-BC	0.55	4.51	651.99	3.81	0.44	2.0	0.57	2.85	2.80	1.12	0.38	2.0
0.5	-0.5	CUE-1	0.58	6.72	29.84	2.77	0.06	1.0	0.52	7.23	37.43	3.03	0.06	1.0
0.5	-0.5	CUE-25	0.48	6.52	34.40	2.92	0.83	25.0	0.48	5.88	28.08	2.62	0.85	25.0
0.5	-0.5	WWA-Hom	0.42	1.79	4.91	0.83	0.17	0.0	0.45	1.66	39.16	1.00	0.14	0.0
0.5	-0.5	WWA	0.42	1.79	5.01	0.83	0.17	0.0	0.45	1.66	39.72	1.00	0.14	0.0
0.5	0	OLS	0.49	0.20	0.25	0.49	1.00	0.0	0.49	0.10	0.25	0.50	1.00	0.0
0.5	0	GMM-1	0.52	2.25	4.63	1.04	0.08	1.0	0.44	2.17	40.49	1.13	0.06	1.0
0.5	0	GMM-25	0.49	0.32	0.25	0.49	0.82	25.0	0.50	0.32	0.26	0.50	0.81	25.0
0.5	0	GMM-Bartlett	0.49	1.54	2.74	0.82	0.21	1.0	0.49	1.57	39.25	0.97	0.19	0.6
0.5	0	GMM-Tuk-Han	0.49	1.52	2.69	0.80	0.30	2.2	0.50	1.56	39.18	0.96	0.24	1.6
0.5	0	GMM-BR	0.49	1.55	2.75	0.82	0.22	1.2	0.48	1.58	39.26	0.97	0.18	0.8
0.5	0	GMM-BRM	0.50	1.69	2.80	0.84	0.41	0.5	0.48	1.77	39.44	1.01	0.36	0.4
0.5	0	GMM-Trunc	0.49	1.70	2.40	0.83	0.24	1.0	0.50	1.71	1.49	0.81	0.24	1.0
0.5	0	GMM-Trunc-BC	0.49	127.56	41372000.00	610.24	0.71	1.0	0.50	71.46	8679400.00	293.23	0.74	1.0
0.5	0	CUE-1	0.50	5.26	31.27	2.61	0.06	1.0	0.39	5.79	29.07	2.58	0.04	1.0
0.5	0	CUE-25	0.46	5.38	23.71	2.36	0.84	25.0	0.45	5.37	27.04	2.47	0.86	25.0
0.5	0	WWA-Hom	0.44	1.82	1.23	0.75	0.17	0.0	0.48	1.56	7.07	0.82	0.16	0.0
0.5	0	WWA	0.44	1.82	1.23	0.75	0.17	0.0	0.48	1.56	7.07	0.82	0.16	0.0
0.5	0.5	OLS	0.46	0.22	0.23	0.47	1.00	0.0	0.47	0.11	0.22	0.47	1.00	0.0
0.5	0.5	GMM-1	0.52	2.92	5.05	1.23	0.08	1.0	0.50	2.85	3.77	1.13	0.07	1.0
0.5	0.5	GMM-25	0.47	0.34	0.24	0.47	0.76	25.0	0.47	0.37	0.25	0.48	0.81	25.0
0.5	0.5	GMM-Bartlett	0.50	1.59	2.00	0.77	0.16	3.4	0.49	1.22	0.65	0.62	0.15	4.1
0.5	0.5	GMM-Tuk-Han	0.47	1.19	0.55	0.58	0.23	5.5	0.47	1.01	0.42	0.54	0.21	6.2
0.5	0.5	GMM-BR	0.51	1.67	2.00	0.78	0.20	2.9	0.51	1.36	0.80	0.66	0.16	3.2
0.5	0.5	GMM-BRM	0.49	1.89	2.60	0.88	0.41	1.3	0.53	1.71	1.85	0.82	0.42	1.5
0.5	0.5	GMM-Trunc	0.51	1.76	1.32	0.80	0.24	2.0	0.54	1.67	1.80	0.85	0.25	2.0
0.5	0.5	GMM-Trunc-BC	0.41	4.51	220.28	2.52	0.41	2.0	0.44	2.76	4.44	1.10	0.31	2.0
0.5	0.5	CUE-1	0.51	7.26	41.99	3.27	0.05	1.0	0.48	6.49	37.14	3.04	0.04	1.0
0.5	0.5	CUE-25	0.41	6.53	32.58	2.80	0.83	25.0	0.39	6.13	31.98	2.75	0.82	25.0
0.5	0.5	WWA-Hom	0.32	2.19	2083100.00	48.07	0.20	0.0	0.36	1.63	423.87	1.63	0.13	0.0
0.5	0.5	WWA	0.32	2.19	460770.00	25.30	0.20	0.0	0.36	1.63	338.35	1.56	0.13	0.0
0.5	0.9	OLS	0.44	0.27	0.21	0.45	0.99	0.0	0.45	0.14	0.21	0.45	1.00	0.0
0.5	0.9	GMM-1	0.52	4.02	16.51	1.59	0.08	1.0	0.50	3.64	4.75	1.34	0.07	1.0
0.5	0.9	GMM-25	0.46	0.44	0.24	0.46	0.64	25.0	0.45	0.46	0.25	0.46	0.69	25.0
0.5	0.9	GMM-Bartlett	0.54	1.47	0.79	0.70	0.17	5.2	0.52	1.08	0.46	0.58	0.18	7.7
0.5	0.9	GMM-Tuk-Han	0.50	1.13	0.45	0.56	0.24	9.2	0.49	0.85	0.34	0.51	0.27	12.4
0.5	0.9	GMM-BR	0.53	1.51	0.81	0.70	0.22	4.8	0.51	1.15	0.50	0.59	0.22	6.5
0.5	0.9	GMM-BRM	0.54	1.99	1.67	0.87	0.48	2.2	0.55	1.60	0.84	0.71	0.52	2.9
0.5	0.9	GMM-Trunc	0.55	1.89	11.44	1.00	0.30	4.0	0.58	1.47	1.75	0.86	0.38	5.0
0.5	0.9	GMM-Trunc-BC	0.22	3.73	1.90E+12	69340.00	0.38	4.0	0.37	2.44	2.00	0.89	0.34	5.0
0.5	0.9	CUE-1	0.44	10.56	49.44	3.85	0.03	1.0	0.48	9.19	45.63	3.61	0.02	1.0
0.5	0.9	CUE-25	0.44	8.81	45.13	3.52	0.81	25.0	0.37	8.48	44.65	3.41	0.81	25.0
0.5	0.9	WWA-Hom	0.19	5.04	951230.00	43.62	0.27	0.0	0.21	2.03	239.92	2.00	0.12	0.0
0.5	0.9	WWA	0.19	5.04	80234.00	21.81	0.27	0.0	0.21	2.03	238.46	1.99	0.12	0.0

Table 1c - homoskedastic DGP, ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.97	0.21	0.95	0.97	1.00	0.0	0.97	0.11	0.95	0.97	1.00	0.0
0.9	-0.9	GMM-1	1.05	2.28	6.82	1.41	0.27	1.0	1.01	2.25	5.88	1.28	0.26	1.0
0.9	-0.9	GMM-25	0.98	0.34	0.98	0.98	1.00	25.0	0.98	0.33	0.98	0.98	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.99	0.82	1.11	1.00	0.78	8.3	0.97	0.63	0.98	0.95	0.83	11.4
0.9	-0.9	GMM-Tuk-Han	0.97	0.73	1.00	0.96	0.86	12.8	0.95	0.58	0.94	0.94	0.91	16.8
0.9	-0.9	GMM-BR	0.97	0.94	1.09	0.97	0.81	6.7	0.92	0.80	0.94	0.92	0.86	8.8
0.9	-0.9	GMM-BRM	1.01	1.22	1.42	1.06	0.87	3.1	1.02	1.04	1.19	1.01	0.92	4.0
0.9	-0.9	GMM-Trunc	1.03	0.91	1.63	1.11	0.75	5.0	1.02	0.71	1.26	1.05	0.82	7.0
0.9	-0.9	GMM-Trunc-BC	1.00	1.70	6.74	1.24	0.70	5.0	0.99	1.22	1.36	1.05	0.78	7.0
0.9	-0.9	CUE-1	1.05	5.68	37.25	3.06	0.21	1.0	0.99	6.02	33.02	2.94	0.21	1.0
0.9	-0.9	CUE-25	1.12	4.31	22.50	2.45	0.86	25.0	1.06	4.30	21.57	2.41	0.85	25.0
0.9	-0.9	WWA-Hom	0.92	1.76	1.78	1.04	0.37	0.0	0.89	1.58	3.47	1.01	0.32	0.0
0.9	-0.9	WWA	0.92	1.76	1.80	1.05	0.36	0.0	0.89	1.58	4.45	1.02	0.32	0.0
0.9	-0.5	OLS	0.94	0.14	0.87	0.93	1.00	0.0	0.94	0.07	0.88	0.94	1.00	0.0
0.9	-0.5	GMM-1	1.00	1.53	2.59	1.18	0.45	1.0	0.99	1.54	2.33	1.12	0.42	1.0
0.9	-0.5	GMM-25	0.93	0.22	0.88	0.94	0.99	25.0	0.93	0.22	0.89	0.94	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.96	0.70	1.10	0.98	0.77	4.5	0.95	0.62	0.96	0.94	0.81	5.4
0.9	-0.5	GMM-Tuk-Han	0.94	0.62	0.96	0.94	0.85	7.0	0.93	0.55	0.92	0.93	0.89	8.0
0.9	-0.5	GMM-BR	0.96	0.74	1.20	1.00	0.79	3.7	0.96	0.68	1.01	0.95	0.83	4.2
0.9	-0.5	GMM-BRM	0.98	1.03	1.56	1.07	0.84	1.6	0.98	0.92	1.13	0.98	0.87	1.9
0.9	-0.5	GMM-Trunc	0.99	1.23	2.38	1.14	0.60	1.0	1.00	1.18	1.77	1.07	0.65	1.0
0.9	-0.5	GMM-Trunc-BC	0.97	1.77	2.75	1.19	0.60	1.0	0.99	1.46	1.82	1.07	0.63	1.0
0.9	-0.5	CUE-1	1.02	3.97	20.94	2.28	0.36	1.0	1.01	3.99	22.77	2.33	0.35	1.0
0.9	-0.5	CUE-25	0.97	3.48	18.10	2.09	0.91	25.0	0.89	3.45	19.97	2.17	0.90	25.0
0.9	-0.5	WWA-Hom	0.87	1.16	2.39	0.97	0.62	0.0	0.86	1.08	1.16	0.91	0.60	0.0
0.9	-0.5	WWA	0.87	1.16	2.25	0.97	0.61	0.0	0.86	1.08	1.17	0.91	0.60	0.0
0.9	0	OLS	0.89	0.10	0.79	0.89	1.00	0.0	0.89	0.05	0.80	0.89	1.00	0.0
0.9	0	GMM-1	0.90	1.22	1.53	0.99	0.49	1.0	0.86	1.19	2.76	1.01	0.44	1.0
0.9	0	GMM-25	0.89	0.16	0.79	0.89	0.97	25.0	0.89	0.17	0.80	0.89	0.97	25.0
0.9	0	GMM-Bartlett	0.89	0.80	1.20	0.95	0.72	1.3	0.88	0.83	2.30	0.97	0.67	0.8
0.9	0	GMM-Tuk-Han	0.89	0.75	1.18	0.95	0.75	2.7	0.88	0.82	1.30	0.94	0.70	1.9
0.9	0	GMM-BR	0.89	0.79	1.21	0.95	0.72	1.4	0.88	0.85	2.30	0.97	0.66	1.0
0.9	0	GMM-BRM	0.89	0.86	1.22	0.96	0.81	0.6	0.87	0.95	2.31	0.97	0.78	0.4
0.9	0	GMM-Trunc	0.89	1.04	1.50	0.99	0.61	1.0	0.87	1.08	1.44	0.95	0.56	1.0
0.9	0	GMM-Trunc-BC	0.90	18.44	14655000.00	154.41	0.80	1.0	0.85	8.56	58939.00	19.54	0.72	1.0
0.9	0	CUE-1	0.90	2.64	13.32	1.75	0.45	1.0	0.81	3.10	20.85	2.02	0.42	1.0
0.9	0	CUE-25	0.86	2.60	13.16	1.71	0.92	25.0	0.85	2.65	20.29	1.98	0.91	25.0
0.9	0	WWA-Hom	0.85	1.01	1.14	0.92	0.76	0.0	0.88	0.85	1.81	0.95	0.77	0.0
0.9	0	WWA	0.85	1.01	1.14	0.92	0.76	0.0	0.88	0.85	1.81	0.95	0.77	0.0
0.9	0.5	OLS	0.85	0.16	0.72	0.85	1.00	0.0	0.85	0.08	0.72	0.85	1.00	0.0
0.9	0.5	GMM-1	0.89	1.85	5.16	1.16	0.28	1.0	0.84	1.77	2.34	1.08	0.27	1.0
0.9	0.5	GMM-25	0.85	0.25	0.73	0.85	0.98	25.0	0.85	0.27	0.74	0.85	1.00	25.0
0.9	0.5	GMM-Bartlett	0.88	0.96	1.26	0.94	0.65	4.4	0.87	0.81	0.90	0.88	0.66	4.8
0.9	0.5	GMM-Tuk-Han	0.85	0.75	0.90	0.87	0.74	7.0	0.84	0.70	0.81	0.85	0.77	7.4
0.9	0.5	GMM-BR	0.88	1.02	1.29	0.95	0.68	3.7	0.88	0.90	0.96	0.89	0.72	3.8
0.9	0.5	GMM-BRM	0.87	1.29	1.94	1.00	0.74	1.7	0.88	1.16	1.35	0.95	0.78	1.7
0.9	0.5	GMM-Trunc	0.92	1.55	2.22	1.06	0.48	1.0	0.90	1.38	1.86	1.01	0.51	1.0
0.9	0.5	GMM-Trunc-BC	0.85	2.56	65.95	1.60	0.51	1.0	0.84	1.83	2.16	1.06	0.51	1.0
0.9	0.5	CUE-1	0.87	4.40	22.30	2.34	0.23	1.0	0.79	5.07	31.10	2.67	0.22	1.0
0.9	0.5	CUE-25	0.78	3.45	16.61	2.00	0.87	25.0	0.86	3.42	18.67	2.09	0.89	25.0
0.9	0.5	WWA-Hom	0.68	1.26	1.47	0.85	0.40	0.0	0.73	1.10	7.24	0.90	0.40	0.0
0.9	0.5	WWA	0.68	1.27	1.52	0.85	0.39	0.0	0.73	1.11	7.64	0.90	0.40	0.0
0.9	0.9	OLS	0.81	0.24	0.67	0.81	1.00	0.0	0.81	0.13	0.66	0.81	1.00	0.0
0.9	0.9	GMM-1	0.85	2.66	3.67	1.29	0.15	1.0	0.86	2.62	6.28	1.33	0.15	1.0
0.9	0.9	GMM-25	0.82	0.37	0.70	0.82	0.99	25.0	0.83	0.41	0.71	0.83	1.00	25.0
0.9	0.9	GMM-Bartlett	0.83	0.96	0.90	0.85	0.54	7.5	0.83	0.77	0.77	0.82	0.62	10.2
0.9	0.9	GMM-Tuk-Han	0.79	0.82	0.77	0.81	0.69	12.0	0.80	0.72	0.72	0.80	0.76	15.5
0.9	0.9	GMM-BR	0.80	1.12	0.93	0.85	0.63	6.2	0.77	0.96	0.74	0.78	0.67	8.1
0.9	0.9	GMM-BRM	0.86	1.62	1.29	0.95	0.76	2.8	0.88	1.36	1.13	0.93	0.82	3.7
0.9	0.9	GMM-Trunc	0.87	1.24	1.54	0.98	0.62	5.0	0.89	0.88	1.19	0.96	0.72	7.0
0.9	0.9	GMM-Trunc-BC	0.74	2.18	2.58	1.11	0.53	5.0	0.77	1.58	1.37	0.96	0.61	7.0
0.9	0.9	CUE-1	0.84	6.86	33.29	3.01	0.11	1.0	0.82	6.64	35.78	3.15	0.10	1.0
0.9	0.9	CUE-25	0.81	4.56	21.97	2.35	0.78	25.0	0.95	4.62	23.95	2.50	0.78	25.0
0.9	0.9	WWA-Hom	0.56	1.64	1.36	0.82	0.14	0.0	0.61	1.55	5.04	0.85	0.13	0.0
0.9	0.9	WWA	0.57	1.64	1.41	0.82	0.14	0.0	0.61	1.55	5.35	0.86	0.13	0.0

Table 2a - homoskedastic DGP, ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.12	0.33	0.03	0.14	0.22	0.0	0.12	0.16	0.02	0.12	0.55	0.0
0.1	-0.9	GMM-1	0.12	3.17	6.76	1.20	0.06	1.0	0.01	2.00	1.10	0.66	0.07	1.0
0.1	-0.9	GMM-25	0.11	0.53	0.05	0.19	0.10	25.0	0.10	0.52	0.05	0.18	0.10	25.0
0.1	-0.9	GMM-Bartlett	0.15	1.66	0.81	0.58	0.04	3.4	0.08	1.25	0.29	0.40	0.04	5.0
0.1	-0.9	GMM-Tuk-Han	0.14	1.27	0.37	0.44	0.04	6.9	0.08	1.02	0.18	0.33	0.04	9.1
0.1	-0.9	GMM-BR	0.15	1.68	0.93	0.59	0.07	3.6	0.07	1.24	0.30	0.41	0.05	4.7
0.1	-0.9	GMM-BRM	0.11	2.07	1.44	0.74	0.34	1.6	0.09	1.54	0.49	0.52	0.38	2.1
0.1	-0.9	GMM-Trunc	0.19	2.11	4.49	0.84	0.07	1.0	0.13	1.49	0.58	0.51	0.09	2.0
0.1	-0.9	GMM-Trunc-BC	0.08	4.34	1.64E+11	1.56E+04	0.40	1.0	0.02	2.37	1.10	0.80	0.33	2.0
0.1	-0.9	CUE-1	0.10	6.86	38.98	3.13	0.02	1.0	-0.03	3.04	16.91	1.61	0.04	1.0
0.1	-0.9	CUE-25	0.02	8.03	40.89	3.29	0.82	25.0	-0.01	6.52	36.18	2.88	0.79	25.0
0.1	-0.9	WWA-Hom	0.06	2.13	29.55	1.06	0.11	0.0	0.01	1.59	116.29	1.20	0.13	0.0
0.1	-0.9	WWA	0.06	2.13	31.36	1.06	0.12	0.0	0.01	1.59	105.45	1.16	0.13	0.0
0.1	-0.5	OLS	0.10	0.27	0.02	0.12	0.23	0.0	0.11	0.14	0.01	0.11	0.61	0.0
0.1	-0.5	GMM-1	0.11	2.55	3.76	0.93	0.05	1.0	0.03	1.61	0.72	0.54	0.03	1.0
0.1	-0.5	GMM-25	0.10	0.43	0.04	0.16	0.09	25.0	0.09	0.43	0.04	0.15	0.11	25.0
0.1	-0.5	GMM-Bartlett	0.08	1.50	1.01	0.55	0.02	2.7	0.06	1.11	0.27	0.35	0.01	3.8
0.1	-0.5	GMM-Tuk-Han	0.08	1.25	0.50	0.43	0.03	4.6	0.06	0.97	0.19	0.32	0.02	5.9
0.1	-0.5	GMM-BR	0.08	1.58	1.14	0.58	0.04	2.4	0.07	1.16	0.30	0.37	0.03	3.0
0.1	-0.5	GMM-BRM	0.09	1.87	2.91	0.71	0.24	1.1	0.06	1.37	0.53	0.46	0.27	1.4
0.1	-0.5	GMM-Trunc	0.10	1.75	1.09	0.61	0.04	2.0	0.08	1.19	0.47	0.41	0.04	2.0
0.1	-0.5	GMM-Trunc-BC	0.06	4.89	79.09	2.47	0.41	2.0	0.04	2.07	2.72	0.82	0.27	2.0
0.1	-0.5	CUE-1	0.10	5.64	27.43	2.51	0.01	1.0	-0.01	2.41	11.62	1.27	0.02	1.0
0.1	-0.5	CUE-25	0.09	6.60	32.94	2.82	0.81	25.0	0.02	5.20	24.23	2.28	0.77	25.0
0.1	-0.5	WWA-Hom	0.06	1.87	6.16	0.80	0.08	0.0	0.05	1.29	0.58	0.45	0.06	0.0
0.1	-0.5	WWA	0.06	1.87	6.36	0.80	0.08	0.0	0.05	1.29	0.58	0.45	0.06	0.0
0.1	0	OLS	0.09	0.21	0.02	0.10	0.18	0.0	0.09	0.11	0.01	0.09	0.58	0.0
0.1	0	GMM-1	0.10	2.23	2.07	0.80	0.02	1.0	0.01	1.24	1.18	0.46	0.01	1.0
0.1	0	GMM-25	0.09	0.34	0.03	0.13	0.05	25.0	0.08	0.34	0.03	0.13	0.08	25.0
0.1	0	GMM-Bartlett	0.08	1.44	1.04	0.54	0.02	2.4	0.04	1.02	1.01	0.36	0.01	3.0
0.1	0	GMM-Tuk-Han	0.07	1.28	0.76	0.47	0.03	4.1	0.04	0.90	0.16	0.29	0.01	4.6
0.1	0	GMM-BR	0.08	1.43	1.06	0.55	0.02	2.1	0.04	1.03	0.93	0.37	0.01	2.4
0.1	0	GMM-BRM	0.09	1.60	1.17	0.60	0.19	1.0	0.03	1.15	1.06	0.41	0.18	1.1
0.1	0	GMM-Trunc	0.09	1.48	0.98	0.54	0.03	2.0	0.04	1.01	0.22	0.33	0.03	2.0
0.1	0	GMM-Trunc-BC	0.10	5.69	32221.00	15.39	0.40	2.0	0.01	1.70	13.60	0.74	0.17	2.0
0.1	0	CUE-1	0.09	4.34	22.64	2.07	0.00	1.0	-0.01	1.86	6.66	0.97	0.01	1.0
0.1	0	CUE-25	0.06	5.78	27.37	2.47	0.81	25.0	0.02	4.44	19.31	1.92	0.77	25.0
0.1	0	WWA-Hom	0.07	1.73	4.37	0.67	0.07	0.0	0.03	1.17	0.30	0.37	0.04	0.0
0.1	0	WWA	0.07	1.74	4.34	0.67	0.07	0.0	0.03	1.17	0.30	0.37	0.04	0.0
0.1	0.5	OLS	0.08	0.21	0.01	0.09	0.10	0.0	0.08	0.11	0.01	0.08	0.36	0.0
0.1	0.5	GMM-1	0.09	2.57	4.19	0.97	0.03	1.0	0.00	1.36	0.87	0.51	0.02	1.0
0.1	0.5	GMM-25	0.08	0.33	0.02	0.12	0.03	25.0	0.07	0.34	0.02	0.12	0.04	25.0
0.1	0.5	GMM-Bartlett	0.07	1.52	1.05	0.55	0.01	3.5	0.04	0.87	0.21	0.30	0.01	5.2
0.1	0.5	GMM-Tuk-Han	0.06	1.28	0.52	0.44	0.01	5.5	0.05	0.83	0.16	0.27	0.00	7.0
0.1	0.5	GMM-BR	0.07	1.57	1.66	0.59	0.02	2.9	0.05	0.93	0.24	0.32	0.01	3.6
0.1	0.5	GMM-BRM	0.07	1.88	2.57	0.71	0.23	1.3	0.04	1.11	0.32	0.37	0.20	1.6
0.1	0.5	GMM-Trunc	0.08	1.63	2.34	0.61	0.02	2.0	0.05	0.97	0.26	0.32	0.01	3.0
0.1	0.5	GMM-Trunc-BC	0.01	4.72	335.19	3.44	0.33	2.0	-0.02	1.55	0.87	0.55	0.13	3.0
0.1	0.5	CUE-1	0.07	6.17	33.43	2.68	0.01	1.0	-0.02	2.36	11.04	1.29	0.01	1.0
0.1	0.5	CUE-25	0.05	6.71	36.85	2.87	0.77	25.0	0.02	5.40	29.16	2.44	0.77	25.0
0.1	0.5	WWA-Hom	0.01	2.35	2416.80	4.70	0.21	0.0	0.01	1.05	2.36	0.44	0.08	0.0
0.1	0.5	WWA	0.01	2.35	2193.70	4.62	0.21	0.0	0.01	1.05	2.36	0.44	0.08	0.0
0.1	0.9	OLS	0.06	0.24	0.01	0.09	0.05	0.0	0.07	0.13	0.01	0.07	0.19	0.0
0.1	0.9	GMM-1	0.12	3.28	9.49	1.25	0.04	1.0	-0.02	1.71	1.43	0.63	0.03	1.0
0.1	0.9	GMM-25	0.07	0.40	0.03	0.14	0.03	25.0	0.07	0.39	0.03	0.14	0.03	25.0
0.1	0.9	GMM-Bartlett	0.08	1.41	0.61	0.48	0.02	5.1	0.05	0.83	0.13	0.27	0.01	8.4
0.1	0.9	GMM-Tuk-Han	0.07	1.11	0.25	0.36	0.02	9.1	0.06	0.69	0.09	0.23	0.00	12.7
0.1	0.9	GMM-BR	0.07	1.48	0.59	0.49	0.04	4.7	0.07	0.88	0.15	0.29	0.02	6.7
0.1	0.9	GMM-BRM	0.07	1.87	1.74	0.69	0.30	2.1	0.06	1.24	0.29	0.39	0.28	3.0
0.1	0.9	GMM-Trunc	0.12	1.64	1.30	0.62	0.04	3.0	0.09	0.85	0.24	0.31	0.02	6.0
0.1	0.9	GMM-Trunc-BC	0.02	3.10	4.72E+11	21755.00	0.34	3.0	-0.02	1.50	0.49	0.52	0.23	6.0
0.1	0.9	CUE-1	0.11	7.76	43.48	3.36	0.01	1.0	-0.06	3.22	19.67	1.74	0.02	1.0
0.1	0.9	CUE-25	0.08	9.01	48.93	3.60	0.78	25.0	0.02	7.53	36.77	2.96	0.76	25.0
0.1	0.9	WWA-Hom	-0.01	39.85	87394.00	36.19	0.54	0.0	0.00	14.08	4633.40	12.86	0.44	0.0
0.1	0.9	WWA	-0.01	39.90	87377.00	36.23	0.54	0.0	0.00	14.08	4641.30	12.86	0.44	0.0

Table 2b - homoskedastic DGP, ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.58	0.29	0.34	0.58	1.00	0.0	0.58	0.14	0.34	0.58	1.00	0.0
0.5	-0.9	GMM-1	0.50	2.94	8.51	1.22	0.12	1.0	0.16	1.92	33.95	0.83	0.10	1.0
0.5	-0.9	GMM-25	0.57	0.46	0.36	0.57	0.84	25.0	0.54	0.45	0.33	0.54	0.86	25.0
0.5	-0.9	GMM-Bartlett	0.58	1.26	0.82	0.69	0.25	5.2	0.43	1.10	0.36	0.50	0.28	7.5
0.5	-0.9	GMM-Tuk-Han	0.56	1.01	0.48	0.60	0.37	9.1	0.47	0.91	0.32	0.49	0.39	12.0
0.5	-0.9	GMM-BR	0.58	1.30	0.87	0.69	0.31	4.7	0.44	1.09	0.37	0.50	0.30	6.2
0.5	-0.9	GMM-BRM	0.59	1.77	1.32	0.81	0.60	2.2	0.46	1.40	0.54	0.59	0.58	2.8
0.5	-0.9	GMM-Trunc	0.64	1.51	2.07	0.85	0.43	4.0	0.50	1.37	33.72	0.81	0.47	5.0
0.5	-0.9	GMM-Trunc-BC	0.43	3.18	2.87E+11	19970.00	0.46	4.0	0.25	2.25	33.90	0.93	0.41	5.0
0.5	-0.9	CUE-1	0.44	7.03	37.29	3.03	0.08	1.0	0.02	3.06	13.03	1.47	0.10	1.0
0.5	-0.9	CUE-25	0.49	8.42	40.73	3.31	0.82	25.0	0.25	6.53	26.56	2.54	0.78	25.0
0.5	-0.9	WWA-Hom	0.30	1.95	4.07	0.87	0.14	0.0	0.16	1.40	78.82	0.84	0.10	0.0
0.5	-0.9	WWA	0.30	1.95	26.57	1.05	0.14	0.0	0.16	1.40	42.09	0.76	0.10	0.0
0.5	-0.5	OLS	0.52	0.23	0.28	0.52	1.00	0.0	0.53	0.12	0.28	0.53	1.00	0.0
0.5	-0.5	GMM-1	0.43	2.30	10.78	1.00	0.11	1.0	0.13	1.47	0.65	0.52	0.08	1.0
0.5	-0.5	GMM-25	0.52	0.37	0.29	0.51	0.86	25.0	0.48	0.37	0.26	0.49	0.88	25.0
0.5	-0.5	GMM-Bartlett	0.47	1.32	0.86	0.64	0.21	2.9	0.31	1.04	0.28	0.43	0.21	4.1
0.5	-0.5	GMM-Tuk-Han	0.48	1.02	0.54	0.56	0.30	5.1	0.35	0.92	0.25	0.41	0.28	6.2
0.5	-0.5	GMM-BR	0.47	1.41	0.93	0.66	0.24	2.7	0.33	1.07	0.32	0.45	0.21	3.3
0.5	-0.5	GMM-BRM	0.48	1.68	10.05	0.84	0.46	1.2	0.30	1.35	0.58	0.53	0.45	1.5
0.5	-0.5	GMM-Trunc	0.49	1.50	0.93	0.69	0.30	2.0	0.34	1.27	0.45	0.50	0.29	2.0
0.5	-0.5	GMM-Trunc-BC	0.37	3.48	41.97	1.86	0.45	2.0	0.18	1.77	0.81	0.61	0.29	2.0
0.5	-0.5	CUE-1	0.37	5.35	24.39	2.36	0.08	1.0	0.00	2.43	12.79	1.33	0.09	1.0
0.5	-0.5	CUE-25	0.46	6.35	32.41	2.76	0.83	25.0	0.23	4.87	25.99	2.28	0.80	25.0
0.5	-0.5	WWA-Hom	0.37	1.63	2.79	0.77	0.16	0.0	0.19	1.20	0.48	0.45	0.10	0.0
0.5	-0.5	WWA	0.37	1.63	3.02	0.78	0.16	0.0	0.19	1.20	0.48	0.45	0.10	0.0
0.5	0	OLS	0.45	0.19	0.21	0.45	1.00	0.0	0.46	0.10	0.21	0.46	1.00	0.0
0.5	0	GMM-1	0.35	1.99	1.65	0.79	0.05	1.0	0.10	1.19	0.77	0.46	0.05	1.0
0.5	0	GMM-25	0.45	0.30	0.22	0.45	0.81	25.0	0.42	0.31	0.20	0.43	0.83	25.0
0.5	0	GMM-Bartlett	0.40	1.37	1.08	0.65	0.19	2.5	0.23	1.07	0.52	0.42	0.19	2.8
0.5	0	GMM-Tuk-Han	0.40	1.13	0.71	0.57	0.29	4.1	0.26	0.91	0.20	0.37	0.24	4.3
0.5	0	GMM-BR	0.39	1.40	1.10	0.65	0.21	2.1	0.20	1.09	0.50	0.42	0.18	2.3
0.5	0	GMM-BRM	0.39	1.47	1.16	0.67	0.40	1.0	0.20	1.20	0.59	0.44	0.34	1.0
0.5	0	GMM-Trunc	0.41	1.42	0.75	0.61	0.25	2.0	0.25	1.01	0.24	0.39	0.24	2.0
0.5	0	GMM-Trunc-BC	0.35	4.54	6215.50	6.78	0.48	2.0	0.11	1.63	2.87	0.61	0.25	2.0
0.5	0	CUE-1	0.30	4.19	19.67	1.99	0.06	1.0	0.01	1.81	7.53	0.97	0.05	1.0
0.5	0	CUE-25	0.37	5.00	25.85	2.33	0.83	25.0	0.20	4.06	19.64	1.90	0.77	25.0
0.5	0	WWA-Hom	0.34	1.49	13.41	0.74	0.15	0.0	0.18	1.09	0.29	0.38	0.09	0.0
0.5	0	WWA	0.34	1.49	13.27	0.75	0.14	0.0	0.18	1.09	0.29	0.38	0.09	0.0
0.5	0.5	OLS	0.38	0.20	0.16	0.39	1.00	0.0	0.39	0.10	0.15	0.39	1.00	0.0
0.5	0.5	GMM-1	0.32	2.49	5.47	0.99	0.05	1.0	0.09	1.33	0.83	0.50	0.02	1.0
0.5	0.5	GMM-25	0.39	0.31	0.16	0.39	0.67	25.0	0.36	0.32	0.15	0.37	0.70	25.0
0.5	0.5	GMM-Bartlett	0.36	1.36	0.77	0.59	0.11	3.6	0.22	0.93	0.22	0.36	0.10	4.4
0.5	0.5	GMM-Tuk-Han	0.34	1.11	0.43	0.48	0.15	5.6	0.24	0.82	0.19	0.34	0.14	6.4
0.5	0.5	GMM-BR	0.37	1.44	0.78	0.61	0.13	2.9	0.24	0.99	0.27	0.39	0.10	3.4
0.5	0.5	GMM-BRM	0.35	1.75	3.93	0.76	0.37	1.3	0.22	1.18	0.39	0.44	0.34	1.5
0.5	0.5	GMM-Trunc	0.38	1.59	3.79	0.73	0.16	2.0	0.26	1.07	0.35	0.43	0.17	2.0
0.5	0.5	GMM-Trunc-BC	0.18	3.56	105.32	1.93	0.31	2.0	0.07	1.54	0.62	0.51	0.18	2.0
0.5	0.5	CUE-1	0.25	5.49	31.35	2.62	0.04	1.0	0.00	2.30	10.70	1.24	0.03	1.0
0.5	0.5	CUE-25	0.34	6.24	33.80	2.78	0.79	25.0	0.17	5.12	24.73	2.24	0.74	25.0
0.5	0.5	WWA-Hom	0.17	1.84	49788.00	11.24	0.17	0.0	0.09	0.96	0.67	0.38	0.07	0.0
0.5	0.5	WWA	0.17	1.84	1919.90	4.46	0.17	0.0	0.09	0.96	0.66	0.38	0.07	0.0
0.5	0.9	OLS	0.33	0.24	0.12	0.33	0.92	0.0	0.33	0.13	0.11	0.33	1.00	0.0
0.5	0.9	GMM-1	0.30	3.21	3.91	1.17	0.05	1.0	0.07	1.69	1.30	0.62	0.02	1.0
0.5	0.9	GMM-25	0.34	0.40	0.14	0.34	0.40	25.0	0.31	0.39	0.13	0.33	0.41	25.0
0.5	0.9	GMM-Bartlett	0.37	1.46	0.65	0.59	0.08	4.9	0.26	0.96	0.22	0.37	0.06	6.8
0.5	0.9	GMM-Tuk-Han	0.33	1.05	0.33	0.44	0.13	8.8	0.28	0.77	0.16	0.33	0.11	11.2
0.5	0.9	GMM-BR	0.36	1.52	0.71	0.59	0.13	4.5	0.27	1.02	0.23	0.39	0.09	5.8
0.5	0.9	GMM-BRM	0.39	1.92	1.32	0.74	0.42	2.1	0.25	1.27	0.39	0.46	0.39	2.6
0.5	0.9	GMM-Trunc	0.39	1.80	1.74	0.77	0.17	3.0	0.33	1.22	0.60	0.50	0.18	3.0
0.5	0.9	GMM-Trunc-BC	-0.01	3.20	5.93E+11	24362.00	0.29	3.0	0.00	1.69	0.84	0.57	0.21	3.0
0.5	0.9	CUE-1	0.24	6.67	34.73	2.97	0.02	1.0	-0.01	3.02	16.60	1.55	0.03	1.0
0.5	0.9	CUE-25	0.34	8.53	40.44	3.33	0.78	25.0	0.13	7.15	36.81	2.87	0.73	25.0
0.5	0.9	WWA-Hom	0.03	5.14	7171.10	11.91	0.28	0.0	0.02	1.14	212.24	1.17	0.10	0.0
0.5	0.9	WWA	0.03	5.33	9371.70	13.37	0.28	0.0	0.02	1.14	219.10	1.17	0.10	0.0

Table 2c - homoskedastic DGP, ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	1.04	0.17	1.08	1.04	1.00	0.0	1.04	0.09	1.09	1.04	1.00	0.0
0.9	-0.9	GMM-1	0.74	2.37	3.53	1.19	0.37	1.0	0.24	1.69	1.10	0.64	0.25	1.0
0.9	-0.9	GMM-25	1.04	0.29	1.08	1.03	1.00	25.0	0.98	0.28	0.97	0.98	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.97	0.72	1.04	0.96	0.86	9.5	0.76	0.78	0.64	0.74	0.77	11.5
0.9	-0.9	GMM-Tuk-Han	0.98	0.63	1.00	0.96	0.91	14.3	0.82	0.65	0.70	0.79	0.87	16.0
0.9	-0.9	GMM-BR	0.96	0.75	1.04	0.95	0.88	7.5	0.78	0.73	0.65	0.75	0.83	8.3
0.9	-0.9	GMM-BRM	1.00	0.96	1.22	1.02	0.91	3.4	0.83	1.02	0.82	0.82	0.87	3.8
0.9	-0.9	GMM-Trunc	1.01	1.05	1.85	1.06	0.78	5.0	0.83	1.34	0.89	0.80	0.68	4.0
0.9	-0.9	GMM-Trunc-BC	0.95	1.34	1.93	1.07	0.76	5.0	0.73	1.47	0.88	0.77	0.67	4.0
0.9	-0.9	CUE-1	0.50	7.04	37.37	2.96	0.24	1.0	-0.03	2.81	15.92	1.52	0.13	1.0
0.9	-0.9	CUE-25	0.96	4.63	25.05	2.46	0.87	25.0	0.35	4.74	25.03	2.22	0.77	25.0
0.9	-0.9	WWA-Hom	0.79	1.66	8.45	1.07	0.50	0.0	0.40	1.28	0.67	0.58	0.36	0.0
0.9	-0.9	WWA	0.79	1.67	4.51	1.04	0.50	0.0	0.40	1.28	0.67	0.58	0.36	0.0
0.9	-0.5	OLS	0.94	0.12	0.89	0.94	1.00	0.0	0.94	0.06	0.89	0.94	1.00	0.0
0.9	-0.5	GMM-1	0.67	1.73	2.75	0.97	0.44	1.0	0.23	1.28	1.65	0.55	0.27	1.0
0.9	-0.5	GMM-25	0.93	0.20	0.88	0.93	0.99	25.0	0.88	0.21	0.79	0.88	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.84	0.79	0.83	0.83	0.77	4.5	0.49	0.89	0.37	0.52	0.57	3.9
0.9	-0.5	GMM-Tuk-Han	0.86	0.64	0.79	0.84	0.85	7.0	0.59	0.72	0.42	0.59	0.71	6.2
0.9	-0.5	GMM-BR	0.86	0.82	0.89	0.85	0.79	3.7	0.51	0.87	0.38	0.54	0.61	3.2
0.9	-0.5	GMM-BRM	0.85	0.98	1.09	0.89	0.82	1.7	0.48	1.13	0.59	0.58	0.69	1.5
0.9	-0.5	GMM-Trunc	0.79	1.39	2.41	0.95	0.59	1.0	0.29	1.39	0.76	0.54	0.39	1.0
0.9	-0.5	GMM-Trunc-BC	0.74	1.66	2.50	0.94	0.56	1.0	0.22	1.46	0.77	0.53	0.36	1.0
0.9	-0.5	CUE-1	0.43	5.53	30.20	2.50	0.30	1.0	-0.04	2.22	11.58	1.26	0.12	1.0
0.9	-0.5	CUE-25	0.84	3.98	24.05	2.29	0.89	25.0	0.29	4.18	24.50	2.06	0.77	25.0
0.9	-0.5	WWA-Hom	0.69	1.18	2.21	0.84	0.64	0.0	0.35	0.94	0.38	0.46	0.46	0.0
0.9	-0.5	WWA	0.69	1.18	2.65	0.84	0.64	0.0	0.35	0.94	0.38	0.46	0.46	0.0
0.9	0	OLS	0.82	0.11	0.67	0.82	1.00	0.0	0.82	0.06	0.67	0.82	1.00	0.0
0.9	0	GMM-1	0.56	1.45	1.11	0.75	0.30	1.0	0.17	1.09	0.63	0.44	0.15	1.0
0.9	0	GMM-25	0.81	0.18	0.66	0.81	0.94	25.0	0.76	0.19	0.60	0.77	0.95	25.0
0.9	0	GMM-Bartlett	0.71	0.98	0.82	0.76	0.62	2.6	0.36	0.98	0.48	0.48	0.45	2.1
0.9	0	GMM-Tuk-Han	0.73	0.75	0.74	0.75	0.69	4.2	0.44	0.81	0.30	0.47	0.50	3.6
0.9	0	GMM-BR	0.70	1.00	0.83	0.76	0.62	2.2	0.36	1.00	0.47	0.48	0.43	1.9
0.9	0	GMM-BRM	0.68	1.06	0.84	0.76	0.71	1.0	0.32	1.10	0.54	0.49	0.52	0.8
0.9	0	GMM-Trunc	0.64	1.23	0.89	0.74	0.46	1.0	0.25	1.08	0.42	0.43	0.27	1.0
0.9	0	GMM-Trunc-BC	0.57	1.98	6.81	1.08	0.48	1.0	0.17	1.23	0.56	0.46	0.25	1.0
0.9	0	CUE-1	0.36	4.34	22.46	2.05	0.26	1.0	-0.02	1.76	13.34	1.19	0.09	1.0
0.9	0	CUE-25	0.68	3.09	19.20	1.94	0.90	25.0	0.28	3.89	20.37	1.81	0.78	25.0
0.9	0	WWA-Hom	0.60	1.11	11.01	0.83	0.58	0.0	0.30	0.88	0.28	0.40	0.42	0.0
0.9	0	WWA	0.60	1.12	38.37	0.94	0.57	0.0	0.30	0.88	0.28	0.40	0.42	0.0
0.9	0.5	OLS	0.70	0.17	0.49	0.70	1.00	0.0	0.70	0.09	0.49	0.70	1.00	0.0
0.9	0.5	GMM-1	0.52	1.90	2.70	0.91	0.12	1.0	0.19	1.31	3.71	0.57	0.08	1.0
0.9	0.5	GMM-25	0.69	0.27	0.49	0.70	0.97	25.0	0.65	0.29	0.45	0.66	0.99	25.0
0.9	0.5	GMM-Bartlett	0.63	1.05	1.43	0.73	0.44	4.1	0.40	0.92	0.29	0.45	0.32	4.0
0.9	0.5	GMM-Tuk-Han	0.63	0.84	0.57	0.66	0.56	6.4	0.45	0.73	0.28	0.46	0.43	6.3
0.9	0.5	GMM-BR	0.62	1.20	1.52	0.75	0.51	3.3	0.43	0.98	0.36	0.50	0.42	3.3
0.9	0.5	GMM-BRM	0.61	1.34	2.15	0.83	0.60	1.5	0.37	1.21	0.44	0.52	0.53	1.5
0.9	0.5	GMM-Trunc	0.64	1.58	1.47	0.82	0.34	1.0	0.28	1.34	3.31	0.58	0.24	1.0
0.9	0.5	GMM-Trunc-BC	0.45	2.56	226.23	2.14	0.37	1.0	0.12	1.57	3.39	0.59	0.22	1.0
0.9	0.5	CUE-1	0.38	4.99	24.24	2.19	0.11	1.0	0.03	2.04	12.79	1.23	0.05	1.0
0.9	0.5	CUE-25	0.49	4.51	19.67	2.14	0.83	25.0	0.29	4.56	26.57	2.23	0.75	25.0
0.9	0.5	WWA-Hom	0.37	1.18	66.51	0.91	0.18	0.0	0.19	0.83	0.52	0.36	0.11	0.0
0.9	0.5	WWA	0.37	1.19	57.07	0.91	0.17	0.0	0.19	0.83	0.92	0.37	0.11	0.0
0.9	0.9	OLS	0.60	0.24	0.37	0.60	1.00	0.0	0.60	0.13	0.36	0.60	1.00	0.0
0.9	0.9	GMM-1	0.45	2.53	3.24	1.05	0.07	1.0	0.17	1.71	2.31	0.67	0.05	1.0
0.9	0.9	GMM-25	0.60	0.37	0.39	0.61	0.97	25.0	0.57	0.39	0.36	0.58	0.97	25.0
0.9	0.9	GMM-Bartlett	0.58	1.08	0.59	0.65	0.31	7.1	0.44	0.82	0.30	0.46	0.29	9.6
0.9	0.9	GMM-Tuk-Han	0.55	0.86	0.46	0.59	0.41	11.2	0.46	0.69	0.28	0.46	0.39	13.9
0.9	0.9	GMM-BR	0.58	1.18	0.63	0.65	0.45	5.8	0.43	0.91	0.32	0.48	0.39	7.2
0.9	0.9	GMM-BRM	0.61	1.79	1.11	0.81	0.62	2.6	0.46	1.37	0.57	0.59	0.59	3.3
0.9	0.9	GMM-Trunc	0.60	1.40	1.65	0.81	0.41	4.0	0.51	1.24	0.68	0.60	0.43	4.0
0.9	0.9	GMM-Trunc-BC	0.33	2.54	1.91E+07	168.37	0.37	4.0	0.30	1.54	0.77	0.58	0.32	4.0
0.9	0.9	CUE-1	0.35	5.52	27.97	2.54	0.06	1.0	0.07	2.60	11.65	1.34	0.04	1.0
0.9	0.9	CUE-25	0.45	5.28	30.52	2.73	0.71	25.0	0.25	5.12	25.85	2.38	0.68	25.0
0.9	0.9	WWA-Hom	0.18	1.53	147.57	1.07	0.06	0.0	0.09	0.99	0.40	0.37	0.04	0.0
0.9	0.9	WWA	0.18	1.55	10.25	0.77	0.06	0.0	0.09	0.99	0.39	0.37	0.04	0.0

Table 3a - homoskedastic DGP, ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.11	0.32	0.03	0.13	0.25	0.0	0.11	0.16	0.02	0.11	0.56	0.0
0.1	-0.9	GMM-1	0.06	1.55	0.78	0.53	0.08	1.0	0.00	0.65	0.07	0.20	0.09	1.0
0.1	-0.9	GMM-25	0.10	0.49	0.05	0.17	0.13	25.0	0.07	0.40	0.03	0.14	0.13	25.0
0.1	-0.9	GMM-Bartlett	0.07	1.14	0.27	0.37	0.07	3.1	0.01	0.58	0.05	0.18	0.08	5.0
0.1	-0.9	GMM-Tuk-Han	0.07	0.94	0.17	0.31	0.06	6.5	0.02	0.59	0.05	0.18	0.09	8.7
0.1	-0.9	GMM-BR	0.08	1.16	0.28	0.38	0.10	3.4	0.01	0.60	0.06	0.19	0.09	4.5
0.1	-0.9	GMM-BRM	0.07	1.34	0.40	0.44	0.37	1.5	0.02	0.69	0.08	0.22	0.39	2.0
0.1	-0.9	GMM-Trunc	0.08	1.27	0.49	0.44	0.09	1.0	0.02	0.59	0.06	0.19	0.09	3.0
0.1	-0.9	GMM-Trunc-BC	-0.06	2.79	4.33E+12	9.65E+04	0.52	1.0	-0.15	1.17	0.23	0.41	0.50	3.0
0.1	-0.9	CUE-1	0.05	2.22	8.05	1.06	0.06	1.0	-0.01	0.68	0.08	0.22	0.10	1.0
0.1	-0.9	CUE-25	0.02	5.36	28.29	2.40	0.77	25.0	-0.03	1.32	3.05	0.60	0.53	25.0
0.1	-0.9	WWA-Hom	0.04	1.30	2.64	0.53	0.10	0.0	0.00	0.61	0.10	0.21	0.08	0.0
0.1	-0.9	WWA	0.04	1.29	3.41	0.53	0.10	0.0	0.00	0.61	0.10	0.21	0.08	0.0
0.1	-0.5	OLS	0.09	0.25	0.02	0.11	0.23	0.0	0.10	0.13	0.01	0.10	0.60	0.0
0.1	-0.5	GMM-1	0.05	1.24	0.53	0.43	0.06	1.0	-0.01	0.51	0.04	0.16	0.07	1.0
0.1	-0.5	GMM-25	0.09	0.39	0.03	0.14	0.12	25.0	0.05	0.32	0.02	0.11	0.12	25.0
0.1	-0.5	GMM-Bartlett	0.07	0.87	0.34	0.32	0.05	4.2	0.01	0.47	0.03	0.15	0.07	6.4
0.1	-0.5	GMM-Tuk-Han	0.05	0.81	0.17	0.27	0.06	6.5	0.01	0.46	0.03	0.15	0.08	8.5
0.1	-0.5	GMM-BR	0.05	0.91	0.36	0.32	0.07	3.4	0.01	0.49	0.04	0.15	0.08	4.4
0.1	-0.5	GMM-BRM	0.06	1.09	0.44	0.38	0.35	1.5	0.01	0.56	0.05	0.17	0.35	2.0
0.1	-0.5	GMM-Trunc	0.07	0.94	0.23	0.32	0.07	3.0	0.01	0.48	0.04	0.15	0.09	5.0
0.1	-0.5	GMM-Trunc-BC	-0.01	1.82	12.74	0.79	0.40	3.0	-0.05	0.73	0.08	0.23	0.25	5.0
0.1	-0.5	CUE-1	0.04	1.69	5.18	0.81	0.06	1.0	-0.01	0.54	0.05	0.17	0.07	1.0
0.1	-0.5	CUE-25	0.05	3.98	20.18	1.93	0.76	25.0	-0.02	1.06	3.33	0.51	0.52	25.0
0.1	-0.5	WWA-Hom	0.02	1.09	1.52	0.40	0.07	0.0	0.00	0.48	0.04	0.15	0.04	0.0
0.1	-0.5	WWA	0.02	1.09	1.52	0.40	0.07	0.0	0.00	0.48	0.04	0.15	0.04	0.0
0.1	0	OLS	0.08	0.19	0.01	0.09	0.16	0.0	0.08	0.10	0.01	0.08	0.51	0.0
0.1	0	GMM-1	0.02	0.95	0.27	0.33	0.03	1.0	-0.01	0.38	0.03	0.12	0.02	1.0
0.1	0	GMM-25	0.07	0.30	0.02	0.11	0.07	25.0	0.05	0.26	0.01	0.09	0.07	25.0
0.1	0	GMM-Bartlett	0.04	0.74	0.15	0.25	0.04	5.6	0.01	0.37	0.02	0.11	0.02	8.8
0.1	0	GMM-Tuk-Han	0.05	0.68	0.11	0.23	0.03	7.6	0.01	0.36	0.02	0.11	0.03	10.1
0.1	0	GMM-BR	0.04	0.73	0.16	0.26	0.03	4.0	0.00	0.37	0.02	0.11	0.03	5.3
0.1	0	GMM-BRM	0.04	0.86	0.18	0.29	0.30	1.8	0.01	0.42	0.03	0.13	0.30	2.4
0.1	0	GMM-Trunc	0.05	0.69	0.13	0.23	0.03	4.0	0.02	0.37	0.02	0.11	0.04	6.0
0.1	0	GMM-Trunc-BC	0.02	1.21	1.05	0.49	0.28	4.0	-0.01	0.47	0.04	0.15	0.12	6.0
0.1	0	CUE-1	0.01	1.24	4.33	0.64	0.03	1.0	-0.01	0.40	0.03	0.13	0.03	1.0
0.1	0	CUE-25	0.06	3.39	15.53	1.63	0.73	25.0	0.00	0.79	2.46	0.38	0.45	25.0
0.1	0	WWA-Hom	0.02	0.91	0.32	0.32	0.06	0.0	0.00	0.39	0.02	0.12	0.04	0.0
0.1	0	WWA	0.02	0.91	0.32	0.32	0.06	0.0	0.00	0.39	0.02	0.12	0.04	0.0
0.1	0.5	OLS	0.06	0.17	0.01	0.07	0.05	0.0	0.06	0.09	0.00	0.06	0.23	0.0
0.1	0.5	GMM-1	0.01	0.98	0.40	0.35	0.01	1.0	-0.01	0.36	0.02	0.11	0.00	1.0
0.1	0.5	GMM-25	0.05	0.27	0.01	0.10	0.02	25.0	0.04	0.23	0.01	0.08	0.02	25.0
0.1	0.5	GMM-Bartlett	0.03	0.72	0.17	0.26	0.02	6.0	0.00	0.33	0.02	0.10	0.01	9.9
0.1	0.5	GMM-Tuk-Han	0.03	0.65	0.11	0.22	0.02	8.2	0.00	0.32	0.02	0.10	0.00	11.1
0.1	0.5	GMM-BR	0.03	0.71	0.20	0.26	0.02	4.3	0.00	0.33	0.02	0.10	0.00	5.7
0.1	0.5	GMM-BRM	0.01	0.83	0.31	0.31	0.22	2.0	0.01	0.36	0.02	0.12	0.17	2.6
0.1	0.5	GMM-Trunc	0.04	0.66	0.25	0.25	0.01	5.0	0.01	0.33	0.02	0.10	0.01	6.0
0.1	0.5	GMM-Trunc-BC	-0.03	1.20	1.07	0.50	0.19	5.0	-0.03	0.43	0.03	0.14	0.05	6.0
0.1	0.5	CUE-1	0.00	1.32	3.83	0.65	0.02	1.0	-0.01	0.38	0.03	0.12	0.01	1.0
0.1	0.5	CUE-25	0.04	4.51	27.87	2.16	0.67	25.0	0.00	0.80	3.31	0.48	0.42	25.0
0.1	0.5	WWA-Hom	-0.01	1.07	530.99	2.85	0.19	0.0	-0.01	0.31	0.02	0.10	0.05	0.0
0.1	0.5	WWA	-0.01	1.07	539.74	2.85	0.19	0.0	-0.01	0.31	0.02	0.10	0.05	0.0
0.1	0.9	OLS	0.04	0.19	0.01	0.07	0.01	0.0	0.04	0.10	0.00	0.05	0.07	0.0
0.1	0.9	GMM-1	0.01	1.13	0.48	0.41	0.01	1.0	-0.01	0.40	0.03	0.13	0.01	1.0
0.1	0.9	GMM-25	0.04	0.30	0.02	0.10	0.01	25.0	0.03	0.25	0.01	0.08	0.02	25.0
0.1	0.9	GMM-Bartlett	0.03	0.79	0.15	0.27	0.02	6.6	0.00	0.36	0.02	0.11	0.01	10.8
0.1	0.9	GMM-Tuk-Han	0.04	0.67	0.09	0.22	0.01	10.3	0.01	0.34	0.02	0.11	0.00	14.4
0.1	0.9	GMM-BR	0.04	0.78	0.15	0.27	0.02	5.3	0.01	0.37	0.02	0.12	0.00	7.6
0.1	0.9	GMM-BRM	0.02	1.01	0.24	0.33	0.24	2.4	0.02	0.42	0.03	0.13	0.15	3.4
0.1	0.9	GMM-Trunc	0.03	0.77	0.24	0.27	0.02	5.0	0.00	0.35	0.02	0.11	0.01	7.0
0.1	0.9	GMM-Trunc-BC	-0.10	1.63	8.02E+10	15886.00	0.30	5.0	-0.10	0.60	0.06	0.21	0.13	7.0
0.1	0.9	CUE-1	0.00	1.72	5.90	0.86	0.02	1.0	-0.01	0.43	0.11	0.15	0.01	1.0
0.1	0.9	CUE-25	0.05	5.84	30.28	2.47	0.69	25.0	-0.01	1.01	5.98	0.64	0.40	25.0
0.1	0.9	WWA-Hom	-0.03	32.02	10106.00	21.61	0.54	0.0	0.00	4.70	5494.20	7.76	0.41	0.0
0.1	0.9	WWA	-0.03	32.22	33962.00	25.04	0.53	0.0	0.00	4.71	5146.40	7.63	0.41	0.0

Table 3b - homoskedastic DGP, ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.55	0.28	0.31	0.54	1.00	0.0	0.55	0.14	0.30	0.55	1.00	0.0
0.5	-0.9	GMM-1	0.13	1.44	2.52	0.57	0.11	1.0	0.00	0.65	0.07	0.21	0.10	1.0
0.5	-0.9	GMM-25	0.50	0.43	0.28	0.50	0.84	25.0	0.35	0.37	0.14	0.35	0.72	25.0
0.5	-0.9	GMM-Bartlett	0.26	1.08	0.74	0.43	0.21	4.2	0.04	0.65	0.06	0.20	0.15	3.5
0.5	-0.9	GMM-Tuk-Han	0.33	0.93	0.23	0.40	0.31	7.8	0.08	0.62	0.06	0.20	0.18	7.1
0.5	-0.9	GMM-BR	0.28	1.06	1.12	0.45	0.28	4.0	0.06	0.65	0.07	0.21	0.17	3.7
0.5	-0.9	GMM-BRM	0.29	1.30	1.19	0.51	0.53	1.8	0.05	0.71	0.08	0.23	0.40	1.7
0.5	-0.9	GMM-Trunc	0.33	1.27	2.25	0.55	0.36	2.0	0.04	0.74	0.09	0.24	0.23	1.0
0.5	-0.9	GMM-Trunc-BC	-0.03	2.45	6.18E+11	35977.00	0.48	2.0	-0.24	0.92	0.21	0.36	0.40	1.0
0.5	-0.9	CUE-1	0.01	2.16	13.04	1.19	0.11	1.0	-0.03	0.70	0.10	0.23	0.09	1.0
0.5	-0.9	CUE-25	0.20	5.11	26.21	2.32	0.77	25.0	-0.01	1.13	4.01	0.57	0.51	25.0
0.5	-0.9	WWA-Hom	0.11	1.20	1.40	0.50	0.12	0.0	0.02	0.58	0.06	0.18	0.08	0.0
0.5	-0.9	WWA	0.11	1.20	2.04	0.51	0.12	0.0	0.02	0.58	0.06	0.18	0.08	0.0
0.5	-0.5	OLS	0.47	0.22	0.23	0.47	1.00	0.0	0.47	0.12	0.22	0.47	1.00	0.0
0.5	-0.5	GMM-1	0.10	1.22	0.66	0.42	0.10	1.0	0.00	0.51	0.05	0.17	0.08	1.0
0.5	-0.5	GMM-25	0.43	0.35	0.21	0.43	0.84	25.0	0.30	0.29	0.10	0.30	0.73	25.0
0.5	-0.5	GMM-Bartlett	0.21	0.87	0.24	0.34	0.20	3.7	0.03	0.50	0.04	0.16	0.12	3.7
0.5	-0.5	GMM-Tuk-Han	0.25	0.76	0.16	0.33	0.26	5.8	0.06	0.49	0.04	0.16	0.15	5.7
0.5	-0.5	GMM-BR	0.22	0.90	0.27	0.36	0.22	3.0	0.04	0.51	0.04	0.16	0.14	3.0
0.5	-0.5	GMM-BRM	0.20	1.10	0.54	0.42	0.44	1.4	0.03	0.57	0.06	0.18	0.35	1.3
0.5	-0.5	GMM-Trunc	0.24	0.99	0.29	0.39	0.29	3.0	0.06	0.53	0.05	0.18	0.19	3.0
0.5	-0.5	GMM-Trunc-BC	0.01	1.59	2.27	0.56	0.31	3.0	-0.07	0.59	0.06	0.19	0.14	3.0
0.5	-0.5	CUE-1	0.01	1.66	10.20	0.97	0.10	1.0	-0.02	0.56	0.06	0.18	0.07	1.0
0.5	-0.5	CUE-25	0.18	3.89	21.92	1.94	0.77	25.0	-0.02	0.91	1.51	0.41	0.48	25.0
0.5	-0.5	WWA-Hom	0.13	1.02	0.41	0.38	0.12	0.0	0.02	0.48	0.04	0.15	0.07	0.0
0.5	-0.5	WWA	0.13	1.02	0.42	0.38	0.12	0.0	0.02	0.48	0.04	0.15	0.07	0.0
0.5	0	OLS	0.37	0.18	0.15	0.38	1.00	0.0	0.38	0.09	0.14	0.38	1.00	0.0
0.5	0	GMM-1	0.05	0.93	0.27	0.32	0.05	1.0	-0.01	0.39	0.03	0.13	0.04	1.0
0.5	0	GMM-25	0.35	0.27	0.13	0.35	0.80	25.0	0.24	0.23	0.07	0.24	0.68	25.0
0.5	0	GMM-Bartlett	0.18	0.77	0.18	0.30	0.18	4.8	0.04	0.40	0.03	0.13	0.10	4.9
0.5	0	GMM-Tuk-Han	0.20	0.68	0.13	0.28	0.24	6.8	0.06	0.38	0.03	0.13	0.12	6.7
0.5	0	GMM-BR	0.17	0.78	0.18	0.30	0.18	3.5	0.04	0.39	0.03	0.13	0.09	3.5
0.5	0	GMM-BRM	0.16	0.86	0.21	0.32	0.36	1.6	0.03	0.44	0.04	0.14	0.28	1.6
0.5	0	GMM-Trunc	0.20	0.74	0.15	0.30	0.26	4.0	0.07	0.39	0.03	0.14	0.15	4.0
0.5	0	GMM-Trunc-BC	0.07	1.20	0.86	0.45	0.27	4.0	0.00	0.44	0.04	0.14	0.11	4.0
0.5	0	CUE-1	0.00	1.28	5.26	0.66	0.05	1.0	-0.02	0.41	0.03	0.13	0.03	1.0
0.5	0	CUE-25	0.14	3.26	18.96	1.73	0.74	25.0	-0.01	0.71	1.51	0.34	0.42	25.0
0.5	0	WWA-Hom	0.09	0.87	0.27	0.31	0.10	0.0	0.01	0.39	0.02	0.12	0.06	0.0
0.5	0	WWA	0.09	0.87	0.27	0.31	0.10	0.0	0.01	0.39	0.02	0.12	0.06	0.0
0.5	0.5	OLS	0.28	0.17	0.09	0.28	0.98	0.0	0.28	0.09	0.08	0.28	1.00	0.0
0.5	0.5	GMM-1	0.05	0.91	0.30	0.33	0.02	1.0	0.00	0.36	0.02	0.12	0.01	1.0
0.5	0.5	GMM-25	0.26	0.26	0.08	0.27	0.49	25.0	0.18	0.22	0.04	0.18	0.32	25.0
0.5	0.5	GMM-Bartlett	0.13	0.73	0.16	0.28	0.05	4.9	0.03	0.34	0.02	0.11	0.03	4.7
0.5	0.5	GMM-Tuk-Han	0.15	0.64	0.13	0.26	0.09	7.0	0.04	0.33	0.02	0.11	0.03	6.7
0.5	0.5	GMM-BR	0.13	0.76	0.19	0.29	0.07	3.7	0.03	0.35	0.02	0.11	0.02	3.5
0.5	0.5	GMM-BRM	0.11	0.93	0.22	0.32	0.27	1.6	0.03	0.37	0.03	0.12	0.17	1.6
0.5	0.5	GMM-Trunc	0.15	0.77	0.16	0.29	0.10	3.0	0.05	0.35	0.02	0.12	0.04	4.0
0.5	0.5	GMM-Trunc-BC	-0.08	1.17	0.72	0.43	0.15	3.0	-0.05	0.38	0.03	0.13	0.02	4.0
0.5	0.5	CUE-1	0.00	1.28	5.77	0.75	0.02	1.0	-0.01	0.38	0.03	0.12	0.01	1.0
0.5	0.5	CUE-25	0.12	4.41	25.05	2.07	0.66	25.0	0.00	0.77	3.22	0.44	0.35	25.0
0.5	0.5	WWA-Hom	0.02	0.85	404.63	1.71	0.13	0.0	-0.01	0.31	0.02	0.10	0.05	0.0
0.5	0.5	WWA	0.02	0.86	520.48	1.81	0.13	0.0	-0.01	0.31	0.02	0.10	0.05	0.0
0.5	0.9	OLS	0.20	0.19	0.05	0.21	0.55	0.0	0.21	0.10	0.04	0.21	1.00	0.0
0.5	0.9	GMM-1	0.03	1.07	0.45	0.40	0.01	1.0	0.00	0.42	0.03	0.13	0.00	1.0
0.5	0.9	GMM-25	0.20	0.32	0.06	0.21	0.13	25.0	0.13	0.25	0.03	0.14	0.09	25.0
0.5	0.9	GMM-Bartlett	0.12	0.85	0.17	0.30	0.04	5.2	0.02	0.41	0.03	0.13	0.01	5.4
0.5	0.9	GMM-Tuk-Han	0.14	0.70	0.12	0.26	0.05	8.7	0.03	0.39	0.03	0.12	0.02	9.3
0.5	0.9	GMM-BR	0.13	0.86	0.17	0.30	0.05	4.5	0.03	0.41	0.03	0.13	0.02	4.8
0.5	0.9	GMM-BRM	0.11	1.07	0.32	0.37	0.27	2.0	0.04	0.44	0.04	0.15	0.17	2.2
0.5	0.9	GMM-Trunc	0.15	0.86	0.26	0.32	0.06	3.0	0.02	0.44	0.03	0.14	0.03	2.0
0.5	0.9	GMM-Trunc-BC	-0.25	1.57	3.88E+11	28839.00	0.26	3.0	-0.18	0.51	0.07	0.22	0.11	2.0
0.5	0.9	CUE-1	0.00	1.53	9.39	0.98	0.02	1.0	-0.01	0.44	0.04	0.14	0.01	1.0
0.5	0.9	CUE-25	0.10	6.36	36.53	2.70	0.67	25.0	0.01	0.92	5.88	0.61	0.34	25.0
0.5	0.9	WWA-Hom	-0.04	6.44	3772.70	11.53	0.33	0.0	-0.01	0.35	136.42	0.77	0.11	0.0
0.5	0.9	WWA	-0.04	6.39	7434.10	13.43	0.33	0.0	-0.01	0.35	121.68	0.75	0.11	0.0

Table 3c - homoskedastic DGP, ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.98	0.16	0.96	0.98	1.00	0.0	0.98	0.08	0.96	0.98	1.00	0.0
0.9	-0.9	GMM-1	0.14	1.31	1.93	0.56	0.22	1.0	0.00	0.64	0.09	0.21	0.14	1.0
0.9	-0.9	GMM-25	0.91	0.26	0.83	0.91	1.00	25.0	0.63	0.25	0.40	0.63	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.47	0.93	0.35	0.51	0.60	6.5	0.07	0.61	0.06	0.20	0.22	3.3
0.9	-0.9	GMM-Tuk-Han	0.57	0.77	0.41	0.58	0.73	10.5	0.15	0.54	0.07	0.21	0.31	6.8
0.9	-0.9	GMM-BR	0.54	0.91	0.40	0.56	0.68	5.4	0.13	0.64	0.07	0.22	0.32	3.5
0.9	-0.9	GMM-BRM	0.52	1.08	0.45	0.58	0.75	2.4	0.09	0.71	0.09	0.24	0.51	1.6
0.9	-0.9	GMM-Trunc	0.40	1.39	0.56	0.58	0.53	2.0	0.01	0.67	0.09	0.22	0.22	1.0
0.9	-0.9	GMM-Trunc-BC	0.31	1.64	12869.00	4.18	0.55	2.0	-0.09	0.88	0.16	0.29	0.34	1.0
0.9	-0.9	CUE-1	-0.07	2.08	14.27	1.25	0.10	1.0	-0.04	0.70	0.15	0.24	0.07	1.0
0.9	-0.9	CUE-25	0.13	4.17	21.17	1.95	0.75	25.0	-0.05	0.81	0.89	0.31	0.36	25.0
0.9	-0.9	WWA-Hom	0.22	1.07	0.41	0.43	0.36	0.0	0.04	0.58	0.06	0.19	0.29	0.0
0.9	-0.9	WWA	0.23	1.07	0.41	0.43	0.36	0.0	0.04	0.58	0.06	0.19	0.28	0.0
0.9	-0.5	OLS	0.84	0.13	0.72	0.85	1.00	0.0	0.85	0.06	0.72	0.85	1.00	0.0
0.9	-0.5	GMM-1	0.12	1.06	1.35	0.45	0.20	1.0	0.01	0.51	0.06	0.17	0.11	1.0
0.9	-0.5	GMM-25	0.78	0.22	0.62	0.78	0.99	25.0	0.54	0.19	0.30	0.54	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.30	0.79	0.24	0.38	0.44	3.3	0.05	0.45	0.04	0.15	0.15	3.2
0.9	-0.5	GMM-Tuk-Han	0.40	0.68	0.23	0.42	0.58	5.7	0.10	0.42	0.04	0.16	0.24	5.5
0.9	-0.5	GMM-BR	0.34	0.82	0.27	0.41	0.51	2.9	0.07	0.47	0.04	0.16	0.20	2.9
0.9	-0.5	GMM-BRM	0.29	1.08	1.00	0.49	0.60	1.3	0.05	0.51	0.05	0.17	0.39	1.3
0.9	-0.5	GMM-Trunc	0.17	1.17	1.28	0.47	0.34	1.0	0.03	0.53	0.05	0.17	0.19	2.0
0.9	-0.5	GMM-Trunc-BC	0.10	1.21	1.30	0.47	0.30	1.0	-0.01	0.52	0.05	0.17	0.14	2.0
0.9	-0.5	CUE-1	-0.05	1.66	9.65	0.97	0.10	1.0	-0.03	0.57	0.09	0.19	0.07	1.0
0.9	-0.5	CUE-25	0.09	3.00	14.73	1.47	0.73	25.0	-0.04	0.65	0.28	0.24	0.34	25.0
0.9	-0.5	WWA-Hom	0.21	0.89	4.77	0.44	0.42	0.0	0.04	0.47	0.04	0.15	0.30	0.0
0.9	-0.5	WWA	0.21	0.89	4.75	0.44	0.42	0.0	0.04	0.47	0.04	0.15	0.30	0.0
0.9	0	OLS	0.68	0.13	0.46	0.68	1.00	0.0	0.68	0.06	0.46	0.68	1.00	0.0
0.9	0	GMM-1	0.09	0.84	0.33	0.32	0.11	1.0	0.00	0.39	0.04	0.13	0.04	1.0
0.9	0	GMM-25	0.62	0.20	0.40	0.63	0.97	25.0	0.43	0.17	0.19	0.43	0.99	25.0
0.9	0	GMM-Bartlett	0.28	0.74	0.23	0.36	0.41	3.9	0.07	0.38	0.03	0.13	0.15	4.6
0.9	0	GMM-Tuk-Han	0.35	0.61	0.19	0.37	0.50	5.9	0.11	0.37	0.03	0.15	0.22	6.4
0.9	0	GMM-BR	0.29	0.75	0.24	0.37	0.44	3.0	0.08	0.38	0.03	0.14	0.17	3.3
0.9	0	GMM-BRM	0.24	0.98	0.29	0.38	0.51	1.4	0.07	0.45	0.05	0.16	0.38	1.5
0.9	0	GMM-Trunc	0.24	0.82	0.21	0.35	0.33	2.0	0.08	0.38	0.03	0.14	0.17	3.0
0.9	0	GMM-Trunc-BC	0.14	0.94	0.34	0.36	0.28	2.0	0.04	0.40	0.03	0.13	0.10	3.0
0.9	0	CUE-1	-0.02	1.22	6.08	0.70	0.07	1.0	-0.02	0.43	0.06	0.15	0.03	1.0
0.9	0	CUE-25	0.14	2.44	15.40	1.46	0.72	25.0	-0.02	0.57	0.80	0.23	0.32	25.0
0.9	0	WWA-Hom	0.15	0.71	0.20	0.29	0.25	0.0	0.02	0.37	0.03	0.12	0.20	0.0
0.9	0	WWA	0.15	0.71	0.20	0.29	0.25	0.0	0.02	0.37	0.03	0.12	0.20	0.0
0.9	0.5	OLS	0.51	0.16	0.27	0.51	1.00	0.0	0.51	0.08	0.26	0.51	1.00	0.0
0.9	0.5	GMM-1	0.08	0.90	0.70	0.34	0.04	1.0	0.01	0.37	0.03	0.12	0.01	1.0
0.9	0.5	GMM-25	0.47	0.25	0.24	0.48	0.96	25.0	0.32	0.20	0.12	0.33	0.91	25.0
0.9	0.5	GMM-Bartlett	0.21	0.68	0.17	0.29	0.16	4.2	0.05	0.32	0.02	0.11	0.04	4.3
0.9	0.5	GMM-Tuk-Han	0.26	0.58	0.16	0.31	0.25	6.5	0.09	0.30	0.02	0.12	0.05	6.6
0.9	0.5	GMM-BR	0.23	0.79	0.21	0.34	0.28	3.4	0.06	0.39	0.03	0.13	0.10	3.4
0.9	0.5	GMM-BRM	0.17	0.98	0.28	0.36	0.39	1.5	0.04	0.39	0.03	0.13	0.26	1.6
0.9	0.5	GMM-Trunc	0.17	0.93	0.26	0.34	0.17	1.0	0.05	0.34	0.03	0.12	0.04	2.0
0.9	0.5	GMM-Trunc-BC	-0.06	1.06	22.57	0.57	0.10	1.0	-0.05	0.33	0.03	0.12	0.02	2.0
0.9	0.5	CUE-1	0.00	1.16	4.64	0.61	0.03	1.0	-0.01	0.40	0.06	0.13	0.01	1.0
0.9	0.5	CUE-25	0.17	4.19	25.46	2.03	0.65	25.0	0.00	0.68	1.88	0.34	0.27	25.0
0.9	0.5	WWA-Hom	0.07	0.67	0.32	0.26	0.08	0.0	0.01	0.31	0.02	0.10	0.05	0.0
0.9	0.5	WWA	0.07	0.67	0.36	0.26	0.07	0.0	0.01	0.32	0.02	0.10	0.05	0.0
0.9	0.9	OLS	0.37	0.20	0.15	0.38	1.00	0.0	0.37	0.10	0.14	0.37	1.00	0.0
0.9	0.9	GMM-1	0.08	1.11	1.62	0.43	0.02	1.0	0.02	0.45	0.04	0.14	0.01	1.0
0.9	0.9	GMM-25	0.34	0.33	0.15	0.36	0.63	25.0	0.24	0.26	0.07	0.25	0.43	25.0
0.9	0.9	GMM-Bartlett	0.20	0.79	0.17	0.31	0.10	6.2	0.06	0.42	0.03	0.14	0.03	5.9
0.9	0.9	GMM-Tuk-Han	0.21	0.65	0.13	0.28	0.14	9.9	0.08	0.39	0.03	0.14	0.05	9.8
0.9	0.9	GMM-BR	0.21	0.91	0.23	0.35	0.24	5.1	0.07	0.50	0.05	0.17	0.12	5.1
0.9	0.9	GMM-BRM	0.20	1.25	0.39	0.45	0.45	2.3	0.06	0.62	0.08	0.20	0.34	2.3
0.9	0.9	GMM-Trunc	0.21	1.01	0.25	0.37	0.16	3.0	0.04	0.52	0.05	0.16	0.06	2.0
0.9	0.9	GMM-Trunc-BC	-0.07	1.24	8.04E+10	12713.00	0.17	3.0	-0.07	0.56	0.06	0.19	0.06	2.0
0.9	0.9	CUE-1	0.03	1.42	5.52	0.73	0.02	1.0	0.00	0.48	0.08	0.16	0.01	1.0
0.9	0.9	CUE-25	0.10	5.68	27.07	2.32	0.58	25.0	0.01	0.73	4.98	0.54	0.28	25.0
0.9	0.9	WWA-Hom	-0.01	0.76	0.52	0.30	0.05	0.0	0.00	0.33	0.02	0.10	0.01	0.0
0.9	0.9	WWA	-0.01	0.76	0.74	0.31	0.05	0.0	0.00	0.33	0.02	0.10	0.01	0.0

Table 4a - Homoskedastic DGP ϕ=.1, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.05	0.23	0.01	0.08	0.04	0.0	0.06	0.11	0.01	0.06	0.11	0.0
0.1	-0.9	GMM-1	0.00	3.27	3.93	1.16	0.03	1.0	0.05	3.02	38.24	1.32	0.03	1.0
0.1	-0.9	GMM-25	0.05	0.40	0.03	0.13	0.02	25.0	0.05	0.39	0.03	0.13	0.02	25.0
0.1	-0.9	GMM-Bartlett	0.03	1.37	0.55	0.46	0.02	5.7	0.06	0.93	0.16	0.30	0.01	8.3
0.1	-0.9	GMM-Tuk-Han	0.05	1.08	0.26	0.35	0.02	9.6	0.06	0.76	0.10	0.25	0.00	12.9
0.1	-0.9	GMM-BR	0.06	1.41	0.57	0.49	0.05	5.0	0.06	1.00	0.19	0.32	0.02	6.7
0.1	-0.9	GMM-BRM	0.04	1.94	1.20	0.66	0.32	2.3	0.05	1.43	0.45	0.47	0.32	3.0
0.1	-0.9	GMM-Trunc	0.05	1.47	1.33	0.55	0.03	4.0	0.08	0.98	32.43	0.57	0.02	6.0
0.1	-0.9	GMM-Trunc-BC	-0.13	2.75	4.69E+11	2.95E+04	0.35	4.0	-0.06	1.82	32.66	0.82	0.29	6.0
0.1	-0.9	CUE-1	-0.06	8.23	35.72	3.14	0.01	1.0	0.00	6.75	32.13	2.75	0.01	1.0
0.1	-0.9	CUE-25	-0.01	10.33	53.03	3.85	0.79	25.0	-0.01	9.38	51.66	3.69	0.78	25.0
0.1	-0.9	WWA-Hom	-0.02	1.29	28.46	0.82	0.11	0.0	0.01	1.09	9.64	0.71	0.13	0.0
0.1	-0.9	WWA	-0.02	1.29	422.60	1.35	0.11	0.0	0.01	1.09	9.67	0.71	0.13	0.0
0.1	-0.5	OLS	0.07	0.21	0.01	0.08	0.09	0.0	0.07	0.10	0.01	0.07	0.29	0.0
0.1	-0.5	GMM-1	0.04	2.80	2.41	0.95	0.03	1.0	0.04	2.36	16.54	1.02	0.03	1.0
0.1	-0.5	GMM-25	0.06	0.33	0.02	0.12	0.03	25.0	0.06	0.32	0.02	0.12	0.04	25.0
0.1	-0.5	GMM-Bartlett	0.04	1.35	0.68	0.46	0.02	4.7	0.07	0.93	0.39	0.34	0.00	6.1
0.1	-0.5	GMM-Tuk-Han	0.06	1.07	0.26	0.35	0.03	7.1	0.06	0.85	0.24	0.29	0.01	8.4
0.1	-0.5	GMM-BR	0.04	1.47	0.66	0.48	0.03	3.8	0.06	1.01	0.39	0.37	0.00	4.4
0.1	-0.5	GMM-BRM	0.04	1.75	1.11	0.60	0.24	1.7	0.08	1.32	1.04	0.51	0.23	2.0
0.1	-0.5	GMM-Trunc	0.06	1.38	0.68	0.47	0.02	3.0	0.06	1.00	0.31	0.34	0.02	4.0
0.1	-0.5	GMM-Trunc-BC	0.00	3.11	16.55	1.28	0.27	3.0	0.01	1.95	1.74	0.69	0.22	4.0
0.1	-0.5	CUE-1	0.02	6.42	28.44	2.61	0.00	1.0	0.02	5.15	25.47	2.30	0.01	1.0
0.1	-0.5	CUE-25	-0.01	8.56	43.54	3.29	0.80	25.0	0.01	7.09	39.11	2.99	0.79	25.0
0.1	-0.5	WWA-Hom	0.03	1.45	4.42	0.70	0.12	0.0	0.03	1.40	7.34	0.71	0.12	0.0
0.1	-0.5	WWA	0.03	1.45	4.63	0.70	0.12	0.0	0.03	1.40	7.30	0.71	0.12	0.0
0.1	0	OLS	0.08	0.22	0.01	0.10	0.18	0.0	0.09	0.10	0.01	0.09	0.57	0.0
0.1	0	GMM-1	0.10	2.33	2.91	0.89	0.03	1.0	0.07	2.13	2.52	0.80	0.02	1.0
0.1	0	GMM-25	0.08	0.33	0.02	0.12	0.06	25.0	0.09	0.33	0.02	0.13	0.07	25.0
0.1	0	GMM-Bartlett	0.08	1.38	1.37	0.55	0.02	4.0	0.09	1.05	0.55	0.38	0.00	5.3
0.1	0	GMM-Tuk-Han	0.07	1.14	0.41	0.39	0.02	6.2	0.09	0.93	0.20	0.31	0.01	7.2
0.1	0	GMM-BR	0.08	1.42	1.36	0.56	0.03	3.3	0.09	1.08	0.63	0.41	0.01	3.8
0.1	0	GMM-BRM	0.08	1.59	1.49	0.62	0.22	1.5	0.08	1.32	1.17	0.53	0.23	1.7
0.1	0	GMM-Trunc	0.09	1.34	0.55	0.44	0.03	2.0	0.10	1.06	0.41	0.39	0.02	3.0
0.1	0	GMM-Trunc-BC	0.07	3.18	89.23	2.27	0.30	2.0	0.10	1.87	3.95	0.79	0.20	3.0
0.1	0	CUE-1	0.11	5.61	25.39	2.39	0.00	1.0	0.06	4.46	20.21	1.94	0.00	1.0
0.1	0	CUE-25	-0.01	5.97	26.56	2.44	0.83	25.0	0.03	5.33	35.60	2.67	0.82	25.0
0.1	0	WWA-Hom	0.05	1.73	200.20	1.23	0.10	0.0	0.05	1.43	3.77	0.63	0.09	0.0
0.1	0	WWA	0.05	1.73	198.95	1.23	0.10	0.0	0.05	1.43	3.77	0.63	0.09	0.0
0.1	0.5	OLS	0.10	0.26	0.02	0.12	0.24	0.0	0.10	0.13	0.01	0.11	0.61	0.0
0.1	0.5	GMM-1	0.13	2.88	4.62	1.03	0.04	1.0	0.11	2.63	9.65	1.05	0.04	1.0
0.1	0.5	GMM-25	0.10	0.41	0.04	0.15	0.08	25.0	0.10	0.41	0.04	0.16	0.09	25.0
0.1	0.5	GMM-Bartlett	0.10	1.61	0.73	0.55	0.02	3.4	0.08	1.28	0.49	0.43	0.01	4.3
0.1	0.5	GMM-Tuk-Han	0.08	1.25	0.36	0.42	0.03	5.7	0.09	1.10	0.23	0.35	0.02	6.6
0.1	0.5	GMM-BR	0.09	1.67	0.77	0.56	0.04	3.0	0.09	1.32	0.49	0.45	0.03	3.4
0.1	0.5	GMM-BRM	0.11	2.05	3.15	0.73	0.29	1.3	0.12	1.75	5.01	0.71	0.29	1.6
0.1	0.5	GMM-Trunc	0.13	1.80	1.16	0.62	0.06	2.0	0.12	1.45	4.81	0.62	0.04	3.0
0.1	0.5	GMM-Trunc-BC	0.15	4.23	11.95	1.63	0.40	2.0	0.12	2.79	6.90	1.09	0.32	3.0
0.1	0.5	CUE-1	0.11	6.57	37.25	2.93	0.02	1.0	0.08	5.59	25.53	2.37	0.01	1.0
0.1	0.5	CUE-25	-0.02	6.49	32.61	2.73	0.82	25.0	0.07	6.08	29.60	2.61	0.83	25.0
0.1	0.5	WWA-Hom	0.04	2.61	3532.70	9.00	0.17	0.0	0.08	1.83	8.38	0.85	0.10	0.0
0.1	0.5	WWA	0.04	2.61	5343.50	10.06	0.17	0.0	0.08	1.83	8.38	0.85	0.10	0.0
0.1	0.9	OLS	0.12	0.32	0.03	0.14	0.21	0.0	0.12	0.16	0.02	0.12	0.56	0.0
0.1	0.9	GMM-1	0.17	3.67	20.10	1.41	0.08	1.0	0.11	3.18	5.03	1.15	0.08	1.0
0.1	0.9	GMM-25	0.12	0.50	0.05	0.18	0.09	25.0	0.12	0.52	0.05	0.19	0.12	25.0
0.1	0.9	GMM-Bartlett	0.14	2.00	0.93	0.67	0.06	2.9	0.12	1.52	0.42	0.49	0.03	4.2
0.1	0.9	GMM-Tuk-Han	0.13	1.37	0.37	0.45	0.05	6.2	0.11	1.12	0.21	0.36	0.04	8.4
0.1	0.9	GMM-BR	0.16	1.86	0.74	0.62	0.10	3.2	0.11	1.47	0.38	0.47	0.07	4.3
0.1	0.9	GMM-BRM	0.16	2.51	15.76	0.97	0.37	1.5	0.13	1.85	0.70	0.61	0.38	2.0
0.1	0.9	GMM-Trunc	0.20	2.84	19.18	1.18	0.11	1.0	0.20	2.37	2.02	0.83	0.10	1.0
0.1	0.9	GMM-Trunc-BC	0.01	5.95	1.10E+14	357640.00	0.48	1.0	0.23	3.39	2.82	1.17	0.35	1.0
0.1	0.9	CUE-1	0.16	7.93	43.37	3.36	0.02	1.0	0.10	6.67	27.10	2.63	0.02	1.0
0.1	0.9	CUE-25	0.00	8.01	39.06	3.21	0.83	25.0	0.05	7.65	37.53	3.13	0.84	25.0
0.1	0.9	WWA-Hom	-0.05	99.81	448140.00	85.85	0.51	0.0	0.04	33.99	27643000.00	200.82	0.47	0.0
0.1	0.9	WWA	-0.05	99.99	194130.00	77.13	0.51	0.0	0.04	34.00	27242000.00	199.67	0.47	0.0

Table 4b - Homoskedastic DGP ϕ=.1, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.27	0.22	0.08	0.28	0.78	0.0	0.28	0.11	0.08	0.28	1.00	0.0
0.5	-0.9	GMM-1	0.30	3.22	4.41	1.21	0.05	1.0	0.25	2.84	4.96	1.10	0.05	1.0
0.5	-0.9	GMM-25	0.28	0.37	0.10	0.28	0.27	25.0	0.27	0.36	0.10	0.28	0.28	25.0
0.5	-0.9	GMM-Bartlett	0.32	1.34	0.44	0.51	0.06	5.3	0.31	0.99	0.29	0.41	0.05	6.6
0.5	-0.9	GMM-Tuk-Han	0.28	1.04	0.26	0.40	0.08	9.4	0.28	0.75	0.17	0.34	0.07	11.1
0.5	-0.9	GMM-BR	0.33	1.41	0.46	0.52	0.10	4.9	0.30	1.06	0.31	0.42	0.07	5.8
0.5	-0.9	GMM-BRM	0.32	1.79	0.92	0.66	0.41	2.2	0.31	1.49	0.64	0.56	0.41	2.6
0.5	-0.9	GMM-Trunc	0.32	1.54	1.46	0.67	0.10	3.0	0.32	1.40	2.88	0.72	0.13	4.0
0.5	-0.9	GMM-Trunc-BC	-0.06	2.98	2.88E+10	5364.00	0.31	3.0	0.04	2.12	3.06	0.77	0.19	4.0
0.5	-0.9	CUE-1	0.26	9.19	42.04	3.47	0.01	1.0	0.16	7.11	36.13	2.89	0.01	1.0
0.5	-0.9	CUE-25	0.27	10.60	52.31	3.84	0.80	25.0	0.19	8.52	49.14	3.54	0.77	25.0
0.5	-0.9	WWA-Hom	0.07	1.29	14.04	0.73	0.10	0.0	0.04	1.17	302.27	1.32	0.12	0.0
0.5	-0.9	WWA	0.08	1.30	16.57	0.77	0.10	0.0	0.05	1.17	103.50	1.08	0.12	0.0
0.5	-0.5	OLS	0.34	0.19	0.12	0.34	1.00	0.0	0.35	0.09	0.12	0.35	1.00	0.0
0.5	-0.5	GMM-1	0.34	2.47	2.65	0.96	0.05	1.0	0.27	2.33	11.47	1.02	0.04	1.0
0.5	-0.5	GMM-25	0.34	0.29	0.13	0.34	0.59	25.0	0.34	0.30	0.13	0.34	0.63	25.0
0.5	-0.5	GMM-Bartlett	0.32	1.26	0.59	0.53	0.10	4.7	0.34	0.88	0.36	0.43	0.08	5.4
0.5	-0.5	GMM-Tuk-Han	0.32	1.00	0.41	0.45	0.15	7.3	0.32	0.78	0.22	0.38	0.13	7.7
0.5	-0.5	GMM-BR	0.34	1.36	0.70	0.56	0.12	3.8	0.34	1.00	0.43	0.46	0.09	4.0
0.5	-0.5	GMM-BRM	0.35	1.71	1.08	0.66	0.37	1.7	0.32	1.30	9.08	0.68	0.39	1.8
0.5	-0.5	GMM-Trunc	0.33	1.45	1.02	0.60	0.15	3.0	0.34	1.01	8.01	0.57	0.16	3.0
0.5	-0.5	GMM-Trunc-BC	0.14	3.01	4036.60	3.64	0.30	3.0	0.18	1.71	8.72	0.73	0.23	3.0
0.5	-0.5	CUE-1	0.27	6.37	34.45	2.83	0.02	1.0	0.23	5.30	24.42	2.28	0.02	1.0
0.5	-0.5	CUE-25	0.30	8.08	43.75	3.28	0.81	25.0	0.26	6.27	36.45	2.88	0.79	25.0
0.5	-0.5	WWA-Hom	0.20	1.29	2.57	0.66	0.15	0.0	0.19	1.25	105.63	1.00	0.14	0.0
0.5	-0.5	WWA	0.20	1.29	7.02	0.72	0.14	0.0	0.19	1.24	109.21	1.01	0.14	0.0
0.5	0	OLS	0.43	0.19	0.19	0.43	1.00	0.0	0.43	0.09	0.19	0.43	1.00	0.0
0.5	0	GMM-1	0.40	2.04	2.75	0.89	0.07	1.0	0.33	1.89	1.83	0.77	0.06	1.0
0.5	0	GMM-25	0.42	0.29	0.19	0.42	0.81	25.0	0.43	0.28	0.19	0.43	0.88	25.0
0.5	0	GMM-Bartlett	0.40	1.18	1.24	0.60	0.22	4.3	0.40	0.85	0.48	0.48	0.24	6.1
0.5	0	GMM-Tuk-Han	0.39	0.97	0.43	0.50	0.31	6.7	0.40	0.79	0.31	0.45	0.33	8.1
0.5	0	GMM-BR	0.40	1.15	1.59	0.61	0.23	3.4	0.40	0.89	0.59	0.50	0.23	4.3
0.5	0	GMM-BRM	0.40	1.38	1.88	0.69	0.44	1.6	0.40	1.12	1.13	0.58	0.46	1.9
0.5	0	GMM-Trunc	0.40	1.23	0.84	0.59	0.29	2.0	0.41	0.93	0.91	0.55	0.32	3.0
0.5	0	GMM-Trunc-BC	0.36	2.54	48.24	1.61	0.39	2.0	0.39	1.56	2.31	0.76	0.39	3.0
0.5	0	CUE-1	0.36	4.81	25.31	2.33	0.06	1.0	0.27	4.03	21.90	2.01	0.05	1.0
0.5	0	CUE-25	0.33	5.55	28.10	2.46	0.83	25.0	0.32	4.94	32.64	2.52	0.83	25.0
0.5	0	WWA-Hom	0.36	1.55	831.43	1.76	0.16	0.0	0.32	1.26	3.39	0.65	0.17	0.0
0.5	0	WWA	0.36	1.55	658.71	1.65	0.15	0.0	0.32	1.26	3.53	0.65	0.17	0.0
0.5	0.5	OLS	0.51	0.23	0.27	0.51	1.00	0.0	0.52	0.11	0.27	0.52	1.00	0.0
0.5	0.5	GMM-1	0.53	2.50	4.85	1.10	0.13	1.0	0.43	2.38	3.20	0.97	0.11	1.0
0.5	0.5	GMM-25	0.51	0.35	0.28	0.51	0.82	25.0	0.51	0.37	0.28	0.51	0.90	25.0
0.5	0.5	GMM-Bartlett	0.50	1.44	0.84	0.70	0.23	3.3	0.47	1.13	0.56	0.58	0.24	4.3
0.5	0.5	GMM-Tuk-Han	0.49	1.10	0.53	0.59	0.33	5.6	0.48	0.94	0.39	0.52	0.35	6.6
0.5	0.5	GMM-BR	0.51	1.42	0.88	0.70	0.26	2.9	0.48	1.18	0.66	0.60	0.26	3.5
0.5	0.5	GMM-BRM	0.51	1.81	2.27	0.84	0.50	1.3	0.48	1.58	2.30	0.78	0.52	1.6
0.5	0.5	GMM-Trunc	0.55	1.51	1.75	0.78	0.33	2.0	0.51	1.27	2.05	0.73	0.39	3.0
0.5	0.5	GMM-Trunc-BC	0.51	3.80	18.53	1.64	0.49	2.0	0.43	2.45	3.46	1.02	0.47	3.0
0.5	0.5	CUE-1	0.51	6.16	32.38	2.74	0.10	1.0	0.37	5.32	25.74	2.37	0.10	1.0
0.5	0.5	CUE-25	0.40	5.54	29.05	2.56	0.84	25.0	0.43	5.47	26.87	2.46	0.84	25.0
0.5	0.5	WWA-Hom	0.43	2.31	4924.10	10.05	0.26	0.0	0.40	1.63	10.08	0.91	0.20	0.0
0.5	0.5	WWA	0.43	2.31	6557.70	10.69	0.26	0.0	0.40	1.63	10.08	0.91	0.20	0.0
0.5	0.9	OLS	0.58	0.28	0.35	0.58	1.00	0.0	0.59	0.14	0.35	0.59	1.00	0.0
0.5	0.9	GMM-1	0.65	3.30	3.90	1.28	0.16	1.0	0.51	2.92	15.47	1.28	0.14	1.0
0.5	0.9	GMM-25	0.59	0.44	0.37	0.58	0.87	25.0	0.58	0.46	0.37	0.58	0.91	25.0
0.5	0.9	GMM-Bartlett	0.63	1.45	0.85	0.75	0.28	4.5	0.57	1.16	0.53	0.62	0.32	6.8
0.5	0.9	GMM-Tuk-Han	0.61	1.05	0.55	0.65	0.40	8.4	0.58	0.87	0.43	0.58	0.46	11.7
0.5	0.9	GMM-BR	0.64	1.35	0.80	0.74	0.35	4.3	0.57	1.13	0.51	0.61	0.36	6.1
0.5	0.9	GMM-BRM	0.60	1.81	1.59	0.86	0.56	2.0	0.57	1.44	0.72	0.70	0.62	2.7
0.5	0.9	GMM-Trunc	0.66	1.90	2.42	0.98	0.45	3.0	0.63	1.50	1.23	0.80	0.54	5.0
0.5	0.9	GMM-Trunc-BC	0.54	4.36	5.91E+12	76885.00	0.52	3.0	0.44	2.51	1.77	0.95	0.48	5.0
0.5	0.9	CUE-1	0.63	7.50	34.78	3.07	0.11	1.0	0.43	6.22	28.41	2.67	0.10	1.0
0.5	0.9	CUE-25	0.49	6.99	34.67	3.00	0.84	25.0	0.48	7.19	34.24	2.91	0.84	25.0
0.5	0.9	WWA-Hom	0.41	36.73	386800.00	63.17	0.44	0.0	0.39	4.19	75479.00	24.32	0.31	0.0
0.5	0.9	WWA	0.41	36.51	252530.00	60.04	0.44	0.0	0.39	4.13	44920.00	20.89	0.31	0.0

Table 4c - Homoskedastic DGP ϕ=.1, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.50	0.19	0.25	0.50	1.00	0.0	0.50	0.10	0.25	0.50	1.00	0.0
0.9	-0.9	GMM-1	0.48	2.53	4.34	1.05	0.05	1.0	0.36	2.41	5.14	1.01	0.06	1.0
0.9	-0.9	GMM-25	0.50	0.31	0.27	0.51	0.92	25.0	0.50	0.32	0.27	0.50	0.94	25.0
0.9	-0.9	GMM-Bartlett	0.53	0.95	0.45	0.58	0.22	6.9	0.48	0.71	0.32	0.50	0.21	8.8
0.9	-0.9	GMM-Tuk-Han	0.48	0.72	0.34	0.51	0.30	11.1	0.44	0.56	0.26	0.46	0.31	13.7
0.9	-0.9	GMM-BR	0.50	1.16	0.56	0.60	0.39	5.8	0.46	0.92	0.38	0.52	0.38	7.1
0.9	-0.9	GMM-BRM	0.50	1.69	0.91	0.72	0.56	2.6	0.45	1.54	0.71	0.65	0.59	3.2
0.9	-0.9	GMM-Trunc	0.53	1.21	0.86	0.68	0.33	4.0	0.50	1.04	3.31	0.67	0.36	6.0
0.9	-0.9	GMM-Trunc-BC	0.21	2.52	5.44E+09	3297.90	0.30	4.0	0.22	1.60	3.42	0.63	0.24	6.0
0.9	-0.9	CUE-1	0.43	6.20	37.44	2.99	0.02	1.0	0.28	5.09	26.80	2.38	0.04	1.0
0.9	-0.9	CUE-25	0.65	6.57	33.90	3.13	0.76	25.0	0.42	6.03	33.50	2.93	0.71	25.0
0.9	-0.9	WWA-Hom	0.15	1.17	5.05	0.57	0.05	0.0	0.14	1.05	352.66	1.24	0.06	0.0
0.9	-0.9	WWA	0.16	1.16	86.28	0.84	0.05	0.0	0.14	1.06	229.92	1.28	0.06	0.0
0.9	-0.5	OLS	0.62	0.14	0.39	0.62	1.00	0.0	0.62	0.07	0.39	0.62	1.00	0.0
0.9	-0.5	GMM-1	0.60	1.93	1.62	0.89	0.11	1.0	0.45	1.83	1.81	0.82	0.11	1.0
0.9	-0.5	GMM-25	0.62	0.22	0.40	0.62	0.96	25.0	0.61	0.23	0.39	0.62	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.62	0.97	0.69	0.69	0.41	4.3	0.59	0.81	0.49	0.60	0.39	4.5
0.9	-0.5	GMM-Tuk-Han	0.61	0.75	0.56	0.65	0.53	7.1	0.58	0.61	0.39	0.57	0.50	7.0
0.9	-0.5	GMM-BR	0.62	1.12	0.78	0.71	0.51	3.7	0.58	0.97	0.55	0.62	0.48	3.6
0.9	-0.5	GMM-BRM	0.61	1.44	0.91	0.75	0.59	1.7	0.54	1.28	1.15	0.71	0.62	1.7
0.9	-0.5	GMM-Trunc	0.62	1.37	0.92	0.74	0.34	1.0	0.57	1.21	1.36	0.72	0.34	1.0
0.9	-0.5	GMM-Trunc-BC	0.45	3.03	707.49	3.07	0.40	1.0	0.37	1.76	1.51	0.73	0.28	1.0
0.9	-0.5	CUE-1	0.55	5.02	26.81	2.44	0.09	1.0	0.34	4.06	20.40	2.01	0.09	1.0
0.9	-0.5	CUE-25	0.59	5.18	28.36	2.65	0.85	25.0	0.53	5.19	24.78	2.43	0.80	25.0
0.9	-0.5	WWA-Hom	0.38	0.93	117.00	0.99	0.18	0.0	0.34	0.85	3.59	0.61	0.17	0.0
0.9	-0.5	WWA	0.38	0.93	19.29	0.80	0.17	0.0	0.34	0.86	5.70	0.65	0.17	0.0
0.9	0	OLS	0.77	0.10	0.60	0.77	1.00	0.0	0.77	0.05	0.60	0.78	1.00	0.0
0.9	0	GMM-1	0.70	1.36	1.60	0.86	0.39	1.0	0.56	1.28	1.16	0.75	0.35	1.0
0.9	0	GMM-25	0.77	0.16	0.60	0.77	0.99	25.0	0.77	0.17	0.59	0.77	1.00	25.0
0.9	0	GMM-Bartlett	0.75	0.58	0.67	0.75	0.75	5.4	0.71	0.52	0.52	0.69	0.78	6.6
0.9	0	GMM-Tuk-Han	0.75	0.49	0.63	0.75	0.83	8.1	0.71	0.44	0.53	0.70	0.87	8.8
0.9	0	GMM-BR	0.75	0.64	0.70	0.76	0.76	4.2	0.70	0.55	0.54	0.69	0.79	4.5
0.9	0	GMM-BRM	0.75	0.78	1.08	0.80	0.84	1.8	0.71	0.68	0.65	0.72	0.86	2.0
0.9	0	GMM-Trunc	0.74	0.96	1.13	0.80	0.58	1.0	0.68	0.86	0.83	0.71	0.59	1.0
0.9	0	GMM-Trunc-BC	0.71	1.34	97.38	1.41	0.60	1.0	0.65	0.98	0.85	0.71	0.58	1.0
0.9	0	CUE-1	0.63	3.48	17.55	1.89	0.32	1.0	0.44	3.32	17.59	1.75	0.28	1.0
0.9	0	CUE-25	0.71	3.34	20.61	2.05	0.90	25.0	0.65	3.51	22.25	2.10	0.90	25.0
0.9	0	WWA-Hom	0.65	0.87	108330.00	11.16	0.64	0.0	0.58	0.78	8.96	0.78	0.61	0.0
0.9	0	WWA	0.65	0.87	2234.10	2.25	0.63	0.0	0.59	0.78	12.99	0.80	0.61	0.0
0.9	0.5	OLS	0.93	0.12	0.86	0.93	1.00	0.0	0.93	0.06	0.87	0.93	1.00	0.0
0.9	0.5	GMM-1	0.90	1.38	1.55	1.03	0.55	1.0	0.72	1.51	2.13	0.94	0.52	1.0
0.9	0.5	GMM-25	0.92	0.18	0.86	0.92	0.98	25.0	0.92	0.19	0.85	0.92	1.00	25.0
0.9	0.5	GMM-Bartlett	0.91	0.75	1.03	0.93	0.80	3.4	0.84	0.71	0.80	0.82	0.79	3.9
0.9	0.5	GMM-Tuk-Han	0.90	0.58	0.87	0.90	0.86	5.9	0.85	0.60	0.75	0.83	0.87	6.3
0.9	0.5	GMM-BR	0.91	0.76	1.02	0.93	0.82	3.1	0.84	0.73	0.82	0.83	0.81	3.2
0.9	0.5	GMM-BRM	0.91	0.94	1.28	0.98	0.85	1.4	0.83	0.95	1.07	0.87	0.83	1.5
0.9	0.5	GMM-Trunc	0.91	1.17	1.39	0.99	0.66	1.0	0.81	1.21	1.33	0.89	0.64	1.0
0.9	0.5	GMM-Trunc-BC	0.88	1.88	2.49	1.10	0.64	1.0	0.74	1.50	1.40	0.88	0.60	1.0
0.9	0.5	CUE-1	0.86	3.54	19.00	2.07	0.46	1.0	0.58	4.16	27.55	2.28	0.36	1.0
0.9	0.5	CUE-25	0.86	2.91	16.03	1.88	0.90	25.0	0.79	3.28	15.00	1.83	0.89	25.0
0.9	0.5	WWA-Hom	0.82	1.47	8928.20	9.68	0.76	0.0	0.70	1.04	9.67	1.02	0.71	0.0
0.9	0.5	WWA	0.82	1.47	11298.00	10.31	0.76	0.0	0.70	1.04	9.44	1.02	0.71	0.0
0.9	0.9	OLS	1.05	0.16	1.11	1.05	1.00	0.0	1.05	0.08	1.11	1.05	1.00	0.0
0.9	0.9	GMM-1	1.03	1.85	3.08	1.24	0.50	1.0	0.85	1.83	4.80	1.11	0.46	1.0
0.9	0.9	GMM-25	1.06	0.24	1.11	1.05	1.00	25.0	1.05	0.27	1.10	1.04	1.00	25.0
0.9	0.9	GMM-Bartlett	1.04	0.61	1.15	1.03	0.89	8.3	1.00	0.56	0.99	0.97	0.91	11.6
0.9	0.9	GMM-Tuk-Han	1.03	0.53	1.09	1.02	0.95	13.2	1.01	0.47	1.01	0.99	0.98	16.9
0.9	0.9	GMM-BR	1.04	0.63	1.22	1.04	0.93	7.0	1.00	0.56	1.00	0.97	0.95	8.9
0.9	0.9	GMM-BRM	1.06	0.84	1.41	1.07	0.95	3.2	1.04	0.74	1.13	1.01	0.96	4.0
0.9	0.9	GMM-Trunc	1.08	0.99	2.29	1.15	0.80	4.0	1.04	0.84	1.21	1.01	0.81	6.0
0.9	0.9	GMM-Trunc-BC	1.01	1.28	2.33	1.14	0.77	4.0	0.97	1.05	1.18	0.98	0.80	6.0
0.9	0.9	CUE-1	1.01	4.46	24.63	2.48	0.40	1.0	0.66	6.06	31.31	2.62	0.33	1.0
0.9	0.9	CUE-25	1.00	3.92	24.28	2.38	0.90	25.0	0.90	4.08	26.14	2.43	0.85	25.0
0.9	0.9	WWA-Hom	0.98	1.22	5927.20	3.61	0.66	0.0	0.86	1.29	69067.00	9.40	0.63	0.0
0.9	0.9	WWA	0.97	1.22	11401.00	4.53	0.66	0.0	0.86	1.29	451830.00	22.36	0.63	0.0

Table 5a - Homoskedastic DGP ϕ=.3, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
					n=128						n=512			
0.1	-0.9	OLS	0.07	0.27	0.02	0.11	0.08	0.0	0.08	0.13	0.01	0.08	0.26	0.0
0.1	-0.9	GMM-1	0.02	3.39	15.61	1.28	0.05	1.0	0.01	2.37	8.90	0.98	0.04	1.0
0.1	-0.9	GMM-25	0.08	0.46	0.04	0.15	0.04	25.0	0.07	0.43	0.04	0.15	0.04	25.0
0.1	-0.9	GMM-Bartlett	0.05	1.55	0.72	0.53	0.03	4.9	0.08	1.07	0.21	0.34	0.01	7.2
0.1	-0.9	GMM-Tuk-Han	0.06	1.19	0.42	0.41	0.03	8.6	0.07	0.90	0.13	0.28	0.01	11.7
0.1	-0.9	GMM-BR	0.05	1.64	0.72	0.55	0.05	4.5	0.09	1.17	0.24	0.37	0.03	6.1
0.1	-0.9	GMM-BRM	0.05	2.04	1.32	0.72	0.34	2.1	0.06	1.44	0.46	0.48	0.33	2.8
0.1	-0.9	GMM-Trunc	0.09	1.85	13.84	0.81	0.05	3.0	0.14	1.23	1.03	0.48	0.05	5.0
0.1	-0.9	GMM-Trunc-BC	-0.13	3.42	1.27E+12	62716.00	0.35	3.0	-0.01	2.01	1.38	0.71	0.26	5.0
0.1	-0.9	CUE-1	-0.02	8.20	40.43	3.28	0.01	1.0	-0.04	5.65	28.69	2.41	0.01	1.0
0.1	-0.9	CUE-25	0.02	9.53	47.56	3.62	0.80	25.0	-0.01	7.96	38.53	3.15	0.78	25.0
0.1	-0.9	WWA-Hom	0.01	1.54	193.13	1.35	0.12	0.0	0.01	1.28	2.65	0.59	0.12	0.0
0.1	-0.9	WWA	0.01	1.54	151.68	1.28	0.12	0.0	0.01	1.28	2.65	0.59	0.12	0.0
0.1	-0.5	OLS	0.08	0.24	0.02	0.10	0.14	0.0	0.09	0.11	0.01	0.09	0.45	0.0
0.1	-0.5	GMM-1	0.05	2.64	26.94	1.15	0.04	1.0	0.03	1.94	1.89	0.70	0.02	1.0
0.1	-0.5	GMM-25	0.08	0.37	0.03	0.13	0.04	25.0	0.08	0.37	0.03	0.13	0.05	25.0
0.1	-0.5	GMM-Bartlett	0.05	1.58	1.10	0.56	0.02	3.6	0.08	1.01	0.38	0.37	0.01	5.0
0.1	-0.5	GMM-Tuk-Han	0.05	1.34	0.55	0.45	0.02	5.7	0.07	0.93	0.22	0.32	0.01	7.0
0.1	-0.5	GMM-BR	0.06	1.68	1.17	0.58	0.03	2.9	0.08	1.11	0.42	0.40	0.01	3.6
0.1	-0.5	GMM-BRM	0.04	1.92	1.65	0.69	0.23	1.3	0.08	1.37	0.64	0.50	0.23	1.6
0.1	-0.5	GMM-Trunc	0.08	1.83	3.08	0.66	0.03	2.0	0.11	1.13	0.51	0.42	0.02	3.0
0.1	-0.5	GMM-Trunc-BC	0.02	6.56	72891.00	17.40	0.35	2.0	0.07	1.90	8.61	0.89	0.18	3.0
0.1	-0.5	CUE-1	-0.01	6.01	33.29	2.72	0.01	1.0	0.00	4.07	22.16	2.02	0.01	1.0
0.1	-0.5	CUE-25	-0.02	8.04	41.08	3.16	0.81	25.0	0.01	6.42	38.01	2.84	0.78	25.0
0.1	-0.5	WWA-Hom	0.04	1.72	119.09	1.09	0.12	0.0	0.04	1.31	137.79	1.10	0.09	0.0
0.1	-0.5	WWA	0.05	1.72	106.20	1.07	0.12	0.0	0.04	1.31	137.68	1.10	0.09	0.0
0.1	0	OLS	0.09	0.23	0.02	0.11	0.20	0.0	0.10	0.11	0.01	0.10	0.61	0.0
0.1	0	GMM-1	0.08	2.43	68.68	1.23	0.03	1.0	0.05	1.73	1.74	0.65	0.02	1.0
0.1	0	GMM-25	0.09	0.35	0.03	0.13	0.06	25.0	0.09	0.35	0.03	0.13	0.07	25.0
0.1	0	GMM-Bartlett	0.08	1.68	2.59	0.67	0.02	2.1	0.08	1.18	0.62	0.45	0.01	3.6
0.1	0	GMM-Tuk-Han	0.07	1.57	1.52	0.59	0.04	3.7	0.08	1.09	0.41	0.39	0.01	5.3
0.1	0	GMM-BR	0.09	1.71	2.60	0.68	0.02	1.9	0.07	1.21	0.63	0.46	0.01	2.8
0.1	0	GMM-BRM	0.08	1.83	2.68	0.71	0.18	0.9	0.06	1.39	0.69	0.50	0.20	1.3
0.1	0	GMM-Trunc	0.10	1.78	1.19	0.61	0.03	1.0	0.10	1.27	0.40	0.43	0.03	2.0
0.1	0	GMM-Trunc-BC	0.08	21.48	89093000.00	437.31	0.49	1.0	0.09	3.91	12235.00	5.70	0.31	2.0
0.1	0	CUE-1	0.07	4.86	27.92	2.33	0.00	1.0	0.03	3.32	17.42	1.68	0.01	1.0
0.1	0	CUE-25	0.01	5.95	26.92	2.49	0.82	25.0	0.04	4.79	28.05	2.36	0.80	25.0
0.1	0	WWA-Hom	0.04	1.81	28.96	0.79	0.07	0.0	0.05	1.27	0.36	0.43	0.04	0.0
0.1	0	WWA	0.04	1.81	28.83	0.79	0.07	0.0	0.05	1.27	0.36	0.43	0.04	0.0
0.1	0.5	OLS	0.10	0.26	0.02	0.12	0.20	0.0	0.11	0.13	0.01	0.11	0.58	0.0
0.1	0.5	GMM-1	0.10	3.18	9.43	1.23	0.04	1.0	0.08	2.13	1.43	0.74	0.04	1.0
0.1	0.5	GMM-25	0.10	0.41	0.04	0.15	0.06	25.0	0.10	0.42	0.04	0.16	0.09	25.0
0.1	0.5	GMM-Bartlett	0.09	1.82	1.63	0.65	0.03	3.2	0.08	1.17	0.38	0.40	0.01	4.4
0.1	0.5	GMM-Tuk-Han	0.08	1.40	0.83	0.49	0.03	5.4	0.08	1.04	0.22	0.34	0.01	6.4
0.1	0.5	GMM-BR	0.10	1.84	1.69	0.67	0.04	2.8	0.09	1.24	0.44	0.43	0.03	3.3
0.1	0.5	GMM-BRM	0.08	2.27	2.55	0.81	0.27	1.3	0.09	1.59	0.73	0.54	0.24	1.5
0.1	0.5	GMM-Trunc	0.13	2.05	1.26	0.68	0.05	2.0	0.11	1.32	0.55	0.46	0.03	2.0
0.1	0.5	GMM-Trunc-BC	0.10	7.00	412880.00	31.91	0.44	2.0	0.06	2.46	7.36	1.09	0.26	2.0
0.1	0.5	CUE-1	0.09	6.45	30.80	2.68	0.01	1.0	0.05	4.21	24.51	2.08	0.02	1.0
0.1	0.5	CUE-25	0.03	6.55	32.26	2.75	0.81	25.0	0.03	5.90	31.18	2.59	0.81	25.0
0.1	0.5	WWA-Hom	0.03	2.44	86105.00	14.58	0.14	0.0	0.05	1.37	0.41	0.45	0.05	0.0
0.1	0.5	WWA	0.03	2.44	86113.00	14.58	0.14	0.0	0.05	1.37	0.41	0.45	0.05	0.0
0.1	0.9	OLS	0.11	0.32	0.03	0.14	0.18	0.0	0.11	0.16	0.02	0.12	0.50	0.0
0.1	0.9	GMM-1	0.15	3.91	12.38	1.52	0.06	1.0	0.08	2.72	2.34	0.94	0.06	1.0
0.1	0.9	GMM-25	0.12	0.50	0.05	0.18	0.06	25.0	0.11	0.52	0.05	0.18	0.09	25.0
0.1	0.9	GMM-Bartlett	0.15	1.92	0.99	0.64	0.05	3.4	0.11	1.28	0.31	0.42	0.03	5.1
0.1	0.9	GMM-Tuk-Han	0.12	1.32	0.41	0.44	0.04	6.9	0.12	1.00	0.18	0.33	0.03	9.3
0.1	0.9	GMM-BR	0.13	1.89	0.95	0.63	0.07	3.6	0.12	1.31	0.30	0.42	0.05	4.8
0.1	0.9	GMM-BRM	0.15	2.45	3.91	0.94	0.37	1.6	0.09	1.58	0.49	0.51	0.33	2.2
0.1	0.9	GMM-Trunc	0.20	2.77	4.94	1.04	0.11	1.0	0.20	1.73	1.22	0.66	0.08	2.0
0.1	0.9	GMM-Trunc-BC	0.03	5.26	1.49E+12	67175.00	0.45	1.0	0.17	2.74	1.78	0.94	0.31	2.0
0.1	0.9	CUE-1	0.14	8.45	41.69	3.29	0.01	1.0	0.06	5.27	24.05	2.31	0.02	1.0
0.1	0.9	CUE-25	0.03	8.14	42.44	3.36	0.81	25.0	0.07	7.01	39.90	3.15	0.82	25.0
0.1	0.9	WWA-Hom	-0.02	62.98	214750.00	58.64	0.50	0.0	0.02	13.86	163470.00	34.19	0.42	0.0
0.1	0.9	WWA	-0.02	63.00	193980.00	57.20	0.50	0.0	0.02	13.86	167660.00	34.40	0.42	0.0

Table 5b - Homoskedastic DGP ϕ=.3, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.39	0.25	0.16	0.39	0.98	0.0	0.39	0.12	0.16	0.39	1.00	0.0
0.5	-0.9	GMM-1	0.41	3.07	3.70	1.18	0.07	1.0	0.23	2.34	1.67	0.82	0.05	1.0
0.5	-0.9	GMM-25	0.39	0.43	0.18	0.39	0.51	25.0	0.38	0.41	0.17	0.39	0.56	25.0
0.5	-0.9	GMM-Bartlett	0.42	1.41	0.59	0.59	0.13	5.2	0.38	1.08	0.31	0.46	0.12	6.9
0.5	-0.9	GMM-Tuk-Han	0.37	1.14	0.37	0.49	0.18	9.2	0.36	0.83	0.22	0.40	0.17	11.5
0.5	-0.9	GMM-BR	0.44	1.43	0.58	0.60	0.17	4.8	0.37	1.08	0.30	0.46	0.15	6.0
0.5	-0.9	GMM-BRM	0.43	1.85	1.06	0.74	0.48	2.2	0.37	1.52	0.58	0.58	0.47	2.7
0.5	-0.9	GMM-Trunc	0.45	1.65	1.68	0.78	0.22	3.0	0.43	1.47	0.95	0.66	0.24	3.0
0.5	-0.9	GMM-Trunc-BC	0.13	3.12	9.60E+11	30987.00	0.33	3.0	0.15	2.14	1.16	0.69	0.23	3.0
0.5	-0.9	CUE-1	0.37	8.41	44.27	3.47	0.03	1.0	0.16	4.62	26.96	2.29	0.02	1.0
0.5	-0.9	CUE-25	0.35	9.00	46.64	3.60	0.80	25.0	0.25	7.10	37.51	3.04	0.77	25.0
0.5	-0.9	WWA-Hom	0.15	1.55	10.56	0.83	0.10	0.0	0.11	1.32	174.69	1.00	0.10	0.0
0.5	-0.9	WWA	0.15	1.58	327.33	1.38	0.10	0.0	0.11	1.32	105.71	0.91	0.10	0.0
0.5	-0.5	OLS	0.43	0.21	0.19	0.43	1.00	0.0	0.43	0.10	0.19	0.43	1.00	0.0
0.5	-0.5	GMM-1	0.40	2.53	3.90	1.01	0.07	1.0	0.25	1.86	2.18	0.72	0.04	1.0
0.5	-0.5	GMM-25	0.42	0.33	0.20	0.42	0.72	25.0	0.41	0.34	0.19	0.42	0.78	25.0
0.5	-0.5	GMM-Bartlett	0.40	1.47	1.35	0.64	0.15	3.7	0.35	0.94	0.34	0.44	0.13	4.8
0.5	-0.5	GMM-Tuk-Han	0.39	1.13	0.64	0.52	0.21	5.9	0.35	0.84	0.25	0.41	0.19	7.0
0.5	-0.5	GMM-BR	0.42	1.46	1.44	0.66	0.17	3.1	0.37	1.06	0.39	0.47	0.14	3.6
0.5	-0.5	GMM-BRM	0.41	1.81	2.04	0.77	0.40	1.4	0.36	1.33	0.98	0.61	0.41	1.6
0.5	-0.5	GMM-Trunc	0.42	1.65	1.31	0.70	0.22	2.0	0.38	1.24	1.21	0.60	0.20	2.0
0.5	-0.5	GMM-Trunc-BC	0.22	4.35	539.95	2.68	0.38	2.0	0.24	1.79	1.69	0.71	0.23	2.0
0.5	-0.5	CUE-1	0.29	6.25	30.12	2.67	0.04	1.0	0.14	3.58	21.46	1.94	0.03	1.0
0.5	-0.5	CUE-25	0.35	6.92	37.08	2.99	0.82	25.0	0.28	5.67	33.42	2.66	0.79	25.0
0.5	-0.5	WWA-Hom	0.27	1.59	13.68	0.91	0.15	0.0	0.22	1.13	63.16	0.81	0.11	0.0
0.5	-0.5	WWA	0.27	1.60	13.55	0.90	0.15	0.0	0.22	1.13	63.45	0.82	0.11	0.0
0.5	0	OLS	0.48	0.20	0.23	0.48	1.00	0.0	0.48	0.09	0.23	0.48	1.00	0.0
0.5	0	GMM-1	0.40	2.14	3.62	0.96	0.07	1.0	0.24	1.63	1.16	0.63	0.06	1.0
0.5	0	GMM-25	0.47	0.30	0.23	0.47	0.81	25.0	0.46	0.30	0.23	0.46	0.86	25.0
0.5	0	GMM-Bartlett	0.43	1.42	1.85	0.70	0.20	2.4	0.37	1.02	0.81	0.52	0.24	4.1
0.5	0	GMM-Tuk-Han	0.42	1.30	1.04	0.63	0.28	4.1	0.38	0.95	0.51	0.48	0.31	5.6
0.5	0	GMM-BR	0.43	1.44	1.86	0.71	0.22	2.1	0.36	1.07	0.82	0.53	0.22	2.9
0.5	0	GMM-BRM	0.44	1.54	1.98	0.74	0.43	1.0	0.35	1.25	0.90	0.56	0.43	1.3
0.5	0	GMM-Trunc	0.44	1.50	1.04	0.69	0.25	1.0	0.37	1.15	0.43	0.51	0.28	2.0
0.5	0	GMM-Trunc-BC	0.38	16.02	6365200.00	112.83	0.60	1.0	0.32	2.78	141.88	2.11	0.44	2.0
0.5	0	CUE-1	0.35	4.74	25.40	2.26	0.07	1.0	0.17	3.03	17.07	1.63	0.06	1.0
0.5	0	CUE-25	0.39	5.23	21.01	2.23	0.83	25.0	0.31	4.69	21.41	2.13	0.83	25.0
0.5	0	WWA-Hom	0.37	1.59	24.58	0.81	0.15	0.0	0.27	1.19	0.36	0.45	0.11	0.0
0.5	0	WWA	0.37	1.59	24.60	0.81	0.15	0.0	0.27	1.19	0.36	0.45	0.11	0.0
0.5	0.5	OLS	0.52	0.23	0.28	0.52	1.00	0.0	0.53	0.11	0.28	0.53	1.00	0.0
0.5	0.5	GMM-1	0.47	2.76	6.89	1.19	0.10	1.0	0.31	1.98	2.08	0.76	0.10	1.0
0.5	0.5	GMM-25	0.52	0.35	0.29	0.52	0.82	25.0	0.51	0.37	0.28	0.51	0.88	25.0
0.5	0.5	GMM-Bartlett	0.50	1.53	2.55	0.76	0.22	3.2	0.42	1.06	0.45	0.51	0.21	4.4
0.5	0.5	GMM-Tuk-Han	0.48	1.14	0.55	0.59	0.31	5.5	0.42	0.96	0.32	0.47	0.30	6.6
0.5	0.5	GMM-BR	0.50	1.55	2.63	0.78	0.25	2.9	0.42	1.14	0.47	0.53	0.22	3.4
0.5	0.5	GMM-BRM	0.49	2.00	3.16	0.88	0.45	1.3	0.41	1.42	0.69	0.61	0.47	1.6
0.5	0.5	GMM-Trunc	0.51	1.74	1.38	0.78	0.30	2.0	0.44	1.32	0.67	0.61	0.32	2.0
0.5	0.5	GMM-Trunc-BC	0.40	4.28	788.28	3.14	0.47	2.0	0.28	2.11	1.33	0.77	0.34	2.0
0.5	0.5	CUE-1	0.43	5.86	29.65	2.62	0.09	1.0	0.20	4.00	24.41	2.09	0.09	1.0
0.5	0.5	CUE-25	0.39	5.93	30.05	2.60	0.83	25.0	0.33	5.09	21.97	2.24	0.82	25.0
0.5	0.5	WWA-Hom	0.34	2.10	111490.00	15.94	0.22	0.0	0.25	1.22	0.43	0.47	0.11	0.0
0.5	0.5	WWA	0.34	2.10	118600.00	16.26	0.22	0.0	0.25	1.22	0.43	0.47	0.11	0.0
0.5	0.9	OLS	0.56	0.29	0.33	0.56	1.00	0.0	0.57	0.15	0.32	0.57	1.00	0.0
0.5	0.9	GMM-1	0.54	3.46	4.21	1.31	0.12	1.0	0.35	2.57	3.36	0.99	0.10	1.0
0.5	0.9	GMM-25	0.56	0.44	0.34	0.56	0.83	25.0	0.55	0.47	0.34	0.55	0.87	25.0
0.5	0.9	GMM-Bartlett	0.59	1.45	0.99	0.74	0.26	5.1	0.51	1.12	0.42	0.56	0.27	7.3
0.5	0.9	GMM-Tuk-Han	0.56	1.07	0.51	0.61	0.34	8.9	0.51	0.88	0.35	0.52	0.38	11.9
0.5	0.9	GMM-BR	0.60	1.40	0.87	0.73	0.31	4.6	0.50	1.10	0.41	0.55	0.29	6.2
0.5	0.9	GMM-BRM	0.58	2.01	1.67	0.87	0.54	2.1	0.52	1.37	0.59	0.63	0.57	2.8
0.5	0.9	GMM-Trunc	0.62	1.89	2.22	0.94	0.41	3.0	0.58	1.55	0.92	0.74	0.47	5.0
0.5	0.9	GMM-Trunc-BC	0.34	3.99	5.75E+12	75859.00	0.46	3.0	0.31	2.56	1.26	0.81	0.38	5.0
0.5	0.9	CUE-1	0.51	7.14	36.79	3.11	0.08	1.0	0.25	4.88	28.63	2.42	0.08	1.0
0.5	0.9	CUE-25	0.45	7.60	42.54	3.25	0.84	25.0	0.37	6.59	32.14	2.80	0.81	25.0
0.5	0.9	WWA-Hom	0.31	8.27	67194.00	27.50	0.32	0.0	0.22	1.63	192780.00	26.48	0.14	0.0
0.5	0.9	WWA	0.32	8.27	82283.00	29.49	0.32	0.0	0.22	1.64	473070.00	29.43	0.14	0.0

Table 5c - Homoskedastic DGP ϕ=.3, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.71	0.22	0.51	0.71	1.00	0.0	0.71	0.11	0.50	0.71	1.00	0.0
0.9	-0.9	GMM-1	0.60	2.34	2.60	1.06	0.09	1.0	0.35	2.09	1.77	0.83	0.10	1.0
0.9	-0.9	GMM-25	0.71	0.35	0.53	0.72	0.99	25.0	0.69	0.36	0.51	0.70	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.69	0.95	0.66	0.71	0.43	7.4	0.60	0.78	0.47	0.60	0.44	9.8
0.9	-0.9	GMM-Tuk-Han	0.66	0.83	0.55	0.67	0.56	11.7	0.60	0.65	0.41	0.59	0.57	14.5
0.9	-0.9	GMM-BR	0.68	1.11	0.73	0.73	0.55	6.1	0.60	0.87	0.50	0.61	0.54	7.6
0.9	-0.9	GMM-BRM	0.69	1.49	1.11	0.81	0.67	2.8	0.62	1.41	0.89	0.74	0.74	3.4
0.9	-0.9	GMM-Trunc	0.73	1.29	1.54	0.87	0.51	4.0	0.65	1.25	0.94	0.73	0.55	6.0
0.9	-0.9	GMM-Trunc-BC	0.52	2.30	1.34E+09	1158.40	0.42	4.0	0.46	1.64	1.08	0.71	0.43	6.0
0.9	-0.9	CUE-1	0.51	6.06	32.77	2.75	0.07	1.0	0.23	3.64	20.78	1.85	0.07	1.0
0.9	-0.9	CUE-25	0.88	5.10	25.61	2.64	0.78	25.0	0.44	4.68	25.69	2.40	0.73	25.0
0.9	-0.9	WWA-Hom	0.34	1.37	12904.00	4.24	0.09	0.0	0.24	1.20	10.49	0.59	0.08	0.0
0.9	-0.9	WWA	0.34	1.36	4131.30	2.68	0.08	0.0	0.24	1.20	12.18	0.60	0.08	0.0
0.9	-0.5	OLS	0.77	0.15	0.61	0.78	1.00	0.0	0.78	0.08	0.60	0.78	1.00	0.0
0.9	-0.5	GMM-1	0.66	1.86	1.77	0.93	0.21	1.0	0.38	1.56	1.38	0.70	0.18	1.0
0.9	-0.5	GMM-25	0.77	0.24	0.61	0.77	0.95	25.0	0.75	0.25	0.58	0.75	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.74	1.02	0.90	0.79	0.55	4.1	0.60	0.90	0.48	0.60	0.48	4.3
0.9	-0.5	GMM-Tuk-Han	0.73	0.79	0.71	0.74	0.64	6.7	0.62	0.72	0.43	0.60	0.59	6.6
0.9	-0.5	GMM-BR	0.77	1.12	0.95	0.81	0.61	3.5	0.61	0.91	0.54	0.63	0.55	3.4
0.9	-0.5	GMM-BRM	0.75	1.33	1.27	0.85	0.67	1.6	0.58	1.27	1.05	0.70	0.66	1.5
0.9	-0.5	GMM-Trunc	0.74	1.35	1.14	0.83	0.41	1.0	0.49	1.40	1.24	0.70	0.37	1.0
0.9	-0.5	GMM-Trunc-BC	0.60	2.54	35.10	1.54	0.44	1.0	0.36	1.69	1.34	0.70	0.33	1.0
0.9	-0.5	CUE-1	0.57	4.87	28.21	2.45	0.18	1.0	0.25	3.18	18.29	1.68	0.15	1.0
0.9	-0.5	CUE-25	0.73	4.38	21.83	2.33	0.86	25.0	0.51	4.14	21.47	2.16	0.79	25.0
0.9	-0.5	WWA-Hom	0.52	1.03	145.05	1.03	0.30	0.0	0.37	0.85	14.23	0.61	0.24	0.0
0.9	-0.5	WWA	0.52	1.03	174.91	1.07	0.29	0.0	0.37	0.85	11.88	0.60	0.24	0.0
0.9	0	OLS	0.86	0.10	0.74	0.86	1.00	0.0	0.86	0.05	0.74	0.86	1.00	0.0
0.9	0	GMM-1	0.69	1.48	1.12	0.83	0.42	1.0	0.43	1.16	1.68	0.64	0.34	1.0
0.9	0	GMM-25	0.85	0.16	0.73	0.85	0.98	25.0	0.83	0.16	0.70	0.83	0.99	25.0
0.9	0	GMM-Bartlett	0.78	0.78	0.78	0.79	0.71	3.8	0.62	0.70	0.47	0.62	0.68	4.3
0.9	0	GMM-Tuk-Han	0.79	0.66	0.75	0.80	0.77	5.8	0.66	0.59	0.48	0.65	0.77	6.1
0.9	0	GMM-BR	0.78	0.77	0.79	0.80	0.71	3.0	0.61	0.74	0.51	0.63	0.67	3.2
0.9	0	GMM-BRM	0.79	0.93	0.90	0.82	0.80	1.4	0.60	0.93	0.94	0.66	0.75	1.4
0.9	0	GMM-Trunc	0.77	1.20	1.00	0.83	0.56	1.0	0.49	1.08	1.67	0.67	0.47	1.0
0.9	0	GMM-Trunc-BC	0.71	3.10	832.22	4.69	0.63	1.0	0.44	1.17	20.54	0.81	0.44	1.0
0.9	0	CUE-1	0.56	4.15	25.11	2.18	0.35	1.0	0.21	2.92	19.45	1.68	0.22	1.0
0.9	0	CUE-25	0.76	3.21	12.45	1.75	0.90	25.0	0.53	3.67	23.07	2.01	0.84	25.0
0.9	0	WWA-Hom	0.69	0.92	6.64	0.84	0.68	0.0	0.49	0.77	0.40	0.53	0.59	0.0
0.9	0	WWA	0.69	0.92	6.87	0.85	0.68	0.0	0.49	0.77	0.40	0.53	0.59	0.0
0.9	0.5	OLS	0.95	0.13	0.90	0.95	1.00	0.0	0.95	0.06	0.90	0.95	1.00	0.0
0.9	0.5	GMM-1	0.82	1.67	2.72	1.01	0.46	1.0	0.48	1.55	1.31	0.74	0.38	1.0
0.9	0.5	GMM-25	0.94	0.20	0.89	0.94	0.97	25.0	0.92	0.22	0.85	0.92	1.00	25.0
0.9	0.5	GMM-Bartlett	0.89	0.89	0.93	0.89	0.73	3.7	0.67	0.85	0.57	0.68	0.67	3.6
0.9	0.5	GMM-Tuk-Han	0.88	0.66	0.83	0.86	0.82	6.2	0.73	0.70	0.58	0.71	0.77	5.9
0.9	0.5	GMM-BR	0.89	0.90	0.97	0.91	0.75	3.2	0.69	0.89	0.68	0.71	0.71	3.1
0.9	0.5	GMM-BRM	0.88	1.14	1.15	0.94	0.80	1.4	0.68	1.13	0.90	0.75	0.77	1.4
0.9	0.5	GMM-Trunc	0.88	1.39	2.47	0.98	0.58	1.0	0.56	1.39	1.17	0.74	0.49	1.0
0.9	0.5	GMM-Trunc-BC	0.79	1.91	2.63	0.98	0.53	1.0	0.44	1.57	1.16	0.71	0.42	1.0
0.9	0.5	CUE-1	0.68	4.28	25.79	2.29	0.36	1.0	0.21	3.69	19.76	1.85	0.24	1.0
0.9	0.5	CUE-25	0.81	3.61	20.71	2.09	0.88	25.0	0.55	3.50	22.01	2.03	0.82	25.0
0.9	0.5	WWA-Hom	0.70	1.15	5.57	0.92	0.63	0.0	0.46	0.96	0.44	0.53	0.50	0.0
0.9	0.5	WWA	0.71	1.15	3.22	0.90	0.63	0.0	0.46	0.97	0.44	0.53	0.50	0.0
0.9	0.9	OLS	1.01	0.19	1.03	1.01	1.00	0.0	1.02	0.10	1.04	1.02	1.00	0.0
0.9	0.9	GMM-1	0.86	2.18	3.92	1.18	0.34	1.0	0.52	1.86	1.89	0.85	0.28	1.0
0.9	0.9	GMM-25	1.01	0.29	1.04	1.01	1.00	25.0	0.99	0.32	0.99	0.99	1.00	25.0
0.9	0.9	GMM-Bartlett	0.96	0.76	1.02	0.95	0.79	7.9	0.87	0.71	0.77	0.83	0.79	10.7
0.9	0.9	GMM-Tuk-Han	0.96	0.64	0.98	0.95	0.89	12.5	0.89	0.62	0.81	0.87	0.88	15.4
0.9	0.9	GMM-BR	0.96	0.79	1.02	0.95	0.84	6.6	0.85	0.69	0.78	0.83	0.85	8.0
0.9	0.9	GMM-BRM	1.00	1.09	1.25	1.02	0.89	3.0	0.94	0.94	0.97	0.91	0.91	3.7
0.9	0.9	GMM-Trunc	1.01	1.17	2.03	1.08	0.74	4.0	0.94	1.15	1.07	0.88	0.70	5.0
0.9	0.9	GMM-Trunc-BC	0.89	1.58	2.33	1.08	0.69	4.0	0.80	1.35	1.02	0.84	0.68	5.0
0.9	0.9	CUE-1	0.73	5.02	25.67	2.50	0.28	1.0	0.23	4.07	23.10	2.04	0.21	1.0
0.9	0.9	CUE-25	0.87	4.38	24.09	2.37	0.85	25.0	0.60	3.91	24.54	2.27	0.78	25.0
0.9	0.9	WWA-Hom	0.80	1.46	1.94	0.94	0.41	0.0	0.54	1.23	0.64	0.64	0.35	0.0
0.9	0.9	WWA	0.80	1.46	1.74	0.94	0.40	0.0	0.55	1.22	0.64	0.65	0.35	0.0

Table 6a - Homoskedastic DGP ϕ=.5, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.10	0.30	0.02	0.12	0.14	0.0	0.10	0.15	0.01	0.10	0.40	0.0
0.1	-0.9	GMM-1	0.14	3.97	24.58	1.53	0.05	1.0	0.06	3.83	12.72	1.46	0.05	1.0
0.1	-0.9	GMM-25	0.10	0.50	0.05	0.17	0.06	25.0	0.10	0.50	0.05	0.17	0.06	25.0
0.1	-0.9	GMM-Bartlett	0.12	1.84	4.78	0.66	0.03	4.1	0.14	1.36	0.37	0.45	0.02	6.0
0.1	-0.9	GMM-Tuk-Han	0.10	1.37	0.41	0.45	0.04	7.8	0.14	1.04	0.19	0.35	0.02	10.4
0.1	-0.9	GMM-BR	0.11	1.82	0.88	0.62	0.05	4.1	0.13	1.42	0.43	0.48	0.03	5.4
0.1	-0.9	GMM-BRM	0.11	2.31	16.59	0.92	0.34	1.9	0.11	1.97	0.85	0.67	0.37	2.5
0.1	-0.9	GMM-Trunc	0.18	2.44	21.70	1.03	0.06	2.0	0.23	1.88	9.02	0.83	0.06	3.0
0.1	-0.9	GMM-Trunc-BC	0.02	4.35	5.42E+12	101870.00	0.38	2.0	0.21	2.90	9.64	1.12	0.31	3.0
0.1	-0.9	CUE-1	0.13	9.24	50.71	3.80	0.01	1.0	0.00	9.36	45.87	3.63	0.00	1.0
0.1	-0.9	CUE-25	-0.03	10.11	49.22	3.69	0.82	25.0	-0.03	9.06	43.17	3.46	0.82	25.0
0.1	-0.9	WWA-Hom	0.03	1.91	11088.00	5.87	0.12	0.0	0.01	1.77	22.98	1.01	0.13	0.0
0.1	-0.9	WWA	0.03	1.91	17822.00	6.95	0.12	0.0	0.01	1.77	23.82	1.02	0.13	0.0
0.1	-0.5	OLS	0.10	0.26	0.02	0.11	0.17	0.0	0.10	0.12	0.01	0.10	0.55	0.0
0.1	-0.5	GMM-1	0.12	3.14	5.91	1.17	0.05	1.0	0.08	3.08	8.75	1.22	0.03	1.0
0.1	-0.5	GMM-25	0.09	0.41	0.03	0.15	0.06	25.0	0.10	0.42	0.04	0.15	0.07	25.0
0.1	-0.5	GMM-Bartlett	0.12	1.79	1.18	0.63	0.02	2.6	0.10	1.53	1.99	0.61	0.01	3.1
0.1	-0.5	GMM-Tuk-Han	0.11	1.47	0.72	0.51	0.03	4.6	0.10	1.27	0.50	0.44	0.01	5.1
0.1	-0.5	GMM-BR	0.11	1.87	1.25	0.66	0.02	2.4	0.08	1.64	2.14	0.66	0.01	2.6
0.1	-0.5	GMM-BRM	0.11	2.14	1.92	0.78	0.23	1.1	0.05	2.21	3.23	0.85	0.22	1.2
0.1	-0.5	GMM-Trunc	0.15	2.17	1.89E+00	0.77	0.04	1.0	0.12	1.93	2.13	0.72	0.03	2.0
0.1	-0.5	GMM-Trunc-BC	0.14	13.35	8.96E+09	3646.90	0.44	1.0	0.14	3.93	1515.00	3.74	0.24	2.0
0.1	-0.5	CUE-1	0.10	7.19	34.64	2.98	0.01	1.0	0.02	7.75	36.34	3.03	0.00	1.0
0.1	-0.5	CUE-25	0.04	7.60	39.15	3.13	0.82	25.0	0.04	7.09	33.54	2.89	0.80	25.0
0.1	-0.5	WWA-Hom	0.07	1.94	13.27	0.96	0.12	0.0	0.07	1.94	20.96	0.97	0.10	0.0
0.1	-0.5	WWA	0.07	1.94	25.45	1.01	0.12	0.0	0.07	1.94	20.89	0.97	0.10	0.0
0.1	0	OLS	0.10	0.23	0.02	0.11	0.19	0.0	0.10	0.11	0.01	0.10	0.63	0.0
0.1	0	GMM-1	0.13	2.78	6.86	1.05	0.02	1.0	0.07	2.64	33.23	1.17	0.01	1.0
0.1	0	GMM-25	0.09	0.36	0.03	0.14	0.06	25.0	0.10	0.37	0.03	0.14	0.07	25.0
0.1	0	GMM-Bartlett	0.08	1.80	2.04	0.67	0.02	0.7	0.09	1.88	29.16	0.84	0.00	0.2
0.1	0	GMM-Tuk-Han	0.09	1.73	2.01	0.66	0.03	1.7	0.10	1.89	29.15	0.84	0.01	0.8
0.1	0	GMM-BR	0.10	1.84	2.05	0.68	0.02	0.9	0.09	1.91	29.18	0.86	0.01	0.4
0.1	0	GMM-BRM	0.11	2.04	2.15E+00	0.73	0.18	0.4	0.08	2.03	3.25E+01	0.98	0.14	0.2
0.1	0	GMM-Trunc	0.11	2.06	5.64	0.79	0.03	1.0	0.11	2.00	5.30	0.79	0.03	1.0
0.1	0	GMM-Trunc-BC	0.11	373.45	4.43E+11	22741.00	0.61	1.0	0.15	1897.23	1.41E+12	82858.00	0.66	1.0
0.1	0	CUE-1	0.11	6.21	35.42	2.82	0.00	1.0	0.00	6.14	33.48	2.77	0.00	1.0
0.1	0	CUE-25	0.05	6.27	29.11	2.63	0.83	25.0	0.08	5.95	30.05	2.63	0.82	25.0
0.1	0	WWA-Hom	0.03	1.93	1.99	0.70	0.08	0.0	0.09	1.81	0.90	0.62	0.05	0.0
0.1	0	WWA	0.03	1.92	1.99	0.70	0.08	0.0	0.09	1.81	0.90	0.62	0.05	0.0
0.1	0.5	OLS	0.10	0.25	0.02	0.11	0.16	0.0	0.10	0.12	0.01	0.10	0.52	0.0
0.1	0.5	GMM-1	0.17	3.35	4.70	1.21	0.04	1.0	0.11	3.24	12.98	1.27	0.03	1.0
0.1	0.5	GMM-25	0.09	0.40	0.03	0.15	0.05	25.0	0.09	0.42	0.04	0.15	0.08	25.0
0.1	0.5	GMM-Bartlett	0.11	1.84	1.95	0.69	0.01	2.8	0.11	1.56	1.25	0.56	0.01	3.4
0.1	0.5	GMM-Tuk-Han	0.10	1.49	0.60	0.50	0.02	4.9	0.10	1.25	0.73	0.43	0.01	5.4
0.1	0.5	GMM-BR	0.10	1.93	2.08	0.72	0.03	2.6	0.10	1.70	1.35	0.61	0.02	2.8
0.1	0.5	GMM-BRM	0.10	2.37	2.75	0.87	0.26	1.2	0.08	2.30	4.10	0.85	0.20	1.3
0.1	0.5	GMM-Trunc	0.13	2.15	2.15	0.75	0.05	1.0	0.14	1.83	1.81	0.69	0.03	2.0
0.1	0.5	GMM-Trunc-BC	0.08	15.33	104380.00	31.56	0.46	1.0	0.13	3.92	17460.00	6.42	0.25	2.0
0.1	0.5	CUE-1	0.17	7.95	40.28	3.29	0.01	1.0	0.06	8.00	40.00	3.23	0.00	1.0
0.1	0.5	CUE-25	0.05	6.93	32.64	2.85	0.81	25.0	0.00	7.12	33.77	2.94	0.84	25.0
0.1	0.5	WWA-Hom	0.03	3.16	4321.20	5.96	0.20	0.0	0.08	1.95	6.40	0.89	0.10	0.0
0.1	0.5	WWA	0.03	3.16	3977.20	5.77	0.20	0.0	0.08	1.95	6.40	0.89	0.10	0.0
0.1	0.9	OLS	0.10	0.30	0.02	0.12	0.13	0.0	0.10	0.16	0.01	0.10	0.40	0.0
0.1	0.9	GMM-1	0.18	4.23	16.70	1.65	0.06	1.0	0.14	3.85	17.29	1.56	0.05	1.0
0.1	0.9	GMM-25	0.11	0.48	0.05	0.17	0.05	25.0	0.11	0.49	0.05	0.18	0.07	25.0
0.1	0.9	GMM-Bartlett	0.15	1.84	0.76	0.61	0.03	4.1	0.12	1.28	0.37	0.44	0.02	6.0
0.1	0.9	GMM-Tuk-Han	0.14	1.28	0.35	0.43	0.03	7.8	0.12	0.95	0.19	0.34	0.03	10.5
0.1	0.9	GMM-BR	0.16	1.90	0.83	0.64	0.06	4.1	0.12	1.39	0.39	0.46	0.05	5.4
0.1	0.9	GMM-BRM	0.15	2.43	3.10	0.90	0.35	1.8	0.15	1.91	0.86	0.64	0.33	2.5
0.1	0.9	GMM-Trunc	0.21	2.38	4.95	0.95	0.08	2.0	0.24	1.91	2.05	0.75	0.07	3.0
0.1	0.9	GMM-Trunc-BC	0.13	4.42	1.56E+12	52538.00	0.38	2.0	0.25	2.85	2.57	1.01	0.29	3.0
0.1	0.9	CUE-1	0.18	10.51	54.51	4.07	0.01	1.0	0.10	10.44	50.82	3.91	0.01	1.0
0.1	0.9	CUE-25	0.06	8.85	44.99	3.55	0.81	25.0	0.05	8.94	46.97	3.60	0.84	25.0
0.1	0.9	WWA-Hom	0.02	50.33	43519.00	38.07	0.51	0.0	0.05	21.78	11032.00	21.00	0.46	0.0
0.1	0.9	WWA	0.03	50.08	40678.00	37.86	0.51	0.0	0.05	21.80	11039.00	21.02	0.46	0.0

Table 6b - Homoskedastic DGP ϕ=.5, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.50	0.28	0.26	0.50	1.00	0.0	0.50	0.14	0.25	0.50	1.00	0.0
0.5	-0.9	GMM-1	0.64	3.49	8.45	1.49	0.10	1.0	0.57	3.42	6.12	1.42	0.08	1.0
0.5	-0.9	GMM-25	0.50	0.46	0.28	0.50	0.72	25.0	0.50	0.45	0.29	0.50	0.76	25.0
0.5	-0.9	GMM-Bartlett	0.57	1.56	3.27	0.77	0.18	5.3	0.58	1.09	0.52	0.62	0.22	7.8
0.5	-0.9	GMM-Tuk-Han	0.52	1.19	0.48	0.59	0.27	9.3	0.53	0.83	0.38	0.54	0.33	12.7
0.5	-0.9	GMM-BR	0.57	1.52	3.29	0.76	0.24	4.9	0.54	1.15	0.52	0.61	0.26	6.6
0.5	-0.9	GMM-BRM	0.57	2.03	4.43	0.93	0.53	2.2	0.59	1.69	0.84	0.75	0.56	3.0
0.5	-0.9	GMM-Trunc	0.60	1.84	4.99	0.98	0.33	4.0	0.61	1.53	2.14	0.91	0.40	6.0
0.5	-0.9	GMM-Trunc-BC	0.35	3.50	5.14E+10	8557.70	0.40	4.0	0.41	2.47	2.43	0.96	0.34	6.0
0.5	-0.9	CUE-1	0.65	8.66	42.28	3.49	0.04	1.0	0.53	8.02	40.45	3.33	0.03	1.0
0.5	-0.9	CUE-25	0.47	8.77	43.82	3.53	0.82	25.0	0.40	7.86	38.29	3.21	0.81	25.0
0.5	-0.9	WWA-Hom	0.28	2.03	463.62	1.85	0.12	0.0	0.22	1.84	4.45	0.84	0.11	0.0
0.5	-0.9	WWA	0.28	2.04	38.06	1.12	0.11	0.0	0.22	1.85	4.55	0.84	0.11	0.0
0.5	-0.5	OLS	0.50	0.23	0.26	0.50	1.00	0.0	0.50	0.11	0.25	0.50	1.00	0.0
0.5	-0.5	GMM-1	0.57	2.73	3.81	1.16	0.09	1.0	0.51	2.67	3.54	1.10	0.07	1.0
0.5	-0.5	GMM-25	0.50	0.36	0.27	0.50	0.80	25.0	0.50	0.36	0.27	0.50	0.86	25.0
0.5	-0.5	GMM-Bartlett	0.52	1.59	1.91	0.79	0.18	3.2	0.53	1.18	1.42	0.67	0.17	4.0
0.5	-0.5	GMM-Tuk-Han	0.50	1.21	0.65	0.63	0.26	5.2	0.49	1.04	0.48	0.57	0.25	6.2
0.5	-0.5	GMM-BR	0.52	1.65	2.10	0.82	0.20	2.7	0.54	1.31	1.77	0.71	0.18	3.2
0.5	-0.5	GMM-BRM	0.55	1.94	2.52	0.91	0.43	1.2	0.55	1.76	2.55	0.88	0.46	1.5
0.5	-0.5	GMM-Trunc	0.56	1.84	1.61	0.86	0.27	2.0	0.57	1.69	1.54	0.84	0.28	2.0
0.5	-0.5	GMM-Trunc-BC	0.49	4.63	831.18	3.69	0.42	2.0	0.52	2.76	10.13	1.18	0.35	2.0
0.5	-0.5	CUE-1	0.57	6.21	30.11	2.76	0.05	1.0	0.49	6.37	36.08	2.95	0.04	1.0
0.5	-0.5	CUE-25	0.43	6.95	37.57	3.03	0.82	25.0	0.46	5.88	28.37	2.66	0.85	25.0
0.5	-0.5	WWA-Hom	0.39	1.76	24.54	1.02	0.16	0.0	0.39	1.62	2.78	0.78	0.14	0.0
0.5	-0.5	WWA	0.39	1.76	15.23	0.96	0.16	0.0	0.39	1.62	2.82	0.78	0.14	0.0
0.5	0	OLS	0.50	0.20	0.25	0.50	1.00	0.0	0.50	0.10	0.25	0.50	1.00	0.0
0.5	0	GMM-1	0.52	2.42	5.48	1.05	0.08	1.0	0.48	2.30	25.49	1.14	0.06	1.0
0.5	0	GMM-25	0.49	0.31	0.26	0.49	0.83	25.0	0.50	0.32	0.27	0.50	0.82	25.0
0.5	0	GMM-Bartlett	0.49	1.57	1.78	0.78	0.20	0.7	0.49	1.64	22.45	0.92	0.19	0.2
0.5	0	GMM-Tuk-Han	0.49	1.51	1.76	0.77	0.30	1.7	0.50	1.64	22.44	0.92	0.26	0.8
0.5	0	GMM-BR	0.50	1.60	1.79	0.79	0.23	0.9	0.49	1.66	22.46	0.93	0.17	0.4
0.5	0	GMM-BRM	0.50	1.78	1.86E+00	0.81	0.41	0.4	0.49	1.77	2.50E+01	1.01	0.36	0.2
0.5	0	GMM-Trunc	0.51	1.78	4.56	0.88	0.25	1.0	0.51	1.73	4.23	0.88	0.24	1.0
0.5	0	GMM-Trunc-BC	0.50	325.05	3.36E+11	19794.00	0.76	1.0	0.54	1651.37	1.07E+12	72119.00	0.81	1.0
0.5	0	CUE-1	0.50	5.33	31.56	2.64	0.06	1.0	0.41	5.42	29.78	2.58	0.05	1.0
0.5	0	CUE-25	0.46	5.46	26.13	2.45	0.84	25.0	0.47	5.24	26.24	2.45	0.86	25.0
0.5	0	WWA-Hom	0.44	1.69	1.96	0.76	0.18	0.0	0.49	1.58	0.94	0.72	0.16	0.0
0.5	0	WWA	0.44	1.69	1.96	0.76	0.18	0.0	0.49	1.58	0.94	0.72	0.16	0.0
0.5	0.5	OLS	0.49	0.23	0.26	0.50	1.00	0.0	0.50	0.11	0.25	0.50	1.00	0.0
0.5	0.5	GMM-1	0.57	2.88	10.38	1.34	0.09	1.0	0.54	2.83	11.37	1.27	0.08	1.0
0.5	0.5	GMM-25	0.50	0.35	0.27	0.50	0.78	25.0	0.50	0.38	0.28	0.51	0.84	25.0
0.5	0.5	GMM-Bartlett	0.53	1.59	1.45	0.78	0.18	3.4	0.53	1.22	0.96	0.65	0.18	4.1
0.5	0.5	GMM-Tuk-Han	0.50	1.17	0.52	0.59	0.26	5.6	0.51	1.03	0.44	0.56	0.26	6.4
0.5	0.5	GMM-BR	0.54	1.64	1.77	0.81	0.23	2.9	0.53	1.33	1.27	0.70	0.18	3.3
0.5	0.5	GMM-BRM	0.52	2.01	2.64	0.92	0.42	1.3	0.53	1.77	2.53	0.87	0.43	1.5
0.5	0.5	GMM-Trunc	0.55	1.82	1.97	0.84	0.26	2.0	0.57	1.66	9.78	0.98	0.27	2.0
0.5	0.5	GMM-Trunc-BC	0.44	4.63	128.60	2.61	0.42	2.0	0.48	2.74	10.79	1.19	0.35	2.0
0.5	0.5	CUE-1	0.54	7.07	39.16	3.19	0.06	1.0	0.51	7.03	36.66	3.05	0.05	1.0
0.5	0.5	CUE-25	0.43	6.39	31.21	2.73	0.84	25.0	0.41	6.21	32.44	2.78	0.83	25.0
0.5	0.5	WWA-Hom	0.35	2.32	3472.00	4.07	0.22	0.0	0.41	1.73	46.50	1.03	0.15	0.0
0.5	0.5	WWA	0.35	2.31	4622.70	4.39	0.22	0.0	0.41	1.73	34.01	0.99	0.15	0.0
0.5	0.9	OLS	0.49	0.28	0.26	0.50	0.99	0.0	0.50	0.15	0.25	0.50	1.00	0.0
0.5	0.9	GMM-1	0.61	3.75	8.80	1.57	0.10	1.0	0.58	3.56	6.09	1.43	0.09	1.0
0.5	0.9	GMM-25	0.51	0.45	0.29	0.50	0.72	25.0	0.51	0.47	0.30	0.51	0.77	25.0
0.5	0.9	GMM-Bartlett	0.58	1.41	0.82	0.72	0.21	5.4	0.57	1.04	0.50	0.62	0.23	8.0
0.5	0.9	GMM-Tuk-Han	0.52	1.13	0.49	0.59	0.28	9.6	0.54	0.85	0.39	0.55	0.33	12.8
0.5	0.9	GMM-BR	0.58	1.47	0.81	0.72	0.26	4.9	0.56	1.16	0.52	0.62	0.25	6.6
0.5	0.9	GMM-BRM	0.61	1.99	1.36	0.87	0.51	2.2	0.58	1.59	0.88	0.74	0.55	3.0
0.5	0.9	GMM-Trunc	0.60	1.92	3.67	1.01	0.35	4.0	0.63	1.37	2.41	0.88	0.43	6.0
0.5	0.9	GMM-Trunc-BC	0.34	3.89	4.64E+11	21536.00	0.44	4.0	0.41	2.34	2.72	0.92	0.37	6.0
0.5	0.9	CUE-1	0.60	9.56	45.69	3.69	0.05	1.0	0.58	8.72	44.96	3.62	0.04	1.0
0.5	0.9	CUE-25	0.47	8.57	41.40	3.36	0.83	25.0	0.41	8.16	46.40	3.47	0.82	25.0
0.5	0.9	WWA-Hom	0.28	4.88	9001.70	13.30	0.27	0.0	0.27	2.18	473.39	2.09	0.12	0.0
0.5	0.9	WWA	0.29	4.93	11471.00	14.59	0.27	0.0	0.27	2.18	482.18	2.08	0.12	0.0

Table 6c - Homoskedastic DGP ϕ=.5, γ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.90	0.22	0.82	0.90	1.00	0.0	0.90	0.12	0.81	0.90	1.00	0.0
0.9	-0.9	GMM-1	0.98	2.41	4.38	1.34	0.19	1.0	0.94	2.38	4.84	1.28	0.19	1.0
0.9	-0.9	GMM-25	0.91	0.35	0.85	0.91	1.00	25.0	0.91	0.37	0.86	0.91	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.92	0.90	1.01	0.93	0.68	8.0	0.91	0.70	0.87	0.89	0.75	10.8
0.9	-0.9	GMM-Tuk-Han	0.90	0.80	0.90	0.89	0.79	12.4	0.88	0.63	0.82	0.87	0.85	16.2
0.9	-0.9	GMM-BR	0.88	1.08	1.00	0.90	0.72	6.4	0.85	0.85	0.83	0.85	0.77	8.4
0.9	-0.9	GMM-BRM	0.94	1.37	1.30	1.00	0.80	2.9	0.93	1.15	1.12	0.96	0.87	3.9
0.9	-0.9	GMM-Trunc	0.96	1.13	1.80	1.08	0.69	5.0	0.97	0.78	1.70	1.04	0.78	7.0
0.9	-0.9	GMM-Trunc-BC	0.89	1.98	2.15E+10	4641.40	0.62	5.0	0.90	1.38	1.81	1.04	0.71	7.0
0.9	-0.9	CUE-1	0.98	5.78	37.56	3.10	0.15	1.0	0.92	6.33	34.97	3.02	0.15	1.0
0.9	-0.9	CUE-25	1.08	4.66	25.78	2.59	0.84	25.0	0.99	4.34	20.14	2.34	0.82	25.0
0.9	-0.9	WWA-Hom	0.77	1.77	2.23	0.96	0.25	0.0	0.73	1.59	1.56	0.89	0.22	0.0
0.9	-0.9	WWA	0.77	1.76	2.42	0.97	0.25	0.0	0.73	1.59	1.58	0.89	0.22	0.0
0.9	-0.5	OLS	0.90	0.15	0.81	0.90	1.00	0.0	0.90	0.07	0.81	0.90	1.00	0.0
0.9	-0.5	GMM-1	0.95	1.73	2.05	1.13	0.36	1.0	0.93	1.57	3.12	1.13	0.35	1.0
0.9	-0.5	GMM-25	0.90	0.23	0.82	0.90	0.98	25.0	0.90	0.24	0.83	0.90	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.93	0.82	1.14	0.96	0.71	4.2	0.92	0.67	0.94	0.92	0.76	5.2
0.9	-0.5	GMM-Tuk-Han	0.90	0.69	0.92	0.91	0.79	6.8	0.90	0.59	0.88	0.90	0.86	7.9
0.9	-0.5	GMM-BR	0.93	0.84	1.17	0.97	0.74	3.5	0.93	0.75	0.99	0.93	0.80	4.1
0.9	-0.5	GMM-BRM	0.93	1.14	1.55	1.03	0.80	1.6	0.95	1.08	1.92	1.00	0.84	1.9
0.9	-0.5	GMM-Trunc	0.95	1.37	1.77	1.08	0.54	1.0	0.96	1.29	2.64	1.08	0.59	1.0
0.9	-0.5	GMM-Trunc-BC	0.93	2.17	3.37	1.23	0.55	1.0	0.93	1.75	2.72	1.10	0.58	1.0
0.9	-0.5	CUE-1	0.95	4.06	24.81	2.41	0.30	1.0	0.94	4.44	30.65	2.63	0.30	1.0
0.9	-0.5	CUE-25	0.95	3.64	19.16	2.15	0.89	25.0	0.88	3.42	22.05	2.23	0.89	25.0
0.9	-0.5	WWA-Hom	0.80	1.19	1.13	0.89	0.51	0.0	0.80	1.08	7.39	0.94	0.49	0.0
0.9	-0.5	WWA	0.80	1.19	1.13	0.89	0.51	0.0	0.80	1.07	8.50	0.94	0.49	0.0
0.9	0	OLS	0.90	0.10	0.81	0.90	1.00	0.0	0.90	0.05	0.81	0.90	1.00	0.0
0.9	0	GMM-1	0.91	1.22	2.17	1.03	0.49	1.0	0.89	1.16	7.26	1.06	0.49	1.0
0.9	0	GMM-25	0.90	0.16	0.81	0.90	0.97	25.0	0.90	0.16	0.82	0.90	0.98	25.0
0.9	0	GMM-Bartlett	0.89	0.79	1.19	0.96	0.73	0.7	0.89	0.83	6.53	1.02	0.67	0.2
0.9	0	GMM-Tuk-Han	0.89	0.76	1.18	0.95	0.74	1.7	0.90	0.83	6.53	1.02	0.69	0.8
0.9	0	GMM-BR	0.90	0.80	1.20	0.96	0.73	0.9	0.89	0.84	6.53	1.02	0.68	0.4
0.9	0	GMM-BRM	0.90	0.89	1.20E+00	0.96	0.82	0.4	0.89	0.89	7.14E+00	1.05	0.79	0.2
0.9	0	GMM-Trunc	0.91	1.03	2.04	1.01	0.62	1.0	0.89	1.05	7.18	1.05	0.58	1.0
0.9	0	GMM-Trunc-BC	0.91	163.60	8.51E+10	9962.80	0.86	1.0	0.89	831.17	2.21E+11	32204.00	0.89	1.0
0.9	0	CUE-1	0.90	2.69	20.13	1.99	0.44	1.0	0.86	2.69	17.71	1.88	0.44	1.0
0.9	0	CUE-25	0.88	2.75	16.89	1.86	0.92	25.0	0.90	2.58	12.36	1.74	0.92	25.0
0.9	0	WWA-Hom	0.87	0.93	1.84	0.95	0.77	0.0	0.89	0.81	1.05	0.93	0.79	0.0
0.9	0	WWA	0.87	0.93	1.84	0.95	0.77	0.0	0.89	0.81	1.05	0.93	0.79	0.0
0.9	0.5	OLS	0.90	0.16	0.81	0.90	1.00	0.0	0.90	0.08	0.81	0.90	1.00	0.0
0.9	0.5	GMM-1	0.96	1.80	1.85	1.09	0.36	1.0	0.94	1.73	2.17	1.13	0.35	1.0
0.9	0.5	GMM-25	0.90	0.24	0.82	0.90	0.99	25.0	0.90	0.25	0.83	0.91	0.99	25.0
0.9	0.5	GMM-Bartlett	0.92	0.84	1.06	0.95	0.71	4.4	0.93	0.73	0.96	0.93	0.75	5.0
0.9	0.5	GMM-Tuk-Han	0.90	0.67	0.90	0.90	0.80	7.1	0.91	0.62	0.88	0.90	0.84	7.6
0.9	0.5	GMM-BR	0.93	0.89	1.14	0.96	0.74	3.7	0.94	0.78	1.05	0.94	0.78	3.9
0.9	0.5	GMM-BRM	0.93	1.15	1.40	1.00	0.79	1.7	0.94	1.04	1.42	1.00	0.84	1.8
0.9	0.5	GMM-Trunc	0.96	1.42	1.57	1.04	0.55	1.0	0.97	1.28	1.70	1.06	0.58	1.0
0.9	0.5	GMM-Trunc-BC	0.90	2.14	2.48	1.17	0.56	1.0	0.95	1.67	2.15	1.10	0.58	1.0
0.9	0.5	CUE-1	0.95	4.43	24.98	2.39	0.31	1.0	0.94	4.18	24.57	2.39	0.28	1.0
0.9	0.5	CUE-25	0.85	3.36	14.05	1.91	0.89	25.0	0.93	3.23	18.52	2.03	0.89	25.0
0.9	0.5	WWA-Hom	0.77	1.22	2.77	0.96	0.52	0.0	0.83	1.11	1.51	0.92	0.53	0.0
0.9	0.5	WWA	0.78	1.23	3.17	0.97	0.51	0.0	0.83	1.11	1.50	0.92	0.53	0.0
0.9	0.9	OLS	0.90	0.23	0.82	0.90	1.00	0.0	0.90	0.12	0.81	0.90	1.00	0.0
0.9	0.9	GMM-1	0.96	2.56	4.13	1.34	0.21	1.0	0.98	2.42	4.59	1.33	0.20	1.0
0.9	0.9	GMM-25	0.92	0.35	0.85	0.91	1.00	25.0	0.92	0.39	0.86	0.91	1.00	25.0
0.9	0.9	GMM-Bartlett	0.91	0.84	1.00	0.93	0.67	7.9	0.91	0.73	0.91	0.91	0.75	10.8
0.9	0.9	GMM-Tuk-Han	0.89	0.78	0.89	0.89	0.80	12.6	0.90	0.65	0.87	0.89	0.86	16.0
0.9	0.9	GMM-BR	0.90	1.03	1.03	0.92	0.74	6.6	0.87	0.87	0.85	0.86	0.78	8.3
0.9	0.9	GMM-BRM	0.94	1.34	1.31	1.00	0.82	3.0	0.94	1.23	1.22	0.99	0.87	3.8
0.9	0.9	GMM-Trunc	0.95	1.11	1.42	1.03	0.70	5.0	0.98	0.79	1.26	1.03	0.79	7.0
0.9	0.9	GMM-Trunc-BC	0.87	1.91	2.08	1.12	0.62	5.0	0.94	1.37	1.43	1.05	0.72	7.0
0.9	0.9	CUE-1	0.95	6.27	31.35	2.94	0.17	1.0	0.97	6.11	33.53	3.04	0.16	1.0
0.9	0.9	CUE-25	0.90	4.52	18.67	2.25	0.81	25.0	1.03	4.36	19.76	2.32	0.80	25.0
0.9	0.9	WWA-Hom	0.72	1.65	4.12	1.02	0.23	0.0	0.80	1.55	1.52	0.93	0.24	0.0
0.9	0.9	WWA	0.73	1.66	4.92	1.04	0.22	0.0	0.80	1.54	1.51	0.93	0.24	0.0

Table 7a - Heteroskedastic DGP with Robust Weight Matrix ϕ=.1, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.10	0.32	0.03	0.13	0.16	0.0	0.11	0.16	0.02	0.11	0.45	0.0
0.1	-0.9	GMM-1	0.17	3.89	19.37	1.58	0.04	1.0	0.08	3.82	6.36	1.41	0.03	1.0
0.1	-0.9	GMM-25	0.09	1.20	0.25	0.38	0.61	25.0	0.09	0.63	0.07	0.21	0.17	25.0
0.1	-0.9	GMM-Bartlett	0.14	1.79	0.87	0.62	0.06	3.8	0.16	1.28	0.33	0.43	0.01	5.4
0.1	-0.9	GMM-Tuk-Han	0.13	1.44	0.50	0.49	0.12	7.6	0.12	1.00	0.18	0.33	0.02	9.8
0.1	-0.9	GMM-BR	0.15	1.86	0.86	0.63	0.08	3.9	0.14	1.42	0.42	0.48	0.03	5.1
0.1	-0.9	GMM-BRM	0.12	2.17	1.42	0.76	0.29	1.8	0.12	1.93	0.91	0.67	0.33	2.3
0.1	-0.9	GMM-Trunc	0.22	2.36	2.13	0.86	0.10	2.0	0.25	2.07	2.22	0.81	0.05	2.0
0.1	-0.9	GMM-Trunc-BC	0.10	4.75	1.80E+10	5542.20	0.39	2.0	0.20	3.02	2.83	1.09	0.26	2.0
0.1	-0.9	CUE-1	0.13	10.95	46.03	3.76	0.01	1.0	-0.06	10.08	44.79	3.62	0.01	1.0
0.1	-0.9	CUE-25	0.04	24.07	120.39	7.27	0.97	25.0	-0.04	9.03	50.01	3.49	0.72	25.0
0.1	-0.9	WWA	0.06	2.22	8.26	0.97	0.12	0.0	0.03	1.98	25.39	1.09	0.13	0.0
0.1	-0.5	OLS	0.10	0.26	0.02	0.12	0.19	0.0	0.10	0.13	0.01	0.10	0.57	0.0
0.1	-0.5	GMM-1	0.15	3.15	11.57	1.23	0.04	1.0	0.06	3.10	3.84	1.13	0.02	1.0
0.1	-0.5	GMM-25	0.10	1.21	0.25	0.39	0.66	25.0	0.10	0.58	0.06	0.20	0.18	25.0
0.1	-0.5	GMM-Bartlett	0.12	1.66	8.16	0.70	0.05	2.9	0.10	1.41	0.86	0.52	0.01	3.5
0.1	-0.5	GMM-Tuk-Han	0.11	1.48	0.77	0.54	0.10	4.9	0.10	1.16	0.50	0.43	0.01	5.5
0.1	-0.5	GMM-BR	0.12	1.78	8.27	0.72	0.05	2.6	0.08	1.64	1.00	0.58	0.02	2.9
0.1	-0.5	GMM-BRM	0.14	2.06	8.61	0.82	0.23	1.2	0.08	2.20	1.87	0.79	0.22	1.3
0.1	-0.5	GMM-Trunc	0.15	2.16	1.88	0.77	0.07	1.0	0.13	1.94	1.69	0.71	0.04	2.0
0.1	-0.5	GMM-Trunc-BC	0.14	10.68	4.3E+06	121.58	0.46	1.0	0.18	3.96	876.63	2.85	0.22	2.0
0.1	-0.5	CUE-1	0.14	8.58	37.27	3.27	0.02	1.0	-0.03	8.65	42.52	3.33	0.01	1.0
0.1	-0.5	CUE-25	0.13	22.90	113.67	6.87	0.97	25.0	0.01	12.31	72.62	4.34	0.80	25.0
0.1	-0.5	WWA	0.07	2.10	93.17	1.18	0.11	0.0	0.09	1.99	1.97	0.75	0.09	0.0
0.1	0	OLS	0.09	0.23	0.02	0.11	0.19	0.0	0.10	0.11	0.01	0.10	0.63	0.0
0.1	0	GMM-1	0.13	2.73	3.34	1.00	0.03	1.0	0.06	2.64	3.83	1.03	0.01	1.0
0.1	0	GMM-25	0.09	1.17	0.23	0.37	0.66	25.0	0.10	0.56	0.06	0.19	0.20	25.0
0.1	0	GMM-Bartlett	0.12	1.85	1.69	0.66	0.07	1.6	0.10	1.84	2.68	0.74	0.01	0.8
0.1	0	GMM-Tuk-Han	0.10	1.78	1.56	0.65	0.15	2.9	0.08	1.85	2.59	0.72	0.03	1.8
0.1	0	GMM-BR	0.10	1.87	1.69	0.67	0.07	1.5	0.10	1.89	2.71	0.76	0.01	1.0
0.1	0	GMM-BRM	0.11	1.96	1.75	0.71	0.22	0.7	0.08	2.04	2.88	0.81	0.15	0.4
0.1	0	GMM-Trunc	0.13	1.99	1.22	0.67	0.07	1.0	0.12	1.96	1.86	0.74	0.03	1.0
0.1	0	GMM-Trunc-BC	0.14	147.06	2.7E+08	1135.50	0.62	1.0	0.16	93.38	7.4E+07	637.04	0.59	1.0
0.1	0	CUE-1	0.17	7.00	38.05	3.02	0.02	1.0	-0.01	7.82	40.82	3.21	0.01	1.0
0.1	0	CUE-25	-0.02	24.67	115.91	6.99	0.98	25.0	0.14	14.34	77.64	5.08	0.93	25.0
0.1	0	WWA	0.06	2.05	1.31	0.68	0.08	0.0	0.10	1.82	4.69	0.71	0.05	0.0
0.1	0.5	OLS	0.09	0.24	0.02	0.11	0.15	0.0	0.09	0.12	0.01	0.10	0.48	0.0
0.1	0.5	GMM-1	0.15	3.37	5.94	1.27	0.04	1.0	0.07	3.35	3.24	1.13	0.02	1.0
0.1	0.5	GMM-25	0.06	1.25	0.25	0.39	0.67	25.0	0.08	0.58	0.06	0.19	0.22	25.0
0.1	0.5	GMM-Bartlett	0.10	1.82	1.59	0.68	0.06	3.2	0.08	1.39	0.78	0.50	0.02	3.7
0.1	0.5	GMM-Tuk-Han	0.09	1.58	0.65	0.53	0.13	5.3	0.09	1.15	0.41	0.40	0.02	5.7
0.1	0.5	GMM-BR	0.09	1.95	1.67	0.72	0.09	2.8	0.09	1.61	0.95	0.56	0.03	3.0
0.1	0.5	GMM-BRM	0.09	2.23	3.40	0.86	0.26	1.2	0.06	2.14	1.72	0.76	0.23	1.4
0.1	0.5	GMM-Trunc	0.14	2.08	1.69	0.73	0.09	1.0	0.12	1.79	1.47	0.65	0.03	2.0
0.1	0.5	GMM-Trunc-BC	0.10	13.63	1.6E+05	33.95	0.49	1.0	0.14	3.47	1679.40	3.41	0.23	2.0
0.1	0.5	CUE-1	0.17	10.16	58.25	4.08	0.02	1.0	-0.03	9.41	46.19	3.58	0.01	1.0
0.1	0.5	CUE-25	-0.32	20.64	98.51	6.43	0.98	25.0	0.04	15.07	75.77	4.76	0.85	25.0
0.1	0.5	WWA	0.03	3.36	1.3E+07	119.09	0.21	0.0	0.08	1.94	51.90	1.29	0.12	0.0
0.1	0.9	OLS	0.08	0.28	0.02	0.11	0.10	0.0	0.09	0.15	0.01	0.09	0.33	0.0
0.1	0.9	GMM-1	0.15	4.22	9.68	1.60	0.05	1.0	0.07	4.24	6.40	1.46	0.03	1.0
0.1	0.9	GMM-25	0.06	1.18	0.23	0.37	0.68	25.0	0.08	0.57	0.06	0.19	0.19	25.0
0.1	0.9	GMM-Bartlett	0.12	1.70	0.72	0.58	0.07	4.2	0.10	1.15	0.25	0.37	0.02	6.5
0.1	0.9	GMM-Tuk-Han	0.11	1.35	0.35	0.43	0.15	7.9	0.08	0.87	0.14	0.28	0.04	11.1
0.1	0.9	GMM-BR	0.14	1.83	0.76	0.61	0.13	4.1	0.10	1.37	0.34	0.43	0.05	5.7
0.1	0.9	GMM-BRM	0.10	2.35	3.46	0.89	0.37	1.9	0.10	1.88	0.78	0.62	0.34	2.6
0.1	0.9	GMM-Trunc	0.18	2.40	4.19	0.96	0.13	2.0	0.17	1.80	1.46	0.68	0.06	4.0
0.1	0.9	GMM-Trunc-BC	0.18	4.41	7.75E+11	47895.00	0.42	2.0	0.11	2.61	1.93	0.94	0.30	4.0
0.1	0.9	CUE-1	0.05	13.78	62.96	4.56	0.02	1.0	-0.03	12.15	54.11	4.10	0.01	1.0
0.1	0.9	CUE-25	-0.12	19.47	95.93	6.55	0.98	25.0	-0.01	12.60	60.92	4.25	0.80	25.0
0.1	0.9	WWA	0.03	45.59	4.5E+07	247.21	0.53	0.0	0.02	18.23	2.8E+04	24.71	0.46	0.0

Table 7b - Heteroskedastic DGP with Robust Weight Matrix ϕ=.1, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.52	0.34	0.30	0.53	1.00	0.0	0.54	0.16	0.29	0.54	1.00	0.0
0.5	-0.9	GMM-1	0.64	3.47	6.81	1.49	0.09	1.0	0.64	3.60	5.60	1.42	0.07	1.0
0.5	-0.9	GMM-25	0.55	1.21	0.56	0.62	0.82	25.0	0.50	0.67	0.33	0.51	0.78	25.0
0.5	-0.9	GMM-Bartlett	0.57	1.53	0.86	0.74	0.31	5.1	0.55	1.17	0.54	0.62	0.22	7.0
0.5	-0.9	GMM-Tuk-Han	0.51	1.25	0.57	0.62	0.45	9.0	0.48	0.93	0.36	0.52	0.32	11.8
0.5	-0.9	GMM-BR	0.58	1.57	0.90	0.74	0.34	4.7	0.54	1.31	0.57	0.63	0.26	6.1
0.5	-0.9	GMM-BRM	0.57	1.97	1.52	0.85	0.54	2.1	0.59	1.74	0.96	0.76	0.53	2.8
0.5	-0.9	GMM-Trunc	0.63	1.98	2.32	0.97	0.40	4.0	0.61	1.74	1.66	0.91	0.38	5.0
0.5	-0.9	GMM-Trunc-BC	0.46	3.74	5.70	1.39	0.46	4.0	0.40	2.70	1.98	0.97	0.34	5.0
0.5	-0.9	CUE-1	0.50	10.06	50.08	3.86	0.04	1.0	0.44	10.05	47.24	3.70	0.05	1.0
0.5	-0.9	CUE-25	0.16	21.78	109.31	6.98	0.96	25.0	-0.03	12.30	67.28	4.24	0.73	25.0
0.5	-0.9	WWA	0.32	2.22	32.98	1.11	0.14	0.0	0.29	2.02	108.44	1.31	0.12	0.0
0.5	-0.5	OLS	0.50	0.28	0.27	0.51	1.00	0.0	0.51	0.14	0.27	0.52	1.00	0.0
0.5	-0.5	GMM-1	0.58	2.83	5.71	1.21	0.09	1.0	0.55	2.74	3.17	1.14	0.07	1.0
0.5	-0.5	GMM-25	0.53	1.17	0.51	0.59	0.80	25.0	0.50	0.56	0.30	0.51	0.84	25.0
0.5	-0.5	GMM-Bartlett	0.55	1.53	1.11	0.75	0.26	3.2	0.54	1.22	0.77	0.65	0.19	4.2
0.5	-0.5	GMM-Tuk-Han	0.52	1.27	0.67	0.64	0.38	5.3	0.50	1.09	0.52	0.58	0.29	6.5
0.5	-0.5	GMM-BR	0.54	1.62	1.23	0.77	0.28	2.8	0.55	1.38	0.89	0.70	0.21	3.4
0.5	-0.5	GMM-BRM	0.54	1.92	4.30	0.92	0.46	1.3	0.55	1.89	1.90	0.87	0.44	1.5
0.5	-0.5	GMM-Trunc	0.59	1.78	1.74	0.88	0.31	2.0	0.58	1.67	1.74	0.87	0.27	2.0
0.5	-0.5	GMM-Trunc-BC	0.54	4.58	605.33	3.49	0.46	2.0	0.58	2.83	3.24	1.15	0.35	2.0
0.5	-0.5	CUE-1	0.51	8.24	39.21	3.20	0.06	1.0	0.44	8.61	37.18	3.18	0.05	1.0
0.5	-0.5	CUE-25	0.25	21.99	107.33	6.67	0.96	25.0	0.24	10.64	61.34	3.96	0.81	25.0
0.5	-0.5	WWA	0.42	1.81	5.87	0.89	0.18	0.0	0.45	1.70	61.68	1.04	0.15	0.0
0.5	0	OLS	0.48	0.24	0.25	0.49	1.00	0.0	0.49	0.12	0.24	0.49	1.00	0.0
0.5	0	GMM-1	0.50	2.33	2.44	0.99	0.10	1.0	0.47	2.45	13.80	1.13	0.06	1.0
0.5	0	GMM-25	0.49	1.11	0.44	0.55	0.79	25.0	0.49	0.52	0.28	0.49	0.77	25.0
0.5	0	GMM-Bartlett	0.51	1.66	1.77	0.82	0.29	1.4	0.48	1.70	12.97	0.94	0.21	0.8
0.5	0	GMM-Tuk-Han	0.50	1.61	1.72	0.80	0.37	2.8	0.48	1.66	12.89	0.93	0.28	1.8
0.5	0	GMM-BR	0.50	1.66	1.78	0.82	0.29	1.5	0.49	1.72	12.98	0.95	0.20	0.9
0.5	0	GMM-BRM	0.51	1.71	1.80	0.83	0.44	0.7	0.47	1.85	13.07	0.97	0.36	0.4
0.5	0	GMM-Trunc	0.51	1.77	1.28	0.81	0.29	1.0	0.51	1.77	1.67	0.84	0.25	1.0
0.5	0	GMM-Trunc-BC	0.54	109.82	4.5E+08	1060.40	0.73	1.0	0.50	59.89	3.5E+14	5.9E+05	0.72	1.0
0.5	0	CUE-1	0.50	6.54	39.07	3.06	0.08	1.0	0.39	7.25	35.47	2.97	0.05	1.0
0.5	0	CUE-25	0.31	24.91	105.65	6.47	0.98	25.0	0.35	12.62	69.03	4.64	0.91	25.0
0.5	0	WWA	0.44	1.89	1.22	0.75	0.18	0.0	0.50	1.62	4.10	0.80	0.17	0.0
0.5	0.5	OLS	0.45	0.26	0.22	0.46	1.00	0.0	0.47	0.13	0.22	0.47	1.00	0.0
0.5	0.5	GMM-1	0.54	3.21	6.21	1.30	0.09	1.0	0.52	3.01	4.18	1.20	0.07	1.0
0.5	0.5	GMM-25	0.43	1.21	0.44	0.54	0.77	25.0	0.43	0.57	0.24	0.44	0.78	25.0
0.5	0.5	GMM-Bartlett	0.50	1.63	1.34	0.77	0.26	3.5	0.45	1.25	0.67	0.61	0.17	4.4
0.5	0.5	GMM-Tuk-Han	0.45	1.33	0.60	0.59	0.36	5.9	0.43	1.08	0.44	0.53	0.26	6.7
0.5	0.5	GMM-BR	0.50	1.68	1.24	0.77	0.28	3.0	0.48	1.39	0.85	0.65	0.18	3.5
0.5	0.5	GMM-BRM	0.50	1.97	1.98	0.89	0.43	1.4	0.49	1.76	2.04	0.83	0.44	1.5
0.5	0.5	GMM-Trunc	0.50	1.90	2.60	0.88	0.29	2.0	0.54	1.64	2.42	0.87	0.26	2.0
0.5	0.5	GMM-Trunc-BC	0.39	5.01	40.21	2.27	0.45	2.0	0.45	2.68	4.41	1.12	0.33	2.0
0.5	0.5	CUE-1	0.53	9.50	44.92	3.58	0.07	1.0	0.42	8.90	43.44	3.43	0.04	1.0
0.5	0.5	CUE-25	-0.55	24.26	108.49	6.75	0.97	25.0	0.20	13.36	63.49	4.38	0.87	25.0
0.5	0.5	WWA	0.30	2.43	3.6E+05	25.55	0.22	0.0	0.37	1.70	28.10	1.18	0.15	0.0
0.5	0.9	OLS	0.43	0.29	0.21	0.44	0.99	0.0	0.45	0.15	0.20	0.45	1.00	0.0
0.5	0.9	GMM-1	0.56	4.24	10.66	1.64	0.08	1.0	0.52	3.97	7.33	1.51	0.06	1.0
0.5	0.9	GMM-25	0.43	1.21	0.48	0.55	0.79	25.0	0.39	0.69	0.24	0.41	0.68	25.0
0.5	0.9	GMM-Bartlett	0.49	1.58	0.84	0.70	0.26	4.9	0.41	1.16	0.45	0.54	0.17	6.3
0.5	0.9	GMM-Tuk-Han	0.39	1.33	0.51	0.56	0.38	8.8	0.36	0.91	0.29	0.44	0.27	11.1
0.5	0.9	GMM-BR	0.48	1.61	0.86	0.70	0.30	4.6	0.42	1.26	0.49	0.56	0.20	5.8
0.5	0.9	GMM-BRM	0.48	2.10	1.89	0.88	0.49	2.1	0.47	1.73	1.04	0.72	0.49	2.6
0.5	0.9	GMM-Trunc	0.56	2.18	2.77	1.01	0.36	3.0	0.53	1.99	2.90	0.94	0.32	3.0
0.5	0.9	GMM-Trunc-BC	0.26	4.27	1.79E+12	43867.00	0.42	3.0	0.27	2.78	3.29	1.02	0.30	3.0
0.5	0.9	CUE-1	0.39	13.18	59.33	4.34	0.06	1.0	0.39	12.17	52.61	4.03	0.03	1.0
0.5	0.9	CUE-25	-1.75	20.63	97.67	6.74	1.00	25.0	-0.21	13.23	67.43	4.71	0.86	25.0
0.5	0.9	WWA	0.17	6.20	6.7E+04	21.04	0.29	0.0	0.21	2.27	1.3E+03	3.38	0.14	0.0

Table 7c - Heteroskedastic DGP with Robust Weight Matrix ϕ=.1, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.95	0.29	0.92	0.95	1.00	0.0	0.96	0.14	0.92	0.96	1.00	0.0
0.9	-0.9	GMM-1	1.15	2.66	4.22	1.48	0.33	1.0	1.08	2.48	8.63	1.46	0.25	1.0
0.9	-0.9	GMM-25	1.09	0.87	1.32	1.10	1.00	25.0	1.06	0.62	1.13	1.04	1.00	25.0
0.9	-0.9	GMM-Bartlett	1.04	1.18	1.37	1.07	0.80	7.1	0.96	0.90	1.08	0.97	0.78	10.1
0.9	-0.9	GMM-Tuk-Han	1.02	1.08	1.23	1.02	0.88	11.9	0.97	0.90	1.03	0.95	0.88	15.5
0.9	-0.9	GMM-BR	0.99	1.23	1.31	1.03	0.82	6.2	0.94	1.06	1.08	0.94	0.81	8.1
0.9	-0.9	GMM-BRM	1.06	1.42	1.82	1.15	0.88	2.8	1.02	1.30	1.30	1.03	0.87	3.7
0.9	-0.9	GMM-Trunc	1.09	1.32	2.23	1.21	0.72	4.0	1.04	1.09	2.31	1.15	0.78	7.0
0.9	-0.9	GMM-Trunc-BC	1.18	2.29	3.22	1.41	0.69	4.0	1.08	1.63	2.60	1.21	0.74	7.0
0.9	-0.9	CUE-1	0.81	8.59	46.97	3.57	0.21	1.0	0.64	10.20	46.71	3.61	0.19	1.0
0.9	-0.9	CUE-25	1.38	34.37	135.11	8.41	0.99	25.0	-0.48	34.78	132.52	7.78	0.89	25.0
0.9	-0.9	WWA	0.91	1.86	3.35	1.11	0.38	0.0	0.92	1.69	3.18	1.05	0.34	0.0
0.9	-0.5	OLS	0.91	0.24	0.85	0.92	1.00	0.0	0.92	0.11	0.86	0.92	1.00	0.0
0.9	-0.5	GMM-1	1.04	1.97	2.48	1.25	0.45	1.0	1.03	1.79	4.86	1.25	0.37	1.0
0.9	-0.5	GMM-25	0.97	0.73	1.05	0.99	0.99	25.0	0.94	0.44	0.90	0.93	1.00	25.0
0.9	-0.5	GMM-Bartlett	0.96	0.95	1.21	1.01	0.78	4.4	0.94	0.78	1.02	0.95	0.76	5.1
0.9	-0.5	GMM-Tuk-Han	0.94	0.83	1.08	0.97	0.85	7.1	0.92	0.69	0.94	0.92	0.85	7.8
0.9	-0.5	GMM-BR	0.96	0.99	1.23	1.01	0.79	3.7	0.95	0.82	1.08	0.98	0.80	4.0
0.9	-0.5	GMM-BRM	0.98	1.19	1.55	1.07	0.83	1.7	0.97	1.04	1.30	1.02	0.84	1.8
0.9	-0.5	GMM-Trunc	1.00	1.51	1.91	1.15	0.60	1.0	1.00	1.35	4.79	1.15	0.59	1.0
0.9	-0.5	GMM-Trunc-BC	1.00	2.35	2.85	1.27	0.62	1.0	0.99	1.70	4.85	1.15	0.57	1.0
0.9	-0.5	CUE-1	0.93	5.38	32.00	2.85	0.33	1.0	0.85	6.56	34.90	2.92	0.29	1.0
0.9	-0.5	CUE-25	1.00	22.68	99.07	6.37	0.96	25.0	0.31	8.35	55.19	3.57	0.84	25.0
0.9	-0.5	WWA	0.89	1.27	1.31	0.98	0.62	0.0	0.89	1.14	1.31	0.95	0.60	0.0
0.9	0	OLS	0.87	0.20	0.77	0.88	1.00	0.0	0.88	0.10	0.78	0.88	1.00	0.0
0.9	0	GMM-1	0.93	1.36	1.62	1.05	0.48	1.0	0.90	1.37	2.53	1.07	0.42	1.0
0.9	0	GMM-25	0.91	0.63	0.88	0.90	0.96	25.0	0.89	0.30	0.81	0.89	0.96	25.0
0.9	0	GMM-Bartlett	0.91	0.94	1.28	0.99	0.73	1.6	0.89	0.94	2.25	1.03	0.67	1.1
0.9	0	GMM-Tuk-Han	0.91	0.88	1.19	0.98	0.76	3.1	0.90	0.88	2.12	1.02	0.69	2.2
0.9	0	GMM-BR	0.91	0.98	1.29	0.99	0.73	1.6	0.90	0.96	2.25	1.03	0.66	1.2
0.9	0	GMM-BRM	0.90	1.03	1.35	1.00	0.82	0.8	0.89	1.02	2.32	1.04	0.77	0.5
0.9	0	GMM-Trunc	0.91	1.19	1.41	1.01	0.61	1.0	0.90	1.20	1.55	1.02	0.53	1.0
0.9	0	GMM-Trunc-BC	0.95	18.70	3.7E+05	59.58	0.80	1.0	0.92	9.56	4.8E+06	82.46	0.73	1.0
0.9	0	CUE-1	0.91	3.64	24.27	2.29	0.41	1.0	0.86	3.66	25.65	2.38	0.38	1.0
0.9	0	CUE-25	0.92	14.76	76.67	5.02	0.93	25.0	0.80	7.02	56.98	3.68	0.90	25.0
0.9	0	WWA	0.88	1.10	1.24	0.95	0.77	0.0	0.93	0.95	1.99	0.99	0.77	0.0
0.9	0.5	OLS	0.83	0.23	0.70	0.83	1.00	0.0	0.83	0.12	0.70	0.84	1.00	0.0
0.9	0.5	GMM-1	0.96	2.06	3.71	1.26	0.33	1.0	0.90	2.21	2.65	1.19	0.29	1.0
0.9	0.5	GMM-25	0.89	0.83	0.89	0.89	0.97	25.0	0.86	0.59	0.78	0.85	0.99	25.0
0.9	0.5	GMM-Bartlett	0.88	1.21	2.02	1.01	0.70	4.0	0.85	1.03	1.02	0.90	0.68	4.6
0.9	0.5	GMM-Tuk-Han	0.84	1.07	1.21	0.91	0.78	6.6	0.83	0.94	0.92	0.87	0.81	7.2
0.9	0.5	GMM-BR	0.88	1.30	2.01	1.02	0.73	3.5	0.86	1.14	1.11	0.92	0.72	3.7
0.9	0.5	GMM-BRM	0.88	1.50	2.48	1.08	0.74	1.6	0.85	1.35	1.37	0.97	0.76	1.7
0.9	0.5	GMM-Trunc	0.92	1.65	2.35	1.11	0.51	1.0	0.89	1.67	1.94	1.06	0.51	1.0
0.9	0.5	GMM-Trunc-BC	0.90	2.89	1.85E+12	43030.00	0.53	1.0	0.87	2.23	2.36	1.14	0.50	1.0
0.9	0.5	CUE-1	0.84	7.17	40.17	3.29	0.24	1.0	0.78	7.93	50.37	3.56	0.22	1.0
0.9	0.5	CUE-25	-2.16	32.98	122.19	7.58	0.99	25.0	-1.10	17.45	90.67	5.98	0.94	25.0
0.9	0.5	WWA	0.70	1.33	1.65	0.89	0.42	0.0	0.74	1.19	39.07	1.07	0.41	0.0
0.9	0.9	OLS	0.79	0.28	0.65	0.80	1.00	0.0	0.80	0.15	0.65	0.80	1.00	0.0
0.9	0.9	GMM-1	0.97	2.97	3.45	1.37	0.23	1.0	0.92	3.22	4.50	1.45	0.19	1.0
0.9	0.9	GMM-25	0.90	1.01	0.96	0.90	0.98	25.0	0.88	0.81	0.82	0.85	0.99	25.0
0.9	0.9	GMM-Bartlett	0.84	1.38	1.15	0.92	0.64	6.5	0.79	1.22	0.93	0.84	0.62	8.5
0.9	0.9	GMM-Tuk-Han	0.79	1.31	1.00	0.86	0.76	11.1	0.74	1.20	0.81	0.78	0.75	13.8
0.9	0.9	GMM-BR	0.82	1.53	1.19	0.91	0.68	5.8	0.74	1.30	0.92	0.82	0.66	7.2
0.9	0.9	GMM-BRM	0.87	1.77	1.45	1.00	0.75	2.6	0.84	1.53	1.28	0.93	0.75	3.2
0.9	0.9	GMM-Trunc	0.94	1.65	1.71	1.09	0.62	4.0	0.91	1.42	1.86	1.07	0.67	5.0
0.9	0.9	GMM-Trunc-BC	0.92	3.15	1.89E+07	139.02	0.57	4.0	0.92	2.18	2.46	1.20	0.62	5.0
0.9	0.9	CUE-1	0.65	11.40	52.70	4.05	0.15	1.0	0.71	10.54	46.73	3.77	0.13	1.0
0.9	0.9	CUE-25	-4.04	33.39	163.55	9.67	1.00	25.0	-5.98	27.25	157.91	9.83	0.98	25.0
0.9	0.9	WWA	0.59	1.75	1.54	0.85	0.15	0.0	0.62	1.64	7.06	0.91	0.16	0.0

Table 8a - Heteroskedastic DGP with Robust Weight Matrix ϕ=.3, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.11	0.33	0.03	0.14	0.21	0.0	0.12	0.17	0.02	0.12	0.54	0.0
0.1	-0.9	GMM-1	0.11	3.14	5.20	1.17	0.04	1.0	0.03	2.05	1.18	0.68	0.02	1.0
0.1	-0.9	GMM-25	0.06	1.15	0.22	0.37	0.58	25.0	0.09	0.63	0.07	0.21	0.15	25.0
0.1	-0.9	GMM-Bartlett	0.14	1.66	0.77	0.58	0.06	3.4	0.09	1.12	0.27	0.38	0.01	5.0
0.1	-0.9	GMM-Tuk-Han	0.14	1.31	0.39	0.44	0.10	7.0	0.10	0.93	0.16	0.31	0.02	9.1
0.1	-0.9	GMM-BR	0.13	1.61	0.79	0.58	0.08	3.6	0.10	1.26	0.31	0.41	0.03	4.7
0.1	-0.9	GMM-BRM	0.13	2.10	1.17	0.72	0.28	1.6	0.09	1.51	0.52	0.52	0.29	2.1
0.1	-0.9	GMM-Trunc	0.17	2.26	3.37	0.83	0.09	1.0	0.12	1.49	0.65	0.51	0.03	2.0
0.1	-0.9	GMM-Trunc-BC	-0.02	4.46	8.96E+10	12047.00	0.35	1.0	0.04	2.35	1.16	0.80	0.23	2.0
0.1	-0.9	CUE-1	0.08	9.44	46.50	3.53	0.02	1.0	-0.02	3.99	26.13	2.15	0.03	1.0
0.1	-0.9	CUE-25	0.01	22.96	105.89	6.77	0.96	25.0	-0.02	8.65	47.45	3.28	0.69	25.0
0.1	-0.9	WWA	0.05	2.14	4.95	0.91	0.11	0.0	0.04	1.63	73.83	1.04	0.13	0.0
0.1	-0.5	OLS	0.10	0.27	0.02	0.12	0.22	0.0	0.11	0.14	0.01	0.11	0.60	0.0
0.1	-0.5	GMM-1	0.11	2.54	2.33	0.92	0.04	1.0	0.00	1.63	0.67	0.54	0.01	1.0
0.1	-0.5	GMM-25	0.05	1.12	0.21	0.36	0.61	25.0	0.09	0.54	0.05	0.18	0.15	25.0
0.1	-0.5	GMM-Bartlett	0.12	1.53	0.97	0.56	0.05	2.7	0.06	1.00	0.25	0.34	0.01	4.2
0.1	-0.5	GMM-Tuk-Han	0.09	1.26	0.47	0.44	0.09	4.9	0.06	0.94	0.18	0.31	0.02	6.3
0.1	-0.5	GMM-BR	0.10	1.59	1.10	0.60	0.06	2.5	0.07	1.07	0.30	0.37	0.02	3.3
0.1	-0.5	GMM-BRM	0.11	1.91	1.49	0.68	0.22	1.1	0.05	1.30	0.49	0.45	0.21	1.5
0.1	-0.5	GMM-Trunc	0.15	1.75	1.00	0.61	0.06	2.0	0.09	1.15	0.44	0.40	0.02	2.0
0.1	-0.5	GMM-Trunc-BC	0.10	4.78	141.20	2.60	0.39	2.0	0.04	2.06	2.85	0.82	0.20	2.0
0.1	-0.5	CUE-1	0.08	7.56	38.68	3.06	0.02	1.0	-0.03	3.61	22.68	1.95	0.04	1.0
0.1	-0.5	CUE-25	-0.37	31.17	121.68	7.21	0.97	25.0	0.03	10.51	56.82	3.84	0.80	25.0
0.1	-0.5	WWA	0.07	1.88	310.73	1.52	0.08	0.0	0.03	1.28	0.76	0.46	0.06	0.0
0.1	0	OLS	0.09	0.22	0.02	0.10	0.18	0.0	0.09	0.11	0.01	0.09	0.57	0.0
0.1	0	GMM-1	0.09	2.24	2.53	0.82	0.03	1.0	0.01	1.31	5.26	0.52	0.01	1.0
0.1	0	GMM-25	0.05	1.04	0.18	0.33	0.65	25.0	0.06	0.47	0.04	0.16	0.20	25.0
0.1	0	GMM-Bartlett	0.08	1.43	2.37	0.61	0.06	2.7	0.03	1.02	0.27	0.34	0.02	3.3
0.1	0	GMM-Tuk-Han	0.08	1.30	1.29	0.53	0.12	4.4	0.03	0.92	0.18	0.30	0.03	4.9
0.1	0	GMM-BR	0.09	1.46	2.39	0.63	0.07	2.3	0.03	1.07	0.29	0.35	0.02	2.5
0.1	0	GMM-BRM	0.10	1.61	2.49	0.66	0.23	1.0	0.03	1.10	0.31	0.37	0.19	1.2
0.1	0	GMM-Trunc	0.10	1.50	0.80	0.54	0.07	2.0	0.05	1.02	0.24	0.34	0.03	2.0
0.1	0	GMM-Trunc-BC	0.10	5.86	4.6E+05	34.88	0.44	2.0	0.00	1.72	1.79	0.65	0.18	2.0
0.1	0	CUE-1	0.10	6.89	35.57	2.88	0.02	1.0	-0.03	3.72	21.84	1.87	0.04	1.0
0.1	0	CUE-25	-0.48	25.22	117.39	7.20	0.98	25.0	0.20	22.43	103.15	6.36	0.97	25.0
0.1	0	WWA	0.07	1.69	26.26	0.76	0.07	0.0	0.03	1.17	0.31	0.38	0.05	0.0
0.1	0.5	OLS	0.07	0.21	0.01	0.09	0.09	0.0	0.08	0.11	0.01	0.08	0.37	0.0
0.1	0.5	GMM-1	0.09	2.64	4.56	1.01	0.04	1.0	-0.01	1.40	1.10	0.53	0.03	1.0
0.1	0.5	GMM-25	0.05	1.10	0.20	0.33	0.66	25.0	0.04	0.45	0.03	0.14	0.19	25.0
0.1	0.5	GMM-Bartlett	0.06	1.47	2.04	0.59	0.07	3.7	0.04	0.82	0.21	0.29	0.03	5.6
0.1	0.5	GMM-Tuk-Han	0.07	1.28	0.55	0.45	0.12	5.9	0.03	0.79	0.15	0.27	0.04	7.4
0.1	0.5	GMM-BR	0.07	1.55	2.10	0.60	0.08	3.1	0.05	0.87	0.24	0.31	0.02	3.8
0.1	0.5	GMM-BRM	0.09	1.79	2.35	0.69	0.27	1.4	0.07	1.03	0.45	0.38	0.27	1.7
0.1	0.5	GMM-Trunc	0.09	1.55	1.04	0.55	0.08	2.0	0.04	0.87	0.37	0.32	0.03	3.0
0.1	0.5	GMM-Trunc-BC	0.04	4.48	6.8E+04	14.35	0.44	2.0	-0.03	1.44	0.95	0.54	0.20	3.0
0.1	0.5	CUE-1	0.07	10.56	52.27	3.73	0.03	1.0	-0.06	7.26	42.42	2.94	0.08	1.0
0.1	0.5	CUE-25	-0.56	17.18	82.06	5.60	0.99	25.0	0.11	18.68	94.49	6.12	0.94	25.0
0.1	0.5	WWA	0.02	2.60	2.2E+03	4.97	0.22	0.0	0.00	1.06	13.72	0.61	0.08	0.0
0.1	0.9	OLS	0.06	0.24	0.01	0.09	0.05	0.0	0.07	0.13	0.01	0.07	0.19	0.0
0.1	0.9	GMM-1	0.11	3.29	9.30	1.26	0.05	1.0	-0.01	1.70	3.47	0.67	0.04	1.0
0.1	0.9	GMM-25	0.04	0.98	0.17	0.31	0.65	25.0	0.04	0.41	0.03	0.13	0.16	25.0
0.1	0.9	GMM-Bartlett	0.08	1.35	0.68	0.48	0.10	5.1	0.04	0.78	0.12	0.25	0.04	8.1
0.1	0.9	GMM-Tuk-Han	0.06	1.10	0.27	0.36	0.15	9.0	0.03	0.62	0.07	0.20	0.04	12.5
0.1	0.9	GMM-BR	0.08	1.51	0.58	0.50	0.11	4.7	0.03	0.89	0.16	0.29	0.06	6.5
0.1	0.9	GMM-BRM	0.06	1.77	4.05	0.70	0.34	2.1	0.06	1.16	0.26	0.37	0.34	3.0
0.1	0.9	GMM-Trunc	0.10	1.73	1.67	0.67	0.12	3.0	0.07	0.90	1.95	0.36	0.07	5.0
0.1	0.9	GMM-Trunc-BC	0.01	3.31	3.29E+12	65068.00	0.49	3.0	-0.08	1.55	2.20	0.56	0.36	5.0
0.1	0.9	CUE-1	0.09	13.42	61.78	4.37	0.03	1.0	-0.08	10.19	45.50	3.30	0.09	1.0
0.1	0.9	CUE-25	-0.74	10.85	53.04	4.45	0.99	25.0	0.02	8.10	41.16	3.60	0.91	25.0
0.1	0.9	WWA	-0.01	34.39	1.2E+04	26.43	0.54	0.0	-0.02	13.43	4.7E+03	12.72	0.44	0.0

Table 8b - Heteroskedastic DGP with Robust Weight Matrix ϕ=.3, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.56	0.34	0.34	0.57	1.00	0.0	0.57	0.17	0.33	0.57	1.00	0.0
0.5	-0.9	GMM-1	0.53	3.01	4.16	1.19	0.08	1.0	0.12	1.94	1.49	0.69	0.04	1.0
0.5	-0.9	GMM-25	0.53	1.13	0.49	0.59	0.81	25.0	0.47	0.60	0.30	0.49	0.76	25.0
0.5	-0.9	GMM-Bartlett	0.56	1.38	0.82	0.70	0.28	5.0	0.38	1.10	0.37	0.49	0.19	6.6
0.5	-0.9	GMM-Tuk-Han	0.53	1.19	0.56	0.61	0.43	8.8	0.41	0.93	0.29	0.45	0.30	11.2
0.5	-0.9	GMM-BR	0.57	1.46	0.81	0.71	0.34	4.6	0.42	1.10	0.38	0.50	0.22	5.8
0.5	-0.9	GMM-BRM	0.58	1.75	1.29	0.82	0.53	2.1	0.42	1.40	0.53	0.58	0.49	2.6
0.5	-0.9	GMM-Trunc	0.61	1.62	2.44	0.88	0.39	3.0	0.45	1.48	1.10	0.64	0.33	3.0
0.5	-0.9	GMM-Trunc-BC	0.41	3.33	2.87E+11	16954.00	0.45	3.0	0.15	2.38	1.48	0.78	0.32	3.0
0.5	-0.9	CUE-1	0.33	9.35	49.39	3.65	0.06	1.0	-0.10	3.92	20.48	1.86	0.04	1.0
0.5	-0.9	CUE-25	-0.06	27.38	112.76	6.99	0.95	25.0	-0.10	8.04	50.06	3.31	0.69	25.0
0.5	-0.9	WWA	0.29	2.00	17.30	1.04	0.15	0.0	0.17	1.46	8.62	0.65	0.12	0.0
0.5	-0.5	OLS	0.51	0.28	0.28	0.51	1.00	0.0	0.52	0.14	0.27	0.52	1.00	0.0
0.5	-0.5	GMM-1	0.43	2.34	1.88	0.92	0.09	1.0	0.11	1.51	0.75	0.54	0.04	1.0
0.5	-0.5	GMM-25	0.45	1.04	0.41	0.52	0.75	25.0	0.42	0.50	0.23	0.43	0.73	25.0
0.5	-0.5	GMM-Bartlett	0.48	1.34	0.74	0.64	0.24	3.0	0.32	1.03	0.29	0.42	0.16	4.5
0.5	-0.5	GMM-Tuk-Han	0.47	1.14	0.58	0.59	0.36	5.2	0.35	0.88	0.24	0.41	0.25	6.6
0.5	-0.5	GMM-BR	0.47	1.42	0.82	0.66	0.27	2.7	0.34	1.11	0.33	0.46	0.16	3.4
0.5	-0.5	GMM-BRM	0.48	1.71	1.05	0.73	0.43	1.2	0.31	1.38	0.60	0.54	0.40	1.5
0.5	-0.5	GMM-Trunc	0.49	1.69	0.99	0.73	0.29	2.0	0.35	1.30	0.44	0.51	0.23	2.0
0.5	-0.5	GMM-Trunc-BC	0.42	3.82	81.11	2.03	0.43	2.0	0.18	1.87	1.00	0.64	0.26	2.0
0.5	-0.5	CUE-1	0.31	7.80	43.89	3.25	0.07	1.0	-0.06	3.44	17.47	1.69	0.06	1.0
0.5	-0.5	CUE-25	-0.04	24.08	104.59	6.44	0.96	25.0	0.12	9.09	57.82	3.71	0.80	25.0
0.5	-0.5	WWA	0.36	1.67	2.27E+03	3.04	0.17	0.0	0.19	1.27	0.50	0.46	0.12	0.0
0.5	0	OLS	0.44	0.23	0.21	0.45	1.00	0.0	0.45	0.12	0.21	0.45	1.00	0.0
0.5	0	GMM-1	0.34	2.03	53.66	1.04	0.09	1.0	0.10	1.25	38.26	0.67	0.05	1.0
0.5	0	GMM-25	0.37	1.02	0.32	0.46	0.76	25.0	0.34	0.43	0.16	0.36	0.71	25.0
0.5	0	GMM-Bartlett	0.40	1.41	53.32	0.93	0.29	2.6	0.23	1.06	0.49	0.43	0.22	3.1
0.5	0	GMM-Tuk-Han	0.40	1.23	14.78	0.74	0.38	4.3	0.25	0.93	0.21	0.37	0.29	4.6
0.5	0	GMM-BR	0.40	1.46	53.33	0.94	0.29	2.2	0.20	1.12	0.49	0.43	0.19	2.4
0.5	0	GMM-BRM	0.42	1.57	53.41	0.96	0.43	1.0	0.20	1.19	0.53	0.45	0.33	1.1
0.5	0	GMM-Trunc	0.42	1.42	0.82	0.63	0.30	2.0	0.25	1.05	0.27	0.40	0.26	2.0
0.5	0	GMM-Trunc-BC	0.33	4.89	1.53E+05	18.93	0.50	2.0	0.14	1.68	3.54	0.65	0.26	2.0
0.5	0	CUE-1	0.30	7.34	38.81	2.98	0.09	1.0	-0.02	3.56	22.95	1.90	0.08	1.0
0.5	0	CUE-25	0.25	28.44	117.47	7.03	0.99	25.0	1.15	19.27	86.74	5.73	0.95	25.0
0.5	0	WWA	0.33	1.57	8.44	0.74	0.15	0.0	0.17	1.13	0.31	0.39	0.11	0.0
0.5	0.5	OLS	0.37	0.23	0.15	0.38	1.00	0.0	0.38	0.12	0.15	0.38	1.00	0.0
0.5	0.5	GMM-1	0.31	2.60	6.91	1.07	0.07	1.0	0.06	1.40	0.86	0.52	0.04	1.0
0.5	0.5	GMM-25	0.32	1.09	0.32	0.44	0.76	25.0	0.25	0.46	0.11	0.28	0.65	25.0
0.5	0.5	GMM-Bartlett	0.32	1.45	0.98	0.60	0.22	3.6	0.17	0.92	0.22	0.34	0.12	4.3
0.5	0.5	GMM-Tuk-Han	0.32	1.22	0.53	0.51	0.31	5.9	0.18	0.81	0.17	0.31	0.17	6.4
0.5	0.5	GMM-BR	0.35	1.54	1.04	0.64	0.25	3.1	0.19	0.96	0.28	0.37	0.13	3.3
0.5	0.5	GMM-BRM	0.34	1.86	4.26	0.79	0.41	1.4	0.16	1.13	0.44	0.43	0.34	1.5
0.5	0.5	GMM-Trunc	0.35	1.79	3.75	0.78	0.24	2.0	0.21	1.06	0.38	0.41	0.18	2.0
0.5	0.5	GMM-Trunc-BC	0.15	3.63	351.21	2.40	0.36	2.0	-0.01	1.50	0.71	0.50	0.18	2.0
0.5	0.5	CUE-1	0.27	9.35	49.11	3.60	0.06	1.0	-0.03	6.96	35.81	2.72	0.10	1.0
0.5	0.5	CUE-25	-1.37	12.40	65.20	4.83	0.99	25.0	0.17	13.48	63.78	4.82	0.95	25.0
0.5	0.5	WWA	0.18	1.98	1.92E+03	4.38	0.18	0.0	0.09	0.96	0.54	0.39	0.08	0.0
0.5	0.9	OLS	0.32	0.26	0.12	0.33	0.91	0.0	0.33	0.13	0.11	0.33	1.00	0.0
0.5	0.9	GMM-1	0.29	3.31	7.44	1.26	0.08	1.0	0.05	1.72	1.76	0.65	0.04	1.0
0.5	0.9	GMM-25	0.25	1.08	0.31	0.41	0.75	25.0	0.18	0.53	0.09	0.23	0.48	25.0
0.5	0.9	GMM-Bartlett	0.29	1.46	0.70	0.56	0.19	4.8	0.15	0.99	0.20	0.34	0.11	5.1
0.5	0.9	GMM-Tuk-Han	0.24	1.16	0.35	0.43	0.29	8.6	0.15	0.74	0.12	0.26	0.14	9.2
0.5	0.9	GMM-BR	0.30	1.57	0.73	0.58	0.24	4.5	0.16	1.02	0.23	0.36	0.14	4.8
0.5	0.9	GMM-BRM	0.29	1.92	2.66	0.77	0.45	2.0	0.14	1.20	0.37	0.42	0.36	2.2
0.5	0.9	GMM-Trunc	0.34	1.85	3.93	0.80	0.28	2.0	0.21	1.31	1.09	0.51	0.17	1.0
0.5	0.9	GMM-Trunc-BC	-0.05	3.44	1.02E+12	46003.00	0.42	2.0	-0.16	1.82	1.36	0.62	0.24	1.0
0.5	0.9	CUE-1	0.25	11.92	52.00	4.05	0.05	1.0	-0.04	10.66	46.96	3.30	0.09	1.0
0.5	0.9	CUE-25	-2.10	9.49	48.27	4.38	0.99	25.0	-2.00	7.11	27.50	3.31	0.95	25.0
0.5	0.9	WWA	0.03	8.05	9.60E+03	14.74	0.31	0.0	0.01	1.22	515.84	2.18	0.12	0.0

Table 8c - Heteroskedastic DGP with Robust Weight Matrix ϕ=.3, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
					n=128						n=512			
0.9	-0.9	OLS	1.01	0.28	1.06	1.02	1.00	0.0	1.03	0.14	1.06	1.03	1.00	0.0
0.9	-0.9	GMM-1	0.78	2.39	3.50	1.23	0.35	1.0	0.21	1.83	2.11	0.72	0.20	1.0
0.9	-0.9	GMM-25	1.05	0.77	1.21	1.06	1.00	25.0	0.92	0.53	0.89	0.92	1.00	25.0
0.9	-0.9	GMM-Bartlett	0.99	0.95	1.15	1.00	0.85	9.1	0.72	0.89	0.63	0.71	0.66	10.2
0.9	-0.9	GMM-Tuk-Han	1.00	0.90	1.11	0.99	0.91	13.9	0.78	0.80	0.69	0.77	0.78	14.8
0.9	-0.9	GMM-BR	0.99	1.00	1.14	0.99	0.86	7.3	0.75	0.86	0.65	0.73	0.71	7.7
0.9	-0.9	GMM-BRM	1.02	1.09	1.27	1.03	0.90	3.3	0.78	1.11	0.81	0.80	0.80	3.5
0.9	-0.9	GMM-Trunc	1.02	1.21	2.03	1.10	0.75	5.0	0.75	1.44	0.88	0.78	0.60	4.0
0.9	-0.9	GMM-Trunc-BC	1.00	1.64	2.78	1.17	0.74	5.0	0.65	1.62	0.92	0.77	0.58	4.0
0.9	-0.9	CUE-1	0.30	8.71	41.74	3.21	0.17	1.0	-0.12	3.10	17.29	1.60	0.07	1.0
0.9	-0.9	CUE-25	0.94	20.58	92.03	6.03	0.94	25.0	-0.26	10.74	61.67	3.82	0.67	25.0
0.9	-0.9	WWA	0.80	1.77	978.49	2.05	0.51	0.0	0.42	1.35	0.73	0.61	0.37	0.0
0.9	-0.5	OLS	0.92	0.22	0.87	0.93	1.00	0.0	0.93	0.11	0.87	0.93	1.00	0.0
0.9	-0.5	GMM-1	0.70	1.85	2.33	1.01	0.37	1.0	0.21	1.41	0.91	0.56	0.19	1.0
0.9	-0.5	GMM-25	0.85	0.67	0.83	0.87	0.98	25.0	0.77	0.34	0.64	0.79	0.99	25.0
0.9	-0.5	GMM-Bartlett	0.84	0.89	0.87	0.85	0.76	4.7	0.48	0.96	0.38	0.53	0.50	3.9
0.9	-0.5	GMM-Tuk-Han	0.86	0.74	0.83	0.85	0.83	7.3	0.58	0.76	0.42	0.58	0.64	6.2
0.9	-0.5	GMM-BR	0.86	0.88	0.91	0.87	0.78	3.8	0.52	0.93	0.42	0.56	0.54	3.2
0.9	-0.5	GMM-BRM	0.85	1.02	1.20	0.91	0.81	1.7	0.49	1.23	0.60	0.59	0.63	1.5
0.9	-0.5	GMM-Trunc	0.79	1.44	1.67	0.94	0.55	1.0	0.28	1.52	0.81	0.57	0.31	1.0
0.9	-0.5	GMM-Trunc-BC	0.75	1.72	1.78	0.94	0.53	1.0	0.19	1.61	0.83	0.56	0.29	1.0
0.9	-0.5	CUE-1	0.34	7.20	40.56	3.05	0.23	1.0	-0.09	2.99	17.15	1.58	0.09	1.0
0.9	-0.5	CUE-25	0.39	21.69	91.05	5.49	0.90	25.0	0.21	5.56	37.49	2.63	0.71	25.0
0.9	-0.5	WWA	0.71	1.24	1.69	0.86	0.64	0.0	0.37	1.02	0.43	0.49	0.47	0.0
0.9	0	OLS	0.80	0.19	0.66	0.81	1.00	0.0	0.81	0.10	0.66	0.81	1.00	0.0
0.9	0	GMM-1	0.57	1.64	1.74	0.84	0.32	1.0	0.17	1.09	1.29	0.50	0.15	1.0
0.9	0	GMM-25	0.70	0.66	0.59	0.72	0.91	25.0	0.67	0.27	0.48	0.68	0.93	25.0
0.9	0	GMM-Bartlett	0.69	1.13	1.49	0.84	0.66	2.9	0.37	0.98	0.78	0.51	0.47	2.2
0.9	0	GMM-Tuk-Han	0.72	0.95	0.86	0.79	0.73	4.8	0.45	0.82	0.33	0.49	0.53	3.7
0.9	0	GMM-BR	0.70	1.16	1.53	0.85	0.66	2.5	0.37	0.99	0.75	0.51	0.45	1.9
0.9	0	GMM-BRM	0.68	1.28	1.57	0.85	0.74	1.1	0.34	1.14	1.01	0.55	0.53	0.9
0.9	0	GMM-Trunc	0.64	1.46	1.15	0.80	0.47	1.0	0.25	1.15	0.66	0.46	0.28	1.0
0.9	0	GMM-Trunc-BC	0.57	2.47	16.93	1.31	0.49	1.0	0.18	1.33	1.21	0.52	0.25	1.0
0.9	0	CUE-1	0.39	6.64	38.39	2.93	0.25	1.0	-0.05	3.20	25.72	1.90	0.13	1.0
0.9	0	CUE-25	-0.04	14.44	72.28	4.86	0.92	25.0	1.37	15.85	80.39	5.23	0.94	25.0
0.9	0	WWA	0.62	1.17	17.88	0.90	0.59	0.0	0.32	0.93	0.31	0.42	0.43	0.0
0.9	0.5	OLS	0.68	0.21	0.48	0.69	1.00	0.0	0.69	0.11	0.48	0.69	1.00	0.0
0.9	0.5	GMM-1	0.52	2.32	2.58	0.99	0.19	1.0	0.11	1.42	1.48	0.55	0.11	1.0
0.9	0.5	GMM-25	0.60	0.84	0.51	0.64	0.91	25.0	0.45	0.48	0.29	0.50	0.96	25.0
0.9	0.5	GMM-Bartlett	0.58	1.43	1.22	0.78	0.54	3.9	0.25	0.95	0.28	0.39	0.34	3.5
0.9	0.5	GMM-Tuk-Han	0.55	1.20	0.89	0.71	0.66	6.3	0.30	0.77	0.26	0.39	0.45	5.8
0.9	0.5	GMM-BR	0.57	1.51	1.32	0.81	0.58	3.3	0.28	1.01	0.33	0.43	0.40	3.0
0.9	0.5	GMM-BRM	0.60	1.77	1.63	0.87	0.64	1.5	0.26	1.20	0.44	0.47	0.49	1.4
0.9	0.5	GMM-Trunc	0.60	1.89	2.07	0.90	0.38	1.0	0.15	1.36	0.81	0.51	0.22	1.0
0.9	0.5	GMM-Trunc-BC	0.41	3.28	1976.50	2.99	0.41	1.0	-0.05	1.61	0.93	0.55	0.20	1.0
0.9	0.5	CUE-1	0.32	10.16	55.33	3.79	0.15	1.0	-0.08	8.13	48.82	3.10	0.12	1.0
0.9	0.5	CUE-25	-1.88	7.54	41.44	3.83	1.00	25.0	-1.98	5.46	25.66	3.27	0.99	25.0
0.9	0.5	WWA	0.38	1.29	14.39	0.83	0.19	0.0	0.20	0.90	0.38	0.37	0.13	0.0
0.9	0.9	OLS	0.58	0.25	0.36	0.59	1.00	0.0	0.59	0.14	0.35	0.59	1.00	0.0
0.9	0.9	GMM-1	0.50	2.89	7.80	1.23	0.11	1.0	0.11	1.80	1.78	0.69	0.06	1.0
0.9	0.9	GMM-25	0.52	1.01	0.49	0.58	0.92	25.0	0.33	0.80	0.26	0.41	0.83	25.0
0.9	0.9	GMM-Bartlett	0.51	1.58	0.98	0.72	0.47	5.7	0.25	1.12	0.34	0.43	0.34	6.2
0.9	0.9	GMM-Tuk-Han	0.43	1.47	0.75	0.63	0.58	9.6	0.24	1.00	0.27	0.38	0.43	10.4
0.9	0.9	GMM-BR	0.50	1.73	0.96	0.72	0.53	5.0	0.27	1.14	0.40	0.45	0.40	5.4
0.9	0.9	GMM-BRM	0.57	1.99	1.38	0.84	0.63	2.3	0.31	1.50	0.68	0.58	0.55	2.5
0.9	0.9	GMM-Trunc	0.61	2.05	2.18	0.95	0.46	2.0	0.31	1.53	0.74	0.60	0.35	1.0
0.9	0.9	GMM-Trunc-BC	0.36	3.78	3.72E+11	28748.00	0.50	2.0	0.06	2.21	1.21	0.75	0.42	1.0
0.9	0.9	CUE-1	0.27	12.11	55.19	4.02	0.09	1.0	-0.09	12.56	61.58	3.93	0.12	1.0
0.9	0.9	CUE-25	-2.99	8.66	54.28	4.97	1.00	25.0	-2.96	3.48	19.18	3.57	1.00	25.0
0.9	0.9	WWA	0.19	1.63	6.81	0.77	0.07	0.0	0.10	1.05	0.49	0.40	0.05	0.0

Table 9a - Heteroskedastic DGP with Robust Weight Matrix ϕ=.5, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	OLS	0.10	0.32	0.03	0.13	0.24	0.0	0.11	0.16	0.02	0.11	0.55	0.0
0.1	-0.9	GMM-1	0.05	1.53	1.01	0.55	0.02	1.0	0.00	0.67	0.07	0.21	0.01	1.0
0.1	-0.9	GMM-25	0.04	0.95	0.16	0.30	0.56	25.0	0.05	0.47	0.04	0.15	0.10	25.0
0.1	-0.9	GMM-Bartlett	0.07	1.11	0.27	0.37	0.05	3.3	0.01	0.58	0.05	0.18	0.01	5.0
0.1	-0.9	GMM-Tuk-Han	0.07	0.99	0.19	0.33	0.08	6.7	0.03	0.56	0.05	0.17	0.02	8.7
0.1	-0.9	GMM-BR	0.08	1.19	0.30	0.39	0.07	3.5	0.02	0.59	0.06	0.19	0.02	4.5
0.1	-0.9	GMM-BRM	0.07	1.32	0.70	0.46	0.27	1.6	0.02	0.67	0.07	0.21	0.21	2.1
0.1	-0.9	GMM-Trunc	0.07	1.28	0.76	0.46	0.07	1.0	0.01	0.61	0.06	0.19	0.02	2.0
0.1	-0.9	GMM-Trunc-BC	-0.10	3.03	3.10E+10	9260.50	0.40	1.0	-0.16	1.19	0.22	0.40	0.25	2.0
0.1	-0.9	CUE-1	0.03	6.13	37.29	2.70	0.03	1.0	-0.02	0.86	16.03	1.10	0.09	1.0
0.1	-0.9	CUE-25	0.05	21.31	96.78	6.37	0.95	25.0	-0.10	12.51	63.32	4.20	0.75	25.0
0.1	-0.9	WWA	0.02	1.29	3.28	0.52	0.09	0.0	0.01	0.62	0.13	0.21	0.09	0.0
0.1	-0.5	OLS	0.09	0.25	0.02	0.11	0.23	0.0	0.10	0.13	0.01	0.10	0.58	0.0
0.1	-0.5	GMM-1	0.03	1.20	0.49	0.42	0.03	1.0	-0.01	0.53	0.04	0.16	0.01	1.0
0.1	-0.5	GMM-25	0.03	0.82	0.12	0.27	0.56	25.0	0.04	0.38	0.02	0.12	0.11	25.0
0.1	-0.5	GMM-Bartlett	0.05	0.91	0.33	0.33	0.08	4.1	0.01	0.47	0.03	0.14	0.03	6.5
0.1	-0.5	GMM-Tuk-Han	0.06	0.85	0.17	0.29	0.10	6.6	0.02	0.47	0.03	0.14	0.03	8.6
0.1	-0.5	GMM-BR	0.05	0.96	0.36	0.35	0.08	3.4	0.01	0.49	0.04	0.15	0.03	4.5
0.1	-0.5	GMM-BRM	0.07	1.12	0.43	0.39	0.28	1.5	0.01	0.55	0.05	0.17	0.24	2.0
0.1	-0.5	GMM-Trunc	0.05	0.97	0.24	0.33	0.08	3.0	0.02	0.48	0.04	0.15	0.02	5.0
0.1	-0.5	GMM-Trunc-BC	0.01	1.82	5.54	0.77	0.34	3.0	-0.04	0.72	0.08	0.23	0.14	5.0
0.1	-0.5	CUE-1	0.03	5.88	40.89	2.76	0.05	1.0	-0.02	0.71	17.39	1.18	0.10	1.0
0.1	-0.5	CUE-25	0.11	18.65	88.70	6.16	0.98	25.0	-0.06	15.34	68.72	4.78	0.85	25.0
0.1	-0.5	WWA	0.02	1.06	126.66	0.73	0.07	0.0	0.01	0.48	0.04	0.15	0.04	0.0
0.1	0	OLS	0.07	0.19	0.01	0.09	0.15	0.0	0.08	0.10	0.01	0.08	0.50	0.0
0.1	0	GMM-1	0.02	0.97	0.28	0.33	0.04	1.0	-0.01	0.39	0.03	0.13	0.02	1.0
0.1	0	GMM-25	0.03	0.66	0.08	0.21	0.60	25.0	0.03	0.30	0.01	0.09	0.14	25.0
0.1	0	GMM-Bartlett	0.03	0.76	0.15	0.26	0.10	5.8	0.01	0.36	0.02	0.11	0.06	9.2
0.1	0	GMM-Tuk-Han	0.04	0.72	0.12	0.24	0.16	7.9	0.01	0.37	0.02	0.11	0.07	10.4
0.1	0	GMM-BR	0.03	0.78	0.17	0.27	0.10	4.1	0.01	0.37	0.02	0.11	0.04	5.4
0.1	0	GMM-BRM	0.04	0.87	0.18	0.29	0.34	1.9	0.01	0.41	0.03	0.13	0.32	2.5
0.1	0	GMM-Trunc	0.04	0.74	0.13	0.24	0.13	4.0	0.01	0.37	0.02	0.11	0.07	6.0
0.1	0	GMM-Trunc-BC	0.01	1.32	1.01	0.50	0.37	4.0	-0.02	0.48	0.04	0.15	0.16	6.0
0.1	0	CUE-1	0.00	8.63	50.71	3.32	0.08	1.0	-0.01	5.66	42.95	2.62	0.26	1.0
0.1	0	CUE-25	-0.21	17.96	86.91	6.03	0.99	25.0	0.29	33.82	133.01	8.16	1.00	25.0
0.1	0	WWA	0.02	0.90	0.28	0.31	0.06	0.0	0.00	0.40	0.02	0.12	0.04	0.0
0.1	0.5	OLS	0.05	0.17	0.01	0.07	0.05	0.0	0.06	0.09	0.00	0.06	0.21	0.0
0.1	0.5	GMM-1	0.01	1.00	0.42	0.35	0.05	1.0	-0.01	0.35	0.02	0.11	0.04	1.0
0.1	0.5	GMM-25	0.02	0.60	0.07	0.19	0.57	25.0	0.01	0.24	0.01	0.08	0.16	25.0
0.1	0.5	GMM-Bartlett	0.02	0.69	0.18	0.25	0.13	6.3	0.00	0.30	0.02	0.10	0.10	10.6
0.1	0.5	GMM-Tuk-Han	0.02	0.66	0.12	0.22	0.18	8.6	0.00	0.30	0.02	0.10	0.10	11.8
0.1	0.5	GMM-BR	0.02	0.72	0.20	0.26	0.11	4.5	0.00	0.31	0.02	0.10	0.08	6.1
0.1	0.5	GMM-BRM	0.02	0.79	0.22	0.28	0.35	2.1	0.00	0.34	0.02	0.11	0.33	2.7
0.1	0.5	GMM-Trunc	0.03	0.69	0.25	0.25	0.13	5.0	0.00	0.31	0.02	0.10	0.09	7.0
0.1	0.5	GMM-Trunc-BC	-0.06	1.23	1.36	0.51	0.44	5.0	-0.04	0.40	0.03	0.13	0.21	7.0
0.1	0.5	CUE-1	0.03	10.26	50.51	3.59	0.13	1.0	-0.05	7.15	46.32	2.97	0.39	1.0
0.1	0.5	CUE-25	-0.53	6.87	36.76	3.16	1.00	25.0	0.21	8.03	36.51	3.51	0.98	25.0
0.1	0.5	WWA	0.00	1.17	619.99	3.61	0.20	0.0	-0.01	0.32	0.02	0.10	0.06	0.0
0.1	0.9	OLS	0.04	0.18	0.01	0.07	0.01	0.0	0.04	0.10	0.00	0.05	0.06	0.0
0.1	0.9	GMM-1	0.00	1.16	0.49	0.41	0.06	1.0	-0.01	0.40	0.03	0.13	0.05	1.0
0.1	0.9	GMM-25	0.02	0.52	0.07	0.17	0.56	25.0	0.01	0.19	0.01	0.06	0.15	25.0
0.1	0.9	GMM-Bartlett	0.01	0.70	0.12	0.24	0.15	6.3	0.00	0.29	0.01	0.09	0.07	10.1
0.1	0.9	GMM-Tuk-Han	0.01	0.64	0.08	0.20	0.20	10.2	0.00	0.26	0.01	0.08	0.08	13.7
0.1	0.9	GMM-BR	0.02	0.76	0.14	0.25	0.15	5.3	0.01	0.31	0.02	0.10	0.08	7.1
0.1	0.9	GMM-BRM	0.01	0.84	0.20	0.29	0.35	2.4	0.00	0.35	0.02	0.11	0.31	3.2
0.1	0.9	GMM-Trunc	0.03	0.70	0.23	0.25	0.15	5.0	0.01	0.27	0.01	0.09	0.08	7.0
0.1	0.9	GMM-Trunc-BC	-0.13	1.63	1.14E+09	1069.50	0.59	5.0	-0.11	0.55	0.06	0.19	0.46	7.0
0.1	0.9	CUE-1	-0.01	12.40	55.32	3.96	0.16	1.0	-0.05	8.10	37.22	2.95	0.46	1.0
0.1	0.9	CUE-25	-0.66	4.19	17.14	2.15	0.97	25.0	-0.04	3.49	7.94	1.64	0.93	25.0
0.1	0.9	WWA	-0.02	31.78	1.7E+04	23.71	0.54	0.0	0.00	4.59	5.1E+03	7.69	0.43	0.0

Table 9b - Heteroskedastic DGP with Robust Weight Matrix ϕ=.5, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	OLS	0.53	0.33	0.30	0.54	1.00	0.0	0.54	0.16	0.30	0.54	1.00	0.0
0.5	-0.9	GMM-1	0.11	1.47	0.94	0.54	0.04	1.0	-0.02	0.67	0.08	0.21	0.02	1.0
0.5	-0.9	GMM-25	0.33	0.89	0.25	0.40	0.68	25.0	0.27	0.44	0.10	0.28	0.46	25.0
0.5	-0.9	GMM-Bartlett	0.24	1.14	0.31	0.42	0.18	3.6	0.03	0.65	0.06	0.20	0.08	3.3
0.5	-0.9	GMM-Tuk-Han	0.30	1.01	0.28	0.41	0.29	7.0	0.07	0.61	0.06	0.20	0.10	6.7
0.5	-0.9	GMM-BR	0.29	1.13	0.33	0.44	0.22	3.7	0.08	0.64	0.07	0.21	0.09	3.5
0.5	-0.9	GMM-BRM	0.26	1.33	0.46	0.50	0.43	1.7	0.06	0.71	0.09	0.23	0.27	1.6
0.5	-0.9	GMM-Trunc	0.27	1.35	0.69	0.52	0.27	2.0	0.02	0.75	0.10	0.24	0.10	1.0
0.5	-0.9	GMM-Trunc-BC	-0.14	2.59	5.05	0.97	0.41	2.0	-0.28	0.95	0.23	0.38	0.18	1.0
0.5	-0.9	CUE-1	-0.09	5.95	35.50	2.60	0.05	1.0	-0.06	1.00	9.94	0.97	0.09	1.0
0.5	-0.9	CUE-25	-0.16	22.33	101.60	6.51	0.96	25.0	-0.40	11.12	57.47	3.90	0.73	25.0
0.5	-0.9	WWA	0.12	1.27	0.85	0.48	0.12	0.0	0.02	0.59	0.06	0.19	0.09	0.0
0.5	-0.5	OLS	0.46	0.27	0.23	0.46	1.00	0.0	0.46	0.14	0.22	0.47	1.00	0.0
0.5	-0.5	GMM-1	0.08	1.25	1.06	0.46	0.05	1.0	-0.01	0.54	0.05	0.17	0.02	1.0
0.5	-0.5	GMM-25	0.23	0.74	0.15	0.31	0.62	25.0	0.22	0.34	0.07	0.23	0.45	25.0
0.5	-0.5	GMM-Bartlett	0.20	0.94	0.49	0.37	0.18	3.8	0.04	0.51	0.04	0.16	0.07	3.8
0.5	-0.5	GMM-Tuk-Han	0.23	0.87	0.19	0.34	0.28	6.2	0.07	0.49	0.04	0.16	0.10	5.9
0.5	-0.5	GMM-BR	0.22	1.01	0.66	0.40	0.20	3.2	0.05	0.52	0.04	0.17	0.08	3.1
0.5	-0.5	GMM-BRM	0.21	1.14	0.96	0.46	0.40	1.5	0.04	0.59	0.06	0.19	0.27	1.4
0.5	-0.5	GMM-Trunc	0.22	1.03	0.65	0.41	0.24	2.0	0.06	0.57	0.06	0.19	0.11	3.0
0.5	-0.5	GMM-Trunc-BC	0.00	1.71	35.52	0.77	0.28	2.0	-0.07	0.61	0.07	0.20	0.08	3.0
0.5	-0.5	CUE-1	-0.03	4.73	35.64	2.52	0.07	1.0	-0.05	0.75	10.14	0.86	0.09	1.0
0.5	-0.5	CUE-25	0.40	16.98	83.29	5.78	0.97	25.0	-0.01	13.94	65.98	4.61	0.84	25.0
0.5	-0.5	WWA	0.13	1.06	86.04	0.72	0.12	0.0	0.02	0.50	0.04	0.15	0.08	0.0
0.5	0	OLS	0.36	0.21	0.15	0.37	1.00	0.0	0.37	0.11	0.14	0.37	1.00	0.0
0.5	0	GMM-1	0.05	0.98	0.29	0.33	0.08	1.0	-0.01	0.40	0.03	0.13	0.04	1.0
0.5	0	GMM-25	0.17	0.62	0.09	0.24	0.64	25.0	0.16	0.26	0.04	0.17	0.48	25.0
0.5	0	GMM-Bartlett	0.13	0.79	0.18	0.30	0.27	4.9	0.04	0.40	0.03	0.13	0.12	5.3
0.5	0	GMM-Tuk-Han	0.17	0.71	0.15	0.28	0.36	6.8	0.06	0.38	0.03	0.13	0.15	7.1
0.5	0	GMM-BR	0.15	0.82	0.19	0.31	0.27	3.5	0.04	0.40	0.03	0.13	0.11	3.7
0.5	0	GMM-BRM	0.13	0.87	0.20	0.32	0.40	1.6	0.03	0.46	0.04	0.15	0.30	1.7
0.5	0	GMM-Trunc	0.17	0.76	0.15	0.29	0.29	4.0	0.07	0.40	0.03	0.14	0.16	4.0
0.5	0	GMM-Trunc-BC	0.03	1.21	0.77	0.44	0.32	4.0	0.00	0.46	0.04	0.15	0.13	4.0
0.5	0	CUE-1	-0.02	5.80	35.93	2.56	0.10	1.0	-0.03	1.93	21.64	1.59	0.20	1.0
0.5	0	CUE-25	-0.08	14.60	72.07	5.10	0.99	25.0	1.41	18.42	91.58	6.32	0.99	25.0
0.5	0	WWA	0.09	0.89	0.25	0.31	0.09	0.0	0.01	0.40	0.03	0.12	0.07	0.0
0.5	0.5	OLS	0.27	0.19	0.08	0.28	0.98	0.0	0.28	0.10	0.08	0.28	1.00	0.0
0.5	0.5	GMM-1	0.03	0.93	0.33	0.34	0.06	1.0	-0.01	0.35	0.03	0.12	0.05	1.0
0.5	0.5	GMM-25	0.13	0.62	0.10	0.23	0.68	25.0	0.09	0.22	0.02	0.10	0.36	25.0
0.5	0.5	GMM-Bartlett	0.07	0.75	0.21	0.27	0.19	4.5	0.01	0.34	0.02	0.11	0.10	4.8
0.5	0.5	GMM-Tuk-Han	0.09	0.65	0.16	0.25	0.26	6.7	0.02	0.31	0.02	0.10	0.11	6.9
0.5	0.5	GMM-BR	0.09	0.77	0.23	0.28	0.21	3.5	0.01	0.33	0.02	0.11	0.10	3.6
0.5	0.5	GMM-BRM	0.08	0.82	0.26	0.31	0.35	1.6	0.01	0.36	0.02	0.12	0.32	1.6
0.5	0.5	GMM-Trunc	0.09	0.74	0.18	0.27	0.21	3.0	0.02	0.32	0.02	0.10	0.11	4.0
0.5	0.5	GMM-Trunc-BC	-0.15	1.15	1.52	0.49	0.35	3.0	-0.08	0.35	0.03	0.13	0.16	4.0
0.5	0.5	CUE-1	-0.06	6.54	37.86	2.78	0.14	1.0	-0.06	3.52	15.03	1.47	0.33	1.0
0.5	0.5	CUE-25	-0.91	3.85	23.16	2.33	0.99	25.0	-1.23	4.10	18.78	2.25	0.98	25.0
0.5	0.5	WWA	0.02	0.87	1.7E+03	3.82	0.14	0.0	0.00	0.32	0.02	0.10	0.05	0.0
0.5	0.9	OLS	0.20	0.20	0.05	0.21	0.53	0.0	0.21	0.10	0.04	0.21	1.00	0.0
0.5	0.9	GMM-1	0.02	1.07	0.55	0.41	0.06	1.0	-0.01	0.40	0.03	0.13	0.06	1.0
0.5	0.9	GMM-25	0.07	0.62	0.10	0.20	0.66	25.0	0.03	0.19	0.01	0.07	0.25	25.0
0.5	0.9	GMM-Bartlett	0.05	0.81	0.24	0.29	0.15	4.5	0.00	0.33	0.02	0.11	0.11	4.7
0.5	0.9	GMM-Tuk-Han	0.06	0.67	0.16	0.24	0.20	8.1	0.01	0.28	0.01	0.09	0.11	8.5
0.5	0.9	GMM-BR	0.06	0.82	0.21	0.29	0.17	4.2	0.01	0.35	0.02	0.11	0.14	4.4
0.5	0.9	GMM-BRM	0.05	0.94	0.30	0.33	0.38	1.9	0.00	0.36	0.02	0.12	0.31	2.0
0.5	0.9	GMM-Trunc	0.07	0.85	0.33	0.32	0.16	2.0	0.01	0.36	0.02	0.11	0.09	2.0
0.5	0.9	GMM-Trunc-BC	-0.33	1.51	9.37E+10	1.03E+04	0.48	2.0	-0.21	0.43	0.07	0.23	0.43	2.0
0.5	0.9	CUE-1	-0.05	8.57	42.98	3.18	0.16	1.0	-0.09	6.26	18.14	2.08	0.45	1.0
0.5	0.9	CUE-25	-1.31	3.67	10.11	1.90	0.99	25.0	-1.48	3.10	3.78	1.52	0.96	25.0
0.5	0.9	WWA	-0.04	9.58	9.3E+03	15.13	0.36	0.0	-0.02	0.41	240.23	1.32	0.14	0.0

Table 9c - Heteroskedastic DGP with Robust Weight Matrix ϕ=.5, δ=.2

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	OLS	0.96	0.25	0.94	0.96	1.00	0.0	0.97	0.12	0.94	0.97	1.00	0.0
0.9	-0.9	GMM-1	0.12	1.39	1.90	0.59	0.13	1.0	-0.01	0.67	0.09	0.22	0.06	1.0
0.9	-0.9	GMM-25	0.69	0.63	0.58	0.72	0.97	25.0	0.51	0.36	0.29	0.52	0.90	25.0
0.9	-0.9	GMM-Bartlett	0.42	1.10	0.38	0.51	0.51	5.1	0.07	0.63	0.07	0.21	0.14	2.9
0.9	-0.9	GMM-Tuk-Han	0.52	0.95	0.43	0.56	0.65	9.2	0.14	0.57	0.07	0.22	0.22	6.4
0.9	-0.9	GMM-BR	0.51	1.11	0.44	0.57	0.59	4.8	0.13	0.71	0.09	0.25	0.24	3.3
0.9	-0.9	GMM-BRM	0.48	1.26	0.48	0.58	0.66	2.1	0.09	0.74	0.10	0.25	0.37	1.5
0.9	-0.9	GMM-Trunc	0.29	1.51	0.78	0.57	0.38	1.0	0.00	0.71	0.10	0.24	0.12	1.0
0.9	-0.9	GMM-Trunc-BC	0.13	1.96	1.73	0.69	0.41	1.0	-0.15	0.96	0.19	0.32	0.20	1.0
0.9	-0.9	CUE-1	-0.22	3.97	23.71	1.96	0.04	1.0	-0.09	0.95	4.62	0.64	0.06	1.0
0.9	-0.9	CUE-25	0.17	17.14	81.07	5.47	0.90	25.0	-0.55	7.87	47.97	3.38	0.65	25.0
0.9	-0.9	WWA	0.23	1.09	0.45	0.45	0.37	0.0	0.05	0.62	0.07	0.20	0.30	0.0
0.9	-0.5	OLS	0.83	0.21	0.70	0.83	1.00	0.0	0.83	0.10	0.70	0.84	1.00	0.0
0.9	-0.5	GMM-1	0.11	1.14	2.22	0.50	0.13	1.0	0.00	0.55	0.07	0.18	0.04	1.0
0.9	-0.5	GMM-25	0.51	0.44	0.32	0.53	0.91	25.0	0.45	0.24	0.20	0.44	0.92	25.0
0.9	-0.5	GMM-Bartlett	0.30	0.84	0.34	0.40	0.43	3.2	0.06	0.50	0.04	0.16	0.11	3.4
0.9	-0.5	GMM-Tuk-Han	0.38	0.75	0.24	0.42	0.55	5.6	0.12	0.47	0.04	0.17	0.19	5.7
0.9	-0.5	GMM-BR	0.35	0.89	0.45	0.44	0.48	2.9	0.10	0.51	0.05	0.17	0.17	3.0
0.9	-0.5	GMM-BRM	0.31	1.13	2.07	0.52	0.56	1.3	0.07	0.56	0.06	0.19	0.33	1.3
0.9	-0.5	GMM-Trunc	0.18	1.21	1.44	0.48	0.26	1.0	0.03	0.58	0.06	0.19	0.10	2.0
0.9	-0.5	GMM-Trunc-BC	0.10	1.26	1.92	0.50	0.23	1.0	-0.01	0.57	0.07	0.19	0.08	2.0
0.9	-0.5	CUE-1	-0.16	3.76	28.38	2.09	0.09	1.0	-0.06	0.88	5.98	0.70	0.09	1.0
0.9	-0.5	CUE-25	0.46	14.02	67.93	4.77	0.93	25.0	-0.09	7.82	46.87	3.41	0.75	25.0
0.9	-0.5	WWA	0.22	0.93	5.44	0.51	0.42	0.0	0.05	0.48	0.04	0.16	0.31	0.0
0.9	0	OLS	0.66	0.18	0.45	0.67	1.00	0.0	0.67	0.09	0.45	0.67	1.00	0.0
0.9	0	GMM-1	0.08	0.91	0.78	0.37	0.15	1.0	0.00	0.43	0.04	0.14	0.07	1.0
0.9	0	GMM-25	0.36	0.39	0.17	0.37	0.84	25.0	0.33	0.20	0.11	0.32	0.92	25.0
0.9	0	GMM-Bartlett	0.25	0.81	0.39	0.37	0.46	4.1	0.07	0.38	0.03	0.14	0.19	4.8
0.9	0	GMM-Tuk-Han	0.30	0.65	0.23	0.36	0.54	6.2	0.11	0.37	0.03	0.15	0.27	6.7
0.9	0	GMM-BR	0.27	0.85	0.54	0.40	0.49	3.2	0.08	0.39	0.03	0.14	0.22	3.5
0.9	0	GMM-BRM	0.22	1.00	0.67	0.42	0.52	1.4	0.07	0.47	0.05	0.17	0.39	1.6
0.9	0	GMM-Trunc	0.21	0.88	0.29	0.35	0.35	2.0	0.08	0.42	0.04	0.15	0.20	3.0
0.9	0	GMM-Trunc-BC	0.11	1.07	1.07	0.41	0.29	2.0	0.04	0.42	0.03	0.14	0.15	3.0
0.9	0	CUE-1	-0.11	3.31	26.97	2.04	0.13	1.0	-0.04	0.67	3.12	0.52	0.10	1.0
0.9	0	CUE-25	-0.32	4.31	31.57	2.40	0.84	25.0	-0.71	3.54	23.42	2.18	0.87	25.0
0.9	0	WWA	0.16	0.76	0.21	0.30	0.27	0.0	0.02	0.40	0.03	0.13	0.21	0.0
0.9	0.5	OLS	0.50	0.18	0.26	0.50	1.00	0.0	0.50	0.09	0.25	0.50	1.00	0.0
0.9	0.5	GMM-1	0.02	0.94	1.44	0.39	0.09	1.0	0.00	0.38	0.03	0.12	0.06	1.0
0.9	0.5	GMM-25	0.24	0.55	0.13	0.29	0.82	25.0	0.16	0.19	0.03	0.16	0.76	25.0
0.9	0.5	GMM-Bartlett	0.12	0.70	0.24	0.28	0.33	4.0	0.02	0.32	0.02	0.10	0.19	4.6
0.9	0.5	GMM-Tuk-Han	0.16	0.61	0.21	0.26	0.43	6.5	0.03	0.29	0.02	0.10	0.22	7.0
0.9	0.5	GMM-BR	0.14	0.77	0.26	0.30	0.39	3.4	0.01	0.37	0.02	0.12	0.25	3.6
0.9	0.5	GMM-BRM	0.13	0.91	0.35	0.34	0.47	1.5	0.03	0.37	0.02	0.12	0.39	1.6
0.9	0.5	GMM-Trunc	0.09	0.94	0.28	0.32	0.22	2.0	0.01	0.35	0.03	0.11	0.14	2.0
0.9	0.5	GMM-Trunc-BC	-0.16	1.13	5.37	0.52	0.23	2.0	-0.08	0.33	0.04	0.13	0.13	2.0
0.9	0.5	CUE-1	-0.15	5.38	27.50	2.29	0.15	1.0	-0.08	2.89	7.02	0.97	0.25	1.0
0.9	0.5	CUE-25	-1.05	1.89	6.22	1.48	1.00	25.0	-1.09	0.88	2.06	1.19	1.00	25.0
0.9	0.5	WWA	0.07	0.72	1.1E+03	1.32	0.08	0.0	0.01	0.33	0.02	0.11	0.07	0.0
0.9	0.9	OLS	0.36	0.21	0.15	0.37	1.00	0.0	0.37	0.11	0.14	0.37	1.00	0.0
0.9	0.9	GMM-1	0.03	1.12	0.96	0.43	0.06	1.0	0.00	0.40	0.03	0.13	0.06	1.0
0.9	0.9	GMM-25	0.12	0.69	0.13	0.24	0.78	25.0	0.05	0.20	0.01	0.08	0.50	25.0
0.9	0.9	GMM-Bartlett	0.09	0.86	0.27	0.31	0.35	5.1	0.00	0.35	0.02	0.11	0.29	4.6
0.9	0.9	GMM-Tuk-Han	0.09	0.75	0.21	0.27	0.40	8.7	0.01	0.30	0.02	0.10	0.32	8.3
0.9	0.9	GMM-BR	0.10	0.92	0.28	0.33	0.41	4.5	0.01	0.41	0.03	0.13	0.37	4.3
0.9	0.9	GMM-BRM	0.12	1.25	0.42	0.42	0.54	2.0	0.02	0.44	0.04	0.14	0.44	2.0
0.9	0.9	GMM-Trunc	0.09	1.03	0.32	0.36	0.27	2.0	0.00	0.41	0.03	0.13	0.13	1.0
0.9	0.9	GMM-Trunc-BC	-0.24	1.53	5.66E+11	31949.00	0.44	2.0	-0.15	0.45	0.07	0.20	0.31	1.0
0.9	0.9	CUE-1	-0.17	8.47	41.96	3.11	0.18	1.0	-0.15	5.65	21.59	2.24	0.43	1.0
0.9	0.9	CUE-25	-1.73	3.78	6.04	1.97	1.00	25.0	-1.72	1.07	3.35	1.72	0.99	25.0
0.9	0.9	WWA	0.00	0.84	991.19	1.57	0.05	0.0	0.00	0.34	0.02	0.11	0.02	0.0

Table 10a - Homoskedastic DGP ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	GMM-Tuk-Han	0.11	1.38	0.41	0.46	0.03	7.5	0.14	1.09	0.21	0.36	0.02	9.9
0.1	-0.9	GMM-BR	0.12	1.87	0.79	0.62	0.06	3.9	0.13	1.51	0.48	0.51	0.04	5.2
0.1	-0.9	GMM-BRM	0.10	2.31	1.63	0.82	0.36	1.8	0.12	2.00	1.00	0.70	0.38	2.3
0.1	-0.9	GMM-Trunc	0.23	2.50	2.69	0.92	0.07	1.0	0.26	2.01	2.06	0.80	0.08	2.0
0.1	-0.9	GMM-Trunc-BC	0.14	4.62	4.26E+11	28,412.00	0.38	1.0	0.28	3.06	2.72	1.10	0.32	2.0
0.1	-0.9	GMM-Tuk-Ha20	0.11	1.39	0.41	0.46	0.03	7.5	0.14	1.09	0.21	0.36	0.02	9.9
0.1	-0.9	GMM-BR20	0.12	1.87	0.79	0.62	0.06	3.9	0.13	1.51	0.48	0.51	0.04	5.2
0.1	-0.9	GMM-BRM20	0.10	2.31	1.63	0.82	0.36	1.8	0.12	2.00	1.00	0.70	0.38	2.3
0.1	-0.9	GMM-Trunc20	0.23	2.50	2.69	0.92	0.07	1.0	0.26	2.01	2.06	0.80	0.08	2.0
0.1	-0.9	GMM-Tr-BC20	0.14	4.62	4.26E+11	28,412.00	0.38	1.0	0.28	3.06	2.72	1.10	0.32	2.0
0.1	-0.5	GMM-Tuk-Han	0.11	1.44	0.65	0.51	0.02	4.5	0.10	1.26	0.47	0.44	0.01	5.0
0.1	-0.5	GMM-BR	0.10	1.91	1.87	0.68	0.03	2.4	0.10	1.64	0.84	0.58	0.01	2.6
0.1	-0.5	GMM-BRM	0.11	2.13	2.66	0.81	0.23	1.1	0.07	2.27	2.62	0.83	0.24	1.2
0.1	-0.5	GMM-Trunc	0.16	2.13	17.58	0.89	0.04	1.0	0.14	2.07	2.27	0.72	0.04	2.0
0.1	-0.5	GMM-Trunc-BC	0.15	12.32	1.2E+07	149.18	0.45	1.0	0.14	4.06	165.95	2.11	0.28	2.0
0.1	-0.5	GMM-Tuk-Ha20	0.11	1.44	0.65	0.51	0.02	4.5	0.10	1.26	0.47	0.44	0.01	5.0
0.1	-0.5	GMM-BR20	0.10	1.92	1.88	0.68	0.03	2.4	0.10	1.64	0.84	0.58	0.01	2.6
0.1	-0.5	GMM-BRM20	0.11	2.13	2.66	0.81	0.23	1.1	0.07	2.27	2.62	0.83	0.24	1.2
0.1	-0.5	GMM-Trunc20	0.16	2.13	17.58	0.89	0.04	1.0	0.14	2.07	2.27	0.72	0.04	2.0
0.1	-0.5	GMM-Tr-BC20	0.15	12.32	1.2E+07	149.18	0.45	1.0	0.14	4.06	165.95	2.11	0.28	2.0
0.1	0	GMM-Tuk-Han	0.09	1.71	1.57	0.64	0.03	2.1	0.09	1.88	13.23	0.78	0.01	1.6
0.1	0	GMM-BR	0.10	1.80	1.64	0.67	0.02	1.1	0.09	1.85	13.31	0.79	0.01	0.8
0.1	0	GMM-BRM	0.11	1.94	1.71	0.71	0.18	0.5	0.08	2.07	13.73	0.87	0.14	0.4
0.1	0	GMM-Trunc	0.11	1.99	1.53	0.69	0.03	1.0	0.12	2.02	1.69	0.71	0.02	1.0
0.1	0	GMM-Trunc-BC	0.08	167.35	6.7E+09	3,383.00	0.59	1.0	0.12	109.30	5.3E+07	623.86	0.60	1.0
0.1	0	GMM-Tuk-Ha20	0.09	1.74	3.47	0.71	0.03	2.1	0.09	1.88	13.23	0.78	0.01	1.6
0.1	0	GMM-BR20	0.10	1.80	1.64	0.67	0.02	1.1	0.09	1.85	13.31	0.79	0.01	0.8
0.1	0	GMM-BRM20	0.11	1.94	1.71	0.71	0.18	0.5	0.08	2.07	13.73	0.87	0.14	0.4
0.1	0	GMM-Trunc20	0.11	1.99	1.53	0.69	0.03	1.0	0.12	2.02	1.69	0.71	0.02	1.0
0.1	0	GMM-Tr-BC20	0.08	167.35	6.7E+09	3,383.00	0.59	1.0	0.12	109.30	5.3E+07	623.86	0.60	1.0
0.1	0.5	GMM-Tuk-Han	0.10	1.50	0.60	0.51	0.03	4.8	0.10	1.24	0.36	0.41	0.01	5.4
0.1	0.5	GMM-BR	0.10	1.97	3.76	0.73	0.03	2.5	0.09	1.62	1.13	0.58	0.02	2.8
0.1	0.5	GMM-BRM	0.08	2.30	4.44	0.88	0.23	1.1	0.09	2.23	1.79	0.76	0.22	1.3
0.1	0.5	GMM-Trunc	0.10	2.06	1.19	0.68	0.04	1.0	0.13	1.74	1.53	0.65	0.03	2.0
0.1	0.5	GMM-Trunc-BC	0.06	14.90	1.0E+05	43.47	0.44	1.0	0.10	3.76	711.28	2.56	0.23	2.0
0.1	0.5	GMM-Tuk-Ha20	0.10	1.51	0.62	0.51	0.03	4.8	0.10	1.24	0.36	0.41	0.01	5.4
0.1	0.5	GMM-BR20	0.10	1.97	3.94	0.74	0.03	2.5	0.09	1.62	1.13	0.58	0.02	2.8
0.1	0.5	GMM-BRM20	0.08	2.30	4.44	0.88	0.23	1.1	0.09	2.23	1.79	0.76	0.22	1.3
0.1	0.5	GMM-Trunc20	0.10	2.06	1.19	0.68	0.04	1.0	0.13	1.74	1.53	0.65	0.03	2.0
0.1	0.5	GMM-Tr-BC20	0.06	14.90	1.0E+05	43.47	0.44	1.0	0.10	3.76	711.28	2.56	0.23	2.0
0.1	0.9	GMM-Tuk-Han	0.11	1.24	0.40	0.42	0.03	8.3	0.12	0.95	0.17	0.32	0.02	11.2
0.1	0.9	GMM-BR	0.15	1.78	0.83	0.61	0.06	4.3	0.11	1.30	0.37	0.44	0.04	5.8
0.1	0.9	GMM-BRM	0.10	2.31	2.04	0.83	0.33	1.9	0.12	1.85	8.2E-01	0.63	0.33	2.6
0.1	0.9	GMM-Trunc	0.19	2.22	2.84	0.86	0.06	2.0	0.21	1.68	1.43	0.67	0.06	4.0
0.1	0.9	GMM-Trunc-BC	0.15	4.01	2.19E+13	2.12E+05	0.37	2.0	0.19	2.56	1.91	0.92	0.27	4.0
0.1	0.9	GMM-Tuk-Ha20	0.11	1.24	0.40	0.42	0.03	8.3	0.12	0.95	0.17	0.32	0.02	11.2
0.1	0.9	GMM-BR20	0.15	1.78	0.83	0.61	0.06	4.3	0.11	1.30	0.37	0.44	0.04	5.8
0.1	0.9	GMM-BRM20	0.10	2.31	2.04	0.83	0.33	1.9	0.12	1.85	0.82	0.63	0.33	2.6
0.1	0.9	GMM-Trunc20	0.19	2.22	2.84	0.86	0.06	2.0	0.21	1.68	1.43	0.67	0.06	4.0
0.1	0.9	GMM-Tr-BC20	0.15	4.01	2.19E+13	2.12E+05	0.37	2.0	0.19	2.56	1.91	0.92	0.27	4.0

Table 10b - Homoskedastic DGP ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	GMM-Tuk-Han	0.55	1.19	0.52	0.61	0.31	9.2	0.57	0.86	0.43	0.58	0.38	12.7
0.5	-0.9	GMM-BR	0.62	1.48	1.10	0.75	0.28	4.8	0.58	1.18	0.55	0.64	0.30	6.6
0.5	-0.9	GMM-BRM	0.61	2.05	3.35	0.93	0.55	2.2	0.62	1.65	0.89	0.77	0.61	3.0
0.5	-0.9	GMM-Trunc	0.64	1.78	4.82	1.01	0.38	3.0	0.66	1.49	1.40	0.89	0.47	6.0
0.5	-0.9	GMM-Trunc-BC	0.43	3.53	1.38E+10	4,258.10	0.42	3.0	0.46	2.42	1.70	0.93	0.39	6.0
0.5	-0.9	GMM-Tuk-Ha20	0.55	1.21	0.56	0.62	0.31	9.2	0.57	0.86	0.43	0.58	0.38	12.7
0.5	-0.9	GMM-BR20	0.62	1.48	1.10	0.75	0.28	4.8	0.58	1.18	0.55	0.64	0.30	6.6
0.5	-0.9	GMM-BRM20	0.61	2.05	3.35	0.93	0.55	2.2	0.62	1.65	0.89	0.77	0.61	3.0
0.5	-0.9	GMM-Trunc20	0.64	1.78	4.82	1.01	0.38	3.0	0.66	1.49	1.40	0.89	0.47	6.0
0.5	-0.9	GMM-Tr-BC20	0.43	3.53	1.38E+10	4,258.10	0.42	3.0	0.46	2.42	1.70	0.93	0.39	6.0
0.5	-0.5	GMM-Tuk-Han	0.52	1.20	0.65	0.63	0.28	5.2	0.52	1.05	0.50	0.58	0.26	6.2
0.5	-0.5	GMM-BR	0.56	1.57	1.35	0.78	0.22	2.7	0.55	1.29	0.81	0.69	0.20	3.2
0.5	-0.5	GMM-BRM	0.57	1.87	5.22	0.96	0.44	1.2	0.56	1.77	1.99	0.87	0.47	1.5
0.5	-0.5	GMM-Trunc	0.58	1.76	2.64	0.87	0.28	2.0	0.60	1.78	1.44	0.85	0.30	2.0
0.5	-0.5	GMM-Trunc-BC	0.55	4.51	651.99	3.81	0.44	2.0	0.57	2.85	2.80	1.12	0.38	2.0
0.5	-0.5	GMM-Tuk-Ha20	0.52	1.20	0.65	0.63	0.28	5.2	0.52	1.05	0.50	0.58	0.26	6.2
0.5	-0.5	GMM-BR20	0.56	1.57	1.35	0.78	0.22	2.7	0.55	1.29	0.81	0.69	0.20	3.2
0.5	-0.5	GMM-BRM20	0.57	1.87	5.22	0.96	0.44	1.2	0.56	1.77	1.99	0.87	0.47	1.5
0.5	-0.5	GMM-Trunc20	0.58	1.76	2.64	0.87	0.28	2.0	0.60	1.78	1.44	0.85	0.30	2.0
0.5	-0.5	GMM-Tr-BC20	0.55	4.51	651.99	3.81	0.44	2.0	0.57	2.85	2.80	1.12	0.38	2.0
0.5	0	GMM-Tuk-Han	0.49	1.52	2.69	0.80	0.30	2.2	0.50	1.56	39.18	0.96	0.24	1.6
0.5	0	GMM-BR	0.49	1.55	2.75	0.82	0.22	1.2	0.48	1.58	39.26	0.97	0.18	0.8
0.5	0	GMM-BRM	0.50	1.69	2.80	0.84	0.41	0.5	0.48	1.77	39.44	1.01	0.36	0.4
0.5	0	GMM-Trunc	0.49	1.70	2.40	0.83	0.24	1.0	0.50	1.71	1.49	0.81	0.24	1.0
0.5	0	GMM-Trunc-BC	0.49	127.56	4.1E+07	610.24	0.71	1.0	0.50	71.46	8.7E+06	293.23	0.74	1.0
0.5	0	GMM-Tuk-Ha20	0.48	1.54	45.18	1.01	0.30	2.2	0.50	1.56	39.18	0.96	0.24	1.6
0.5	0	GMM-BR20	0.49	1.56	2.87	0.83	0.22	1.2	0.48	1.58	39.26	0.97	0.18	0.8
0.5	0	GMM-BRM20	0.50	1.69	2.80	0.84	0.41	0.5	0.48	1.77	39.44	1.01	0.36	0.4
0.5	0	GMM-Trunc20	0.49	1.70	2.40	0.83	0.24	1.0	0.50	1.71	1.49	0.81	0.24	1.0
0.5	0	GMM-Tr-BC20	0.49	127.56	4.1E+07	610.24	0.71	1.0	0.50	71.46	8.7E+06	293.23	0.74	1.0
0.5	0.5	GMM-Tuk-Han	0.47	1.19	0.55	0.58	0.23	5.5	0.47	1.01	0.42	0.54	0.21	6.2
0.5	0.5	GMM-BR	0.51	1.67	2.00	0.78	0.20	2.9	0.51	1.36	0.80	0.66	0.16	3.2
0.5	0.5	GMM-BRM	0.49	1.89	2.60	0.88	0.41	1.3	0.53	1.71	1.85	0.82	0.42	1.5
0.5	0.5	GMM-Trunc	0.51	1.76	1.32	0.80	0.24	2.0	0.54	1.67	1.80	0.85	0.25	2.0
0.5	0.5	GMM-Trunc-BC	0.41	4.51	220.28	2.52	0.41	2.0	0.44	2.76	4.44	1.10	0.31	2.0
0.5	0.5	GMM-Tuk-Ha20	0.47	1.20	0.57	0.58	0.23	5.5	0.47	1.01	0.42	0.54	0.21	6.2
0.5	0.5	GMM-BR20	0.51	1.67	2.00	0.78	0.20	2.9	0.51	1.36	0.80	0.66	0.16	3.2
0.5	0.5	GMM-BRM20	0.49	1.89	2.60	0.88	0.41	1.3	0.53	1.71	1.85	0.82	0.42	1.5
0.5	0.5	GMM-Trunc20	0.51	1.76	1.32	0.80	0.24	2.0	0.54	1.67	1.80	0.85	0.25	2.0
0.5	0.5	GMM-Tr-BC20	0.41	4.51	220.28	2.52	0.41	2.0	0.44	2.76	4.44	1.10	0.31	2.0
0.5	0.9	GMM-Tuk-Han	0.50	1.13	0.45	0.56	0.24	9.2	0.49	0.85	0.34	0.51	0.27	12.4
0.5	0.9	GMM-BR	0.53	1.51	0.81	0.70	0.22	4.8	0.51	1.15	0.50	0.59	0.22	6.5
0.5	0.9	GMM-BRM	0.54	1.99	1.67	0.87	0.48	2.2	0.55	1.60	0.84	0.71	0.52	2.9
0.5	0.9	GMM-Trunc	0.55	1.89	11.44	1.00	0.30	4.0	0.58	1.47	1.75	0.86	0.38	5.0
0.5	0.9	GMM-Trunc-BC	0.22	3.73	1.90E+12	69,340.00	0.38	4.0	0.37	2.44	2.00	0.89	0.34	5.0
0.5	0.9	GMM-Tuk-Ha20	0.50	1.14	0.67	0.57	0.24	9.2	0.49	0.85	0.34	0.51	0.27	12.4
0.5	0.9	GMM-BR20	0.53	1.51	0.81	0.70	0.22	4.8	0.51	1.15	0.50	0.59	0.22	6.5
0.5	0.9	GMM-BRM20	0.54	1.99	1.67	0.87	0.48	2.2	0.55	1.60	0.84	0.71	0.52	2.9
0.5	0.9	GMM-Trunc20	0.55	1.89	11.44	1.00	0.30	4.0	0.58	1.47	1.75	0.86	0.38	5.0
0.5	0.9	GMM-Tr-BC20	0.22	3.73	1.90E+12	69,340.00	0.38	4.0	0.37	2.44	2.00	0.89	0.34	5.0

Table 10c - Homoskedastic DGP ϕ=.1

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	GMM-Tuk-Han	0.97	0.73	1.00	0.96	0.86	12.8	0.95	0.58	0.94	0.94	0.91	16.8
0.9	-0.9	GMM-BR	0.97	0.94	1.09	0.97	0.81	6.7	0.92	0.80	0.94	0.92	0.86	8.8
0.9	-0.9	GMM-BRM	1.01	1.22	1.42	1.06	0.87	3.1	1.02	1.04	1.19	1.01	0.92	4.0
0.9	-0.9	GMM-Trunc	1.03	0.91	1.63	1.11	0.75	5.0	1.02	0.71	1.26	1.05	0.82	7.0
0.9	-0.9	GMM-Trunc-BC	1.00	1.70	6.74	1.24	0.70	5.0	0.99	1.22	1.36	1.05	0.78	7.0
0.9	-0.9	GMM-Tuk-Ha20	0.97	0.74	8.83	1.08	0.86	12.8	0.95	0.58	0.94	0.94	0.91	16.8
0.9	-0.9	GMM-BR20	0.97	0.94	1.09	0.97	0.81	6.7	0.92	0.80	0.94	0.92	0.86	8.8
0.9	-0.9	GMM-BRM20	1.01	1.22	1.42	1.06	0.87	3.1	1.02	1.04	1.19	1.01	0.92	4.0
0.9	-0.9	GMM-Trunc20	1.03	0.91	1.63	1.11	0.75	5.0	1.02	0.71	1.26	1.05	0.82	7.0
0.9	-0.9	GMM-Tr-BC20	1.00	1.70	6.7E+00	1.24	0.70	5.0	0.99	1.22	1.4E+00	1.05	0.78	7.0
0.9	-0.5	GMM-Tuk-Han	0.94	0.62	0.96	0.94	0.85	7.0	0.93	0.55	0.92	0.93	0.89	8.0
0.9	-0.5	GMM-BR	0.96	0.74	1.20	1.00	0.79	3.7	0.96	0.68	1.01	0.95	0.83	4.2
0.9	-0.5	GMM-BRM	0.98	1.03	1.56	1.07	0.84	1.6	0.98	0.92	1.13	0.98	0.87	1.9
0.9	-0.5	GMM-Trunc	0.99	1.23	2.38	1.14	0.60	1.0	1.00	1.18	1.77	1.07	0.65	1.0
0.9	-0.5	GMM-Trunc-BC	0.97	1.77	2.75	1.19	0.60	1.0	0.99	1.46	1.82	1.07	0.63	1.0
0.9	-0.5	GMM-Tuk-Ha20	0.94	0.62	2.16	0.98	0.85	7.0	0.93	0.55	0.92	0.93	0.89	8.0
0.9	-0.5	GMM-BR20	0.96	0.74	1.20	1.00	0.79	3.7	0.96	0.68	1.01	0.95	0.83	4.2
0.9	-0.5	GMM-BRM20	0.98	1.03	1.56	1.07	0.84	1.6	0.98	0.92	1.13	0.98	0.87	1.9
0.9	-0.5	GMM-Trunc20	0.99	1.23	2.38	1.14	0.60	1.0	1.00	1.18	1.77	1.07	0.65	1.0
0.9	-0.5	GMM-Tr-BC20	0.97	1.77	2.75	1.19	0.60	1.0	0.99	1.46	1.82	1.07	0.63	1.0
0.9	0	GMM-Tuk-Han	0.89	0.75	1.18	0.95	0.75	2.7	0.88	0.82	1.30	0.94	0.70	1.9
0.9	0	GMM-BR	0.89	0.79	1.21	0.95	0.72	1.4	0.88	0.85	2.30	0.97	0.66	1.0
0.9	0	GMM-BRM	0.89	0.86	1.22	0.96	0.81	0.6	0.87	0.95	2.31	0.97	0.78	0.4
0.9	0	GMM-Trunc	0.89	1.04	1.50	0.99	0.61	1.0	0.87	1.08	1.44	0.95	0.56	1.0
0.9	0	GMM-Trunc-BC	0.90	18.44	1.5E+07	154.41	0.80	1.0	0.85	8.56	5.9E+04	19.54	0.72	1.0
0.9	0	GMM-Tuk-Ha20	0.89	0.76	1.23	0.96	0.75	2.7	0.88	0.82	1.30	0.94	0.70	1.9
0.9	0	GMM-BR20	0.89	0.80	1.21	0.96	0.72	1.4	0.88	0.85	2.30	0.97	0.66	1.0
0.9	0	GMM-BRM20	0.89	0.86	1.22	0.96	0.81	0.6	0.87	0.95	2.31	0.97	0.78	0.4
0.9	0	GMM-Trunc20	0.89	1.04	1.50	0.99	0.61	1.0	0.87	1.08	1.44	0.95	0.56	1.0
0.9	0	GMM-Tr-BC20	0.90	18.44	1.5E+07	154.41	0.80	1.0	0.85	8.56	5.9E+04	19.54	0.72	1.0
0.9	0.5	GMM-Tuk-Han	0.85	0.75	0.90	0.87	0.74	7.0	0.84	0.70	0.81	0.85	0.77	7.4
0.9	0.5	GMM-BR	0.88	1.02	1.29	0.95	0.68	3.7	0.88	0.90	0.96	0.89	0.72	3.8
0.9	0.5	GMM-BRM	0.87	1.29	1.94	1.00	0.74	1.7	0.88	1.16	1.35	0.95	0.78	1.7
0.9	0.5	GMM-Trunc	0.92	1.55	2.22	1.06	0.48	1.0	0.90	1.38	1.86	1.01	0.51	1.0
0.9	0.5	GMM-Trunc-BC	0.85	2.56	65.95	1.60	0.51	1.0	0.84	1.83	2.16	1.06	0.51	1.0
0.9	0.5	GMM-Tuk-Ha20	0.85	0.76	1.30	0.89	0.74	7.0	0.84	0.70	0.81	0.85	0.77	7.4
0.9	0.5	GMM-BR20	0.88	1.02	1.29	0.95	0.68	3.7	0.88	0.90	0.96	0.89	0.72	3.8
0.9	0.5	GMM-BRM20	0.87	1.29	1.94	1.00	0.74	1.7	0.88	1.16	1.35	0.95	0.78	1.7
0.9	0.5	GMM-Trunc20	0.92	1.55	2.22	1.06	0.48	1.0	0.90	1.38	1.86	1.01	0.51	1.0
0.9	0.5	GMM-Tr-BC20	0.85	2.56	65.95	1.60	0.51	1.0	0.84	1.83	2.16	1.06	0.51	1.0
0.9	0.9	GMM-Tuk-Han	0.79	0.82	0.77	0.81	0.69	12.0	0.80	0.72	0.72	0.80	0.76	15.5
0.9	0.9	GMM-BR	0.80	1.12	0.93	0.85	0.63	6.2	0.77	0.96	0.74	0.78	0.67	8.1
0.9	0.9	GMM-BRM	0.86	1.62	1.29	0.95	0.76	2.8	0.88	1.36	1.13	0.93	0.82	3.7
0.9	0.9	GMM-Trunc	0.87	1.24	1.54	0.98	0.62	5.0	0.89	0.88	1.19	0.96	0.72	7.0
0.9	0.9	GMM-Trunc-BC	0.74	2.18	2.58	1.11	0.53	5.0	0.77	1.58	1.37	0.96	0.61	7.0
0.9	0.9	GMM-Tuk-Ha20	0.79	0.84	0.86	0.82	0.69	12.0	0.80	0.72	0.72	0.80	0.76	15.5
0.9	0.9	GMM-BR20	0.80	1.13	1.06	0.86	0.63	6.2	0.77	0.96	0.73	0.78	0.67	8.1
0.9	0.9	GMM-BRM20	0.86	1.62	1.29	0.95	0.76	2.8	0.88	1.36	1.13	0.93	0.82	3.7
0.9	0.9	GMM-Trunc20	0.87	1.24	1.54	0.98	0.62	5.0	0.89	0.88	1.19	0.96	0.72	7.0
0.9	0.9	GMM-Tr-BC20	0.74	2.18	2.58	1.11	0.53	5.0	0.77	1.58	1.37	0.96	0.61	7.0

Table 11a - Homoskedastic DGP ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	GMM-Tuk-Han	0.14	1.27	0.37	0.44	0.04	6.9	0.08	1.02	0.18	0.33	0.04	9.1
0.1	-0.9	GMM-BR	0.15	1.68	0.93	0.59	0.07	3.6	0.07	1.24	0.30	0.41	0.05	4.7
0.1	-0.9	GMM-BRM	0.11	2.07	1.44	0.74	0.34	1.6	0.09	1.54	0.49	0.52	0.38	2.1
0.1	-0.9	GMM-Trunc	0.19	2.11	4.49	0.84	0.07	1.0	0.13	1.49	0.58	0.51	0.09	2.0
0.1	-0.9	GMM-Trunc-BC	0.08	4.34	1.64E+11	1.56E+04	0.40	1.0	0.02	2.37	1.10	0.80	0.33	2.0
0.1	-0.9	GMM-Tuk-Ha20	0.14	1.27	0.37	0.43	0.04	6.9	0.08	1.02	0.18	0.33	0.04	9.1
0.1	-0.9	GMM-BR20	0.15	1.68	0.93	0.59	0.07	3.6	0.07	1.24	0.30	0.41	0.05	4.7
0.1	-0.9	GMM-BRM20	0.11	2.07	1.44	0.74	0.34	1.6	0.09	1.54	0.49	0.52	0.38	2.1
0.1	-0.9	GMM-Trunc20	0.19	2.11	4.49	0.84	0.07	1.0	0.13	1.49	0.58	0.51	0.09	2.0
0.1	-0.9	GMM-Tr-BC20	0.08	4.34	1.64E+11	1.56E+04	0.40	1.0	0.02	2.37	1.10	0.80	0.33	2.0
0.1	-0.5	GMM-Tuk-Han	0.08	1.25	0.50	0.43	0.03	4.6	0.06	0.97	0.19	0.32	0.02	5.9
0.1	-0.5	GMM-BR	0.08	1.58	1.14	0.58	0.04	2.4	0.07	1.16	0.30	0.37	0.03	3.0
0.1	-0.5	GMM-BRM	0.09	1.87	2.91	0.71	0.24	1.1	0.06	1.37	0.53	0.46	0.27	1.4
0.1	-0.5	GMM-Trunc	0.10	1.75	1.09	0.61	0.04	2.0	0.08	1.19	0.47	0.41	0.04	2.0
0.1	-0.5	GMM-Trunc-BC	0.06	4.89	7.9E+01	2.47	0.41	2.0	0.04	2.07	2.72	0.82	0.27	2.0
0.1	-0.5	GMM-Tuk-Ha20	0.08	1.26	0.52	0.43	0.03	4.6	0.06	0.97	0.19	0.32	0.02	5.9
0.1	-0.5	GMM-BR20	0.08	1.58	1.14	0.58	0.04	2.4	0.07	1.16	0.30	0.37	0.03	3.0
0.1	-0.5	GMM-BRM20	0.09	1.87	2.91	0.71	0.24	1.1	0.06	1.37	0.53	0.46	0.27	1.4
0.1	-0.5	GMM-Trunc20	0.10	1.75	1.09	0.61	0.04	2.0	0.08	1.19	0.47	0.41	0.04	2.0
0.1	-0.5	GMM-Tr-BC20	0.06	4.89	7.9E+01	2.47	0.41	2.0	0.04	2.07	2.72	0.82	0.27	2.0
0.1	0	GMM-Tuk-Han	0.07	1.28	0.76	0.47	0.03	4.1	0.04	0.90	0.16	0.29	0.01	4.6
0.1	0	GMM-BR	0.08	1.43	1.06	0.55	0.02	2.1	0.04	1.03	0.93	0.37	0.01	2.4
0.1	0	GMM-BRM	0.09	1.60	1.17	0.60	0.19	1.0	0.03	1.15	1.06	0.41	0.18	1.1
0.1	0	GMM-Trunc	0.09	1.48	0.98	0.54	0.03	2.0	0.04	1.01	0.22	0.33	0.03	2.0
0.1	0	GMM-Trunc-BC	0.10	5.69	3.2E+04	15.39	0.40	2.0	0.01	1.70	1.4E+01	0.74	0.17	2.0
0.1	0	GMM-Tuk-Ha20	0.07	1.27	0.77	0.47	0.03	4.1	0.04	0.90	0.16	0.29	0.01	4.6
0.1	0	GMM-BR20	0.08	1.43	1.06	0.55	0.02	2.1	0.04	1.03	0.93	0.37	0.01	2.4
0.1	0	GMM-BRM20	0.09	1.60	1.17	0.60	0.19	1.0	0.03	1.15	1.06	0.41	0.18	1.1
0.1	0	GMM-Trunc20	0.09	1.48	0.98	0.54	0.03	2.0	0.04	1.01	0.22	0.33	0.03	2.0
0.1	0	GMM-Tr-BC20	0.10	5.69	3.2E+04	15.39	0.40	2.0	0.01	1.70	1.4E+01	0.74	0.17	2.0
0.1	0.5	GMM-Tuk-Han	0.06	1.28	0.52	0.44	0.01	5.5	0.05	0.83	0.16	0.27	0.00	7.0
0.1	0.5	GMM-BR	0.07	1.57	1.66	0.59	0.02	2.9	0.05	0.93	0.24	0.32	0.01	3.6
0.1	0.5	GMM-BRM	0.07	1.88	2.57	0.71	0.23	1.3	0.04	1.11	0.32	0.37	0.20	1.6
0.1	0.5	GMM-Trunc	0.08	1.63	2.34	0.61	0.02	2.0	0.05	0.97	0.26	0.32	0.01	3.0
0.1	0.5	GMM-Trunc-BC	0.01	4.72	3.4E+02	3.44	0.33	2.0	-0.02	1.55	0.87	0.55	0.13	3.0
0.1	0.5	GMM-Tuk-Ha20	0.06	1.29	0.53	0.44	0.01	5.5	0.05	0.83	0.16	0.27	0.00	7.0
0.1	0.5	GMM-BR20	0.07	1.59	1.66	0.60	0.02	2.9	0.05	0.93	0.24	0.32	0.01	3.6
0.1	0.5	GMM-BRM20	0.07	1.88	2.57	0.71	0.23	1.3	0.04	1.11	0.32	0.37	0.20	1.6
0.1	0.5	GMM-Trunc20	0.08	1.63	2.34	0.61	0.02	2.0	0.05	0.97	0.26	0.32	0.01	3.0
0.1	0.5	GMM-Tr-BC20	0.01	4.72	3.4E+02	3.44	0.33	2.0	-0.02	1.55	0.87	0.55	0.13	3.0
0.1	0.9	GMM-Tuk-Han	0.07	1.11	0.25	0.36	0.02	9.1	0.06	0.69	0.09	0.23	0.00	12.7
0.1	0.9	GMM-BR	0.07	1.48	0.59	0.49	0.04	4.7	0.07	0.88	0.15	0.29	0.02	6.7
0.1	0.9	GMM-BRM	0.07	1.87	1.74	0.69	0.30	2.1	0.06	1.24	2.9E-01	0.39	0.28	3.0
0.1	0.9	GMM-Trunc	0.12	1.64	1.30	0.62	0.04	3.0	0.09	0.85	0.24	0.31	0.02	6.0
0.1	0.9	GMM-Trunc-BC	0.02	3.10	4.72E+11	21,755.00	0.34	3.0	-0.02	1.50	0.49	0.52	0.23	6.0
0.1	0.9	GMM-Tuk-Ha20	0.07	1.12	0.77	0.38	0.02	9.1	0.06	0.69	0.09	0.23	0.00	12.7
0.1	0.9	GMM-BR20	0.07	1.48	0.59	0.49	0.04	4.7	0.07	0.88	0.15	0.29	0.02	6.7
0.1	0.9	GMM-BRM20	0.07	1.87	1.74	0.69	0.30	2.1	0.06	1.24	0.29	0.39	0.28	3.0
0.1	0.9	GMM-Trunc20	0.12	1.64	1.30	0.62	0.04	3.0	0.09	0.85	0.24	0.31	0.02	6.0
0.1	0.9	GMM-Tr-BC20	0.02	3.10	4.72E+11	21,755.00	0.34	3.0	-0.02	1.50	0.49	0.52	0.23	6.0

Table 11b - Homoskedastic DGP ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	GMM-Tuk-Han	0.56	1.01	0.48	0.60	0.37	9.1	0.47	0.91	0.32	0.49	0.39	12.0
0.5	-0.9	GMM-BR	0.58	1.30	0.87	0.69	0.31	4.7	0.44	1.09	0.37	0.50	0.30	6.2
0.5	-0.9	GMM-BRM	0.59	1.77	1.32	0.81	0.60	2.2	0.46	1.40	0.54	0.59	0.58	2.8
0.5	-0.9	GMM-Trunc	0.64	1.51	2.07	0.85	0.43	4.0	0.50	1.37	33.72	0.81	0.47	5.0
0.5	-0.9	GMM-Trunc-BC	0.43	3.18	2.87E+11	19,970.00	0.46	4.0	0.25	2.25	33.90	0.93	0.41	5.0
0.5	-0.9	GMM-Tuk-Ha20	0.56	1.01	1.06	0.63	0.37	9.1	0.47	0.91	0.32	0.49	0.39	12.0
0.5	-0.9	GMM-BR20	0.58	1.30	0.87	0.69	0.31	4.7	0.44	1.09	0.37	0.50	0.30	6.2
0.5	-0.9	GMM-BRM20	0.59	1.78	1.32	0.81	0.60	2.2	0.46	1.40	0.54	0.59	0.58	2.8
0.5	-0.9	GMM-Trunc20	0.64	1.51	2.07	0.85	0.43	4.0	0.50	1.37	33.72	0.81	0.47	5.0
0.5	-0.9	GMM-Tr-BC20	0.43	3.18	2.87E+11	19,970.00	0.46	4.0	0.25	2.25	33.90	0.93	0.41	5.0
0.5	-0.5	GMM-Tuk-Han	0.48	1.02	0.54	0.56	0.30	5.1	0.35	0.92	0.25	0.41	0.28	6.2
0.5	-0.5	GMM-BR	0.47	1.41	0.93	0.66	0.24	2.7	0.33	1.07	0.32	0.45	0.21	3.3
0.5	-0.5	GMM-BRM	0.48	1.68	10.05	0.84	0.46	1.2	0.30	1.35	0.58	0.53	0.45	1.5
0.5	-0.5	GMM-Trunc	0.49	1.50	0.93	0.69	0.30	2.0	0.34	1.27	0.45	0.50	0.29	2.0
0.5	-0.5	GMM-Trunc-BC	0.37	3.48	41.97	1.86	0.45	2.0	0.18	1.77	0.81	0.61	0.29	2.0
0.5	-0.5	GMM-Tuk-Ha20	0.48	1.02	0.55	0.56	0.30	5.1	0.35	0.92	0.25	0.41	0.28	6.2
0.5	-0.5	GMM-BR20	0.47	1.41	0.93	0.66	0.24	2.7	0.33	1.07	0.32	0.45	0.21	3.3
0.5	-0.5	GMM-BRM20	0.48	1.68	10.05	0.84	0.46	1.2	0.30	1.35	0.58	0.53	0.45	1.5
0.5	-0.5	GMM-Trunc20	0.49	1.50	0.93	0.69	0.30	2.0	0.34	1.27	0.45	0.50	0.29	2.0
0.5	-0.5	GMM-Tr-BC20	0.37	3.48	41.97	1.86	0.45	2.0	0.18	1.77	0.81	0.61	0.29	2.0
0.5	0	GMM-Tuk-Han	0.40	1.13	0.71	0.57	0.29	4.1	0.26	0.91	0.20	0.37	0.24	4.3
0.5	0	GMM-BR	0.39	1.40	1.10	0.65	0.21	2.1	0.20	1.09	0.50	0.42	0.18	2.3
0.5	0	GMM-BRM	0.39	1.47	1.16	0.67	0.40	1.0	0.20	1.20	0.59	0.44	0.34	1.0
0.5	0	GMM-Trunc	0.41	1.42	0.75	0.61	0.25	2.0	0.25	1.01	0.24	0.39	0.24	2.0
0.5	0	GMM-Trunc-BC	0.35	4.54	6.2E+03	6.78	0.48	2.0	0.11	1.63	2.9E+00	0.61	0.25	2.0
0.5	0	GMM-Tuk-Ha20	0.40	1.14	0.75	0.58	0.29	4.1	0.26	0.91	0.20	0.37	0.24	4.3
0.5	0	GMM-BR20	0.39	1.41	1.13	0.66	0.21	2.1	0.20	1.09	0.50	0.42	0.18	2.3
0.5	0	GMM-BRM20	0.39	1.47	1.16	0.67	0.40	1.0	0.20	1.20	0.59	0.44	0.34	1.0
0.5	0	GMM-Trunc20	0.41	1.42	0.75	0.61	0.25	2.0	0.25	1.01	0.24	0.39	0.24	2.0
0.5	0	GMM-Tr-BC20	0.35	4.54	6.2E+03	6.78	0.48	2.0	0.11	1.63	2.9E+00	0.61	0.25	2.0
0.5	0.5	GMM-Tuk-Han	0.34	1.11	0.43	0.48	0.15	5.6	0.24	0.82	0.19	0.34	0.14	6.4
0.5	0.5	GMM-BR	0.37	1.44	0.78	0.61	0.13	2.9	0.24	0.99	0.27	0.39	0.10	3.4
0.5	0.5	GMM-BRM	0.35	1.75	3.93	0.76	0.37	1.3	0.22	1.18	0.39	0.44	0.34	1.5
0.5	0.5	GMM-Trunc	0.38	1.59	3.79	0.73	0.16	2.0	0.26	1.07	0.35	0.43	0.17	2.0
0.5	0.5	GMM-Trunc-BC	0.18	3.56	105.32	1.93	0.31	2.0	0.07	1.54	0.62	0.51	0.18	2.0
0.5	0.5	GMM-Tuk-Ha20	0.34	1.11	0.46	0.49	0.15	5.6	0.24	0.82	0.19	0.34	0.14	6.4
0.5	0.5	GMM-BR20	0.37	1.44	0.78	0.61	0.13	2.9	0.24	0.99	0.27	0.39	0.10	3.4
0.5	0.5	GMM-BRM20	0.35	1.75	3.93	0.76	0.37	1.3	0.22	1.18	0.39	0.44	0.34	1.5
0.5	0.5	GMM-Trunc20	0.38	1.59	3.79	0.73	0.16	2.0	0.26	1.07	0.35	0.43	0.17	2.0
0.5	0.5	GMM-Tr-BC20	0.18	3.56	105.32	1.93	0.31	2.0	0.07	1.54	0.62	0.51	0.18	2.0
0.5	0.9	GMM-Tuk-Han	0.33	1.05	0.33	0.44	0.13	8.8	0.28	0.77	0.16	0.33	0.11	11.2
0.5	0.9	GMM-BR	0.36	1.52	0.71	0.59	0.13	4.5	0.27	1.02	0.23	0.39	0.09	5.8
0.5	0.9	GMM-BRM	0.39	1.92	1.32	0.74	0.42	2.1	0.25	1.27	0.39	0.46	0.39	2.6
0.5	0.9	GMM-Trunc	0.39	1.80	1.74	0.77	0.17	3.0	0.33	1.22	0.60	0.50	0.18	3.0
0.5	0.9	GMM-Trunc-BC	-0.01	3.20	5.93E+11	24,362.00	0.29	3.0	0.00	1.69	0.84	0.57	0.21	3.0
0.5	0.9	GMM-Tuk-Ha20	0.33	1.05	0.36	0.45	0.13	8.8	0.28	0.77	0.16	0.33	0.11	11.2
0.5	0.9	GMM-BR20	0.36	1.52	0.71	0.59	0.13	4.5	0.27	1.02	0.23	0.39	0.09	5.8
0.5	0.9	GMM-BRM20	0.39	1.92	1.32	0.74	0.42	2.1	0.25	1.27	0.39	0.46	0.39	2.6
0.5	0.9	GMM-Trunc20	0.39	1.80	1.74	0.77	0.17	3.0	0.33	1.22	0.60	0.50	0.18	3.0
0.5	0.9	GMM-Tr-BC20	-0.01	3.20	5.93E+11	24,362.00	0.29	3.0	0.00	1.69	0.84	0.57	0.21	3.0

Table 11c - Homoskedastic DGP ϕ=.3

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
					n=128						n=512			
0.9	-0.9	GMM-Tuk-Han	0.98	0.63	1.00	0.96	0.91	14.3	0.82	0.65	0.70	0.79	0.87	16.0
0.9	-0.9	GMM-BR	0.96	0.75	1.04	0.95	0.88	7.5	0.78	0.73	0.65	0.75	0.83	8.3
0.9	-0.9	GMM-BRM	1.00	0.96	1.22	1.02	0.91	3.4	0.83	1.02	0.82	0.82	0.87	3.8
0.9	-0.9	GMM-Trunc	1.01	1.05	1.85	1.06	0.78	5.0	0.83	1.34	0.89	0.80	0.68	4.0
0.9	-0.9	GMM-Trunc-BC	0.95	1.34	1.93	1.07	0.76	5.0	0.73	1.47	0.88	0.77	0.67	4.0
0.9	-0.9	GMM-Tuk-Ha20	0.98	0.64	2.58	1.01	0.91	14.3	0.82	0.65	0.70	0.79	0.87	16.0
0.9	-0.9	GMM-BR20	0.96	0.76	1.10	0.96	0.88	7.5	0.78	0.73	0.65	0.75	0.83	8.3
0.9	-0.9	GMM-BRM20	1.00	0.96	1.22	1.02	0.91	3.4	0.83	1.02	0.82	0.82	0.87	3.8
0.9	-0.9	GMM-Trunc20	1.01	1.05	1.85	1.06	0.78	5.0	0.83	1.34	0.89	0.80	0.68	4.0
0.9	-0.9	GMM-Tr-BC20	0.95	1.34	1.9E+00	1.07	0.76	5.0	0.73	1.47	8.8E-01	0.77	0.67	4.0
0.9	-0.5	GMM-Tuk-Han	0.86	0.64	0.79	0.84	0.85	7.0	0.59	0.72	0.42	0.59	0.71	6.2
0.9	-0.5	GMM-BR	0.86	0.82	0.89	0.85	0.79	3.7	0.51	0.87	0.38	0.54	0.61	3.2
0.9	-0.5	GMM-BRM	0.85	0.98	1.09	0.89	0.82	1.7	0.48	1.13	0.59	0.58	0.69	1.5
0.9	-0.5	GMM-Trunc	0.79	1.39	2.41	0.95	0.59	1.0	0.29	1.39	0.76	0.54	0.39	1.0
0.9	-0.5	GMM-Trunc-BC	0.74	1.66	2.5E+00	0.94	0.56	1.0	0.22	1.46	7.7E-01	0.53	0.36	1.0
0.9	-0.5	GMM-Tuk-Ha20	0.86	0.64	0.82	0.85	0.85	7.0	0.59	0.73	0.42	0.59	0.71	6.2
0.9	-0.5	GMM-BR20	0.86	0.82	0.98	0.86	0.79	3.7	0.51	0.87	0.38	0.54	0.61	3.2
0.9	-0.5	GMM-BRM20	0.85	0.98	1.09	0.89	0.82	1.7	0.48	1.13	0.59	0.58	0.69	1.5
0.9	-0.5	GMM-Trunc20	0.79	1.39	2.41	0.95	0.59	1.0	0.29	1.39	0.76	0.54	0.39	1.0
0.9	-0.5	GMM-Tr-BC20	0.74	1.66	2.50	0.94	0.56	1.0	0.22	1.46	0.77	0.53	0.36	1.0
0.9	0	GMM-Tuk-Han	0.73	0.75	0.74	0.75	0.69	4.2	0.44	0.81	0.30	0.47	0.50	3.6
0.9	0	GMM-BR	0.70	1.00	0.83	0.76	0.62	2.2	0.36	1.00	0.47	0.48	0.43	1.9
0.9	0	GMM-BRM	0.68	1.06	0.84	0.76	0.71	1.0	0.32	1.10	0.54	0.49	0.52	0.8
0.9	0	GMM-Trunc	0.64	1.23	0.89	0.74	0.46	1.0	0.25	1.08	0.42	0.43	0.27	1.0
0.9	0	GMM-Trunc-BC	0.57	1.98	6.8E+00	1.08	0.48	1.0	0.17	1.23	5.6E-01	0.46	0.25	1.0
0.9	0	GMM-Tuk-Ha20	0.73	0.76	0.75	0.76	0.69	4.2	0.44	0.81	0.30	0.47	0.50	3.6
0.9	0	GMM-BR20	0.70	1.00	0.83	0.76	0.62	2.2	0.36	1.00	0.47	0.48	0.43	1.9
0.9	0	GMM-BRM20	0.68	1.06	0.86	0.76	0.71	1.0	0.32	1.10	0.54	0.49	0.52	0.8
0.9	0	GMM-Trunc20	0.64	1.23	0.89	0.74	0.46	1.0	0.25	1.08	0.42	0.43	0.27	1.0
0.9	0	GMM-Tr-BC20	0.57	1.98	6.8E+00	1.08	0.48	1.0	0.17	1.23	5.6E-01	0.46	0.25	1.0
0.9	0.5	GMM-Tuk-Han	0.63	0.84	0.57	0.66	0.56	6.4	0.45	0.73	0.28	0.46	0.43	6.3
0.9	0.5	GMM-BR	0.62	1.20	1.52	0.75	0.51	3.3	0.43	0.98	0.36	0.50	0.42	3.3
0.9	0.5	GMM-BRM	0.61	1.34	2.15	0.83	0.60	1.5	0.37	1.21	0.44	0.52	0.53	1.5
0.9	0.5	GMM-Trunc	0.64	1.58	1.47	0.82	0.34	1.0	0.28	1.34	3.31	0.58	0.24	1.0
0.9	0.5	GMM-Trunc-BC	0.45	2.56	226.23	2.14	0.37	1.0	0.12	1.57	3.39	0.59	0.22	1.0
0.9	0.5	GMM-Tuk-Ha20	0.63	0.84	2.96	0.72	0.56	6.4	0.45	0.73	0.28	0.46	0.43	6.3
0.9	0.5	GMM-BR20	0.62	1.20	1.68	0.76	0.51	3.3	0.43	0.98	0.36	0.50	0.42	3.3
0.9	0.5	GMM-BRM20	0.61	1.34	2.15	0.83	0.60	1.5	0.37	1.21	0.44	0.52	0.53	1.5
0.9	0.5	GMM-Trunc20	0.64	1.58	1.47	0.82	0.34	1.0	0.28	1.34	3.31	0.58	0.24	1.0
0.9	0.5	GMM-Tr-BC20	0.45	2.56	226.23	2.14	0.37	1.0	0.12	1.57	3.39	0.59	0.22	1.0
0.9	0.9	GMM-Tuk-Han	0.55	0.86	0.46	0.59	0.41	11.2	0.46	0.69	0.28	0.46	0.39	13.9
0.9	0.9	GMM-BR	0.58	1.18	0.63	0.65	0.45	5.8	0.43	0.91	0.32	0.48	0.39	7.2
0.9	0.9	GMM-BRM	0.61	1.79	1.11	0.81	0.62	2.6	0.46	1.37	0.57	0.59	0.59	3.3
0.9	0.9	GMM-Trunc	0.60	1.40	1.65	0.81	0.41	4.0	0.51	1.24	0.68	0.60	0.43	4.0
0.9	0.9	GMM-Trunc-BC	0.33	2.54	1.91E+07	168.37	0.37	4.0	0.30	1.54	0.77	0.58	0.32	4.0
0.9	0.9	GMM-Tuk-Ha20	0.54	0.86	0.46	0.59	0.41	11.2	0.46	0.69	0.28	0.46	0.39	13.9
0.9	0.9	GMM-BR20	0.58	1.18	0.93	0.67	0.45	5.8	0.43	0.91	0.32	0.48	0.39	7.2
0.9	0.9	GMM-BRM20	0.61	1.79	1.11	0.81	0.62	2.6	0.46	1.37	0.57	0.59	0.59	3.3
0.9	0.9	GMM-Trunc20	0.60	1.40	1.65	0.81	0.41	4.0	0.51	1.24	0.68	0.60	0.43	4.0
0.9	0.9	GMM-Tr-BC20	0.33	2.54	1.91E+07	168.37	0.37	4.0	0.30	1.54	0.77	0.58	0.32	4.0

Table 12a - Homoskedastic DGP ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.1	-0.9	GMM-Tuk-Han	0.07	0.94	0.17	0.31	0.06	6.5	0.02	0.59	0.05	0.18	0.09	8.7
0.1	-0.9	GMM-BR	0.08	1.16	0.28	0.38	0.10	3.4	0.01	0.60	0.06	0.19	0.09	4.5
0.1	-0.9	GMM-BRM	0.07	1.34	0.40	0.44	0.37	1.5	0.02	0.69	0.08	0.22	0.39	2.0
0.1	-0.9	GMM-Trunc	0.08	1.27	0.49	0.44	0.09	1.0	0.02	0.59	0.06	0.19	0.09	3.0
0.1	-0.9	GMM-Trunc-BC	-0.06	2.79	4.33E+12	9.65E+04	0.52	1.0	-0.15	1.17	0.23	0.41	0.50	3.0
0.1	-0.9	GMM-Tuk-Ha20	0.07	0.94	0.17	0.31	0.06	6.5	0.02	0.59	0.05	0.18	0.09	8.7
0.1	-0.9	GMM-BR20	0.08	1.16	0.28	0.38	0.10	3.4	0.01	0.60	0.06	0.19	0.09	4.5
0.1	-0.9	GMM-BRM20	0.07	1.34	0.40	0.44	0.37	1.5	0.02	0.69	0.08	0.22	0.39	2.0
0.1	-0.9	GMM-Trunc20	0.08	1.27	0.49	0.44	0.09	1.0	0.02	0.59	0.06	0.19	0.09	3.0
0.1	-0.9	GMM-Tr-BC20	-0.06	2.79	4.33E+12	9.65E+04	0.52	1.0	-0.15	1.17	0.23	0.41	0.50	3.0
0.1	-0.5	GMM-Tuk-Han	0.05	0.81	0.17	0.27	0.06	6.5	0.01	0.46	0.03	0.15	0.08	8.5
0.1	-0.5	GMM-BR	0.05	0.91	0.36	0.32	0.07	3.4	0.01	0.49	0.04	0.15	0.08	4.4
0.1	-0.5	GMM-BRM	0.06	1.09	0.44	0.38	0.35	1.5	0.01	0.56	0.05	0.17	0.35	2.0
0.1	-0.5	GMM-Trunc	0.07	0.94	0.23	0.32	0.07	3.0	0.01	0.48	0.04	0.15	0.09	5.0
0.1	-0.5	GMM-Trunc-BC	-0.01	1.82	1.3E+01	0.79	0.40	3.0	-0.05	0.73	0.08	0.23	0.25	5.0
0.1	-0.5	GMM-Tuk-Ha20	0.05	0.81	0.17	0.27	0.06	6.5	0.01	0.46	0.03	0.15	0.08	8.5
0.1	-0.5	GMM-BR20	0.05	0.92	0.38	0.33	0.07	3.4	0.01	0.49	0.04	0.15	0.08	4.4
0.1	-0.5	GMM-BRM20	0.06	1.09	0.44	0.38	0.35	1.5	0.01	0.56	0.05	0.17	0.35	2.0
0.1	-0.5	GMM-Trunc20	0.07	0.94	0.23	0.32	0.07	3.0	0.01	0.48	0.04	0.15	0.09	5.0
0.1	-0.5	GMM-Tr-BC20	-0.01	1.82	1.3E+01	0.79	0.40	3.0	-0.05	0.73	0.08	0.23	0.25	5.0
0.1	0	GMM-Tuk-Han	0.05	0.68	0.11	0.23	0.03	7.6	0.01	0.36	0.02	0.11	0.03	10.1
0.1	0	GMM-BR	0.04	0.73	0.16	0.26	0.03	4.0	0.00	0.37	0.02	0.11	0.03	5.3
0.1	0	GMM-BRM	0.04	0.86	0.18	0.29	0.30	1.8	0.01	0.42	0.03	0.13	0.30	2.4
0.1	0	GMM-Trunc	0.05	0.69	0.13	0.23	0.03	4.0	0.02	0.37	0.02	0.11	0.04	6.0
0.1	0	GMM-Trunc-BC	0.02	1.21	1.0E+00	0.49	0.28	4.0	-0.01	0.47	3.8E-02	0.15	0.12	6.0
0.1	0	GMM-Tuk-Ha20	0.05	0.68	0.11	0.23	0.04	7.6	0.01	0.36	0.02	0.11	0.03	10.1
0.1	0	GMM-BR20	0.04	0.73	0.16	0.26	0.03	4.0	0.00	0.37	0.02	0.11	0.03	5.3
0.1	0	GMM-BRM20	0.04	0.86	0.18	0.29	0.30	1.8	0.01	0.42	0.03	0.13	0.30	2.4
0.1	0	GMM-Trunc20	0.05	0.69	0.13	0.23	0.03	4.0	0.02	0.37	0.02	0.11	0.04	6.0
0.1	0	GMM-Tr-BC20	0.02	1.21	1.0E+00	0.49	0.28	4.0	-0.01	0.47	3.8E-02	0.15	0.12	6.0
0.1	0.5	GMM-Tuk-Han	0.03	0.65	0.11	0.22	0.02	8.2	0.00	0.32	0.02	0.10	0.00	11.1
0.1	0.5	GMM-BR	0.03	0.71	0.20	0.26	0.02	4.3	0.00	0.33	0.02	0.10	0.00	5.7
0.1	0.5	GMM-BRM	0.01	0.83	0.31	0.31	0.22	2.0	0.01	0.36	0.02	0.12	0.17	2.6
0.1	0.5	GMM-Trunc	0.04	0.66	0.25	0.25	0.01	5.0	0.01	0.33	0.02	0.10	0.01	6.0
0.1	0.5	GMM-Trunc-BC	-0.03	1.20	1.1E+00	0.50	0.19	5.0	-0.03	0.43	0.03	0.14	0.05	6.0
0.1	0.5	GMM-Tuk-Ha20	0.03	0.66	0.16	0.23	0.02	8.2	0.00	0.32	0.02	0.10	0.00	11.1
0.1	0.5	GMM-BR20	0.03	0.71	0.20	0.26	0.02	4.3	0.00	0.33	0.02	0.10	0.00	5.7
0.1	0.5	GMM-BRM20	0.01	0.83	0.31	0.31	0.22	2.0	0.01	0.36	0.02	0.12	0.17	2.6
0.1	0.5	GMM-Trunc20	0.04	0.66	0.25	0.25	0.01	5.0	0.01	0.33	0.02	0.10	0.01	6.0
0.1	0.5	GMM-Tr-BC20	-0.03	1.20	1.1E+00	0.50	0.19	5.0	-0.03	0.43	0.03	0.14	0.05	6.0
0.1	0.9	GMM-Tuk-Han	0.04	0.67	0.09	0.22	0.01	10.3	0.01	0.34	0.02	0.11	0.00	14.4
0.1	0.9	GMM-BR	0.04	0.78	0.15	0.27	0.02	5.3	0.01	0.37	0.02	0.12	0.00	7.6
0.1	0.9	GMM-BRM	0.02	1.01	0.24	0.33	0.24	2.4	0.02	0.42	0.03	0.13	0.15	3.4
0.1	0.9	GMM-Trunc	0.03	0.77	0.24	0.27	0.02	5.0	0.00	0.35	0.02	0.11	0.01	7.0
0.1	0.9	GMM-Trunc-BC	-0.10	1.63	8.02E+10	1.59E+04	0.30	5.0	-0.10	0.60	0.06	0.21	0.13	7.0
0.1	0.9	GMM-Tuk-Ha20	0.04	0.67	0.09	0.22	0.01	10.3	0.01	0.34	0.02	0.11	0.00	14.4
0.1	0.9	GMM-BR20	0.04	0.78	0.15	0.27	0.02	5.3	0.01	0.37	0.02	0.12	0.00	7.6
0.1	0.9	GMM-BRM20	0.02	1.01	0.24	0.33	0.24	2.4	0.02	0.42	0.03	0.13	0.15	3.4
0.1	0.9	GMM-Trunc20	0.03	0.77	0.24	0.27	0.02	5.0	0.00	0.35	0.02	0.11	0.01	7.0
0.1	0.9	GMM-Tr-BC20	-0.10	1.63	8.02E+10	1.59E+04	0.30	5.0	-0.10	0.60	0.06	0.21	0.13	7.0

Table 12b - Homoskedastic DGP ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.5	-0.9	GMM-Tuk-Han	0.33	0.93	0.23	0.40	0.31	7.8	0.08	0.62	0.06	0.20	0.18	7.1
0.5	-0.9	GMM-BR	0.28	1.06	1.12	0.45	0.28	4.0	0.06	0.65	0.07	0.21	0.17	3.7
0.5	-0.9	GMM-BRM	0.29	1.30	1.19	0.51	0.53	1.8	0.05	0.71	0.08	0.23	0.40	1.7
0.5	-0.9	GMM-Trunc	0.33	1.27	2.25	0.55	0.36	2.0	0.04	0.74	0.09	0.24	0.23	1.0
0.5	-0.9	GMM-Trunc-BC	-0.03	2.45	6.18E+11	3.60E+04	0.48	2.0	-0.24	0.92	0.21	0.36	0.40	1.0
0.5	-0.9	GMM-Tuk-Ha20	0.33	0.93	0.23	0.40	0.31	7.8	0.08	0.62	0.06	0.20	0.18	7.1
0.5	-0.9	GMM-BR20	0.28	1.06	1.12	0.45	0.28	4.0	0.06	0.65	0.07	0.21	0.17	3.7
0.5	-0.9	GMM-BRM20	0.29	1.30	1.19	0.51	0.53	1.8	0.05	0.71	0.08	0.23	0.40	1.7
0.5	-0.9	GMM-Trunc20	0.33	1.27	2.25	0.55	0.36	2.0	0.04	0.74	0.09	0.24	0.23	1.0
0.5	-0.9	GMM-Tr-BC20	-0.03	2.45	6.18E+11	3.60E+04	0.48	2.0	-0.24	0.92	0.21	0.36	0.40	1.0
0.5	-0.5	GMM-Tuk-Han	0.25	0.76	0.16	0.33	0.26	5.8	0.06	0.49	0.04	0.16	0.15	5.7
0.5	-0.5	GMM-BR	0.22	0.90	0.27	0.36	0.22	3.0	0.04	0.51	0.04	0.16	0.14	3.0
0.5	-0.5	GMM-BRM	0.20	1.10	0.54	0.42	0.44	1.4	0.03	0.57	0.06	0.18	0.35	1.3
0.5	-0.5	GMM-Trunc	0.24	0.99	0.29	0.39	0.29	3.0	0.06	0.53	0.05	0.18	0.19	3.0
0.5	-0.5	GMM-Trunc-BC	0.01	1.59	2.27	0.56	0.31	3.0	-0.07	0.59	0.06	0.19	0.14	3.0
0.5	-0.5	GMM-Tuk-Ha20	0.25	0.76	0.64	0.35	0.26	5.8	0.06	0.49	0.04	0.16	0.15	5.7
0.5	-0.5	GMM-BR20	0.22	0.90	0.27	0.36	0.22	3.0	0.04	0.51	0.04	0.16	0.14	3.0
0.5	-0.5	GMM-BRM20	0.20	1.10	0.54	0.42	0.44	1.4	0.03	0.57	0.06	0.18	0.35	1.3
0.5	-0.5	GMM-Trunc20	0.24	0.99	0.29	0.39	0.29	3.0	0.06	0.53	0.05	0.18	0.19	3.0
0.5	-0.5	GMM-Tr-BC20	0.01	1.59	2.27	0.56	0.31	3.0	-0.07	0.59	0.06	0.19	0.14	3.0
0.5	0	GMM-Tuk-Han	0.20	0.68	0.13	0.28	0.24	6.8	0.06	0.38	0.03	0.13	0.12	6.7
0.5	0	GMM-BR	0.17	0.78	0.18	0.30	0.18	3.5	0.04	0.39	0.03	0.13	0.09	3.5
0.5	0	GMM-BRM	0.16	0.86	0.21	0.32	0.36	1.6	0.03	0.44	0.04	0.14	0.28	1.6
0.5	0	GMM-Trunc	0.20	0.74	0.15	0.30	0.26	4.0	0.07	0.39	0.03	0.14	0.15	4.0
0.5	0	GMM-Trunc-BC	0.07	1.20	8.6E-01	0.45	0.27	4.0	0.00	0.44	3.7E-02	0.14	0.11	4.0
0.5	0	GMM-Tuk-Ha20	0.20	0.69	6.88	0.38	0.24	6.8	0.06	0.38	0.03	0.13	0.12	6.7
0.5	0	GMM-BR20	0.17	0.78	0.18	0.30	0.18	3.5	0.04	0.39	0.03	0.13	0.09	3.5
0.5	0	GMM-BRM20	0.16	0.86	0.21	0.32	0.36	1.6	0.03	0.44	0.04	0.14	0.28	1.6
0.5	0	GMM-Trunc20	0.20	0.74	0.15	0.30	0.26	4.0	0.07	0.39	0.03	0.14	0.15	4.0
0.5	0	GMM-Tr-BC20	0.07	1.20	8.6E-01	0.45	0.27	4.0	0.00	0.44	3.7E-02	0.14	0.11	4.0
0.5	0.5	GMM-Tuk-Han	0.15	0.64	0.13	0.26	0.09	7.0	0.04	0.33	0.02	0.11	0.03	6.7
0.5	0.5	GMM-BR	0.13	0.76	0.19	0.29	0.07	3.7	0.03	0.35	0.02	0.11	0.02	3.5
0.5	0.5	GMM-BRM	0.11	0.93	0.22	0.32	0.27	1.6	0.03	0.37	0.03	0.12	0.17	1.6
0.5	0.5	GMM-Trunc	0.15	0.77	0.16	0.29	0.10	3.0	0.05	0.35	0.02	0.12	0.04	4.0
0.5	0.5	GMM-Trunc-BC	-0.08	1.17	0.72	0.43	0.15	3.0	-0.05	0.38	0.03	0.13	0.02	4.0
0.5	0.5	GMM-Tuk-Ha20	0.15	0.64	0.16	0.26	0.09	7.0	0.04	0.33	0.02	0.11	0.03	6.7
0.5	0.5	GMM-BR20	0.13	0.76	0.19	0.29	0.07	3.7	0.03	0.35	0.02	0.11	0.02	3.5
0.5	0.5	GMM-BRM20	0.11	0.93	0.22	0.32	0.27	1.6	0.03	0.37	0.03	0.12	0.17	1.6
0.5	0.5	GMM-Trunc20	0.15	0.77	0.16	0.29	0.10	3.0	0.05	0.35	0.02	0.12	0.04	4.0
0.5	0.5	GMM-Tr-BC20	-0.08	1.17	0.72	0.43	0.15	3.0	-0.05	0.38	0.03	0.13	0.02	4.0
0.5	0.9	GMM-Tuk-Han	0.14	0.70	0.12	0.26	0.05	8.7	0.03	0.39	0.03	0.12	0.02	9.3
0.5	0.9	GMM-BR	0.13	0.86	0.17	0.30	0.05	4.5	0.03	0.41	0.03	0.13	0.02	4.8
0.5	0.9	GMM-BRM	0.11	1.07	0.32	0.37	0.27	2.0	0.04	0.44	0.04	0.15	0.17	2.2
0.5	0.9	GMM-Trunc	0.15	0.86	0.26	0.32	0.06	3.0	0.02	0.44	0.03	0.14	0.03	2.0
0.5	0.9	GMM-Trunc-BC	-0.25	1.57	3.88E+11	2.88E+04	0.26	3.0	-0.18	0.51	0.07	0.22	0.11	2.0
0.5	0.9	GMM-Tuk-Ha20	0.14	0.70	0.17	0.27	0.05	8.7	0.03	0.39	0.03	0.12	0.02	9.3
0.5	0.9	GMM-BR20	0.13	0.86	0.17	0.30	0.05	4.5	0.03	0.41	0.03	0.13	0.02	4.8
0.5	0.9	GMM-BRM20	0.11	1.07	0.32	0.37	0.27	2.0	0.04	0.44	0.04	0.15	0.17	2.2
0.5	0.9	GMM-Trunc20	0.15	0.86	0.26	0.32	0.06	3.0	0.02	0.44	0.03	0.14	0.03	2.0
0.5	0.9	GMM-Tr-BC20	-0.25	1.57	3.88E+11	2.88E+04	0.26	3.0	-0.18	0.51	0.07	0.22	0.11	2.0

Table 12c - Homoskedastic DGP ϕ=.5

rho	Theta	Estimator	Med	DR	MSE	MAE	Size	Inst	Med	DR	MSE	MAE	Size	Inst
			n=128						n=512					
0.9	-0.9	GMM-Tuk-Han	0.57	0.77	0.41	0.58	0.73	10.5	0.15	0.54	0.07	0.21	0.31	6.8
0.9	-0.9	GMM-BR	0.54	0.91	0.40	0.56	0.68	5.4	0.13	0.64	0.07	0.22	0.32	3.5
0.9	-0.9	GMM-BRM	0.52	1.08	0.45	0.58	0.75	2.4	0.09	0.71	0.09	0.24	0.51	1.6
0.9	-0.9	GMM-Trunc	0.40	1.39	0.56	0.58	0.53	2.0	0.01	0.67	0.09	0.22	0.22	1.0
0.9	-0.9	GMM-Trunc-BC	0.31	1.64	12,869.00	4.18	0.55	2.0	-0.09	0.88	0.16	0.29	0.34	1.0
0.9	-0.9	GMM-Tuk-Ha20	0.57	0.77	50.98	0.81	0.73	10.5	0.15	0.54	0.07	0.21	0.31	6.8
0.9	-0.9	GMM-BR20	0.54	0.91	0.40	0.56	0.68	5.4	0.13	0.64	0.07	0.22	0.32	3.5
0.9	-0.9	GMM-BRM20	0.52	1.08	0.45	0.58	0.75	2.4	0.09	0.71	0.09	0.24	0.51	1.6
0.9	-0.9	GMM-Trunc20	0.40	1.39	0.56	0.58	0.53	2.0	0.01	0.67	0.09	0.22	0.22	1.0
0.9	-0.9	GMM-Tr-BC20	0.31	1.64	1.3E+04	4.18	0.55	2.0	-0.09	0.88	1.6E-01	0.29	0.34	1.0
0.9	-0.5	GMM-Tuk-Han	0.40	0.68	0.23	0.42	0.58	5.7	0.10	0.42	0.04	0.16	0.24	5.5
0.9	-0.5	GMM-BR	0.34	0.82	0.27	0.41	0.51	2.9	0.07	0.47	0.04	0.16	0.20	2.9
0.9	-0.5	GMM-BRM	0.29	1.08	1.00	0.49	0.60	1.3	0.05	0.51	0.05	0.17	0.39	1.3
0.9	-0.5	GMM-Trunc	0.17	1.17	1.28	0.47	0.34	1.0	0.03	0.53	0.05	0.17	0.19	2.0
0.9	-0.5	GMM-Trunc-BC	0.10	1.21	1.3E+00	0.47	0.30	1.0	-0.01	0.52	5.5E-02	0.17	0.14	2.0
0.9	-0.5	GMM-Tuk-Ha20	0.40	0.68	0.23	0.42	0.58	5.7	0.10	0.42	0.04	0.16	0.24	5.5
0.9	-0.5	GMM-BR20	0.34	0.82	0.27	0.41	0.51	2.9	0.07	0.47	0.04	0.16	0.20	2.9
0.9	-0.5	GMM-BRM20	0.29	1.08	1.00	0.49	0.60	1.3	0.05	0.51	0.05	0.17	0.39	1.3
0.9	-0.5	GMM-Trunc20	0.17	1.17	1.28	0.47	0.34	1.0	0.03	0.53	0.05	0.17	0.19	2.0
0.9	-0.5	GMM-Tr-BC20	0.10	1.21	1.30	0.47	0.30	1.0	-0.01	0.52	0.05	0.17	0.14	2.0
0.9	0	GMM-Tuk-Han	0.35	0.61	0.19	0.37	0.50	5.9	0.11	0.37	0.03	0.15	0.22	6.4
0.9	0	GMM-BR	0.29	0.75	0.24	0.37	0.44	3.0	0.08	0.38	0.03	0.14	0.17	3.3
0.9	0	GMM-BRM	0.24	0.98	0.29	0.38	0.51	1.4	0.07	0.45	0.05	0.16	0.38	1.5
0.9	0	GMM-Trunc	0.24	0.82	0.21	0.35	0.33	2.0	0.08	0.38	0.03	0.14	0.17	3.0
0.9	0	GMM-Trunc-BC	0.14	0.94	3.4E-01	0.36	0.28	2.0	0.04	0.40	3.1E-02	0.13	0.10	3.0
0.9	0	GMM-Tuk-Ha20	0.35	0.61	0.19	0.37	0.50	5.9	0.11	0.37	0.03	0.15	0.22	6.4
0.9	0	GMM-BR20	0.29	0.75	0.31	0.38	0.44	3.0	0.08	0.38	0.03	0.14	0.17	3.3
0.9	0	GMM-BRM20	0.24	0.98	0.29	0.38	0.51	1.4	0.07	0.45	0.05	0.16	0.38	1.5
0.9	0	GMM-Trunc20	0.24	0.82	0.21	0.35	0.33	2.0	0.08	0.38	0.03	0.14	0.17	3.0
0.9	0	GMM-Tr-BC20	0.14	0.94	3.4E-01	0.36	0.28	2.0	0.04	0.40	3.1E-02	0.13	0.10	3.0
0.9	0.5	GMM-Tuk-Han	0.26	0.58	0.16	0.31	0.25	6.5	0.09	0.30	0.02	0.12	0.05	6.6
0.9	0.5	GMM-BR	0.23	0.79	0.21	0.34	0.28	3.4	0.06	0.39	0.03	0.13	0.10	3.4
0.9	0.5	GMM-BRM	0.17	0.98	0.28	0.36	0.39	1.5	0.04	0.39	0.03	0.13	0.26	1.6
0.9	0.5	GMM-Trunc	0.17	0.93	0.26	0.34	0.17	1.0	0.05	0.34	0.03	0.12	0.04	2.0
0.9	0.5	GMM-Trunc-BC	-0.06	1.06	22.57	0.57	0.10	1.0	-0.05	0.33	0.03	0.12	0.02	2.0
0.9	0.5	GMM-Tuk-Ha20	0.26	0.58	3.09	0.38	0.25	6.5	0.09	0.30	0.02	0.12	0.05	6.6
0.9	0.5	GMM-BR20	0.23	0.79	0.24	0.34	0.28	3.4	0.06	0.39	0.03	0.13	0.10	3.4
0.9	0.5	GMM-BRM20	0.17	0.98	0.28	0.36	0.39	1.5	0.04	0.39	0.03	0.13	0.26	1.6
0.9	0.5	GMM-Trunc20	0.17	0.93	0.26	0.34	0.17	1.0	0.05	0.34	0.03	0.12	0.04	2.0
0.9	0.5	GMM-Tr-BC20	-0.06	1.06	22.57	0.57	0.10	1.0	-0.05	0.33	0.03	0.12	0.02	2.0
0.9	0.9	GMM-Tuk-Han	0.21	0.65	0.13	0.28	0.14	9.9	0.08	0.39	0.03	0.14	0.05	9.8
0.9	0.9	GMM-BR	0.21	0.91	0.23	0.35	0.24	5.1	0.07	0.50	0.05	0.17	0.12	5.1
0.9	0.9	GMM-BRM	0.20	1.25	0.39	0.45	0.45	2.3	0.06	0.62	0.08	0.20	0.34	2.3
0.9	0.9	GMM-Trunc	0.21	1.01	0.25	0.37	0.16	3.0	0.04	0.52	0.05	0.16	0.06	2.0
0.9	0.9	GMM-Trunc-BC	-0.07	1.24	8.04E+10	1.27E+04	0.17	3.0	-0.07	0.56	0.06	0.19	0.06	2.0
0.9	0.9	GMM-Tuk-Ha20	0.21	0.65	0.26	0.30	0.14	9.9	0.08	0.39	0.03	0.14	0.05	9.8
0.9	0.9	GMM-BR20	0.21	0.91	0.23	0.35	0.24	5.1	0.07	0.50	0.05	0.17	0.12	5.1
0.9	0.9	GMM-BRM20	0.20	1.25	0.39	0.44	0.45	2.3	0.06	0.62	0.08	0.20	0.34	2.3
0.9	0.9	GMM-Trunc20	0.21	1.01	0.25	0.37	0.16	3.0	0.04	0.52	0.05	0.16	0.06	2.0
0.9	0.9	GMM-Tr-BC20	-0.07	1.24	8.04E+10	1.27E+04	0.17	3.0	-0.07	0.56	0.06	0.19	0.06	2.0