A foundation for strategic agenda voting ☆

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A R T I C L E   I N F O

A B S T R A C T

We offer complete characterizations of the equilibrium outcomes of two prominent agenda voting institutions that are widely used in the democratic world: the amendment, also known as the Anglo-American procedure, and the successive, or equivalently the Euro-Latin procedure. Our axiomatic approach provides a proper understanding of these voting institutions, and allows comparisons between them, and with other voting procedures.

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1. Introduction

A proper understanding of the democratic institutions that are used in practice is a prime concern in the social sciences. In this paper we focus on two prominent voting procedures that are used extensively in parliamentary, legislative, and committee decision-making world-wide: the amendment and the successive agenda procedures. In a nutshell, in this paper we provide for the first time axiomatic foundations to enhance our understanding of these key voting institutions.

Both, the amendment and the successive procedures are voting institutions used to collectively select one alternative from a set of alternatives. They represent natural extensions of simple majority voting to cases where there are more than two alternatives. In both cases, the alternatives are ordered, forming an agenda, and are considered sequentially, taking at each step in the sequence binary decisions using majority voting. In the particular case of the amendment procedure, two alternatives are jointly considered at each step, and the binary choice consists in deciding by majority voting which alternative is eliminated, and hence which alternative is confronted with the next one in the agenda. For the sake of illustration consider three alternatives, ordered as (a, b, c). Then in this case a is voted against b, and the winner against c. The winner of this last confrontation is declared elected. See Fig. 1(a) for a graphical representation.
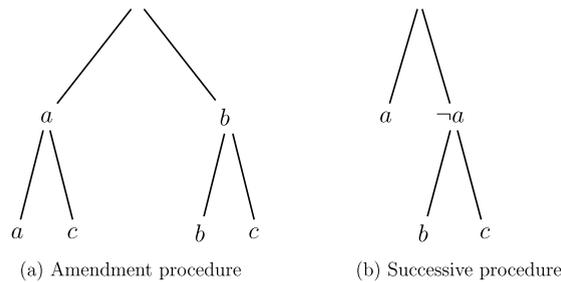
Fig. 1. Structure of amendment procedure and successive procedure with the agenda (a, b, c).

Table 1
Agenda voting procedures by country.

Source: Rasch (2000)

Amendment	Successive
Anglo-American	Euro-Latin
USA	Austria, Belgium, Czech Republic, Denmark
Canada	France, Germany, Greece, Hungary, Iceland
UK	Ireland, Italy, Luxembourg, Netherlands
Sweden	Norway, Poland, Portugal
Finland	Slovakia, Slovenia, Spain
Switzerland	European Parliament

In the successive procedure, an alternative is considered at each step in the sequence, and the binary choice made by majority voting is whether to select it, or to reject it, and in the event of rejection, to consider the next alternative in the sequence. For instance, in the above example, voters must in fact decide between accepting a or rejecting it, in which case they confront the problem of selecting an alternative from $\{b, c\}$. If they prefer a to the alternatives in $\{b, c\}$ then the voting ends. Otherwise they compare the next alternative in the sequence, which is b, with the remaining one, alternative c. Fig. 1(b) provides a graphical representation.

It turns out that versions of these voting procedures are extensively used in committee decision-making, as well as in parliamentary institutions world-wide. Moreover, there seems to be a geographical concentration of the type of agenda voting institution in use. While the amendment procedure is prevalent in the Anglo-American world, European countries adopt the successive procedure (see Table 1 for an illustration). For this reason, from now we will refer by the Anglo-American procedure to the amendment one, and by the Euro-Latin procedure to the successive one.[1]

Given the practical importance of these voting procedures, it is not surprising that they have been subject to substantial theoretical and empirical research, that has led to the clarification of important aspects. Specifically, there is now a good understanding of the nature of the elected outcomes when voters vote either strategically or naively (see Farquharson, 1969; Miller, 1977, 1980; McKelvey and Niemi, 1978; Plott and Levine, 1978; Moulin, 1979, 1986; Shepsle and Weingast, 1984; Banks, 1985; Eckel and Holt, 1989; and Bag et al., 2009). The effect of the agenda on the final elected outcome has also been the subject of intense research (see McKelvey, 1981; Shepsle and Weingast, 1982; Ferejohn et al., 1987; Dutta et al., 2002; and Bernheim et al., 2006). Finally, there are papers that study which voting procedure maximizes in expectation the well-being of society (see Moser, 2007; see also Apesteguia et al., 2011). These questions are of prime importance for the understanding of the voting institutions with which we are concerned.

Here we take a different approach, and offer, for the first time, complete characterizations of these voting institutions. We establish the sets of properties, that when imposed on a decision rule, give the same outcome as the one obtained with strategic voters in the corresponding voting procedure. We show that the two procedures are characterized by two systems of three properties each, sharing a common intuition. The two systems of three properties are formed by (i) a Condorcet-type property, (ii) a property that guides the election in the presence of cycles generated by binary majority voting, and (iii) a consistency property imposing structure on the elections across related sets of alternatives. These characterizations allow a deep understanding of the properties satisfied by the voting institutions, and facilitate comparisons between them, and with other voting procedures. Furthermore, the identification of the characterizing properties of the voting institutions allows us to evaluate their normative appeal, and encourages the study of the consequences of relaxing or strengthening some of these properties.[2]

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[1] Black (1948) represents the first formal treatment of agenda voting institutions. Riker (1958) and Farquharson (1969) study the agenda voting procedures actually used in the US Congress. See also Shepsle and Weingast (1982), Ordeshook and Schwartz (1987), and Schwartz (2008). See Rasch (2000) for a treatment of the European case.

[2] In Section 5.2 we illustrate the sort of voting institutions that emerge when we relax each of the characterization properties, one at a time.

The rest of the paper is organized as follows. Section 2 formally presents the environment, gives the definitions of the voting procedures, and introduces the equilibrium notion used thereafter. Section 3 is devoted to the characterization of the Euro-Latin procedure, while Section 4 does the same for the Anglo-American procedure. Finally, Section 5 discusses the nature of the properties used in the characterizations of the procedures, shows the independence of the axioms by studying alternative voting institutions, and establishes the connection between our exercise and implementation theory and choice theory. All the proofs are contained in Appendix A.

2. Basic definitions

Let X be a finite set of m alternatives and let n denote the number of voters. For convenience, we assume that n is odd. A *decision problem* is a pair (P, A), where $P = (P_1, \ldots, P_n)$ is a profile of preferences, with each P_i being a complete, transitive, and asymmetric binary relation on X, and $A \subseteq X$ is a set of alternatives to vote for. A decision rule v assigns to each decision problem (P, A) an outcome $v(P, A) \in A$.[3]

An agenda $\vec{X} = (x_1, \ldots, x_m)$ is an ordered list of all the elements in X. Given the agenda $\vec{X}$, the associated *Euro-Latin procedure* assigns to any set of alternatives A, the alternative that survives the following process. The first alternative in A according to the agenda is voted for approval. If the alternative is approved by a majority of individuals, the process stops and this alternative is implemented. If the alternative is rejected, the second alternative of A in the agenda is voted for approval. If the alternative is approved by a majority of individuals, the process stops and this alternative is implemented. Otherwise, the next alternative of A in the agenda is considered, and the process is repeated. If the final alternative in the agenda is reached, it is approved without voting. Consider the extensive game representation of the Euro-Latin procedure restricted to the set of alternatives A when voters have preferences P, and denote by $\gamma_{EL}(P, \vec{X}, A)$ its strategic form representation.

Given the agenda $\vec{X}$, the associated *Anglo-American procedure* assigns to any set of alternatives A the alternative that survives the following process. The first pair of alternatives of A in the agenda is voted upon with the one obtaining a majority of votes advancing to the next stage. There, it is paired against the next alternative of A in the agenda, and the process is repeated until the final alternative in the agenda is reached. Consider the extensive game representation of the Anglo-American procedure restricted to the set of alternatives A, and denote by $\gamma_{AA}(P, \vec{X}, A)$ its strategic form representation.

Clearly, both procedures are subject to strategic manipulation by sophisticated voters. In the Euro-Latin procedure, voters may approve an early alternative in the agenda in order to avoid the selection of a later one. In the Anglo-American procedure voters may pass an alternative only because it can defeat a posterior one, and not because it is preferred to the one with which it is competing. Hence, the characterization of these two procedures signifies a challenge that we address in this paper.

One drawback with sophisticated behavior is that Nash equilibrium may yield absurd outcomes. For example, in a simple two-alternative setting, if all players vote for the same alternative, independently of their preferences, we have a Nash equilibrium. The refinement that rules out this sort of behavior is the use of undominated strategies. Clearly, in the former two-alternative example, voting for the less preferred alternative is weakly dominated, and hence would be eliminated. It is well known that binary voting procedures like the one we study here are dominance solvable. That is, the iterated elimination of weakly dominated strategies in the strategic form representation leads to a unique Nash equilibrium outcome (see Moulin, 1979; McKelvey and Niemi, 1978, and Austen-Smith and Banks, 2005). The use of Nash equilibrium in undominated strategies is, therefore, standard practice in voting settings like ours, and this is the one we adopt here. We denote by $UNE[\gamma_{EL}(P, \vec{X}, A)]$ and $UNE[\gamma_{AA}(P, \vec{X}, A)]$ the corresponding equilibria in undominated strategies of the Euro-Latin and Anglo-American procedures respectively.

3. Characterization of the Euro-Latin procedure

Given a decision problem (P, A), a Condorcet winner is an alternative in A such that, for any other alternative in A, a majority of voters ranks the former above the latter. In other words, the Condorcet winner majority dominates all other alternatives in A. The properties of a Condorcet winner make it highly desirable as the social outcome of any political problem. This leads us to consider the classical Condorcet Consistency property.

Condorcet Consistency (CC). The decision rule selects the Condorcet winner whenever this alternative exists.

It is well known that Condorcet winners do not always exist. The simplest situation in which Cordorcet winners fail to exist involves three alternatives x, y and z, and a preference profile P_{xyz}, with x majority dominating y, y majority dominating z, and z majority dominating x. We refer to this three-alternative situation as a Condorcet cycle. The presence of a Condorcet cycle immediately raises the problem of selecting one alternative from the triple forming the cycle. A possible approach to this problem entails identifying an alternative that for certain reasons is given priority over the rest of the

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[3] Dutta et al. (2001, 2002) also consider decision rules in the domain of all the subsets of X.

alternatives. This alternative is always selected in every Condorcet cycle that includes it. More formally, we say that a is *prioritarian* in A if $v(P_{abc}, \{a, b, c\}) = a$, for every P_{abc} and every $b, c \in A$.

Condorcet Priority (CP). The decision rule admits a prioritarian alternative for any set of alternatives.

The presence of Condorcet cycles represents a major challenge, namely, how to break them down and select an alternative. It seems reasonable to assume that political institutions ideally confront this challenge in both: (i) a systematic way, such that the process is coherent and predictable, and (ii) a simple way, such that the procedure of decision making is understandable, transparent and efficient in practice. This is exactly the main features of CP; it provides a simple and systematic method by signifying an alternative over all the others as the candidate for election in case of conflict. The prioritarian alternative may represent for example the status quo, or some other salient alternative.

It is advisable to explore in more detail two simple implications of CP that contribute to a better understanding of the property. First, note that CP implies that for every A with at least three alternatives there is a *unique* prioritarian alternative. This is easy to see. Suppose on the contrary that a and b are prioritarian in A. Then, there exist an alternative $c \in A$ and a preference profile P_{abc}. It follows that whatever the outcome from $v(P_{abc}, \{a, b, c\})$ it leads to a contradiction. Second, it is also immediate that if alternative a is prioritarian in A, then a is prioritarian in every $B \subseteq A$ such that $a \in B$. That is, there is a great deal of consistency across menus of options in the determination of prioritarian alternatives.

Let us now consider the third and last characterizing property of the Euro-Latin procedure. Suppose that the collectivity of voters has to select an alternative from a set A. A natural process involves dividing the scrutiny of alternatives in A into three stages. In stage 1 the collectivity of voters, say the committee, decides over a subset of A, say B, in stage 2 the committee decides over the remaining alternatives in A, say C, and in stage 3, the committee decides over the selection from B and that from C. More formally, we say that a non-empty partition (B, C) of A constitutes a *division* of A if, for any $D \subseteq A$ and for any preference profile P, $v(P, D) = v(P, \{v(P, D \cap B), v(P, D \cap C)\})$.[4]

Division Consistency (DC). The decision rule admits a division for any set of alternatives.

DC allows one to confront all the alternatives at stake by dividing them in reduced problems, simplifying the complexity of the decision. The final election can then be constructed on the basis of elections done in different sessions or by different committees, representative of the preferences of the parliament. This is a desirable feature for institutions dealing with complex options. We now show that the three properties described above completely and uniquely characterize the equilibrium outcome of the Euro-Latin procedure.

Theorem 1. *A decision rule v satisfies CC, CP and DC if and only if there exists an agenda such that v is the undominated Nash equilibrium outcome of the Euro-Latin procedure.*

The intuition of the 'if' part is simple. The main step in the 'only if' part of the proof involves proving that for any set of alternatives A and a prioritarian alternative h of A, the sets $A \setminus \{h\}$ and $\{h\}$ form the unique division of A. That is, we show that the prioritarian alternative of a set can always be separated from the set, without further consequences for the selection. Then, we show that the construction of the agenda places the prioritarian alternative in X, x_1, first in the ordered list, followed by the prioritarian alternative in $X \setminus \{x_1\}$, and so on. Intuitively, therefore, under sophisticated voting in the Euro-Latin procedure, the strongest alternatives, those that are selected in each Condorcet cycle, are considered first in the agenda. The weakest alternatives are only selected when they are Condorcet winners, are considered last in the agenda.

4. Characterization of the Anglo-American procedure

Given a decision problem (P, A), a Condorcet loser is an alternative in A such that, for any other alternative in A, a majority of voters places the former below the latter. Notice that the Condorcet loser, whenever it exists, is unique. The Condorcet loser is a highly undesirable alternative and hence not only would it be absurd to select it as the outcome of a decision problem, but also it might be desired that any sensible voting procedure be robust to the presence or absence of such an alternative. This leads us to the following consistency property.[5]

Condorcet Loser Consistency (CLC). $v(P, A) = v(P, A \setminus \{a\})$ whenever a is a Condorcet loser in (P, A).

In the presence of a Condorcet cycle in the triple $\{x, y, z\}$, our first approach above identifies an element that is given priority. An alternative approach consists in identifying an alternative that is never prioritized in such situations. More formally, we say that a is *antiprioritarian* in A if $v(P_{abc}, \{a, b, c\}) = c$, for every P_{abc} and every $b, c \in A$. That is, alternative a is antiprioritarian if (i) it is never selected when being part of a Cordorcet cycle, and (ii) the alternative selected from the triple $\{a, b, c\}$ is precisely the one that is preferred to a by the majority in the profile P_{abc}, namely c. That is, the alternative a that is considered as antiprioritarian identifies in each case the alternative to be selected.

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[4] For notational purposes, let $v(P, \emptyset) = \emptyset$.

[5] Indeed, in the Discussion section below we argue that Euro-Latin and Anglo-American procedures satisfy both this consistency property and Condorcet Consistency.

Condorcet Antipriority (CA). The decision rule admits an antiprioritarian alternative for any set of alternatives.

CA has the same flavor than CP: it suggests a systematic and simple method to break down cycles by signaling an alternative over the others. While CP signifies an alternative to be chosen in such situations, CA highlights an alternative not to be chosen. This may encourage changes if the alternative is the default one, or avoid risky outcomes when decisions are unclear or controversial. The property naturally suggests the alternative that dominates the antiprioritarian one as the one to be selected.

As in the case of prioritarian alternatives, in every A with $|A| \geq 3$, there is a *unique* antiprioritarian alternative, and also if $a \in B \subseteq A$ is antiprioritarian in A, a is also antiprioritarian in B. One relevant distinction in the structure of the two properties is the importance of the direction of Condorcet cycles. For Condorcet Priority, if alternative a is prioritarian in A and a is part of a Condorcet cycle involving alternatives $\{a, b, c\}$ in A, the direction of the cycle in the preference profile is immaterial for the outcome selected from $\{a, b, c\}$. For Condorcet Antipriority the direction of the cycle is however relevant. Clearly, if a is antiprioritarian and, for the preference profile a is majority preferred to b, b majority preferred to c, and c majority preferred to a the outcome from $\{a, b, c\}$ is option c. However, if the direction of the cycle is the opposite, then the outcome is b.

Our final property establishes a consistency requirement across sets of alternatives. Having identified one antiprioritarian alternative in a set A, one should be able to remove those alternatives dominated by the antiprioritarian one, since one can argue that they are not natural candidates for election, and concentrate on the rest. The election in this subset must coincide with the election in the original set.

Elimination Consistency (EC). If a is an antiprioritarian alternative in A and $v(P, \{a, y\}) = a$, then $v(P, A) = v(P, A \setminus \{y\})$.

EC focuses on the subset of available alternatives that are most relevant, namely, on those that can dominate the alternative that the institution considers as unchoosable. This is consistent with the antiprioritarian idea and helps in the process of decision making especially when multiple and diverse options are available.

We now show that Condorcet Loser Consistency, Condorcet Antipriority and Elimination Consistency completely and uniquely characterize the equilibrium outcome of Anglo-American procedures.

Theorem 2. *A decision rule v satisfies CLC, CA and EC if and only if there exists an agenda such that v is the undominated Nash outcome of the Anglo-American procedure.*

We provide the intuition for the 'only if.' We first construct the agenda $\vec{X}$ as follows. Consider the antiprioritarian alternative x_1 in X, and place it at the end of the agenda. Then, consider the antiprioritarian alternative x_2 in $X \setminus \{x_1\}$ and place it in the second to last position in the agenda, and so on. In the Anglo-American procedure, the most controversial alternatives are placed at the very end of the agenda. These alternatives are selected only when they are Condorcet winners. The second step in the proof involves showing that for every P and A, the election from a decision rule satisfying the properties is the limit of a sequence of stepping stones in the agenda. The proof of this step uses the following idea. The last alternative is antiprioritarian, and thus, by EC and CLC, we can concentrate exclusively on those alternatives that dominate it. A recursive argument concludes the proof. Finally, we show through an inductive argument, along the lines of Shepsle and Weingast (1984), that the $UNE[\gamma_{AA}(P, \vec{X}, A)]$ is exactly the limit of this sequence.

5. Discussion

5.1. Comments on the axiomatic structures of the Euro-Latin and Anglo-American procedures

It is illuminating to note the shared structure of the two systems of properties characterizing the two voting procedures. First, both Condorcet Consistency for Euro-Latin procedures and Condorcet Loser Consistency for Anglo-American procedures follow the fundamental principle of Condorcet-type reasoning. Given the desirability of the two properties, one may wonder whether Condorcet Loser Consistency is satisfied by Euro-Latin procedures and the same is true for Condorcet Consistency and Anglo-American procedures. The answer to both questions is clearly yes. In fact we can prove that we can replace Condorcet Consistency by Condorcet Loser Consistency in our characterization of the Euro-Latin procedure. When Division Consistency holds, then both properties are indeed equivalent.

Proposition 1. *A decision rule v satisfying DC satisfies CC if and only if it satisfies CLC.*

Second, properties Condorcet Priority in the Euro-Latin case and Condorcet Antipriority in the Anglo-American case follow the same type of logic, with opposite directions. Both properties apply in the presence of cycles, and while the former identifies an alternative that gains prevalence whenever it is present, the latter identifies an alternative as unchoosable. One may entertain that while Condorcet Priority is a conservative property in the sense of always selecting the same alternative in the presence of cycles, Condorcet Antipriority entails more diversity and changes by selecting the different alternatives that may defeat the antiprioritarian one. These two properties shape the order of the agenda. While in the Euro-Latin case

the prioritarian alternatives come first in the agenda, in the Anglo-American case the antiprioritarian alternatives come last in the agenda.

Finally, Division Consistency and Elimination Consistency both impose structure on the outcomes selected by the voting procedures across sets of alternatives. Division Consistency makes more sense in political institutions with a marked political party discipline. If party discipline is present, subcommittees that analyze the different options are likely to be representative of the whole parliament, reducing the informational burden of the decision. If party discipline is not so prevalent, all members of the parliament need to process all the alternatives. Here, Elimination Consistency becomes attractive, since it helps to focus the attention only over the subset of relevant alternatives.

5.2. Other voting procedures and the independence of the axioms

We now suggest other voting institutions than the Anglo-American and Euro-Latin ones. The purpose of the exercise is to explore the consequences of relaxing each of the properties we have studied, one at a time. This illustrates the power of the axioms, that is, the structure they are imposing. At the same time, it shows the independence of the properties.

- The *virtue book* (CP, DC, not CC): The decision rule selects in each decision problem the maximal alternative according to a predetermined linear order over the set of alternatives. The *modified virtue book* (CA, EC, not CLC): It selects the alternative with the largest margin with respect to the least virtuous alternative (and, say, it respects the selection of the virtue book in the event of a tie).
- The *Condorcet virtue book* (CC, CP, not DC): The rule would select the Condorcet winner whenever it exists, and would follow the virtue book rule otherwise. The *Condorcet modified virtue book* (CLC, CA, not EC): The rule would apply successively the property of Condorcet Loser Consistency, whenever this is possible, and behaving like the modified virtue book when Condorcet losers do not exist.
- The *generalized agenda rule* (CC, CLC, DC, EC, not CA, not CP). Consider a binary game tree where (i) all alternatives appear once as terminal nodes of the tree and (ii) there exists at least one node with two non-terminal successors.[6] Then the rule selects the undominated Nash equilibrium of the normal form game induced by the binary game.

The rules described in the first two cases are easily checked in terms of their properties and are left to the reader. The generalized agenda rule is especially interesting and can be connected to other similar game tree models in the choice theory literature (Xu and Zhou, 2007). To illustrate why such *tree* agendas fail to satisfy the prioritarian properties, consider one of the special nodes in point (ii) and four alternatives that are successors of it, two from each of the two non-terminal successors. It is immediate to see that for any subset of three alternatives involving a Condorcet cycle, the selection of the rule is the alternative that branches away on its own from the other two. Consequently, there is no prioritarian alternative in the set of four alternatives. Hence, Condorcet Priority does not hold in general. A similar reasoning shows that Condorcet Antipriority is also violated.

5.3. Connection to implementation theory and choice theory

Regarding implementation, our results can be related to the Condorcet Set and the Banks Set. Given a profile of preferences P these sets correspond to the union, *across agendas*, of the equilibrium outcome of the Euro-Latin and the Anglo-American procedures respectively.[7] Suppose one wants to implement a particular social choice function that selects, for every preference profile P, an outcome either in the Condorcet Set or in the Banks Set. Our paper provides an answer to when this can be done by using the Euro-Latin or the Anglo-American procedures, respectively. Namely, the social choice function must satisfy the properties discussed in the paper. The main difference between our approach and the one typically used in implementation theory is that, in our case, a decision rule is defined not only over all possible preference profiles but also over all possible subsets of alternatives, while implementation theory typically works over the grand set of alternatives only.[8]

Ehlers and Sprumont (2008) characterizes the Condorcet Set in the context of choice theory. There are also recent models where a single decision-maker chooses from a set of alternatives by successively comparing alternatives according to a fixed order of alternatives (see Apesteguia and Ballester, 2009; Rubinstein and Salant, 2006; Horan, 2012; and Yildiz, 2012). The main difference between our exercise and these papers is that a decision rule in choice theory is defined only over all possible subsets of alternatives, while in our case a decision rule is defined, in addition, over the profile of preferences. This makes that the characterizations are completely different.

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[6] Some of the conclusions on the independence of the axioms require at least four alternatives, such as the argument that follows.

[7] See Miller (1977) and Banks (1985).

[8] See Jackson (2001) for a survey on implementation theory. An alternative approach to our framework would consider a fixed set of alternatives X and would work with subsets of preference profiles. Instead of $v(P, A)$ one may consider $v(P^A, X)$, where P^A is a preference profile that places alternatives $X \setminus A$ at the bottom of preferences but otherwise coincides with P.

Appendix A. Proofs

Proof of Theorem 1. We first prove that the Nash equilibrium in undominated strategies of a Euro-Latin procedure satisfies the properties. Clearly, CC is immediate. To check CP, we just need to observe that the first alternative of the set according to the agenda is prioritarian. The reason for this is that, if such an alternative is involved in a Condorcet cycle, its rejection would imply the election of the majoritarian winner alternative from among the other two. Because of the Condorcet cycle structure, that alternative is majority inferior to the first one. Finally, any set of alternatives admits a division in which all the alternatives but the first one, according to the agenda, are separated from the first alternative. We now prove the converse statement using a lemma.

Lemma 1. *For any A with $|A| \geq 3$ let h be the prioritarian alternative of A, then $(A \setminus \{h\}, \{h\})$ is the unique division of A.*

Proof of Lemma 1. We proceed by contradiction. Suppose that (B, C) is a division of A, with $h \in B$ and $|B| \geq 2$. Let $b \in B \setminus \{h\}$ and $c \in C$ and consider a profile P_{hbc} where: (i) alternatives b, c and h are preferred to any other alternative in A by every individual, and (ii) alternatives b, c and h form a Condorcet cycle such that h is majority preferred to b, b to c and c to h. By DC, since $\{b, h\}$ is a subset of B and $\{c\}$ is a subset of C, we have $v(P_{hbc}, \{b, c, h\}) = v(P_{hbc}, \{v(P_{hbc}, \{b, h\}), c\})$. Given the structure of P_{hbc}, CC implies $v(P_{hbc}, \{b, h\}) = h$ and $v(P_{hbc}, \{b, c, h\}) = v(P_{hbc}, \{h, c\}) = c$. This is a clear contradiction with h being prioritarian in A. □

Now we define an agenda $\vec{X}$. By CP, there exists a prioritarian alternative in X. Let x_n be such an alternative and let it be placed at the end of the agenda. Recursively, given $x_{k+1}, \ldots, x_n$, we can use CP to define x_k as a prioritarian alternative of $X \setminus \{x_{k+1}, \ldots, x_n\}$.[9]

Next, we prove that v is the outcome of Anglo-American voting over the agenda $\vec{X}$ when voters vote sincerely, that is truthfully reporting their preferences. We prove it by induction on the cardinality of set A. By CC, if $|A| = 2$, the decision rule selects the majoritarian alternative, and hence the claim follows. Suppose the claim is true for any set of alternatives with cardinality lower than or equal to $p \geq 2$. We now prove that the claim is true for any set of alternatives A with $|A| = p + 1$. Let x_m be the last element of A in the agenda. Given the induction hypothesis, $v(P, A \setminus \{x_m\})$ is the outcome of sincere voting in the Anglo-American procedure $\gamma_{AA}(P, \vec{X}, A \setminus \{x_m\})$. Clearly, by CC $v(P, \{v(P, A \setminus \{x_m\}), x_m\})$ is the outcome of sincere voting in the Anglo-American procedure restricted to this pair of alternatives. Given that x_m is the last element of A, it is straightforward that $v(P, \{v(P, A \setminus \{x_m\}), x_m\})$ is also the outcome of sincere voting for the Anglo-American procedure $\gamma_{AA}(P, \vec{X}, A)$. To conclude the claim, we only need to show that $v(P, \{v(P, A \setminus \{x_m\}), x_m\}) = v(P, A)$. By construction, x_m is the prioritarian alternative of $\{x_1, x_2, \ldots, x_m\}$ and hence it is also the prioritarian alternative of A. Hence, by Lemma 1 we know that $(A \setminus \{x_m\}, \{x_m\})$ is the only division of A and hence by DC $v(P, \{v(P, A \setminus \{x_m\}), x_m\}) = v(P, A)$.

To conclude the proof, we use the well-known result on the equivalence between the outcomes of sincere voting in Anglo-American procedures with a given agenda $\vec{X}$ and the outcome of the Nash equilibrium in undominated strategies of the Euro-Latin procedure where the order of the agenda $\vec{X}$ is reversed, given the preference profile P (see Miller, 1977; see also Moulin, 1979). □

Proof of Theorem 2. We first prove that the Nash equilibrium in undominated strategies of an Anglo-American procedure satisfies the properties. CLC is immediate. To check CA, we just need to observe that the last alternative of the set according to the agenda is antiprioritarian. Suppose there exists a Condorcet cycle involving alternatives x, y and a, where x majority dominates y, y majority dominates the last alternative in the triple a and a majority dominates x. Since the outcomes from $\{x, a\}$ and $\{y, a\}$ are a and y respectively, and y majority dominates a, the outcome of the first election, namely between x and y, will be y, which will be confirmed in the final election. Hence, a is antiprioritarian, as announced. Finally, we prove that the Nash equilibrium in undominated strategies of an Anglo-American procedure satisfies EC. The claim is trivial for sets involving two alternatives. Notice that for any set A with at least three alternatives, the antiprioritarian alternative must be the last alternative of A in the agenda, say a. This follows immediately from our previous argument on CA. Let y be majority dominated by a. Clearly, y cannot be the selected alternative in A since y reaching the final election against a would entail the election of a. Then, it follows that at equilibrium the outcome from A must be the same as the outcome from $A \setminus \{y\}$, as desired. We now prove the converse statement.

Let v satisfy CLC, CA and EC. We first construct an agenda $\vec{X}$. By CA, there exists one alternative which is antiprioritarian in X. Denote it by x_1. Suppose we have defined $x_1, \ldots, x_k$. By CA, there exists one alternative which is antiprioritarian in $X \setminus \{x_1, \ldots, x_k\}$. Denote it by x_{k+1}. This process defines an ordered list of alternatives $x_1, \ldots, x_n$. We construct the agenda by setting $\vec{X} = (x_n, x_{n-1}, \ldots, x_1)$.[10]

Now, given the agenda $\vec{X}$, we associate with each pair (P, A) a sequence of alternatives in A, $t_1(P, A), t_2(P, A), \ldots$ as follows. $t_1(P, A)$ is the last alternative in A according to the agenda $\vec{X}$. $t_2(P, A)$ is the last alternative in A, according to the agenda $\vec{X}$, that majority dominates alternative $t_1(P, A)$, if it exists. Otherwise, $t_2(P, A) = t_1(P, A)$. Given

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[9] Notice that this agenda is uniquely defined except for the first two alternatives in the agenda, that can be placed in any order.

[10] Notice again that this agenda is uniquely defined except for the last two alternatives in the agenda, that can be placed in any order.

$t_1(P, A), \ldots, t_s(P, A)$, denote by $t_{s+1}(P, A)$ the first alternative from A in the agenda $\vec{X}$ that majority dominates all alternatives $t_1(P, A), \ldots, t_s(P, A)$, if it exists. Otherwise, $t_{s+1}(P, A) = t_s(P, A)$.

Lemma 2. *Given A and P, $v(P, A) = \lim t_i(P, A)$.*

Proof of Lemma 2. Notice that by construction, $t_1(P, A)$ is antiprioritarian in $A = A_1$. Thus, by EC, $v(P, A_1) = v(P, A'_1)$, where A'_1 contains exactly the alternative $t_1(P, A)$ and the set of alternatives that majority dominate $t_1(P, A)$ in A_1. If $A'_1 = \{t_1(P, A)\}$, we are done. Otherwise, $t_1(P, A)$ is a Condorcet loser in A'_1 by construction. By CLC, $v(P, A'_1) = v(P, A_2)$ where $A_2 = A'_1 \setminus \{t_1(P, A)\}$. By construction, $t_2(P, A)$ is the last alternative in A_2 according to the agenda $\vec{X}$. Hence, $t_2(P, A)$ is antiprioritarian in A_2, and by EC, $v(P, A_2) = v(P, A'_2)$ where A'_2 is the set containing $t_2(P, A)$ and all the alternatives in A_2 that majority dominate $t_2(P, A)$. If $A'_2 = \{t_2(P, A)\}$, we are done. Otherwise, $t_2(P, A)$ is a Condorcet loser in A'_2 by construction. Then, again $t_3(P, A)$ is the last alternative in $A_3 = A'_2 \setminus \{t_2(P, A)\}$ according to the agenda and given the finiteness of X, the iteration of this process proves the lemma. $\square$

Lemma 3. *Given A and P, $UNE[\gamma_{AA}(P, \vec{X}, A)] = \lim t_i(P, A)$.*[11]

Proof of Lemma 3. We prove this lemma by induction on the cardinality of A. If A contains two alternatives, the claim follows immediately. Suppose that the claim is true for sets containing up to k alternatives, and consider A with $k + 1$ alternatives. Take the first and second alternatives in A with respect to the agenda, say x and y. If the alternative x (resp. y) is voted off, then by the induction hypothesis, the outcome of the equilibrium in undominated strategies is the limit $l_1 = \lim t_i(P, A \setminus \{x\})$ (resp., $l_2 = \lim t_i(P, A \setminus \{y\})$). By construction, the outcome of the Nash equilibrium in undominated strategies in A is the alternative that is majority preferred among these two alternatives, l_1 and l_2. We now show that this coincides with the $\lim t_i(P, A)$. We distinguish between the following cases:

- $l_1 \neq y$ and $l_2 \neq x$. Then $l_1 = l_2$ and this element is different from x and y. By construction of l_1, l_1 is majority preferred to both x and y. Hence, l_1 is the limit of the sequence associated with A.
- $l_1 = y$, $l_2 \neq x$. By construction, y majority dominates l_2 and hence y is the outcome of the Nash equilibrium in undominated strategies in A. But clearly, y belongs to the sequence in A, and since x does not belong to the sequence in $A \setminus \{y\}$, it does not belong to the sequence in A either. Hence, y is the limit of the sequence of A, as desired.
- $l_1 \neq y$, $l_2 = x$. By construction, x majority dominates l_1 and hence x is the outcome of the Nash equilibrium in undominated strategies in A. But clearly, y does not belong to the sequence in A since y does not belong to the sequence in $A \setminus \{x\}$. As a consequence, x belongs to it, and it is the limit of the sequence in A.
- $l_1 = y$ and $l_2 = x$. The outcome of the Nash equilibrium in undominated strategies in A is the alternative in $\{x, y\}$ that majority dominates the other. But notice that y belongs to the sequence in A, and x is in the sequence if and only if x majority dominates y. Hence, the limit of the sequence is the alternative in $\{x, y\}$ that majority dominates the other, as desired. $\square$

The two lemmata together conclude the proof. $\square$

Proof of Proposition 1. We first prove that a decision rule v satisfying DC and CC also satisfies CLC. Consider (P, A) and suppose that the Condorcet loser $cl(P, A)$ exists. We need to prove that $v(P, A) = v(P, A \setminus \{cl(P, A)\})$. We define iteratively a finite family of nested sets A_i each containing the Condorcet loser $cl(P, A)$. Let $A_1 = A$. Given the sets $A_1, A_2, \ldots, A_i$ containing $cl(P, A)$, by DC, there exists a division of A_i, B_i (which without loss of generality we assume to contain $cl(P, A)$) and C_i. Define A_{i+1} as B_i whenever B_i contains some alternative in addition to the Condorcet loser. Otherwise, the family stops at A_i.

Now, from the constructed family, we know that:

(1) $v(P, A_i) = v(P, \{v(P, B_i), v(P, C_i)\})$ and
(2) $v(P, A_i \setminus \{cl(P, A)\}) = v(P, \{v(P, B_i \setminus \{cl(P, A)\}), v(P, C_i)\})$.

Hence, $v(P, A_i) = v(P, A_i \setminus \{cl(P, A)\})$ if $v(P, B_i) = v(P, B_i \setminus \{cl(P, A)\})$. That is, $v(P, A_i) = v(P, A_i \setminus \{cl(P, A)\})$ if $v(P, A_{i+1}) = v(P, A_{i+1} \setminus \{cl(P, A)\})$. The conjunction of all these relationships leads to $v(P, A) = v(P, A \setminus \{cl(P, A)\})$ if $v(P, A_k) = v(P, A_k \setminus \{cl(P, A)\})$, where A_k is the last set in the family. In this case, $B_k = \{cl(P, A)\}$ and given that $cl(P, A)$ is majority dominated by all alternatives in $A_k \setminus \{cl(P, A)\}$ we have that $v(P, A_k) = v(P, \{v(P, B_k), v(P, C_k)\}) = v(P, \{cl(P, A), v(P, A_k \setminus \{cl(P, A)\})\}) = v(P, A_k \setminus \{cl(P, A)\})$, as desired.

We now prove that a decision rule v satisfying DC and CLC also satisfies CC. We prove it by induction on the cardinality of the set of alternatives. Suppose that the set has two alternatives. Then, the Condorcet winner is selected by

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[11] Shepsle and Weingast (1984) proved an analogous result. We include Lemma 3 here for completeness.

direct application of CLC. Now suppose that, if they exist, Condorcet winners are always selected for the corresponding sets, which are supposed to have cardinality smaller than or equal to k. Let (P, A) with cardinality of A be equal to $k + 1$ and suppose that the Condorcet winner $cw(P, A)$ exists. By DC, there exists a division (B, C) of A. Suppose without loss of generality that $cw(P, A) \in B$. Hence, $v(P, A) = v(P, B \cup C) = v(P, \{v(P, B), v(P, C)\})$. Since $cw(P, A) = cw(P, B) = cw(P, v(P, C) \cup \{cw(P, A)\})$, by the inductive hypothesis we know that $v(P, \{v(P, B), v(P, C)\}) = v(P, \{cw(P, A), v(P, C)\}) = cw(P, A)$. Hence, $v(P, A) = cw(P, A)$, and CC holds. $\square$

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