Attention Overload

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Introduction

So many options

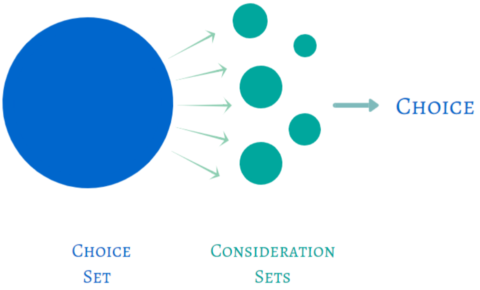
Attention is **scarce**

- Consumer simply cannot pay attention to everything

Competitions over consumer's attention are **fierce**

- Global spending on advertising is around $674 billion every year

Random Attention



- Random attention
- Recent literature
  - Manzini and Mariotti (2014), Brady and Rehbeck (2016), Aguiar (2017), Cattaneo et al (2020)

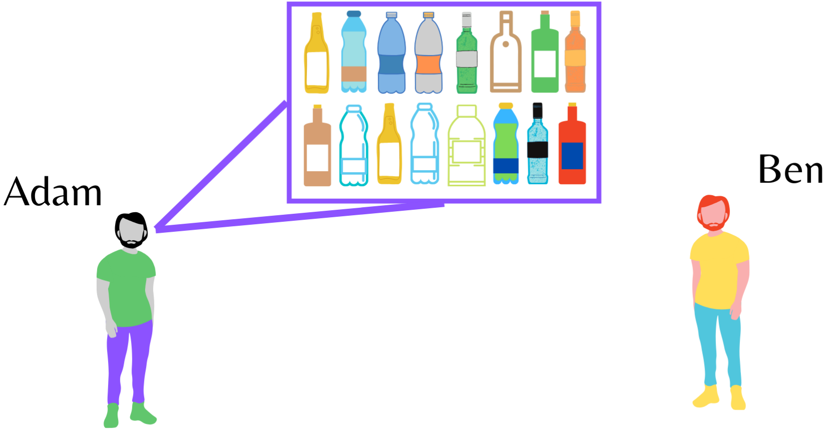
$$\pi(a|S) = \sum_{A \subseteq S} \mathbb{1}(a \text{ is } \succ\text{-best in } A) \cdot \mu(A|S)$$

Attention Overload

Each alternative gets less attention when there are more alternatives

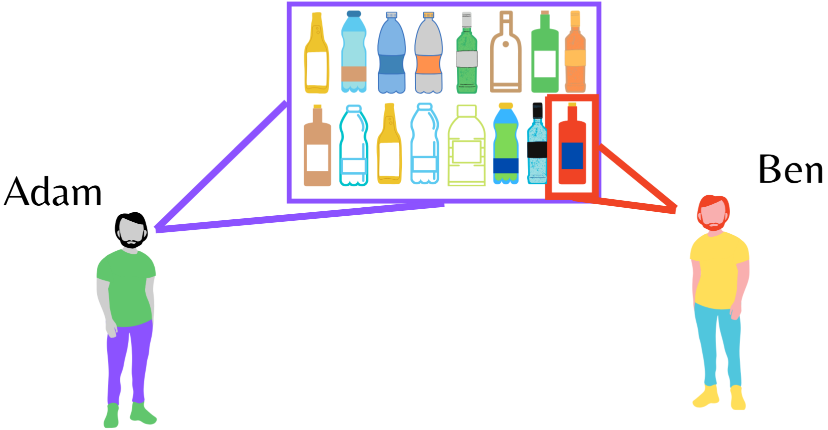
Attention Overload

Attention Overload



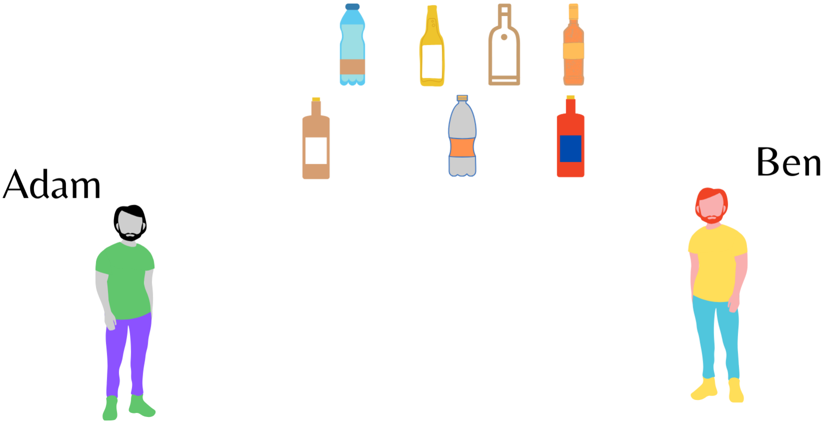
Attention Overload

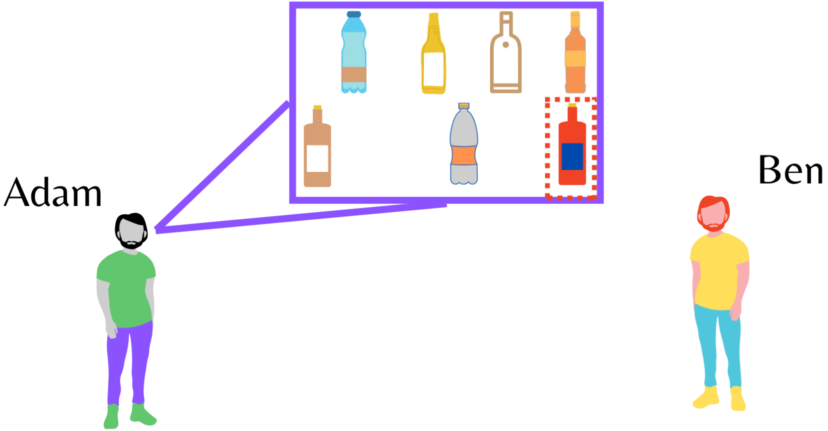
Attention Overload



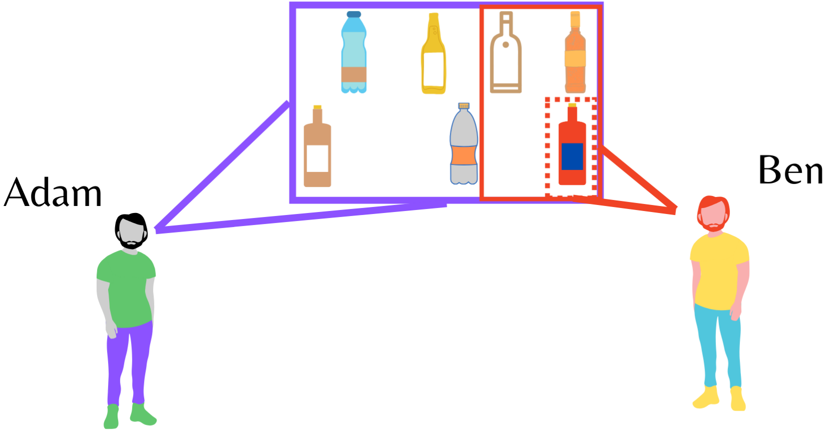
Attention Overload

How is attention affected when some alternatives are eliminated?




Adam
Ben

Attention Overload



Limited Attention in the literature

Adam: Full attention in S: $\mu_{\text{Adam}}(S|S) = 1$
Ben: (Extreme) Limited attention in S: $\mu_{\text{Ben}}(\{a\}|S) = 1$

What will they do in a smaller set $T \subseteq S$?

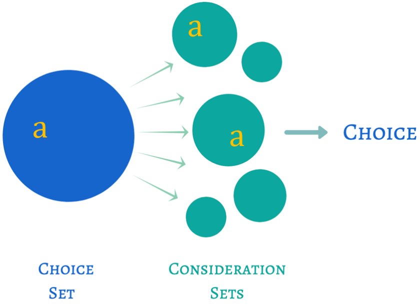
	$\mu_{\text{Adam}}(T\|T)$	$\mu_{\text{Ben}}(\{a\}\|T)$
Attention Overload	1	≤ 1
Tversky 1972 Aguiar 2017	1	1
Brady-Rehbeck 2016	No restriction	1
Cattaneo et al 2020	No restriction	1
Demirkan-Kimya 2020	No restriction	No restriction

This paper

Using a non-parametric approach:

- Attention Overload Model (AOM):
  - Single Preference + Random Attention Rule
- Heterogeneous Preference AOM (HAOM)
  - Random Preference + Random Consideration Set Mapping

Attention Frequency

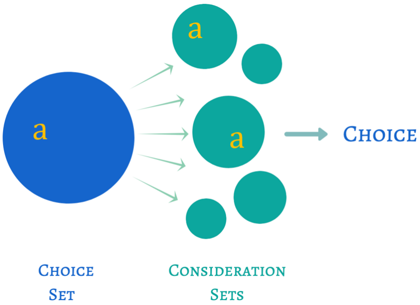


$$\underbrace{\phi(a|S)}_{\text{Attention frequency of } a} := \sum_{a \in A \subseteq S} \mu(A|S)$$

Attention Overload

$\phi(a|S) \leq \phi(a|T)$ for $a \in T \subseteq S$

Attention Overload



Attention Overload

$$\phi(a|S) \leq \phi(a|T) \text{ for } a \in T \subseteq S$$

Model

π is a *Attention Overload Model* if there exists $\succ$, μ satisfying attention overload s.t.

$$\pi(a|S) = \sum_{A \subseteq S} \mathbb{1}(a \text{ is } \succ\text{-best in } A) \cdot \mu(A|S)$$

- Require testing against
  - All $\succ$
  - All μ

Characterization

Let $U_{\succ}(a)$ be the weak upper contour set of a.

Axiom *($\succ$-Regularity)* $\pi(U_{\succ}(a)|T) \geq \pi(a|S)$ *for all* $a \in T \subseteq S$

- Weaker than Regularity: $\pi(a|T) \geq \pi(a|S)$

Characterization

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Characterization

π has an AOM representation with $\succ$ if and only if π satisfies $\succ$-Regularity.

- Bypass the construction of μ

Revealed Preference-1

Lemma *(Revealed Preference-1) Let π be an AOM with $\succ$.*

If $\pi(b|S) > \pi(b|\{a,b\})$ and $\{a,b\} \subseteq S$, then it must be $a \succ b$.

Lemma *(Revealed Preference-1) Let π be an AOM with $\succ$. If $\pi(b|S) > \pi(b|\{a,b\})$ and $\{a,b\} \subseteq S$, then it must be $a \succ b$.*

Proof:

$$\phi(b|\{a,b\}) \geq \phi(b|S)$$

$$\sum_{b \in J \subseteq \{a,b\}} \mu(J|\{a,b\}) \geq \sum_{b \in J \subseteq S} \mu(J|S)$$

$$\pi(b|\{a,b\}) + \sum_{\substack{b \in J \subseteq \{a,b\} \\ b \text{ is not } \succ\text{-best}}} \mu(J|\{a,b\}) \geq \pi(b|S) + \sum_{\substack{b \in J \subseteq S \\ b \text{ is not } \succ\text{-best}}} \mu(J|S)$$

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Every representation must say that $a \succ b$.

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Every representation must say that $a \succ b$.

Revealed Preference-2

Lemma *(Revealed Preference-2) Let π be an AOM with $\succ$.*

*If $\pi(b|S) > \pi(b|T)$ for $T \subseteq S$, then **some** alternatives in T are better than b.*

An Illustration

$\pi(\cdot\|S)$	a	b	c	d
$\{a,b,c,d\}$	0.05	0.1	0.1	0.75
$\{a,b,c\}$	0.8	0.2	0	–
$\{b,c,d\}$	–	0.7	0.3	0
$\{a,b\}$	0.9	0.1	–	–

- Initially, there are $24(=4!)$ possible preferences
- Regularity violations: $\{a,b,c\} \to \{a,b\}$, $a \succ b$
  - Only 12 possible preferences
- Regularity violations: $\{a,b,c,d\} \to \{a,b,c\}$, either $a \succ c$ or $b \succ c$
  - Only 8 possible preferences
- Regularity violations: $\{a,b,c,d\} \to \{b,c,d\}$, either $b \succ d$ or $c \succ d$
  - Only 4 possible preferences
- Applying $\succ$-Regularity
  - Only two left: $a \succ b \succ c \succ d$ and $a \succ c \succ b \succ d$

Revealed Attention

Revealed Attention

Let π be an AOM and $(\mu, \succ)$ represent π. Then, for every a and S such that $a \in S$,

$$\max_{R \supseteq S} \pi(a|R) \;\leq\; \phi(a|S) \;\leq\; \min_{T \subseteq S:\ a \in T} \pi(U_{\succeq}(a)|T)$$

- New revealed attention in the literature
- Lower bound is independent of preference
- The bound is “tight”

An Illustration

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- Focus on the choice set $\{a, b, c, d\}$
- $\phi(a|\{a, b, c, d\})$ must be 0.05
- $\phi(b|\{a, b, c, d\})$ and $\phi(c|\{a, b, c, d\})$ must be between 0.1 and 0.25
- $\phi(d|\{a, b, c, d\})$ must be between 0.75 and 1

heterogeneous preferences

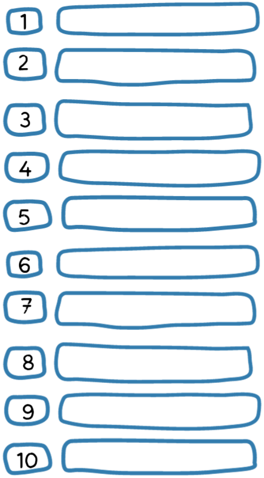
Heterogeneous Preference

- AOM assumes a single preference
  - Nest Random Utility Model in terms of choice behavior
  - $\succ$ is not always fully revealed
- Allowing all preferences
  - No hope in identification
- Heterogenous Preference AOM (HAOM$_{\triangleright}$)
  - Limiting variability of preference and attention through a list, $\triangleright$
  - Assume an item's placement on a list *means something* for attention and evaluation.

A list (linear order) $\triangleright$ over X

- Amazon's product list
- Google's search results
- A ballot for a specific election

denoted by $\langle a_1, a_2, \ldots, a_{|X|} \rangle$



Types

- Each type - $(\succ, \Gamma)$
  - $\succ$: preference
  - Γ: deterministic attention rule
- Both preferences and attention are based on the underlying list $\triangleright$

List-based Attention

- Γ: list-based attention if
  - $\emptyset \neq \Gamma(S) \subseteq S$ (limited consideration)
  - $a_k \in \Gamma(S)$ implies $a_j \in \Gamma(S)$ if $j \leq k$ (following list)
  - $a_k \in \Gamma(S)$ implies $a_k \in \Gamma(T)$ if $a_k \in T \subseteq S$ (attention overload)
  - $\Gamma(S) = S$ whenever $|S| = 2$ (full attention at binaries)
- All list-based attention denoted by $\mathcal{AO}_{\triangleright}$

Single Improvement Preferences

- The list and possible preferences are the same except for one alternative
- For all $j < k$, define $\succ_{kj}$ as a linear order where the kth alternative in $\rhd$ is moved to the jth position.
  - $\succ_{21} = \langle a_2, a_1, a_3, a_4, \ldots, a_{|X|} \rangle$
  - $\succ_{42} = \langle a_1, a_4, a_2, a_3, \ldots, a_{|X|} \rangle$
  - $\succ_{11} = \langle a_1, a_2, a_3, a_4, \ldots, a_{|X|} \rangle = \rhd$
- $\mathcal{P}_{\rhd}$: all such preferences and $|\mathcal{P}_{\rhd}| = \frac{n(n-1)}{2} + 1 < n!$

HAOM$_\triangleright$

Heterogeneous Preference Attention Overload

We say that a probabilistic choice function π has a *Heterogeneous Preference Attention Overload* representation with respect to $\triangleright$ (HAOM$_\triangleright$) if there exists τ on $\mathcal{AO}_\triangleright \times \mathcal{P}_\triangleright$ such that

$$\pi(a|S) = \tau\Big(\Big\{(\Gamma, \succ) \in \mathcal{AO}_\triangleright \times \mathcal{P}_\triangleright : a \text{ is } \succ\text{-best in } \Gamma(S)\Big\}\Big).$$

- Assume the list is observable
- Later, we allow for unobservable lists

Characterization

Axiom *(List-Regularity)* *For all* $a_j, a_k \in T \subset S$ *with* $j < k$, $\pi(a_k|T) \geq \pi(a_k|S)$.

Axiom *(List-Monotonicity)* *For all* a_j, a_k, a_ℓ *such that* $j < k < \ell$, $\pi(a_\ell|a_k) \geq \pi(a_\ell|a_j)$.

Axiom *(List-Boundedness)* $\sum_{j=2}^{|X|} \pi(a_j|a_{j-1}) \leq 1$.

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Axiom *(List-Boundedness)* $\sum_{j=2}^{|X|} \pi(a_j|a_{j-1}) \leq 1$.

Characterization

Given $\triangleright$, a choice rule π satisfies the above three axioms if and only if π has an $\text{HAOM}_{\triangleright}$ representation.

Identification

Preference Types

Let τ be a HAOM$_\triangleright$ representation of π. Then
(i) $\tau(\succ_{kj}) = \pi(a_k|a_j) - \pi(a_k|a_{j-1})$ for $k > j > 1$,
(ii) $\tau(\succ_{k1}) = \pi(a_k|a_1)$, and
(iii) $\tau(\succ_{11}) = 1 - \sum_{k=2}^{|X|} \pi(a_k|a_{k-1})$.

Preference Types

Let τ be a HAOM$_\triangleright$ representation of π. Fix S and let a_{s_1} be its top-listed item. Then, for $k > s_1$,

$$\pi(a_k|S) \leq \tau(\{(\Gamma, \succ) : \Gamma \in \mathcal{AO}_\triangleright \text{ and } a_k \succ a_{s_1}\})$$

- Non-binary choice data provides bounds on types

Revealed Attention

Let τ be a HAOM$_{\triangleright}$ representation of π. Fix S and let a_{s_1} be its top-listed item. Then, (i) $\phi(a_{s_1}|S) = 1$; (ii) for $a_k \in S$ and $k > s_1$

$$\max_{R \supseteq S} \sum_{\ell \geq k} \pi(a_\ell|R) \ \leq \ \phi(a_k|S) \ \leq \ 1 - \sum_{s_1 < j \leq k : a_j \in S} \left(\max_{R \supseteq \{a_{s_1}, a_j\}} \pi(a_j|R) - \min_{\{a_{s_1}, a_j\} \subseteq T \subseteq S} \pi(a_j|T) \right).$$

Unknown List

Let $a \mathsf{L}_\pi b$ if
(i) there exists $\{a, b\} \subseteq S \subseteq T$ such that $\pi(a|S) < \pi(a|T)$, or
(ii) there exists c such that $\pi(c|b) > \pi(c|a)$ and $\pi(b|c) > \pi(b|a)$.

Revealed List

If a strict π has a $\text{HAOM}_\triangleright$ representation, the list is uniquely identified up to the last two elements by L_π.

Summary

- Attention Overload
  - A missing piece in the random attention literature
- Two models: AOM and $\text{HAOM}_{\triangleright}$
  - Applicable in different circumstances

I hope I did not cause Attention Overload