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A Canonical Model of Choice with Initial Endowments

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We use the revealed preference method to derive a model of individual decision making when the endowment of an agent provides a reference point that may influence her choices. This model generalizes the classical rational choice model, which views choice as a consequence of "utility maximization". Instead, our model sees choice as arising from "psychologically constrained utility maximization", where the constraints are induced by one's initial endowment. In particular, this model produces status quo bias as a natural consequence (but not necessarily the endowment effect). A range of economic applications identify the predictive and explanatory strength of the model. In particular, we demonstrate that the status quo bias phenomenon reduces the size of the substitution effect in problems of consumption choice.

Key words: Incomplete preferences, Status quo bias, Endowment effect, Procedural choice, Consumer demand

JEL Codes: D11, D81

1. INTRODUCTION

Many empirical studies, both within economics and psychology, have established that decision makers settle various types of choice problems in a *reference-dependent* manner. In particular, there is now a widespread agreement among both behavioural economists and rational decision theorists that decision makers behave in the same choice situation markedly differently, depending on what "reference" they are given in the form of an initial entitlement, endowment and/or default option. Indeed, in a myriad of experimental and field studies, the relative "value" of an alternative is found to be enhanced for agents who possess that alternative as current endowment. In the literature on choice theory, this effect is commonly referred to as the *status quo bias* phenomenon—a concept first identified in the economics literature by Samuelson and Zeckhauser (1988). It is also well documented that the status quo position of a decision maker affects the behaviour of the agent even if the agent chooses to move away from her status quo (as in *reason-based* decision making, or what is called the *attraction effect*).[1]

1. The empirical/experimental literature on reference-dependent individual decision making is too large to be cited here. We refer the reader to Camerer (1995) and Sugden (1999) for insightful surveys on this matter. For discussions of the attraction effect and related phenomena, see Simonson (1989), Shafir *et al.* (1993), Sen (1998), Malaviya and Sivakumar (2002), Ok *et al.* (2011), and references cited therein.

Despite these developments, the literature on individual decision theory does not provide a canonical model of choice that applies to problems with initial endowments that may serve as reference points. In the literature on behavioural economics, such problems are typically treated by means of the Tversky-Kahneman model of loss averse preferences. Yet, the scope of such models are limited because they properly apply only to consumption choice problems, but not, for instance, to job search and/or voting problems (in which relevant dimensions of the problem are not exogenously given). In addition, as we shall see, their predictions may at times be at odds with the status quo phenomenon.

While there seems to be a consensus that the canonical model of rational choice leaves much to be desired from the descriptive point of view, it is undeniable that this model has served economics well due to its universal nature. After all, this model is *canonical* in the sense that it applies to any type of a choice situation (with or without an initial endowment) and hence provides a unifying perspective. It thus seems desirable to extend the rational choice model in a way that incorporates the experimental findings on reference-dependent choice, while preserving its canonical, and thereby unifying, nature.

This article extends the rational choice model to incorporate the widely documented status quo bias phenomenon, among other types of endowment-based reference effects. Recall that the classical model of choice maintains that an agent chooses from a feasible set S by maximizing a utility function on S. (In Figure 1, where S is a subset of $\mathbb{R}^2$ and the utility function is taken as continuous and quasiconcave, this model declares y as the choice from S.) By contrast, the model developed here says that an agent, whose initial endowment is x, chooses from S by maximizing a utility function on S *subject to a* (*psychological*) *constraint induced by* x. (In the absence of an initial endowment, our model reduces to the standard one.) Put more explicitly, the rational choice model maintains that an agent with utility function U (deduced from binary choice situations) deems an alternative y in S as "choosable" whenever

$$U(y) \geq U(\omega) \qquad \text{for every } \omega \text{ in } S,$$

ignoring the potential presence of an initial endowment x. In turn, our choice model maintains this only in the absence of an initial endowment. When there is a status quo option in the problem, say x, our model says instead that an alternative y in S is "choosable" when

"y is itself appealing from the perspective of x",

and

"$U(y) \geq U(\omega)$ for every ω in S that is appealing from the perspective of x".

Just like U is deduced in revealed preference theory from the choice behaviour of the agent, in our analysis we derive U *and* the psychological constraint relation of "being more appealing from the perspective of x". Put differently, while the classical rational choice model captures "rationality in decision-making" by the notion of *utility maximization*, our model captures "bounded rationality in decision-making" by the notion of (psychologically) *constrained utility maximization*. (In Figure 1, for instance, this model declares z as the choice from S.)

The structure of this article is as follows. Section 2 introduces three simple choice axioms and discusses their behavioural basis in some detail. Our first axiom derives from the classical *weak axiom of revealed preference* (WARP). Put precisely, this axiom imposes WARP across all problems with the same initial endowment. (Thus, an agent must act rationally within such problems, but across choice problems with different endowments, her behaviour is not restrained.) Our second axiom is simply a translation of the experimental findings on status quo bias to the language of choice theory. We call this property the *weak status quo bias axiom* (WSQB), for

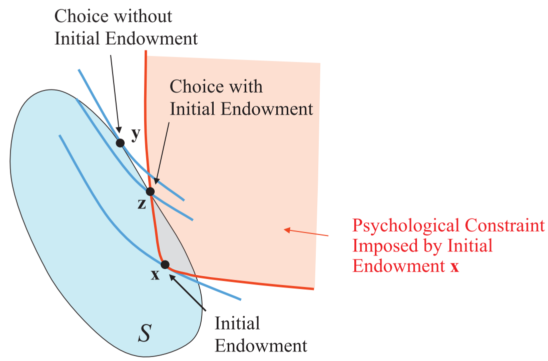

FIGURE 1

Psychologically constrained utility maximization

it says that the choosability of an alternative in a pairwise choice situation cannot possibly deteriorate, and is possibly enhanced, when that alternative is itself the status quo option. Finally, our third axiom, which we call the *status quo irrelevance* (SQI), says that if a status quo option is the least desirable option in a feasible set despite being a status quo (as would be decided by pairwise comparisons), it does not affect one's final choice in that set. Our main result, Theorem 1, characterizes a canonical choice model by means of these axioms (and an additional continuity requirement). Several special cases of this model are also discussed in Section 2.

The model derived in Theorem 1 may be thought of as a bit too general, for it does not say anything about the structure of the psychological constraint relation we mentioned above. In particular, it imposes no discipline on the structure of this constraint across different status quo options. Section 3 shows that replacing WSQB with a stronger status quo bias axiom would yield further structure in this regard. This axiom, which we call the *strong status quo bias* axiom (SSQB), says that, for any choice situation, if an alternative is chosen when it is not a status quo, it should be chosen uniquely when it is itself a status quo, other things being equal. Theorem 2 establishes the fact that with this axiom the psychological constraint relation of the model acts transitively across endowments. As a consequence, we characterize this relation in Theorem 3 as a *dominance* relation with respect to an (endogenously found) collection of criteria, thereby giving the model we study here the flavor of a multi-criteria choice model.

Section 4 looks at the implications of our model(s) in terms of the *endowment effect*—the phenomenon that the minimum compensation demanded by an agent for a good that she owns is more than the maximum price she is willing to pay for the same good (even if one controls for the income effects). We find that the choice behaviour induced by these models need not exhibit this phenomenon, even though they are compatible with status quo bias by construction. We thus conclude here that the common practice of viewing status quo bias and endowment effect as one and the same phenomenon is not really justified.

As we have noted above, a popular way of capturing the status quo bias phenomenon in the literature is through the loss aversion model of Tversky and Kahneman (1991) and its variants. Consequently, if only to clarify the nature of the present contribution, Section 5 provides a brief review of that model and contrasts its structure to that of the choice model developed in the body of the text. In particular, regarding the endowment effect, we find that the qualitative predictions of these two models are generally different. While the loss aversion model entails the endowment effect, our model of choice with status quo bias may be free of this effect.

Section 6 presents four economic applications to illustrate the potential use of our canonical choice model(s) and to contrast them further with the loss aversion model(s). In particular, Section 6.1 uses the model of Theorem 1 to show that the classical Law of Compensated Demand extends to the case of consumers with status quo bias (but not with loss aversion). However, we find (only on the basis of behavioural postulates, and with minimal restrictions on the specification of utility functions) that the presence of the status quo bias reduces the magnitude of the substitution effect. This suggests in the context of labour markets that the backward-bending labour supply phenomenon is likely to be more pronounced in the presence of status quo biased workers. Sections 6.2 and 6.3 consider two financial applications, and show that particular specifications of the choice model of Theorem 2 may be used to explain the equity-premium puzzle, and the tendency of investors to hold on to stocks that have lost value too long while being prone to selling stocks that have gained value. Similarly, Section 6.4 shows that certain specifications of the choice model of Theorem 1 matches the empirical finding that home-owners who sell their houses below their purchase prices tend to set their prices too high, and as a result, such houses remain in the market more than others. In these respects, the implications of the model are similar to those of the loss aversion model.

Finally, Section 7 provides some concluding comments, and the Appendix presents the proofs of our main results.

2. CHOICE WITH INITIAL ENDOWMENTS

In what follows, we designate an arbitrary compact metric space X to act as the universal set of all mutually exclusive alternatives. The set X is thus viewed as the grand alternative space and is kept fixed throughout the exposition. The members of X are denoted as $x,y,z,$ etc.. For reasons that will become clear shortly, we designate the symbol $\diamond$ to denote an object that does not belong to X. We shall use the symbol σ to denote a generic member of $X\cup\{\diamond\}$.

We let Ω_X denote the set of all nonempty closed subsets of X. By a **choice problem**, we mean a list (S,σ) where $S\in\Omega_X$ and either $\sigma\in S$ or $\sigma=\diamond$. The set of all choice problems is denoted by $\mathcal{C}(X)$.

Given any $x\in X$ and $S\in\Omega_X$ with $x\in S$, the choice problem (S,x) is called a **choice problem with a status quo**. (The set of all such choice problems is denoted as $\mathcal{C}_{\mathrm{sq}}(X)$.) The interpretation is that the decision maker is confronted with the problem of choosing an alternative from the feasible set S while she is currently endowed with the alternative x or her default option is x. Viewed this way, choosing an alternative $y\in S\backslash\{x\}$ means that the decision maker gives up her status quo x and switches to y. [2]

On the other hand, many real-life choice situations do not have a natural status quo alternative. Within the formalism of this article, the choice problems of the form $(S,\diamond)$ model such situations. Formally, then, we define a **choice problem without a status quo** as the list $(S,\diamond)$ for any set S in Ω_X. (While the use of the symbol $\diamond$ is clearly redundant here, it will prove convenient in the foregoing analysis.)

By a **choice correspondence** on $\mathcal{C}(X)$ in the present setup, we mean a function $\mathbf{c}:\mathcal{C}(X)\to\Omega_X$, such that

$$\mathbf{c}(S,\sigma)\subseteq S \qquad \text{for every } (S,\sigma)\in\mathcal{C}(X).$$

(Notice that a choice correspondence on $\mathcal{C}(X)$ must be non-empty-valued by definition.)

2. In the language of Rubinstein and Salant (2008), any (S,x) in $\mathcal{C}_{\mathrm{sq}}(X)$ is a choice problem with a frame, where initial endowment x provides the "frame" for the problem. We assume throughout this article that this frame is observable.

2.1. *Axioms for choice with initial endowments*

We begin our axiomatic development by introducing a rationality property familiar from the classical theory of revealed preference. As in that theory, this property warrants that some type of "utility maximization" does take place in the decision-making procedure.

Weak Axiom of Revealed Preference (WARP). *For any* (S,σ) *and* (T,σ) *in* $\mathcal{C}(X)$,

$$\mathbf{c}(S,\sigma)\cap T=\mathbf{c}(T,\sigma),$$

provided that $T\subseteq S$ *and* $\mathbf{c}(S,\sigma)\cap T\neq\varnothing$.

This property conditions the behaviour of a decision maker across two choice problems whose endowment structures are *identical*. In this sense, it is merely a reflection of the classical weak axiom of revealed preference to the framework of individual choice in the (potential) presence of an exogenously given reference alternative. (When $\sigma=\diamond$, our formulation of WARP reduces to the classical formulation of this property.)

We next introduce a way of modelling the status quo bias phenomenon by means of a behavioural postulate.

Weak Axiom of Status Quo Bias (WSQB). *For any* $x,y\in X$,

$$y\in\mathbf{c}(\{x,y\},x) \quad \textit{implies} \quad y\in\mathbf{c}(\{x,y\},\diamond)$$

and

$$y\in\mathbf{c}(\{x,y\},\diamond) \quad \textit{implies} \quad y\in\mathbf{c}(\{x,y\},y).$$

Consider an agent who is vulnerable to the status quo phenomenon. Suppose also that this agent moves away from her status quo option, x, in favour of another alternative y. It seems reasonable that a necessary condition for this to happen is that y is at least as good as x in a reference-free sense. After all, if y was less appealing than x without referential considerations, then, because our agent is in general (weakly) reluctant to move away from her status quo options, she would have never chosen y over x when endowed with x initially.

The interpretation of the second part of WSQB is similar. The idea is that if the decision maker reveals the superiority of y over x in a reference-free setting, then, when the endowment of the agent is y itself, the position of this alternative can only be stronger relative to x, and thus it would surely be choosable over x in that case. This property, which is referred to as "conservatism" by Munro and Sugden (2003), preconditions a choice correspondence to exhibit a bias towards the status quo alternatives in a straightforward manner.

WSQB property seems quite appealing for a rational choice theory that aims at modelling the status quo bias phenomenon. Indeed, versions of this property have recently been adopted in other studies as well.[3] Furthermore, the experimental literature on individual choice provides direct verifications of WSQB.[4]

3. See, for instance, Sugden (2003), Munro and Sugden (2003), Masatlioglu and Ok (2005), Sagi (2006), Apesteguia and Ballester (2009), Ortoleva (2010), Riella and Teper (2010), and Masatlioglu and Nakajima (2013).

4. For instance, Knetsch (1989) has conducted two comparable treatments of a choice experiment involving two goods. In one treatment, participants were simply offered the same choice without initial entitlement. Here, almost half of the participants selected the first good. In the other treatment, the first good was given as an initial entitlement and subjects had the opportunity to exchange their endowment for the other good. Now, the vast majority of students kept

On the other hand, while the status quo bias phenomenon seems to be a fact of life, it would be unreasonable to presume that its effects would be felt regardless of the nature of status quo position. In particular, when the initial endowment is so bad that it is really irrelevant for the final choices, it is likely that it would not exert much influence in the final choices. This leads us to our next postulate, which identifies those situations in which **c** disregards the presence of a status quo alternative.

Status Quo Irrelevance (SQI). *For any given* $(S,x)\in\mathcal{C}_{\mathrm{sq}}(X)$, *suppose that* $\{x\}\neq\mathbf{c}(T,x)$ *for every non-singleton subset* T *of* S *with* $x\in T$ *and that* $\{y\}=\mathbf{c}(\{x,y\},\diamond)$ *for some* $y\in S$. *Then,* $\mathbf{c}(S,x)=\mathbf{c}(S,\diamond)$.

Consider a feasible set S of alternatives and an alternative x in S that is unambiguously "undesirable" in S in the sense that all other alternatives in S are deemed at least as good as x, even when x acts as the status quo option. Furthermore, even if x may be deemed choosable against some alternatives in S when it possesses the power of being the status quo, it is strictly worse than at least one alternative in S in the reference-free sense. Insofar as the desirability of a status quo option is the only arbiter of its reference effects, it seems reasonable in this case to expect the decision maker to view the presence of x as a status quo irrelevant to her final choice. Thus SQI says that in such a situation, she would settle her problem by comparing the alternatives in S in a reference-free manner (*i.e.*, as if there was no status quo in the problem).

Let us illustrate the content of SQI by means of a purposely extreme example. Consider the following alternatives: $z:=$ "accepting the job offer of a Boston company", $y:=$ "accepting the job offer of a San Diego company", and $x:=$ "declining all offers and becoming homeless". In this case, it seems indeed irrelevant that x is given as a default option to the agent. The question is simply choosing between which of the two job offers to accept. Put differently, it seems quite reasonable that $\mathbf{c}(\{x,y,z\},x)=\mathbf{c}(\{x,y,z\},\diamond)$ in this instance, and this is exactly what is posited by SQI.

On the critical side, however, we should note this example overstates the appeal of the SQI property. The only way the choice data at hand can qualify an alternative x as being "very bad" in a given feasible set is by verifying that this alternative would never be chosen in any feasible subset even if it is the status quo. But, in reality, this may be an inadequate method of "irrelevance" of a status quo option. To wit, consider the example above and let $w:=$ "being employed by a New York company". In this case, if w is the status quo option of the agent, but for whatever reason, one that she would surely opt out of in favor of either (inclusive) of her job offers, it is less clear that we would have $\mathbf{c}(\{x,y,w\},w)=\mathbf{c}(\{x,y,w\},\diamond)$. After all, because it is in a big city on the East Coast of the United States, her current job in New York may influence the relative desirability of x and y, and acting as some sort of reference option, it may lead the decision maker to choose x over y (or otherwise) even though she would act differently had w not been an option.

To summarize, let us note that one can think of three basic effects that a person's initial endowment can have on her choice behaviour. First, due to status quo bias, this endowment may create an attraction towards itself. Second, it may force the agent to search for alternatives better than the endowment, thereby altering the ranking of alternatives—albeit only through how

their endowments (around 90 percent versus initial 50 percent). For a critical evaluation of such experiments and more careful takes, see Plott and Zeiler (2005, 2007) and Knetsch and Wong (2009).

There is also plenty of (indirect) field evidence that support the (weak) status quo bias hypothesis. Particularly striking in this regard are the bias found towards default 401(k) plans, various other savings plans, and asset allocation contributions; see, for instance, Madrian and Shea (2001), Choi *et al*. (2004), and Beshears *et al*. (2008).

they compare to the status quo. Third, it may act as a reference point in a more subtle way by changing the relative ranking of the options directly. In our axiomatic development, WSQB is meant to capture the first effect and SQI the second. But, we note that SQI does this at the expense of the third effect. In the next section, we will show that this property still leaves some room for utilizing one's initial endowment as a reference option but allows one to retain the spirit of the standard paradigm of utility maximization, thereby preserving a good amount of predictive power. However, depending on the context, the limits imposed by SQI may well be descriptively untenable, and it will surely be of interest to investigate ways of relaxing this property in future work.[5]

The final property that we shall use is a straightforward reflection of the standard continuity property for choice correspondences. This condition ensures that the choices of a decision maker for "similar" choice problems are "similar".

Continuity (C). *The map* $\mathbf{c}(\cdot,\diamond)$ *is upper hemicontinuous on* Ω_X. *Moreover, for each* (S,x) *in* $\mathcal{C}_{\text{sq}}(X)$ *and convergent sequence* (y_m) *in* X *with* $y_m\in\mathbf{c}(S\cup\{y_m\},x)$ *for each* m, *we have* $\lim y_m\in\mathbf{c}(S\cup\{\lim y_m\},x)$.

Needless to say, this property is automatically satisfied by a choice correspondence $\mathbf{c}$ when the set X of alternatives is finite. Thus, when X is finite, any one of the axioms introduced in this section has a behavioural basis, and hence, it is falsifiable by direct experimentation.

2.2. *Characterization of the main model*

Due to the richness of the present choice domain—we owe this to initial endowments being a part of the description of the choice problems—the class of choice correspondences that satisfy WARP is quite large and includes choice models that can be viewed as *boundedly rational* at best. In fact, even the collection of all choice correspondences on $\mathcal{C}(X)$ that satisfy WARP and the three properties we considered in the previous subsection may correspond to somewhat unorthodox choice behaviour. Fortunately, there is an easy way to think about all such correspondences in a unified manner. In particular, the following theorem shows that any such choice correspondence can be rationalized by a fairly simple two-stage choice procedure.

Theorem 1. *Let* X *be a compact metric space and* $\mathbf{c}$ *a choice correspondence on* $\mathcal{C}(X)$. *Then,* $\mathbf{c}$ *satisfies WARP, WSQB, SQI, and C if and only if, there exist a continuous (utility) function* $U:X\to\mathbb{R}$ *and a closed-valued self-correspondence* $\mathcal{Q}$ *on* X *such that*

$$\mathbf{c}(S,\diamond)=\arg\max U(S), \tag{1}$$

and

$$\mathbf{c}(S,x)=\arg\max U(S\cap\mathcal{Q}(x)) \tag{2}$$

for every $(S,x)\in\mathcal{C}_{sq}(X)$.

5. We are not aware of any empirical evidence against the SQI property. There is some indirect evidence for it in the context of attraction effect experiments, where it is found that a decoy option that is undesirable relative to all other alternatives in a feasible set does not have referential effects; cf. Wedell (1991) and Wedell and Pettibone (1996). (This evidence is indirect because the decoy option is not assigned as the initial endowment in these experiments.) On the other hand, Masatlioglu and Uler (2013) provide some direct evidence for this property by showing that, in a three different treatments, the percentage of the subjects that satisfy SQI vary between 75% and 85%.

This result identifies a canonical choice model for choice problems with exogenously given endowments. To understand its basic nature, let **c** be a choice correspondence on $\mathcal{C}(X)$, U be a continuous real function on X, and Q be a closed-valued self-correspondence on X, and suppose that (1) and (2) hold for any $(S,\sigma)\in\mathcal{C}(X)$. When dealing with a choice problem *without* an initial endowment, an agent whose choice behaviour is modelled through **c** makes her decisions by maximizing the (ordinal) utility function U. That is, in this case, her final choice is realized by solving the problem:

$$\text{Maximize } U(\omega) \text{ subject to } \omega\in S.$$

This is, of course, in complete accord with the weak axiom of revealed preference.

In turn, this agent deals with a choice problem with an initial endowment, say, (S,x), by means of a two-stage procedure. In the first stage, she uses a psychological constraint set $\mathcal{Q}(x)$ and eliminates all feasible alternatives that do not belong to this constraint set. That is, in this stage, the agent identifies the set $S\cap\mathcal{Q}(x)$. (This set is non-empty, because (2) and the fact that $\mathbf{c}(\{x\},x)=\{x\}$ imply $x\in\mathcal{Q}(x)$, for every x in X.)

Intuitively, this set consists of all feasible alternatives that are unambiguously superior to the initial entitlement of the decision maker. Put differently, if $y\in\mathcal{Q}(x)$, then her status quo bias would not prevent a move from x to y. If $y\in S\cap\mathcal{Q}(x)$, then y not only satisfies the physical constraint of the problem (*i.e.* $y\in S$), but it also satisfies the psychological constraint entailed by the status quo position of the agent (*i.e.* $y\in\mathcal{Q}(x)$).[6]

Once $S\cap\mathcal{Q}(x)$ is determined, the agent moves to the second stage of her choice procedure. At this stage, she puts her "rational" hat back on, and among the feasible alternatives that meet her psychological constraint, she chooses those that maximize her utility function. If x is the only element in both $\mathcal{Q}(x)$ and S, this stage leads her to stay with her endowment x trivially, which is a realization of her status quo bias. If there are other alternatives in $S\cap\mathcal{Q}(x)$ as well, then the decision maker may or may not ultimately stay with her status quo. Her final choice is realized by solving the problem:

$$\text{Maximize } U(\omega) \text{ subject to } \omega\in S\cap\mathcal{Q}(x).$$

Notice that we can impose the property $U(y)\geq U(x)$ for every $y\in\mathcal{Q}(x)$ and $x\in X$ in this model (and in the statement of Theorem 1) without loss of generality. The bite of the second stage of the model arises from the fact that the converse of this need not be true; that is, there may be alternatives y outside $\mathcal{Q}(x)$ that may have strictly higher utility value than x. These alternatives are dropped from consideration when the status quo choice of the agent is x – this is how our canonical model accommodates the status quo bias phenomenon (see Figure 1).

In summary, the choice model given by Theorem 1 combines the elements of rationality and the phenomena of status quo bias and reference dependence. An agent whose choice behaviour abides by this model is indistinguishable from a standard utility maximizer in the context of a choice problem without a status quo option. Moreover, even in a choice problem with a given initial entitlement x, among the alternatives that pass the tests imposed by the presence of x, the final choice is determined by maximizing a reference-independent utility function. Thus, while the standard choice model is a utility maximization model, the canonical choice model we derived in Theorem 1 is a constrained utility maximization model, where the constraint is induced by one's default option.

6. Normatively speaking, one may think of the elimination of those alternatives in $S\setminus\mathcal{Q}(x)$ (other than x) from consideration as a simplification the agent uses to settle her possibly complex choice problem (which is similar to the approach used by Manzini and Mariotti (2007)).

Remark. The uniqueness structure of the pair $(U,\mathcal{Q})$ found in Theorem 1 is as follows: Let U and V be two continuous real functions on X, and let $\mathcal{Q}$ and $\mathcal{P}$ be two closed-valued self-correspondences on X such that (1) and (2) hold for every $(S,x)\in\mathcal{C}_{\mathrm{sq}}(X)$ and for each $(U,\mathcal{Q})$ and $(V,\mathcal{P})$. Then, and only then, there is a strictly increasing map $f:U(X)\to\mathbb{R}$ such that $V=f\circ U$ and $\mathcal{Q}(x)\cap x^{\uparrow}=\mathcal{P}(x)\cap x^{\uparrow}$, where $x^{\uparrow}$ is the set of all $y\in X$ with $y\in\mathbf{c}(\{x,y\},\diamond)$, for every $x\in X$.

Remark. The four axioms used in Theorem 1 are logically independent; we provide a proof for this in the Appendix.

In passing, we note that the choice model of Theorem 1 provides a natural way of tracking the magnitude of one's status quo bias, which may be useful for performing certain types of comparative statics exercises. Intuitively, the larger one's psychological constraint set, the less status quo bias she exhibits. This can indeed be obtained as a comparative measure on the basis of a behavioural definition. To wit, for any two choice correspondences $\mathbf{c}_1$ and $\mathbf{c}_2$ on $\mathcal{C}(X)$, we say that $\mathbf{c}_2$ is **less status quo biased than** $\mathbf{c}_1$ if for every x and y in X, (i) $\mathbf{c}_1(\{x,y\},\diamond)=\mathbf{c}_2(\{x,y\},\diamond)$ and (ii) $y\in\mathbf{c}_1(\{x,y\},x)$ implies $y\in\mathbf{c}_2(\{x,y\},x)$. Then, provided that $\mathbf{c}_1$ and $\mathbf{c}_2$ are represented by $(U,\mathcal{Q}_1)$ and $(U,\mathcal{Q}_2)$, respectively, as in Theorem 1, $\mathbf{c}_2$ is less status quo biased than $\mathbf{c}_1$ iff

$$\mathcal{Q}_1(x)\cap\{\omega\in X:U(\omega)\geq U(x)\}\subseteq\mathcal{Q}_2(x)\cap\{\omega\in X:U(\omega)\geq U(x)\}$$

for every $x\in X$. We will make use of this comparative notion in some of the applications presented below.

2.3. *Examples*

This section presents some examples that illustrate the sort of behaviour that is allowed by the canonical choice model outlined above.

Example 1. (*Choice without Status Quo Bias*) A decision maker who is not vulnerable to status quo bias and/or reference effects that may be induced by her initial holdings is captured by the choice model of Theorem 1. Indeed, by setting $\mathcal{Q}(x):=X$ for every $x\in X$, the model becomes $\mathbf{c}(S,\sigma)=\arg\max\{U(\omega):\omega\in S\}$ for every $(S,\sigma)\in\mathcal{C}(X)$. In the language introduced above, this $\mathbf{c}$ is less status quo biased than any choice correspondence that is represented as in Theorem 1 with the utility function U. ■

Example 2. (*Extreme Status Quo Bias*) Consider a decision maker whose choice behaviour is vulnerable to status quo bias at the highest level. Such an agent is captured by the model derived in Theorem 1 by setting $\mathcal{Q}(x):=\{x\}$ for every $x\in X$. With this specification, the canonical model entails $\mathbf{c}(S,x)=\{x\}$ for every $(S,x)\in\mathcal{C}_{\mathrm{sq}}(X)$. An agent whose choice behaviour is modelled as such never moves away from her initial endowment, no matter how good her other options are. In the language introduced above, any choice correspondence that is represented as in Theorem 1 is less status quo biased than this $\mathbf{c}$, regardless of the involved utility function. ■

Example 3. (*No Cycles*) The choice model of Theorem 1 does *not* allow behaviour that exhibits cycles. For instance, for any distinct alternatives x, y and z, the following situation is incompatible with this model: $\{y\}=\mathbf{c}(\{x,y\},x)$, $z\in\mathbf{c}(\{y,z\},y)$ and $x\in\mathbf{c}(\{x,z\},z)$.[7] For, by the representation of $\mathbf{c}$ derived in Theorem 1, these statements would entail $U(y)>U(x)$, $U(z)\geq U(y)$ and $U(x)\geq U(z)$,

7. Notice that our model allows $c(\{x,y\},x)=x$, $c(\{y,z\},y)=y$, and $c(\{x,z\},z)=z$, as in Example 2. This is not a cycle because the decision maker always stays with the initial endowment. Cycles occur when the decision maker moves away from the initial endowment in each choice problem and comes back to where she started.

yielding a contradiction. This shows that, for better or worse, the model at hand still embodies a considerable degree of rationality. ■

Example 4. (*The Attraction Effect*) The model derived in Theorem 1 is suitable for modelling behaviour that exhibits the attraction effect relative to one's initial endowment. That is, it allows an agent to choose x over y when z is the status quo and choose y over x when x is the status quo. To see this, note that "choosing x over y when z is the status quo" is captured in our setup by the statement $\{x\}=\mathbf{c}(\{x,y,z\},z)$ and "choosing y over x when x is the status quo" by $\{y\}=\mathbf{c}(\{x,y\},x)$. But, if $\mathbf{c}$ is represented as in Theorem 1, these two statements would hold provided that $U(y)>U(x)>U(z)$, $x\in\mathcal{Q}(z)$, and $y\in\mathcal{Q}(x)\backslash\mathcal{Q}(z)$. As Figure 4 illustrates, these conditions are duly compatible. ■

In passing, we note that Example 4 points to two essential traits of the choice model of Theorem 1. First, we see that this model allows for using a status quo option as an alternative whose value is somewhat enhanced for the agent, but also, it allows for it to be used as a *reference point*. (In Example 4, we have $\{y\}=\mathbf{c}(\{x,y\},x)$, which means that y is superior to x unambiguously. Yet we have $\{x\}=\mathbf{c}(\{x,y,z\},z)$, which means that "from the viewpoint of z" the option x is deemed better than y, a telltale sign of reference-dependent decision making.)

Second, Example 4 points to the fact that there is a limit to the status quo bias allowed by the choice model of Theorem 1. In particular, if $x\in\mathbf{c}(S,z)$, then we do not need $x\in\mathbf{c}(S,x)$ as well. (Notice that WSQB requires this only for the case $S=\{x,z\}$.) After all, for $S=\{x,y,z\}$ in Example 4, we have $\{x\}=\mathbf{c}(S,z)$ and $\{y\}=\mathbf{c}(S,x)$.

In fact, these two points are closely related. It turns out that a property like "$x\in\mathbf{c}(S,z)$ implies $x\in\mathbf{c}(S,x)$"—this is even weaker than what is dubbed the "Status Quo Bias Axiom" in Masatlioglu and Ok (2005)—robs the involved choice model from attributing a referential status to one's initial endowment. Allowing for reference dependence (on the initial endowment) and status-quo bias phenomenon jointly, then, one needs an axiom that is weaker than the "Status Quo Bias Axiom". This is achieved here by means of WSQB.

The final two examples we shall consider in this section illustrate parametric specializations of the choice model of Theorem 1.

Example 5. Let X stand for the commodity space $\mathbb{R}^n_+$ and take any continuous and strictly increasing utility function $U:X\rightarrow\mathbb{R}$. Let B be a non-empty finite set of n-vectors, and α a nonnegative n-vector. We define the self-correspondence $\mathcal{Q}_{B,\alpha}$ on X as follows:

$$\mathcal{Q}_{B,\alpha}(x):=\{\omega\in X:U(\omega)\geq U(x)\text{ and }\beta\omega\geq\beta(x-\alpha)\text{ for all }\beta\in B\}.$$

With this specification, the choice model of Theorem 1 becomes

$$\mathbf{c}_{B,\alpha}(S,x)=\arg\max\{U(\omega):\beta\omega\geq\beta(x-\alpha)\text{ for all }\beta\in B\}$$

for every non-empty compact subset S of X and every initial endowment x in S (see Figure 2a). For instance, suppose B consists exactly of the unit n-vectors. Then, as illustrated in Figure 2b, the consumption choice behaviour implied by $\mathbf{c}_{B,\alpha}$ does not tolerate a loss of more than α_i units of good $i=1,...,n$. In the extreme case where $\alpha_i=0$ for each i, the agent does not tolerate *any* loss relative to any of the components of her initial holdings.

This parametric formulation facilitates making comparative statics by means of the less status quo biased relation introduced in Section 2.3. For instance, we see here that $\mathbf{c}_{B,\alpha'}$ is less status quo biased than $\mathbf{c}_{B,\alpha}$ iff $\alpha\leq\alpha'$ (component-wise). As each component of α converges to ∞, $\mathbf{c}_{B,\alpha}$ converges to the choice correspondence of a person who maximizes U in a reference-free manner, so we recover the standard rational choice model in the limit. ■

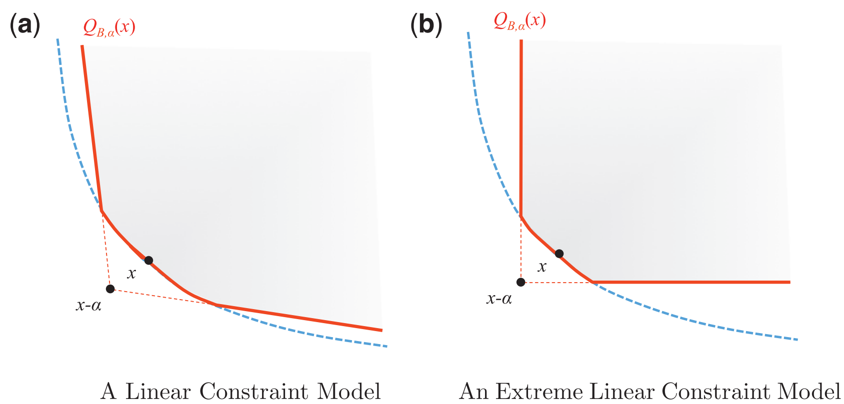

FIGURE 2
Linear constraint models (Example 5)

Example 6. Let X stand for the commodity space $\mathbb{R}^2_{++}$, and assume that the decision maker has a Cobb–Douglas utility function U, defined by $U(x):=\sqrt{x_1x_2}$, on X. Let ρ be a non-positive function on X, and define the self-correspondence $\mathcal{Q}_\rho$ on X as

$$\mathcal{Q}_\rho(x):=\{y\in X:\ v_x(y)\geq v_x(x)\},$$

where $v_x:X\to\mathbb{R}$ is defined by $v_x(y):=\left(a_x{y_1}^{\rho(x)}+b_x{y_2}^{\rho(x)}\right)^{1/\rho(x)}$ with $a_x:={x_2}^{\rho(x)}/2$ and $b_x:={x_1}^{\rho(x)}/2$. (The elasticity of substitution of v_x is equal to $1/(1-\rho(x))$.

With these specifications, the choice model of Theorem 1 becomes

$$\mathbf{c}_\rho(S,x)=\arg\max\{\sqrt{\omega_1\omega_2}:\omega\in S \text{ and } v_x(\omega)\geq v_x(x)\}.$$

It is easy to check that if ρ_1 and ρ_2 are such that $\rho_1\leq\rho_2$ (pointwise), then $\mathbf{c}_{\rho_2}$ is less status quo biased than $\mathbf{c}_{\rho_1}$. Furthermore, in the special case of constant elasticity of substitution, that is, when ρ_1 and ρ_2 are constant functions—Figure 3 depicts this situation—$\mathbf{c}_{\rho_2}$ is less status quo biased than $\mathbf{c}_{\rho_1}$ iff $\rho_1\leq\rho_2$. ■

3. MULTI-CRITERIA DECISION MAKING WITH STATUS QUO BIAS

The choice model characterized in Theorem 1 does not impose any structure on how the psychological constraints induced by various status quo options may relate to each other. According to this model, regardless of the relation between two choice prospects x and y, one may seem extremely status quo biased when x is the status quo option (the case of small $\mathcal{Q}(x)$) but may even act reference-free when her status quo is y (the case of large $\mathcal{Q}(y)$). While this provides strong explanatory power, it limits the predictive power of this model.[8]

In applications, one can of course impose a suitable structure on $\mathcal{Q}$ to avoid these sorts of issues. However, it is not clear if, and how, one can sharpen the behavioural/axiomatic basis of

8. To illustrate, consider an agent whose choice correspondence is represented by U and $\mathcal{Q}$ as in Theorem 1. If $U(y)>U(x)$ and $\mathcal{Q}(x)=\{x,y\}$, then this agent would leave her status quo x only for y. It then stands to reason that it would be even harder for this agent to leave y when y is itself the status quo, thereby exhibiting at least as much status quo bias as she does when x is her endowment. Yet, the model allows even for $\mathcal{Q}(y)=X$.

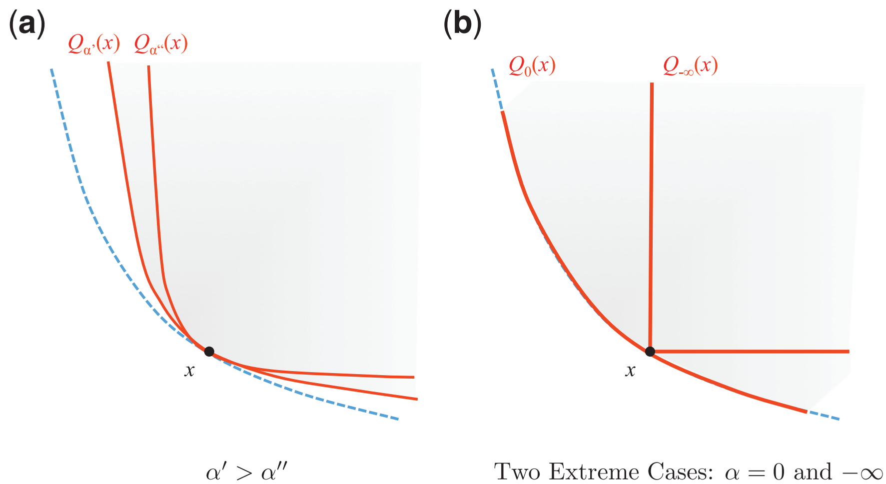

FIGURE 3

A comparative statics for the non-linear constraint model (Example 6)

the present choice model in order to obtain such structural restrictions on $\mathcal{Q}$. In this section, our goal is to show that one can in fact do this on the basis of alternative formulations of the status quo bias phenomenon. Clearly, this requires using a behavioural postulate that is stronger than WSQB. We will focus on the following strengthening of this property.

Strong Axiom of Status Quo Bias (SSQB). *For any $x,y \in X$,*

$$y \in \mathbf{c}(\{x,y\},x) \quad \textit{implies} \quad \{y\} = \mathbf{c}(\{x,y\},\diamond)$$

and for any $S \in \Omega_X$ with $x,y \in S$,

$$y \in \mathbf{c}(S,x) \quad \textit{implies} \quad \{y\} = \mathbf{c}(S,y).$$

The first part of this postulate alone implies WSQB. However, conceptually speaking, this part is only mildly stronger than WSQB. It says that if an agent were to move away from her status quo option x in favor of an alternative y, then this must be because she deems y strictly better than x in a reference-free sense. Clearly, what this adds to WSQB is that ties be broken in favor of the status quo options. On the other hand, the second part of SSQB, which is called the "status quo bias axiom" in Masatlioglu and Ok (2005), says that if y is found choosable in a feasible set S in the presence of a status quo option x, then, the agent would opt for y alone, had y been the status quo option to begin with.

While SSQB seems like a fairly innocuous strengthening of WSQB, we will see below that it has strong implications when combined with WARP and SQI. Let us first give a concrete example of a choice correspondence that satisfies WSQB but not SSQB.

Example 6[Continued]**.** Consider the choice correspondence $\mathbf{c}_\rho$ of Example 6 which is sure to satisfy WSQB. If the elasticity of substitution ρ is constant here, then $\mathbf{c}_\rho$ satisfies SSQB as well. If ρ is not a constant function, however, $\mathbf{c}_\rho$ may well fail to satisfy SSQB, as illustrated in Figure 4. ■

The following result shows how Theorem 1 would change if WSQB were replaced by SSQB.

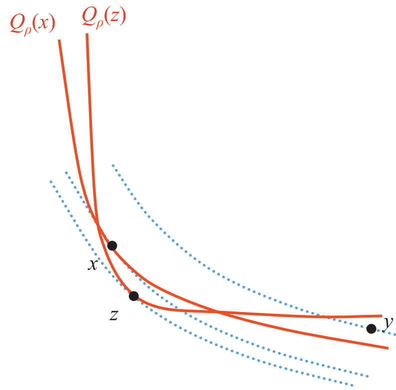

FIGURE 4
An Example satisfying WSQB but not SSQB

Theorem 2. *Let X be a compact metric space and* $\mathbf{c}$ *be a choice correspondence on* $\mathcal{C}(X)$*. Then,* $\mathbf{c}$ *satisfies WARP, SSQB, SQI, and C if and only if, there exist a continuous (utility) function* $U : X \to \mathbb{R}$ *and a closed-valued self-correspondence* $\mathcal{Q}$ *on* X *such that* $\mathcal{Q} \circ \mathcal{Q} \subseteq \mathcal{Q}$ *and*

$$U(y) > U(x) \qquad \text{for every } x \in X \text{ and } y \in \mathcal{Q}(x) \backslash \{x\} \tag{3}$$

while (1) and (2) hold for every $(S,x) \in \mathcal{C}_{sq}(X)$*.*

As expected, replacing WSQB with SSQB in Theorem 1 amounts to imposing further structure on the psychological constraint correspondence $\mathcal{Q}$. In particular, any alternative in $\mathcal{Q}(x)$ other than x itself must now be strictly better than x in the reference-free sense (according to U). While this goes along with the interpretation that $\mathcal{Q}(x)$ consists of "unambiguously better options than x", it implies also that an initial endowment can never be declared indifferent to any other alternative. In particular, the rational choice model (Example 1) is not a special case of the choice model of Theorem 2.[9]

A more important restriction on $\mathcal{Q}$ that is guaranteed by imposing SSQB on top of the postulates of Theorem 1 is the *transitivity* of $\mathcal{Q}$; that is, $\mathcal{Q} \circ \mathcal{Q} \subseteq \mathcal{Q}$. This property says that if $y \in \mathcal{Q}(x)$ and $z \in \mathcal{Q}(y)$, then $z \in \mathcal{Q}(x)$, for any $x, y, z \in X$. In other words, if z is not eliminated from the viewpoint of the status quo y, and y itself is not eliminated from the viewpoint of x, then z is not be eliminated from the perspective of x. This adds quite a bit of structure to the choice model of Theorem 1. In particular, it entails that this model is incompatible with the attraction effect type behaviour we have discussed in Example 4.

On the positive side, Theorem 2 builds a bridge between the psychologically constrained choice model of Theorem 1 and the multi-criteria choice models that are studied elsewhere. This is because, due to the transitivity of $\mathcal{Q}$, the graph of this correspondence act as a preorder. (This preorder is, in fact, a partial order in the present setup; that is, it is antisymmetric.) As any preorder equals the intersection of all of its completions, we find that an expression like $y \in \mathcal{Q}(x)$ can in this case be expressed as saying that y dominates x with respect to all of these completions, and we arrive at a choice model that is based on multi-criteria decision making. This argument becomes

9. In particular, the choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$ defined by $\mathbf{c}(S,\sigma) := S$ is rational (in the sense that it arises from maximization of a reference-free utility function), but it fails SSQB so long as $|X| > 1$.

particularly strong when X is finite. In that case, we can represent each of the completions by means of utility functions, and furthermore, using (3) we may express the reference-free utility function of the model as an aggregation of these criteria. Our final representation result provides a precise description of this situation.

Theorem 3. *Let X be a non-empty finite set and* $\mathbf{c}$ *be a choice correspondence on $\mathcal{C}(X)$. Then,* $\mathbf{c}$ *satisfies WARP, SSQB, and SQI if and only if, there exist a positive integer k and an injection* $\mathbf{u}: X \to \mathbb{R}^k$ *(whose ith component map is denoted by u_i) such that*

$$\mathbf{c}(S,\diamond) = \underset{\omega \in S}{\arg\max} \sum_{i=1}^{k} u_i(\omega),$$

and

$$\mathbf{c}(S,x) = \underset{\omega \in S \text{ and } \mathbf{u}(\omega) \geq \mathbf{u}(x)}{\arg\max} \sum_{i=1}^{k} u_i(\omega)$$

for every $S \in \Omega_X$ and $x \in S$.

Consider a decision maker whose choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$ is represented by finitely many real maps $u_1,...,u_k$ on X as in Theorem 3. We may think of each u_i as measuring how a given alternative fares with respect to a criterion that the decision maker deems relevant for her choices. (Thus, the decision maker evaluates alternatives on the basis of k, subjectively determined, criteria.) When dealing with a choice problem *without* an initial endowment, she makes her decisions by maximizing a utility function that is obtained by the additive aggregation of these criteria. If, on the other hand, the decision maker has a status quo choice in a problem S, say, x, then she proceeds to settle that problem by comparing every feasible alternative to x *with respect to all criteria*. If none of the feasible alternatives y in S dominates x with respect to all objectives—that is, for any feasible y there is a rationale i such that $u_i(x) > u_i(y)$—then she chooses to stay with her status quo. If, however, at least one feasible alternative "beats" x with respect to all criteria, then she concentrates (only) on such alternatives, and chooses among them the one(s) that yield the highest satisfaction in terms of the additive aggregation of her choice criteria.

This choice model relates closely to those heuristically suggested by Simon (1955) and Bewley (1986). There are also more recent contributions in the literature on rational choice theory where related models are axiomatically characterized. In particular, there is a close connection between this result and Theorem 1 of Masatlioglu and Ok (2005). Given the finiteness of X, the systems of axioms used in these results are in fact equivalent. However, the present system of axioms is significantly simpler. Furthermore, Theorem 3 also shows that the multiple objectives found in that paper can be aggregated *additively* into a utility function that governs the behaviour of the decision maker in choice problems without initial endowments.[10]

Remark. The finiteness of X plays an essential role in Theorem 3. First, without this assumption, we cannot ensure that the completions of the preorder induced by Q of Theorem 2 have utility representations. Even more pressing is that, when X is infinite, this partial order need not be the

10. For other multi-criteria choice models that have a similar flavor to that of Theorem 3, see Bossert and Sprumont (2001), Sugden (2003), Munro and Sugden (2003), Giraud (2004), Mandler (2005), Sagi (2006), Houy (2008), Apesteguia and Ballester (2009), Ortoleva (2010), and Riella and Teper (2010).

intersection of *finitely* many of its completions. (In the jargon of order theory, the problem is that the dimension of this partial order need not be finite.) This would make it impossible to obtain finitely many attribute functions $u_1, \ldots, u_k$ in general. Finally, even in those cases where finitely many such objectives can be found, we may not be able to guarantee the linear aggregation of these objectives as in Theorem 3 when X is not finite.[11]

Remark. The axioms used in Theorem 2 (and hence in Theorem 3) are logically independent; we provide a proof for this in the Appendix.

4. THE ENDOWMENT EFFECT VERSUS STATUS QUO BIAS

There are numerous experimental studies that provide evidence to the effect that the minimum compensation demanded by an agent for a good that she owns may be more than the maximum price she is willing to pay for the same good (even if one controls for the income effect). This phenomenon, that is, the negative gap between one's *willingness to pay* (WTP) and *willingness to accept* (WTA), is known as the *endowment effect*—see Thaler (1980)—and it is commonly argued to be a consequence of the status quo bias phenomenon.[12] However, insofar as the notion of status quo bias is modelled through the WSQB property, it does not, in fact, entail the endowment effect. This fact, which seems to have been overlooked in the literature, sits square with the recent empirical findings that in certain environments—for instance in the context of the market interactions of experienced traders—the endowment effect is likely to be absent.[13] In this section we provide a brief demonstration of this point by using an instance of the canonical choice model derived in Theorem 1.

To conform with the previous experimental literature, we adopt a framework that involves only money and a single unit of a commodity, say, a mug. Consequently, we designate $X := I \times \{0,1\}$ as the alternative space for this environment, where I is an interval of the form $[0,\alpha]$ with $\alpha > 0$ being a positive number. (Here a pair like $(a,1)$ in X is interpreted as the agent possessing a dollars and the mug, while $(b,0)$ in X corresponds to the bundle that contains b dollars and no mug.)

Consider a decision maker whose initial monetary endowment is w_o dollars, where $0 \leq w_0 \leq \alpha/2$. We define the *willingness to accept* of this agent (for the mug) as

$$\text{wta}(\mathbf{c}) := \inf\{a \geq 0 : (w_o + a, 0) \in \mathbf{c}(\{(w_o + a, 0), (w_o, 1)\}, (w_o, 1))\},$$

where $\mathbf{c}$ corresponds to her choice correspondence on $\mathcal{C}(X)$. That is, wta$(\mathbf{c})$ is the smallest amount of money that this agent, *if endowed with the mug*, would charge to sell the mug. The formulation of WTP is less straightforward. WTP can be defined as the maximum amount of money that she would be willing to give up in return for the mug when she is endowed with w_o and no mug. That is, WTP is the largest a such that $(w_o - a, 1) \in \mathbf{c}(\{(w_o - a, 1), (w_o, 0)\}, (w_o, 0))$. With this definition, even the standard theory implies that WTP is always less WTA as long as the good is normal (due to income effect). Therefore, we must control income effect when we define WTP so that the standard model would imply WTP=WTA. This leads to defining the *willingness to pay* of this agent (for the mug) as

$$\text{wtp}(\mathbf{c}) := \sup\{w_o \geq a \geq 0 : (w_o, 1) \in \mathbf{c}(\{(w_o, 1), (w_o + a, 0)\}, (w_o + a, 0))\}.$$

11. We could instead use $F \circ \mathbf{u}$ instead of $u_1 + \cdots + u_k$ in Theorem 2, provided that F is a strictly increasing real function on $\mathbf{u}(X)$.

12. See Kahneman *et al.* (1991) and Camerer (1995, pp. 665–670) for detailed surveys on this matter.

13. See Shogren *et al.* (1994), List (2003, 2004), and Plott and Zeiler (2005).

That is, wtp($\mathbf{c}$) is the largest amount of monetary increment the agent would be willing to give up to be able obtain the mug. In what follows, we assume $w_o >$ wtp($\mathbf{c}$); that is, the agent is not willing to give up all of her wealth to receive the mug. Note that if $\mathbf{c}$ arises from the maximization of a strictly increasing utility function on X, we would surely have wta($\mathbf{c}$) = wtp($\mathbf{c}$).

Despite the commonly held contention that "status quo bias implies the endowment effect"—this is indeed a formal implication of various models in the literature; see Example 8 below—certain special cases of the canonical choice model of Theorem 1 (but not of Theorem 2) allow for wta($\mathbf{c}$) = wtp($\mathbf{c}$). This equality may still hold for choice correspondences that exhibit status quo bias in a strict sense. In particular, for a choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$ that is represented by a utility function U on X and a closed self-correspondence $\mathcal{Q}$ on X as in Theorem 1, the absence of the endowment effect is characterized as follows: Let p stand for the unique positive number with $U(w_0+p,0)=U(w_0,1)$; this is the reference-free *fair price* of the mug. Then, $\mathbf{c}$ does not exhibit the endowment effect; that is,

$$\text{wta}(\mathbf{c})=\text{wtp}(\mathbf{c}) \quad \text{iff} \quad (w_0+p,0)\in\mathcal{Q}(w_0,1) \text{ and } (w_0,1)\in\mathcal{Q}(w_0+p,0).$$

To illustrate, we exhibit in Figure 5 two instances, one in which wta($\mathbf{c}$) = wtp($\mathbf{c}$) and one in which wta($\mathbf{c}$) > wtp($\mathbf{c}$). In the left part of this figure, we depict a specification of the choice model introduced in Example 5 such that $(w_0+p,0)$ and $(w_0,1)$ belong to the psychological constraints at $(w_0,1)$ and $(w_0+p,0)$, respectively. In view of the discussion above, the associated choice correspondence is free of the endowment effect. The right part of Figure 5 uses a specification of the choice model introduced in Example 6 in which $(w_0+p,0)$ does not belong to the psychological constraint at $(w_0,1)$. This choice model exhibits the endowment effect.

The choice model of Theorem 1 is thus general enough to allow for the presence and absence of the endowment effect while preserving the status quo bias (in the sense of WSQB) at all times. In such a model, it is even possible that the endowment effect is present for some trades but not others. For instance, it is easy to modify the graphical illustrations in Figure 5 so that no gap is present in a unit sale of the mug but a gap is present in the sale of two or more mugs.

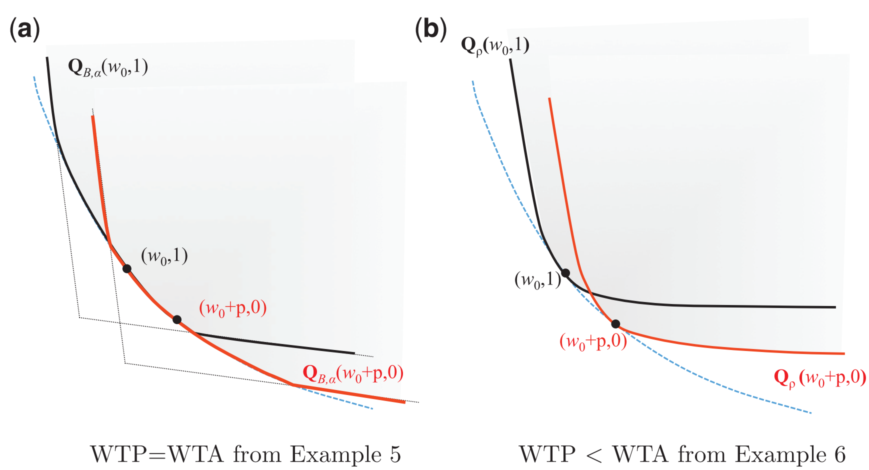

FIGURE 5

WTA versus WTP

5. LOSS AVERSION VERSUS STATUS QUO BIAS

In behavioural economics, *loss aversion* refers to a decision maker's tendency to put more emphasis on avoiding losses than on acquiring gains. While its underlying intuition is unambiguous, it appears difficult to define this concept formally in a context-free fashion. This draws a stark contrast with the status quo bias phenomenon which can be formulated as WSQB (or a variant of it) independently of the context of choice. Nevertheless, loss aversion phenomenon appears very much related to that of status quo bias, so much that it is often argued in behavioural economics that status quo bias is but a manifestation of one's loss aversion. Following this, the most common way of modelling the decision-making procedure of a decision maker with status quo bias is by using a loss aversion model. In this section, we point to some difficulties with this practice and provide a comparison of the choice model we introduced above to those that would be rationalized by loss averse preferences.[14]

To this end, we compare our model of status quo bias with the prototypical specification of the loss aversion model of Tversky and Kahneman (1991). This model is viewed within the behavioural economics folklore as the standard model of reference-dependent decision making. It is defined in a framework where the objects of choice have multiple, say $n \geq 2$, attributes, all of which are observable. It is thus particularly suitable to study choice decisions of individuals over feasible bundles of n goods. Indeed, this is primarily how the model is used in practice.

The basic premise of the Tversky–Kahneman (TK) model, which we shall henceforth refer to as the TK-model, is that an agent, whose initial entitlement is some non-negative n-vector $\mathbf{x}$, chooses those alternatives from a given feasible subset of $\mathbb{R}^n_+$ by maximizing a utility function $U_{\mathbf{x}} : \mathbb{R}^n_+ \to \mathbb{R}$ of the following form:

$$U_{\mathbf{x}}(\mathbf{y}) := \sum_{i=1}^{n} u_i(y_i - x_i). \tag{4}$$

Here, for each i, $u_i : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function that satisfies the following two properties:

(1) $u_i(0) = 0$ and $u_i(t) < -u_i(-t)$ for each $t \neq 0$;
(2) u_i is concave on $\mathbb{R}_+$ and u_i is convex on $\mathbb{R}_-$.

Property (1) captures the phenomenon of *loss aversion*, which says that, in terms of the ith commodity, a gain is less important to the agent than a loss of equal size.[15] In turn, Property (2) says that the marginal value of such gains and losses are decreasing, and hence corresponds to the so-called *diminishing sensitivity* effect.

A number of implications of this model is found to be consistent with experiments in which the presence of an initial entitlement affects one's choice behaviour. There are also a good number of market-related anomalies that are found to be duly consistent with the TK-model. For instance, in the context of all three of the applications we have considered in Sections 6.2–6.4, it is known that the predictions of the TK-model match the empirical regularities quite well. Moreover, the TK-model has a particularly simple mathematical structure that makes it amenable to applications, an

14. We concentrate here exclusively on works in which one's reference is modelled exogenously as her (observable) initial holdings and/or default option. Of course, the case where one's reference is not observable—*e.g.* one's aspirations, as in the work of Köszegi and Rabin (2006), and endogenously formed reference points, as in Ok *et al.* (2011)—is at least as interesting. However, we have little to say about these forms of reference dependence in the present article.

15. This definition was suggested by Wakker and Tversky (1993).

appealing property. Despite its incipient popularity, there are several difficulties that surround this model. First of all, it is not a canonical choice model in that it does not readily apply to individual choice problems in which the objects of choice are not consumption bundles. Indeed, it is difficult to see how to make use of this model, say, in the context of voting over political candidates, choosing between insurance or retirement policies, comparing job offers, etc.[16] The TK-model thus seems best situated within the context of consumption decisions alone. Considering, in addition, the fact that this model is based on an additive aggregation hypothesis, it becomes transparent that it cannot serve as a canonical model of reference-dependent choice.

Even in the context of choosing among consumption bundles, there are some issues with the TK-model, at least insofar as one is interested in capturing the choice behaviour of a decision maker with status quo bias. A major reason for this is that this model does not leave room for studying the decisions of an agent who does *not* have a status quo position. This, in turn, hampers the ability of the model to deal with the *status quo bias phenomenon* properly, contrary to the commonly held view in behavioural economics.

To clarify this point, let us recall that a basic premise of the status quo bias phenomenon is this:

> A person who exhibits status quo bias may choose an alternative x over another alternative y when she is originally endowed with x, even though she would in fact prefer y over x absent any reference effects.

Put differently, the status quo bias phenomenon maintains that, being a status quo may not decrease—in fact it may increase—the value of an alternative (in the eyes of a decision maker) relative to the reference-free value of this alternative. This is precisely the second part of the WSQB axiom, and it is indeed the nature of the status quo bias phenomenon discovered in the experimental studies. And yet, the TK-model is ill-prepared to model this premise, precisely because this model does not apply to problems without status quo points, and hence does not specify when an agent prefers a bundle over another "absent any reference effects".

In applications, this difficulty is either ignored (by not considering those choice problems without reference points) or is dealt with by designating a particular commodity bundle to act as the no reference point. (More often than not, this point is chosen to be the least preferable bundle, namely, the one that has 0 units of all goods.) Unfortunately, this does not solve the problem. For no matter which bundle is chosen as the no reference point, the formalism of the TK-model sees that point as a reference, and hence, it predicts behaviour that may well be *inconsistent* with the status quo bias phenomenon. We illustrate this next.

Example 7. (*The TK–Model and the Status Quo Bias Phenomenon*) Consider an environment in which an agent needs to choose among pairs (M,m), where M and m stand for the units of mugs and money, respectively. We shall use a particular specification of the TK-model in

16. A typical response to this, say, in the context of job offers, is that every job contract is a multidimensional object—the dimensions being, for instance, work location, salary, job quality, etc.—and hence, once the dimensions are specified, the TK-model becomes applicable to this context. The difficulty here is that these dimensions are not observable; the dimensions that are deemed relevant to the problem are private information of the decision maker. Furthermore, even when they are pre-specified, these dimensions need not be quantifiable (think of location, or reputation, that comes with the job), so one has to view $\mathbf{x}$ in this context as the utility profile that the agent derives from her current job. In turn, $y_i - x_i$ corresponds to the gain/loss of *utility* (of switching from $\mathbf{x}$ to $\mathbf{y}$) with respect to the ith dimension. This leads one to subscribe to disconcertingly strong cardinality-of-utility assumptions just to be able to view $y_i - x_i$ as a meaningful expression, and surely makes the quantity $\sum u_i(y_i - x_i)$ rather difficult to interpret.

which the utility of the bundle (M,m) for an agent whose initial (status quo) endowment is (M^*,m^*), is

$$U_{(M^*,m^*)}(M,m) = u(M-M^*)+v(m-m^*),$$

where

$$u(t) := \begin{cases} t, & \text{if } t \geq 0 \\ 2t, & \text{if } t < 0 \end{cases} \quad \text{and} \quad v(t) := \begin{cases} t^{0.8}, & \text{if } t \geq 0 \\ -2|t|^{0.8}, & \text{if } t < 0 \end{cases}.$$

This specification, while robust, is the same as that estimated by Tversky and Kahneman (1992).[17]

Now suppose the agent initially has no holdings of mugs and money, and hence designate $(0,0)$ as the no reference point (but note that the present example can be adapted to the case where any 2-vector is designated to serve as the no reference point). Consequently, when offered to choose between "one mug and \$100" and "no mug and \$103", this agent would choose the first bundle because

$$U_{(0,0)}(1,100) = 1+100^{0.8} > 103^{0.8} = U_{(0,0)}(0,103).$$

Now, right after the agent made her choice, in which she declared that $(1,100)$ is strictly better than $(0,103)$ in a reference-free manner, let us offer her the option of choosing "no mug and \$103" again. Given that her current status quo is $(1,100)$, and we know already that this bundle is better than $(0,103)$ for her even when $(1,100)$ is not a status quo, the status quo bias phenomenon (in the form considered above) requires that the decision maker remains with her current endowment. Surprisingly, the model at hand predicts the opposite of this:

$$U_{(1,100)}(1,100) = 0 < 2(-1)+3^{0.8} = U_{(1,100)}(0,103).$$

Thus, here the TK-model predicts that being a status quo *decreases* the value of the bundle $(1,100)$ in the eyes of our agent relative to the reference-free value of this alternative. Clearly, this prediction goes against the basic premise of the status quo bias phenomenon.[18] ■

Example 7 shows that the TK-model is, in general, not consistent with the basic premise of status quo bias, and certain specifications of it may even exhibit status quo *averse* behaviour. While this model may provide a good way of capturing the choice behaviour of loss averse decision makers, the same does not appear to be true for the choice behaviour of decision-makers with status quo bias. More generally, it seems like the commonly held contention that "loss aversion implies status quo bias" is suspect, at least insofar as loss aversion is modelled through the classical TK-model.[19]

Remark. The observation above stems from the fact the diminishing sensitivity (for losses) may counteract loss aversion. It is, however, generic in the sense that for any TK-model such that $u_i|_{\mathbb{R}_+}$ is strictly concave on some interval (for some i) we can find such a violation of the status

17. Here u, which measures the significance of gain or loss of mugs, exhibits constant loss aversion, while v, which measures that of gain or loss of money, exhibits strictly diminishing sensitivity.

18. While the content of Example 7 seems novel, it was Sagi (2006) who first found that the loss aversion models cannot avoid such adverse consequences (due to the kinks they involve in the associated indifference curves). A related difficulty, namely, the possibility of cyclical choices under the TK-model, was pointed out by Munro and Sugden (2003).

19. The type of behaviour we demonstrated in Example 7 cannot occur according to the model derived in Theorem 1. To see this, note that "choosing x over y in the absence of an initial entitlement" is captured in our setup by the statement $\{x\} = \mathbf{c}(\{x,y\},\diamond)$, and "choosing y over x when x is the status quo" is even stronger than saying $y \in \mathbf{c}(\{x,y\},x)$. But, if $\mathbf{c}$ were represented as in Theorem 1, the first of these statements would entail $U(x) > U(y)$, and the second $U(y) \geq U(x)$.

quo bias phenomenon. In other words, the only special case of the TK-model that is globally consistent with the basic premise of the status quo bias phenomenon is the one of *constant loss aversion* (in which each $u_i|_{\mathbb{R}_+}$ and $u_i|_{\mathbb{R}_-}$ are assumed to be linear). But, clearly, this version of the TK-model is simply too restrictive to serve as a canonical model of choice with status quo bias. In particular, it fails the law of diminishing marginal utility (for every good involved). Indeed, all parametric estimations of this model (that we are aware of) exhibit some degree of diminishing sensitivity.[20]

This observation suggests that the behavioural implications of the phenomena of loss aversion and status quo bias may well be distinct. The following example provides a more concrete demonstration.

Example 8. (*The TK–Model and the Endowment Effect*) We consider the setting introduced in Section 4 when comparing the status quo bias phenomenon with the endowment effect. Let **c** be the choice correspondence in this context that comes from the maximization of an endowment-dependent utility function as in the TK-model. Then, labelling u_1 and u_2 as u and v, respectively, we have

$$\mathrm{wta}(\mathbf{c}) = \inf\{a \geq 0 : u(w_o + a - w_o) + v(-1) \geq 0\}$$

(since $u(0) = 0 = v(0)$). Because u is continuous and strictly increasing, $u(\mathrm{wta}(\mathbf{c})) = -v(-1)$. Similarly,

$$\mathrm{wtp}(\mathbf{c}) = \sup\{w_o \geq a \geq 0 : u(w_o - (w_o + a)) + v(1) \geq 0\},$$

and it follows that $u(-\mathrm{wtp}(\mathbf{c})) = -v(1)$. Consequently, $u(\mathrm{wta}(\mathbf{c})) = -v(-1) > v(1) = -u(-\mathrm{wtp}(\mathbf{c})) > u(\mathrm{wtp}(\mathbf{c}))$, where the strict inequalities follow from the hypothesis of loss aversion. Since u is strictly increasing, we find that $\mathrm{wta}(\mathbf{c}) > \mathrm{wtp}(\mathbf{c})$. Conclusion: every choice correspondence induced by the TK-model is subject to the endowment effect (with respect to every trade). ■

We have seen in Section 4 that status quo bias phenomenon (as formulated by WSQB) does not necessarily entail the endowment effect. Therefore, in light of Example 8, it appears that loss aversion, but not the status quo bias phenomenon, provides sound behavioural foundations for the endowment effect. In particular, the finding that the endowment effect disappears in environments with repeated trades—see, for instance, List (2003, 2004)—entails the same for loss aversion but not for status quo bias.

Remark. The observation of the previous example can be used to devise tests that compare the loss aversion model with, say, the rational choice model of Theorem 1. If, in an experimental setting, a subject does not exhibit the endowment effect, but her behaviour is consistent with WSQB, then her choices may not be represented by the T-K model while they may or may not be represented by the model of Theorem 1. If these choices are, in addition, consistent with WARP and SQI, then they constitute a data point for the latter model. Of course, this is only one way of testing the two models comparatively. (In Section 6.1 we will point to another method in the context of consumption problems.) The recent paper by Masatlioglu and Uler (2013) provides several experimental tests that distinguishes the TK-model and the rational choice models of Theorems 1 and 2.

20. In recent experimental work, Masatlioglu and Uler (2013) have directly compared the explanatory power of the constant loss aversion model with that of the general T-K model. The discrepancy that they have found is substantial.

6. ECONOMIC APPLICATIONS

In this section, we present four applications of the canonical choice models we introduced in Sections 2 and 3 in certain types of economic contexts. On the one hand, these applications demonstrate the applicability of these models, and on the other, they provide further comparison of some of the concrete implications of our models with those of the loss aversion models.

6.1. *Size of the substitution effect in demand theory*

Consider an economy in which a decision maker may consume two goods, the second one being a composite (*numeraire*) good (standing for everything beyond the first good that the consumer may wish to consume). The utility function of this decision maker is given by a continuous, strictly increasing, and strictly quasiconcave map $U:\mathbb{R}^2_+\to\mathbb{R}$, and her income is denoted by $w>0$. When the prices in the economy are given by $\mathbf{p}\in\mathbb{R}^2_{++}$ with $p_2=1$, the optimal consumption choice is found by maximizing $U(\mathbf{x})$ subject to the budget constraint $\mathbf{px}\le w$. (At this stage of the analysis, we treat the decision maker as acting without a status quo option; more on this shortly.) Let us denote this choice by $\mathbf{x}(\mathbf{p},w)$, the demand of the decision maker at prices $\mathbf{p}$ and income w.

Assume now that, perhaps even before the agent exercised her choice, the price of the first good decreases, so that the new price in the economy is $\mathbf{q}\in\mathbb{R}^2_{++}$, where $p_1>q_1$ and $p_2=q_2=1$. In this case, it seems reasonable to model the consumer's problem as making a choice from the new budget set $\{\mathbf{x}\in\mathbb{R}^2_+:\mathbf{qx}\le w\}$ with $\mathbf{x}(\mathbf{p},w)$ acting as her status quo option. Consequently, if we posit that the agent behaves in accordance with the choice model given in Theorem 1, her problem is:

$$\text{Maximize } U(\mathbf{x}) \text{ subject to } \mathbf{qx}\le w \text{ and } \mathbf{x}\in\mathcal{Q}(\mathbf{x}(\mathbf{p},w))$$

where $\mathcal{Q}$ is a closed-valued self-correspondence on $\mathbb{R}^2_+$. Here, of course, $\mathcal{Q}(\mathbf{x}(\mathbf{p},w))$ corresponds to the consumption bundles that look particularly appealing "relative to the status quo choice $\mathbf{x}(\mathbf{p},w)$"; that is, this set consists of all feasible alternatives that are unambiguously superior to $\mathbf{x}(\mathbf{p},w)$.[21] Given that the (reference-free) preferences of the agent are monotonic, it is natural to assume that $\mathcal{Q}$ has increasing values; that is, $\mathbf{x}+\mathbb{R}^2_+\subseteq\mathcal{Q}(\mathbf{x})$. Similarly, the convexity of the preferences of the agent makes it reasonable to posit that $\mathcal{Q}$ is convex-valued.

We now perform a classical Slutsky analysis to understand whether the law of compensated demand holds for this agent, and if so, how the substitution effect in this case relates to that in the standard case where the agent acts without status quo considerations. To this end, let us set $w':=\mathbf{qx}(\mathbf{p},w)$, the income level that makes the earlier consumption $\mathbf{x}(\mathbf{p},w)$ exhaust one's budget in the new prices $\mathbf{q}$. (Here $w'-w<0$ is the Slutsky compensation.) The (hypothetical) choice problem of the agent (in which the income effect of the change of prices from $\mathbf{p}$ to $\mathbf{q}$ is eliminated) is

$$\text{Maximize } U(\mathbf{x}) \text{ subject to } \mathbf{qx}\le w' \text{ and } \mathbf{x}\in\mathcal{Q}(\mathbf{x}(\mathbf{p},w)). \tag{5}$$

We denote the solution of this problem by $\mathbf{x}(\mathbf{q},w')$.

The *Slutsky substitution effect*, or the *compensated own-price effect*, is defined as the magnitude of $x_1(\mathbf{p},w)-x_1(\mathbf{q},w')$. In turn, the classical *Law of Compensated Demand* maintains that this effect is always negative, provided that $\mathcal{Q}=\mathbb{R}^2_+$; that is, when the consumer is a standard utility-maximizer. As we show by revealed preference analysis, this law remains intact in the present framework in which the consumer acts in a status quo biased manner.

21. In the industrial organization literature, researchers often posit the existence of switching costs to account for the stickiness in purchase decisions by consumers. Klemperer (1995) suggests about six different reasons for this phenomenon, one of which is the "psychological costs of switching". Given the foundations we have provided for our choice model in Theorem 1, the present formulation of consumer choice behaviour seems thus well motivated.

Fact 1. (The Law of Compensated Demand) $x_1(\mathbf{q},w') \geq x_1(\mathbf{p},w)$.

Proof. Since $\mathbf{x}(\mathbf{p},w) \in \mathcal{Q}(\mathbf{x}(\mathbf{p},w))$ and $\mathbf{qx}(\mathbf{p},w) = w'$, we have $U(\mathbf{x}(\mathbf{q},w')) \geq U(\mathbf{x}(\mathbf{p},w))$. So, if $\mathbf{px}(\mathbf{p},w) > \mathbf{px}(\mathbf{q},w')$, then, as U is strictly increasing, $\mathbf{x}(\mathbf{p},w)$ would not have been the optimal choice for the initial problem of the agent. Thus:

$$\mathbf{px}(\mathbf{p},w) \leq \mathbf{px}(\mathbf{q},w'). \tag{6}$$

On the other hand, $\mathbf{qx}(\mathbf{q},w') = w'$ (by the monotonicity of U). In view of the choice of w',

$$\mathbf{qx}(\mathbf{p},w) = \mathbf{qx}(\mathbf{q},w'). \tag{7}$$

As $p_2 = q_2$, combining (6) and (7) yields

$$p_1(x_1(\mathbf{p},w) - x_1(\mathbf{q},w')) \leq q_1(x_1(\mathbf{p},w) - x_1(\mathbf{q},w')).$$

As $p_1 > q_1$, this proves our assertion.

Let us now pose the following problem. How does the magnitude of the substitution effect we found above compare with the substitution effect the consumer would exhibit if she did not have a status quo bias? To formulate this question formally, we denote the optimal solution of the problem

$$\text{Maximize } U(\mathbf{x}) \text{ subject to } \mathbf{qx} \leq w' \text{ and } \mathbf{x} \in \mathbb{R}^2_+$$

by $\mathbf{x}^*(\mathbf{q},w')$; that is, $\mathbf{x}^*(\mathbf{q},w')$ is the demand of the consumer at prices $\mathbf{q}$ and income w', provided that she acts according to the standard consumer choice model. The classical *Law of Compensated Demand* can then be stated as saying that $x_1^*(\mathbf{q},w') \geq x_1(\mathbf{p},w)$. In view of Fact 1, then, our question concerns the comparison of $x_1(\mathbf{q},w')$ and $x_1^*(\mathbf{q},w')$. The answer is given in the next result.

Fact 2. (The Comparative Law of Compensated Demand) $x_1^*(\mathbf{q},w') \geq x_1(\mathbf{q},w')$.

Proof. Suppose, to derive a contradiction, we have $x_1^*(\mathbf{q},w') < x_1(\mathbf{q},w')$. By the classical Law of Compensated Demand,

$$x_1(\mathbf{p},w) \leq x_1^*(\mathbf{q},w') < x_1(\mathbf{q},w').$$

Since $\mathbf{x}(\mathbf{p},w)$, $\mathbf{x}^*(\mathbf{q},w')$, and $\mathbf{x}(\mathbf{q},w')$ all lie on the budget line $\{\mathbf{x} \in \mathbb{R}^2_+ : \mathbf{qx} = w'\}$, $\mathbf{x}^*(\mathbf{q},w')$ can be written as a convex combination of the 2-vectors $\mathbf{x}(\mathbf{p},w)$ and $\mathbf{x}(\mathbf{q},w')$. Because both of these 2-vectors belong to $\mathcal{Q}(\mathbf{x}(\mathbf{p},w))$ and $\mathcal{Q}$ is convex-valued, we have $\mathbf{x}^*(\mathbf{q},w') \in \mathcal{Q}(\mathbf{x}(\mathbf{p},w))$ as well. It follows that $U(\mathbf{x}(\mathbf{q},w')) \geq U(\mathbf{x}^*(\mathbf{q},w'))$, while the converse of this inequality is obviously true by the choice of $\mathbf{x}^*(\mathbf{q},w')$. But then $\mathbf{x}(\mathbf{q},w')$ and $\mathbf{x}^*(\mathbf{q},w')$ are two distinct optimal solutions for the problem (5), which contradicts the strict quasiconcavity of U.

The gist of the two facts we proved above can be summarized as follows:

$$\begin{array}{c} \text{Substitution Effect} \\ \text{without} \\ \text{Status Quo Bias} \end{array} \leq \begin{array}{c} \text{Substitution Effect} \\ \text{with} \\ \text{Status Quo Bias} \end{array} \leq 0.$$

Thus, insofar as it is modelled by the canonical choice model of Theorem 1, we see here that the compensated own-price effect is less pronounced for consumers that are subject to status quo bias. This suggests that "backward-bending labour supply" type phenomena may be more prevalent, because of the status quo bias phenomenon, than one may initially suspect.

Remark. The analysis above presumes that the initial demand $\mathbf{x}(\mathbf{p},w)$ of the consumer is obtained in the absence of a status quo option. If we relax this hypothesis, Fact 1 ceases to be correct unless we put further structure on the psychological constraint correspondence $\mathcal{Q}$. In particular, if we assume that the agent is strongly status quo-biased—that is, her choice behaviour is consistent with SSQB—then, as we have seen in Theorem 2, $\mathcal{Q}\circ\mathcal{Q}\subseteq\mathcal{Q}$ holds in the context of the choice model of Theorem 1, and this property ensures the validity of both Facts 1 and 2 even when the initial problem is solved relative to any (fixed) status quo option.

While the above results are obtained from the choice model of Theorem 1 with minimal assumptions on U and $\mathcal{Q}$, the situation may read quite differently in the case of reasonable specifications of the loss aversion model. The following example explores this matter.

Example 9. (*The TK–Model and the Law of Demand*) We consider the framework of Section 5 and as in Example 7, work with a particular specification of the TK-model in which the utility of the bundle $\mathbf{y}\in\mathbb{R}^2_+$ for an agent whose initial endowment is $\mathbf{x}$, is $U_{\mathbf{x}}(\mathbf{y})=u(y_1-x_1)+v(y_2-x_2)$, where

$$u(t):=\begin{cases} t, & \text{if } t\geq 0\\ 2t, & \text{if } t<0\end{cases} \qquad \text{and} \qquad v(t):=\begin{cases} t^\alpha, & \text{if } t\geq 0\\ -2|t|^\alpha, & \text{if } t<0\end{cases}$$

with $0<\alpha<1$. We consider here a parametric model to ensure that the following analysis does not pertain to a knife-edge scenario. When the prices in the economy are given by $\mathbf{p}\in\mathbb{R}^2_{++}$ with $p_2=1$, we assume that the optimal consumption choice is found by maximizing $U_{(0,0)}(\mathbf{x})$ over the budget set $\{\mathbf{x}\in\mathbb{R}^2_+:\mathbf{px}\leq w\}$.[22] In what follows, we shall adopt the following parametric restriction: $p_1<2<w^{1-\alpha}$. Under this condition, the solution to this problem, denoted by $\mathbf{x}(\mathbf{p},w)$, is $\mathbf{x}(\mathbf{p},w)=\left(\frac{1}{p_1}(w-(\alpha p_1)^\theta),(\alpha p_1)^\theta\right)$, where $\theta:=1/(1-\alpha)$.

Consider next a decline in the price of good 1; the new price in the economy is $\mathbf{q}\in\mathbb{R}^2_{++}$, where $p_1>q_1$ and $p_2=q_2=1$. As in Section 4.5, we set $\mathbf{x}(\mathbf{p},w)$ as the reference point of the decision maker. In the present context, this means that the consumer's new choice problem is maximizing $U_{\mathbf{x}(\mathbf{p},w)}(\mathbf{x})$ over the budget set $\{\mathbf{x}\in\mathbb{R}^2_+:\mathbf{qx}\leq w\}$. We denote this solution by $\hat{\mathbf{x}}(\mathbf{q},w)$. Next, to perform the Slutsky analysis, we set $w':=\mathbf{qx}(\mathbf{p},w)$ so that the (hypothetical) choice problem of the agent (in which the income effect of the change of prices from $\mathbf{p}$ to $\mathbf{q}$ is eliminated) becomes maximizing $U_{\mathbf{x}(\mathbf{p},w)}(\mathbf{x})$ over all $\mathbf{x}\in\mathbb{R}^2_+$ with $\mathbf{qx}\leq w'$. We denote the solution of this problem as $\hat{\mathbf{x}}(\mathbf{q},w')$. Clearly, what is required by the Law of Compensated Demand in this context is $\hat{x}_1(\mathbf{q},w')\geq x_1(\mathbf{p},w)$, meaning that in the absence of the income effect, a decrease in the price of good 1 results in increase of the consumption of that good. Yet, under a wide specification of the parameters at hand this does not happen. Indeed, whenever $1<q_1<p_1<2<w^{1-\alpha}$, it can be shown by calculus that $\hat{x}_1(\mathbf{q},w')<x_1(\mathbf{p},w)$; that is, the Law of Compensated Demand fails.[23]

It can further be shown that the Law of Demand may fail in this context, at least locally. Put more precisely, there is an $\varepsilon>0$ such that $\hat{x}_1(\mathbf{q},w')<\hat{x}_1(\mathbf{q},w)<x_1(\mathbf{p},w)$ whenever $1<p_1-\varepsilon<$

22. As in Example 7, we treat the decision maker as acting without a status quo option at this point, which we try to capture within the TK-model upon setting the reference point as $(0,0)$. The substance of the following demonstration would, however, remain intact (albeit, with a different parametric restriction) with any other choice of a reference point for the consumer.

23. To derive a contradiction, suppose $\hat{x}_1(\mathbf{q},w')\geq x_1(\mathbf{p},w)$ under the said parametric specification. Since $w'=q_1x_1(\mathbf{p},w)+x_2(\mathbf{p},w)$, we have $x_1-x_1(\mathbf{p},w)=\frac{1}{q_1}(x_2(\mathbf{p},w)-x_2)$ for any $\mathbf{x}\in\mathbb{R}^2_+$ with $\mathbf{qx}\leq w'$. Thus, by definition of $U_{\mathbf{x}(\mathbf{p},w)}$,

$q_1 < p_1 < 2 < w^{1-\alpha}$. Thus, by consuming less of good 1 at the lower price q_1, the consumer treats this commodity as a *Giffen good* in this price range. Yet, the income effect is positive here—that is, $\hat{x}_1(\mathbf{q},w') < \hat{x}_1(\mathbf{q},w)$—so good 1 is not inferior for the consumer. Thus, the TK-model maintains that the Law of Demand fails in this instance *because* of the substitution effect, not *despite* it, contrary to the intuition behind the "Giffen good phenomenon". ■

Remark. As in the case of Example 7, the reason why the Law of Compensated Demand may fail with the TK-model stems from the interaction of loss aversion and diminishing sensitivity across different dimensions. In prospect theory, the multidimensional aggregation of these traits does not cause a problem, as there is only one dimension of goods, namely, money. By contrast, in the context of consumption bundles, the loss aversion model runs into the difficulty of aggregating convex and concave functions across multiple dimensions. In Example 7, for instance, the shape of the indifference curves depends on whether we are in the gain–loss region (a gain in the first good and loss in the second) or in the loss–gain region. In particular, in the gain–loss region, the optimal point is either the status quo or a corner solution; that is, the agent either exhibits extreme status quo bias or spends all her money in the first good. Under our specification, the status quo is better for the agent than the latter solution, so, in the gain–loss region, the best thing is to stay with the status quo bundle. But this cannot be the overall optimum because an infinitesimal increase in the second consumption increases the utility of the agent infinitely. Thus, the overall optimum must be located in the loss–gain region, which means that the substitution effect here is positive.

We conclude from Example 9 that the TK-model is not compatible with the Law of Compensated Demand in general, and it may well entail a positive substitution effect. Comparing this finding with Fact 1 shows that the implications of the status quo bias phenomenon (as captured by the choice model of Theorem 1) and that of loss aversion (as modelled by the TK-model) may well be quite distinct in certain economic environments.

6.2. *Equity premium puzzle*

One of the well-known puzzles in finance is based on the empirical observation that equity returns are, on average, significantly better than those of bonds. The simultaneous presence of bonds and equities in the market can thus be justified by the utility maximization model only if one presumes unrealistically high risk aversion on the part of the investors. This is dubbed the *equity premium puzzle* (Mehra and Prescott, 1985). It is well-known that this puzzle can be explained by using a (constant) loss aversion model (cf. Barberis *et al.* 2001). We now illustrate how one may utilize the canonical choice model of Section 2.3 to do the same.

Consider a financial market in which there is a bond with price $p_b > 0$ and a stock with price $p_s > 0$. Annually, the bond yields B dollars with certainty, and the stock yields M dollars

and since $\hat{x}_1(\mathbf{q},w') \geq x_1(\mathbf{p},w)$, the number $\hat{x}_2(\mathbf{q},w')$ must solve

$$\text{Maximize } \frac{1}{q_1}(x_2(\mathbf{p},w)-x_2)-2(x_2(\mathbf{p},w)-x_2)^{\alpha} \text{ subject to } x_2(\mathbf{p},w) \geq x_2 \geq 0.$$

(Call this problem (P).) Equivalently, $x_2(\mathbf{p},w)-\hat{x}_2(\mathbf{q},w')$ maximizes $\frac{1}{2q_1}a - a^{\alpha}$ over all a with $(\alpha p_1)^{\theta} \geq a \geq 0$. Because the objective function is strictly convex here, the solution must be either 0 or $(\alpha p_1)^{\theta}$. But, as $1 < q_1 < p_1 < 2$, we have $p_1 < 2q_1$. Using this and the fact that $\alpha\theta = \theta - 1$, we find $\frac{1}{2q_1}(\alpha p_1)^{\theta} - (\alpha p_1)^{\alpha\theta} < 0$. It follows that the unique solution to this problem is 0, which means $x_2(\mathbf{p},w)$ is the unique solution to the problem (P); that is, $\hat{x}_2(\mathbf{q},w') = x_2(\mathbf{p},w)$. This implies that $U_{\mathbf{x}(\mathbf{p},w)}(\hat{\mathbf{x}}(\mathbf{q},w')) = U_{\mathbf{x}(\mathbf{p},w)}(\mathbf{x}(\mathbf{p},w)) = 0$. But this is impossible, for, since the right-derivative of the map $t \mapsto t^{\alpha}$ at zero is ∞, it is clear from the definition of $U_{\mathbf{x}(\mathbf{p},w)}$ that $U_{\mathbf{x}(\mathbf{p},w)}(x_1(\mathbf{p},w)-\varepsilon, x_2(\mathbf{p},w)+q\varepsilon) > 0$ for $\varepsilon > 0$ small enough; that is, $\mathbf{x}(\mathbf{p},w)$ does not maximize $U_{\mathbf{x}(\mathbf{p},w)}(\mathbf{x})$ under the condition $\mathbf{qx} \leq w'$.

with probability α and m dollars with probability $1-\alpha$. Naturally, we assume that $B>p_b$ and $M>B>m$. To ensure comparability, we also posit that the expected per period value of the stock is equal to the return on the bond—that is, $\alpha M+(1-\alpha)m=B$. Now consider a representative agent in this economy who is risk neutral. If this agent is an expected utility maximizer, an equilibrium in which both the stock and bond are purchased can be justified only if this agent is indifferent between buying the stock and the bond—when $\alpha(M-p_s)+(1-\alpha)(m-p_s)=B-p_b$. Since $\alpha M+(1-\alpha)m=B$, this happens only if $p_s=p_b$, that is, in the absence of any equity premium.

Let us now model the decisions of our agent through the choice model of Theorem 3. Let X be the set of all probability distributions on $\mathbb{R}$. In what follows, we denote the expectation of a lottery μ in X by $\mathbb{E}_\mu$, and by δ_a we mean the degenerate lottery that pays a dollars with probability one. Consequently, the stock option corresponds to the lottery $\mu_s:=\alpha\delta_{M-p_s}+(1-\alpha)\delta_{m-p_s}$ and the bond option is the degenerate lottery $\mu_b:=\delta_{B-p_b}$. Now take any non-empty collection $\mathcal{W}$ of continuous and stochastically strictly increasing real functions on X, and assume that there is a $V\in\mathcal{W}$ such that $V(\delta_{\mathbb{E}_\mu})>V(\mu)$ for every non-degenerate $\mu\in X$. (To wit, V represents the preferences of a risk averse decision maker who may not be an expected utility maximizer.) We define $U(\mu):=\mathbb{E}_\mu$ and $Q(\mu):=\{\nu\in X: W(\nu)\geq W(\mu) \text{ for all } W\in\mathcal{W}\}$ for each $\mu\in X$. Consequently, when her default option is μ, the choice of the agent between stocks and bonds is found as

$$\mathbf{c}(S,\mu)=\arg\max\{\mathbb{E}_\nu:\nu\in S \text{ and } W(\nu)\geq W(\mu) \text{ for all } W\in\mathcal{W}\},$$

where $S:=\{\mu_s,\mu_b,\mu\}$. But then, if the default option of this decision maker is to buy the bonds, that is, $\mu=\mu_b$, the stock belongs to the psychologically constrained set if $V(\mu_s)\geq V(\mu_b)$. By the risk aversion property of V, $V(\delta_{\alpha\delta_{M-p_s}+(1-\alpha)\delta_{m-p_s}})>V(\delta_{B-p_b})$, and so, since V is stochastically strictly increasing, $\alpha(M-p_s)+(1-\alpha)(m-p_s)>B-p_b$; that is, $p_b>p_s$. In words, the positive demand to the stock in the market is possible only if the bond sells at a higher price than the stock, even though the representative agent acts as a risk neutral expected utility maximizer in the absence of any status quo effects and the expected values of the bond and the stock are the same.[24] It is also readily checked that the same conclusion holds when the default option of the agent is not to participate in the market, that is, $\mu=\delta_0$, provided that V is risk averse enough that $V(\delta_0)>V(\alpha\delta_{M-p_b}+(1-\alpha)\delta_{m-p_b})$.[25]

The intuition behind this illustration is not unlike how loss aversion theory explains the equity premium puzzle. In the present formulation, the agent behaves as a risk neutral expected utility maximizer, and hence her choices are not compatible with the presence of positive risk premium (and hence with the equity premium puzzle) when she acts in the absence of a status quo option. If, on the other hand, the agent has a default option and she is reluctant to move away from this alternative unless a dominant investment option is available, the situation is different. In particular, if she views buying bonds as her default option and compares alternatives to this option through several ranking criteria in her mind—this is captured by the set $\mathcal{W}$ above—and at least one of these criteria is that of a mildly risk averse person—this is captured by V above—then her behaviour may well entail positive risk premium. The same holds if the default option of the agent is not to invest at all, provided that one of the criteria she uses to check for dominance is sufficiently risk averse. It is also true that if an agent is less status quo biased than someone else, her willingness to pay for the stock is higher.

24. As $p_b>p_s$, we have $(\mathbb{E}_{\mu_s}-p_s)/p_s>(B-p_b)/p_b$, that is, the expected per-dollar return on the stock exceeds that of the bond; there is positive risk premium in the economy.

25. The stock should yield higher V-value than the default option, that is, $V(\mu_s)\geq V(\delta_0)>V(\alpha\delta_{M-p_b}+(1-\alpha)\delta_{m-p_b})$, which implies $p_b>p_s$.

6.3. *The disposition effect*

Another famous behavioural puzzle in the context of finance concerns the tendency of investors to hold on to stocks that have lost value too long while being prone to selling stocks that have gained value. This behaviour is called the *disposition effect*; it was predicted by Shefrin and Statman (1985), and it was later corroborated empirically by several studies (cf. Weber and Camerer, 1998; Odean, 1998; and Ivkovich *et al.* 2005). It is known that the disposition effect can be explained through loss aversion models. We next illustrate how one may utilize the canonical choice model of Section 3 to account for this effect as well.

Consider a stock that yields, per period, \$200 with probability α and $-\$100$ with probability $1-\alpha$, where $0<\alpha<1$. In this simplified context, the disposition effect is captured by the selling price of this stock *after it has produced a gain* being strictly less than that of the stock *after it has yielded a loss.* We wish to show that this may well be the case if the owner of the stock is a rational decision maker with a status quo bias whose behaviour is represented by an instance of the choice model of Section 3.

Let X denote the collection of all probability distributions on $\mathbb{R}$, and define $U:X\to\mathbb{R}$ by $U(\mu):=\mathbb{E}_\mu(u)$, where u is a strictly increasing and strictly concave von Neumann–Morgenstern utility function. (Here $\mathbb{E}_\mu(u)$ is the expected value of u with respect to the lottery μ.) Next, take any continuous and strictly increasing $f:\mathbb{R}\to\mathbb{R}$ that is affine (or concave) on the interval $(-\infty,100]$ and strictly concave on the interval $[100,\infty)$, and define the self-correspondence Q on X by $Q(\mu):=\{\nu\in X:\mathbb{E}_\nu(f)\geq\mathbb{E}_\mu(f)\}$. Then, according to the choice model of Theorem 2, the choices from a given non-empty (compact) subset S of X when a lottery μ is the status quo are

$$\mathbf{c}(S,\mu)=\arg\max\{\mathbb{E}_\nu(u):\nu\in S \text{ and } \mathbb{E}_\nu(f)\geq\mathbb{E}_\mu(f)\}.$$ [26]

Now, consider two lotteries that yield

$$\begin{cases}100 & \text{with probability } \alpha\\ -200 & \text{with probability } 1-\alpha\end{cases} \quad\text{and}\quad \begin{cases}400 \text{ with probability } \alpha\\ 100 \text{ with probability } 1-\alpha,\end{cases}$$

respectively. The first lottery here—we denote it by μ_L—is what our stock looks like after it has yielded a loss in its first period. (Given that a loss of \$100 is incurred in the first period, at the start of the second period, this stock yields $-100+200$ with probability α, and $-100-100$ with probability $1-\alpha$.) Given the loss in the first period, if the agent sells this stock in the second period for \$$p$, the gain is then $p-100$ (disregarding discounting effects). Thus, the minimum selling price of the stock in this case is the smallest $p\geq 0$ such that $\delta_{p-100}\in\mathbf{c}(\{\delta_{p-100},\mu_L\},\mu_L)$.

Similarly, the second lottery above—we denote it by μ_G—corresponds to our stock in the second period after this stock has produced a gain of \$200. The minimum selling price of the stock in this case is the smallest $p\geq 0$ such that $\delta_{p+200}\in\mathbf{c}(\{\delta_{p+200},\mu_G\},\mu_G)$. Let us denote this selling price by p_*. Then, by definition of $\mathbf{c}$, p_*+200 must be equal to the larger of the certainty equivalents of μ_G with respect to the expectation of u and f. But, as u and f are strictly concave on an interval that contains the support of μ_G, both of these certainty equivalents are strictly less than the expected value of μ_G, which is $300\alpha+100$. It follows that $p_*<300\alpha-100$. On the other hand, the certainty equivalent of μ_L with respect to f is simply the expected value of μ_L, that is, $300\alpha-200$. Thus p_*-100 is strictly less than the certainty equivalent of μ_L with respect to f, that is, δ_{p_*-100} does not belong to $Q(\mu_L)$, and hence, $\delta_{p_*-100}\notin\mathbf{c}(\{\delta_{p-100},\mu_L\},\mu_L)$. It follows

26. For instance, in the case of lotteries whose supports are included in $[100,\infty)$, this decision maker may not leave her status quo for a riskier lottery even if the latter yields a higher expected utility in terms of u.

that the minimum selling price for μ_L must exceed p_* in accordance with the disposition effect. Furthermore, the gap between the minimum selling price of μ_L and that of μ_G increases as the agent gets more status quo biased.

The intuition behind this illustration lies in the structure of Q that we have chosen above. This constraint correspondence utilizes the strict concavity of f to posit risk averse behaviour for winner stocks—in this example, a winner stock is one that yields a lottery whose support is contained in $[100,\infty)$—in addition to the one entailed by the concavity of u. It, however, does not do this for loser stocks, because f is affine on the interval $(-\infty,100]$. In this sense, the channel through which the present model explains the disposition effect is analogous to that of prospect theory (cf. DellaVigna, 2009).

6.4. *House pricing*

An analogue of the disposition effect has been observed in the context of housing markets. Genovese and Meyer (2001) have shown empirically that home-owners who sell their houses below their purchase prices tend to set their prices too high, and as a result, such houses remain in the market more than others. One can use a loss aversion model to explain this phenomenon; see, for instance, DellaVigna (2009). The same can be done, in this case in an extremely simple fashion, by means of the canonical choice model of Section 3. We illustrate this next.

Consider an agent who has bought her house for p_0 dollars. Following the illustration given in DellaVigna (2009), we imagine that setting sales price for this house has a direct effect through one's utility and an indirect effect through the probability of being able to sell the house. To be specific, let us denote the contingency of "not selling the house" by h, and fix two functions, $u:\mathbb{R}_+\cup\{h\}\to\mathbb{R}$ and $\alpha:\mathbb{R}_+\to[0,1]$. For any number $p\geq 0$, we interpret $u(p)$ as the utility of the sale of the house at price p, and $\alpha(p)$ as the probability that the house would actually sell at price p. In addition, $u(h)$ is the reservation utility of the agent (from keeping the house). It is thus reasonable to assume that u is strictly increasing and strictly concave on $\mathbb{R}_+$ and that α is decreasing and concave. We also assume that 0 belongs to the range of α so that $\bar{p}:=\inf\{p\geq 0:\alpha(p)=0\}$ exists. Notice that $\underline{p}:=u^{-1}(u(h))$ is the largest price at which selling the house yields less utility than the reservation utility of the owner. To avoid trivialities, we assume henceforth that $\bar{p}>\underline{p}$. The expected utility of the owner in this setting is thus given by the function $U:\mathbb{R}_+\to\mathbb{R}$ defined by $U(p):=\alpha(p)u(p)+(1-\alpha(p))u(h)$ if $\bar{p}>p>\underline{p}$, and $U(p):=u(h)$ otherwise. Our assumptions guarantee that U is strictly concave on $[\underline{p},\bar{p}]$, and hence there is a unique maximizer of this function, say, p^*, and this price satisfies $\bar{p}>p^*>\underline{p}$.

Suppose the home-owner is in the mind-set that she would never sell her house at more than a θ percent loss. It is straightforward to capture this person through the canonical choice model of Theorem 1 by setting $X:=\mathbb{R}_+\cup\{h\}$ and extending U to X by defining $U(h):=u(h)$.[27] In addition, we choose any closed-valued self-correspondence Q on X such that $Q(h)=[\theta p_0,\infty)\cup\{h\}$. Then, the choice model of Theorem 1 maintains that

$$\mathbf{c}(X,h)=\begin{cases}\{p^*\}, & \text{if } p^*\geq\theta p_0\\ \{\theta p_0\}, & \bar{p}>\theta p_0>p^*\\ [\theta p_0,\infty)\cup\{h\} & \text{if } \theta p_0>\bar{p}.\end{cases}$$

In particular, if the home-owner is expected to make a nominal loss, that is, when $p_0>p^*$, then she may well charge a price higher than the optimal price p^*; this actually happens when the loss

27. Formally, we can think of X as a metric space by first metrizing $\{0\}\times\mathbb{R}_+$ by a bounded metric, and then adjoining h (an object that does not belong to $\mathbb{R}_+$) to this set as an isolated point.

is substantial enough so that $\theta p_0 > p^*$. Here, θ provides a measure of the size of the status quo bias of a home-owner. Lower θ corresponds to being less status quo-biased and asking for lower selling prices.

7. CONCLUSION

In this article, we adopted the revealed preference method to derive an individual decision making model that allows for an agent to make use of her status quo option as a reference point (if such an option exists). This model is of "constrained utility maximization" form, where constraints are psychologically induced by one's initial endowments. As the applications we have considered throughout the article suggest, this model may prove useful in economic contexts in which one wishes to investigate the potential implications of an agent's status quo bias for her choice behaviour.

To conclude, let us note that there are several directions in which the present work should be extended. First, we have assumed here that one's status quo option is always feasible. For example, our model cannot be used to formulate the job search problem of a person who has lost her current (status quo) job. Second, we have not considered here how choice under risk and ambiguity can be modelled in the presence of initial endowments. This is problematic, because one main reason that is often suggested for status quo bias is that status quo options are more familiar than others, which may be captured by modelling such options as less ambiguous than other alternatives (in the context of choice under uncertainty). Third, our model is entirely static, while the very nature of status quo is dynamic. After all, an alternative has to be chosen in an earlier period for it to act as a status quo today. There is certainly room for developing a dynamic choice model that would generalize our model. Finally, several empirical queries arise from the present work. It would certainly be useful to run tests comparing the explanatory power of the loss aversion model with that of our rational choice model with status quo bias. Similarly, the empirical validity of the SSQB and SQI properties should be critically investigated. Finally, it will be interesting to see if there is any behavioural evidence for status quo biased choice behaviour that does not cause the endowment effect, which is at present only a theoretical possibility (Section 4).

APPENDIX

Proof of Theorem 1

We first prove the "if" part of the theorem. Let $U: X \to \mathbb{R}$ be a continuous function and $\mathcal{Q}$ be a closed-valued self-correspondence on X, and take any choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$ that satisfies (1) and (2) for any $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$. It is obvious that $\mathbf{c}$ satisfies WARP. To show that $\mathbf{c}$ also satisfies WSQB and SQI, we need to make the following simple observation:

Claim 1.1. $x \in \mathcal{Q}(x)$ for every $x \in X$.

Proof of Claim 1.1. Since $\mathbf{c}$ is a choice correspondence on $\mathcal{C}(X)$, we must have $\mathbf{c}(\{x\},x) = \{x\}$ for any x in X. Our claim thus follows from (2).

Now take any $x, y \in X$, and suppose that $y \in \mathbf{c}(\{x,y\},x)$. By (2) and Claim 1.1, we must have $U(y) \geq U(x)$, which, by (1), is equivalent to $y \in \mathbf{c}(\{x,y\},\diamond)$ as we sought. On the other hand, if $y \in \mathbf{c}(\{x,y\},\diamond)$, that is, $U(y) \geq U(x)$, then, since $y \in \mathcal{Q}(y)$ by Claim 1.1, (2) entails $y \in \mathbf{c}(\{x,y\},y)$, as we sought. Conclusion: $\mathbf{c}$ satisfies WSQB.

Let us now show that $\mathbf{c}$ satisfies SQI. Take any $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$, and suppose that $\{x\} \neq c(T,x)$ for every non-singleton subset T of S with $x \in T$. If S is itself a singleton, then $S = \{x\}$, and we get $\mathbf{c}(S,x) = \{x\} = \mathbf{c}(S,\diamond)$ by virtue of $\mathbf{c}$ being a choice correspondence. If S is not a singleton, then, by hypothesis, $y \in \mathbf{c}(\{x,y\},x)$; that is, $y \in \mathcal{Q}(x)$, for every $y \in S \backslash \{x\}$. Combining this fact with Claim 1.1, we find $S \subseteq \mathcal{Q}(x)$, that is, $S \cap \mathcal{Q}(x) = S$, and it follows from (1) and (2) that $\mathbf{c}(S,x) = \mathbf{c}(S,\diamond)$, as we sought.

It remains to prove that $\mathbf{c}$ satisfies C. But this is a straightforward consequence of the Berge Maximum Theorem and the closed-valuedness of $\mathcal{Q}$.

We now move to prove the “only if” part of Theorem 1. Let $\mathbf{c}$ be a choice correspondence on $\mathcal{C}(X)$ that satisfies WARP, WSQB, SQI, and C. Define the binary relation $\succsim$ on X by

$$y \succsim x \qquad \text{if and only if} \qquad y \in \mathbf{c}(\{x,y\},\diamond).$$

A standard argument, based on WARP, shows that $\succsim$ is a complete preorder on X. It is also easy to see that C ensures that $\succsim$ is continuous. Indeed, for any $x \in X$ and any convergent sequence (y_m) in X such that $y_m \succsim x$ for each m, we have $\{x, y_m\} \to \{x,y\}$, where $y := \lim y_m$. Thus, it follows from C that $y \in \mathbf{c}(\{x,y\},\diamond)$, that is, $\lim y_m \succsim x$. Since x is arbitrarily chosen in X in this argument, we may conclude that $\succsim$ is upper semicontinuous. The lower semicontinuity of $\succsim$ is established similarly.

Now, a standard argument shows that WARP implies

$$\mathbf{c}(S,\diamond) = \{\omega \in S : \omega \succsim x \text{ for all } \omega \in S\} \qquad \text{for every } S \in \Omega_X.$$

Furthermore, given that $\succsim$ is continuous, and X is a compact, and hence separable, metric space, we may apply the Debreu Representation Theorem to find a continuous real map U on X such that $y \succsim x$ iff $U(y) \geq U(x)$ for any $x, y \in X$. Therefore:

$$\mathbf{c}(S,\diamond) = \operatorname{arg\,max}\{U(\omega) : \omega \in S\} \qquad \text{for every } S \in \Omega_X. \tag{A.1}$$

We now define the self-correspondence $\mathcal{Q}$ on X by

$$\mathcal{Q}(x) := \{y \in X : y \in \mathbf{c}(\{x,y\},x)\}.$$

Claim 1.2. $\mathcal{Q}$ is closed-valued.

Proof of Claim 1.2. Pick any $x \in X$, and let (y_m) be a sequence in $\mathcal{Q}(x)$ with $y_m \to y$ for some $y \in X$. Then, $y_m \in \mathbf{c}(\{x, y_m\}, x)$ for each m, so, by C, we find $y \in \mathbf{c}(\{x,y\},x)$, that is, $y \in \mathcal{Q}(x)$.

Claim 1.3. For every (S,x) in $\mathcal{C}_{\text{sq}}(X)$, we have

$$\mathbf{c}(S,x) = \mathbf{c}(S \cap \mathcal{Q}(x), x).$$

Proof of Claim 1.3. Let $T := S \cap \mathcal{Q}(x)$, and pick any $y \in \mathbf{c}(S,x)$. By WARP, we have $y \in \mathbf{c}(\{x,y\},x)$, which means that $y \in \mathcal{Q}(x)$, and hence $y \in T$. Conclusion: $\mathbf{c}(S,x) \subseteq T$. Therefore, $\mathbf{c}(S,x) \cap T$ equals $\mathbf{c}(S,x)$, which ensures that it is a nonempty set. We may thus apply WARP to conclude that $\mathbf{c}(S,x) = \mathbf{c}(S,x) \cap T = \mathbf{c}(T,x)$, and we are done.

Claim 1.4. For every (S,x) in $\mathcal{C}_{\text{sq}}(X)$, we have

$$\mathbf{c}(S \cap \mathcal{Q}(x), x) = \mathbf{c}(S \cap \mathcal{Q}(x), \diamond). \tag{A.2}$$

Proof of Claim 1.4. Fix an arbitrary (S,x) in $\mathcal{C}_{\text{sq}}(X)$. Suppose first that

$$x \in \mathbf{c}(\{x,y\},\diamond) \qquad \text{for every } y \in S \cap \mathcal{Q}(x). \tag{A.3}$$

Now, for any $y \in \mathcal{Q}(x)$, we have $y \in \mathbf{c}(\{x,y\},x)$, so, by WSQB, $y \in \mathbf{c}(\{x,y\},\diamond)$. Combining this with (A.3), we get $\{x,y\} = \mathbf{c}(\{x,y\},\diamond)$ for every $y \in S \cap \mathcal{Q}(x)$. It is easy to check that, in view of WARP, we must have

$$S \cap \mathcal{Q}(x) = \mathbf{c}(S \cap \mathcal{Q}(x), \diamond).$$

On the other hand, by (A.3) and WSQB, we have $x \in \mathbf{c}(\{x,y\},x)$ for every $y \in S \cap \mathcal{Q}(x)$. Combining this with the definition of $\mathcal{Q}$, $\{x,y\} = \mathbf{c}(\{x,y\},x)$ for every $y \in S \cap \mathcal{Q}(x)$, and hence, applying WARP again, we get

$$S \cap \mathcal{Q}(x) = \mathbf{c}(S \cap \mathcal{Q}(x), x).$$

Conclusion: (A.2) is valid when (A.3) holds.

Now we assume that (A.3) is false. Let $\mathcal{T}$ be the collection of all non-singleton subsets T of $S \cap \mathcal{Q}(x)$ that contains x. We wish to show that $\{x\} \neq \mathbf{c}(T,x)$ for every $T \in \mathcal{T}$. Indeed, for any given $T \in \mathcal{T}$, there exists a w in T, distinct from x, such that $w \in \mathcal{Q}(x)$, that is, $w \in \mathbf{c}(\{x,w\},x)$. Therefore, if $\mathbf{c}(T) = \{x\}$ were the case, WARP would imply

$$\{x\} = \mathbf{c}(T) \cap \{x,w\} = \mathbf{c}(\{x,w\},x) \ni w;$$

that is, $x = w$, a contradiction. Conclusion: $\mathbf{c}(T,x) \neq \{x\}$ for any $T \in \mathcal{T}$. Because (A.3) is false, we have $\{y\} = \mathbf{c}(\{x,y\},\diamond)$ for some $y \in S \cap \mathcal{Q}(x)$. Thus we may apply SQI to conclude that (A.2) is valid.

Combining Claims 1.3 and 1.4, we find $\mathbf{c}(S,x) = \mathbf{c}(S \cap \mathcal{Q}(x), \diamond)$, and hence, by (A.1),

$$\mathbf{c}(S,x) = \operatorname{arg\,max}\{U(\omega) : \omega \in S \cap \mathcal{Q}(x)\}, \tag{A.4}$$

for every $(S,x) \in \mathcal{C}_{\text{sq}}(X)$. In view of Claim 1.2, the proof of Theorem 1 is complete.

Proofs of Theorems 2 and 3

Given Theorem 1, the "if" parts of Theorems 2 and 3 are fairly straightforward, so we solely concentrate on the "only if" parts of these result. Let $\mathbf{c}$ be a rational choice correspondence on $\mathcal{C}(X)$ that satisfies WARP, SSQB and SQI. Define the map U and the self-correspondence $\mathcal{Q}$ on X as in the proof of Theorem 1.

Claim 2.1. For any x in X,

$$U(y) > U(x) \qquad \text{for every } y \in \mathcal{Q}(x) \backslash \{x\}.$$

Proof of Claim 2.1. Take any distinct x and y in X with $y \in \mathcal{Q}(x)$. Then, $y \in \mathbf{c}(\{x,y\},x)$, and hence we have $\{y\} = \mathbf{c}(\{x,y\},\diamond)$ by SSQB. The definition of U entails that $U(y) > U(x)$.

Claim 2.2. $\mathcal{Q} \circ \mathcal{Q} \subseteq \mathcal{Q}$.

Proof of Claim 2.2. Take any $x,y,z \in X$ with $y \in \mathcal{Q}(x)$ and $z \in \mathcal{Q}(y)$. We wish to show that $z \in \mathcal{Q}(x)$. We may assume that x, y, and z are distinct, otherwise $z \in \mathcal{Q}(x)$ obtains trivially. In that case, by Claim 2.1, we have

$$U(z) > U(y) > U(x). \tag{A.5}$$

Now, set $S := \{x,y,z\}$, and suppose $x \in \mathbf{c}(S,x)$. By (A.4), this implies that $U(x) \geq U(\omega)$ for every $\omega \in S \cap \mathcal{Q}(x)$. In particular, $U(x) \geq U(y)$, contradicting (A.5). On the other hand, if $y \in \mathbf{c}(S,x)$, then $\{y\} = \mathbf{c}(S,y)$ by SSQB. By WARP, then, $\{y\} = \mathbf{c}(\{y,z\},y)$, contradicting $z \in \mathcal{Q}(y)$. Conclusion: $\{z\} = \mathbf{c}(S,x)$. By WARP, $z \in \mathbf{c}(\{x,y\},x)$; that is, $z \in \mathcal{Q}(x)$, as we sought.

The proof of Theorem 2 is complete at this point. To proceed with the proof of Theorem 3, we assume henceforth that X is finite and define the binary relation $\succcurlyeq$ on X as follows:

$$y \succcurlyeq x \qquad \text{if and only if} \qquad y \in \mathcal{Q}(x).$$

The following claim identifies the main properties of this binary relation.

Claim 2.3. $\succcurlyeq$ is a partial order on X such that, for any x and y in X,

$$y \succcurlyeq x \neq y \qquad \text{implies} \qquad U(y) > U(x). \tag{A.6}$$

Proof of Claim 2.3. As $x \in \mathcal{Q}(x)$ for every $x \in X$, the binary relation $\succcurlyeq$ is reflexive. By Claim 2.2, it is transitive as well. The antisymmetry of $\succcurlyeq$ follows from (A.6), while (A.6) is a restatement of Claim 2.1.

We now recall the following result from order theory.

Lemma A. Let X be a nonempty finite set, and let $\trianglerighteq$ be a complete preorder on Y. Suppose $\succsim$ is a reflexive binary relation on X such that

$$y \succsim x \neq y \qquad \text{implies} \qquad \text{not}(x \trianglerighteq y)$$

for any $x,y \in X$. Then, there exists a map $\alpha : X \to \mathbb{R}$ such that $\alpha(\omega) < 0$ for every $\omega \in X$, and

$$y \trianglerighteq x \qquad \text{if and only if} \qquad \sum \{\alpha(\omega) : \omega \succsim y\} \geq \sum \{\alpha(\omega) : \omega \succsim x\}$$

for every $x,y \in X$.

Proof of Lemma A. This is Lemma 3 of Kreps (1979), and is easily proved by mathematical induction. ■

In view of Claim 2.3, we may apply Lemma A to the complete preorder represented by U and the partial order $\succcurlyeq$ on X to find a map $\alpha : X \to \mathbb{R}_{--}$ such that

$$U(y) \geq U(x) \qquad \text{if and only if} \qquad \sum \{\alpha(\omega) : \omega \succcurlyeq y\} \geq \sum \{\alpha(\omega) : \omega \succcurlyeq x\}.$$

Now let k be the number of elements of X, and enumerate X as $\{x_1,\ldots,x_k\}$. We define the function $u_i : X \to \mathbb{R}$ by

$$u_i(z) := \begin{cases} \alpha(x_i), & \text{if } x_i \succcurlyeq z \\ 0, & \text{otherwise} \end{cases}$$

for each $i = 1,\ldots,k$. It follows from this definition that

$$\sum_{i=1}^{k} u_i(z) = \sum \{\alpha(\omega) : \omega \succcurlyeq z\} \qquad \text{for every } z \in X.$$

Therefore, we have

$$U(y) \geq U(x) \qquad \text{if and only if} \qquad \sum_{i=1}^{k} u_i(y) \geq \sum_{i=1}^{k} u_i(x), \tag{A.7}$$

for any x and y in X.

To complete the proof of Theorem 3, take any two alternatives x and y in X. Suppose $y \succcurlyeq x$ and take any $i \in \{1,...,k\}$. If $x_i \succcurlyeq y$, then $x_i \succcurlyeq x$ as well (by transitivity of $\succcurlyeq$), and hence $u_i(y) = u_i(x)$. If $x_i \succcurlyeq y$ does not hold, then $u_i(y) = 0 \geq u_i(x)$. Conclusion:

$$y \succcurlyeq x \qquad \text{implies} \qquad u_i(y) \geq u_i(x) \text{ for every } i = 1,...,k.$$

Conversely, suppose $u_i(y) \geq u_i(x)$ holds for every $i = 1,...,k$, but, to derive a contradiction, assume that $y \succcurlyeq x$ is false. Suppose y corresponds to the jth alternative in the enumeration $x_1,...,x_k$, that is, $y = x_j$. Then, since $y \succcurlyeq x$ is false, we have $u_j(x) = 0$, and hence, as $x_j \succcurlyeq y$ holds by reflexivity of $\succcurlyeq$, we find $u_j(y) = \alpha(x_j) < 0 = u_j(x)$, a contradiction. Conclusion:

$$y \succcurlyeq x \qquad \text{if and only if} \qquad u_i(y) \geq u_i(x) \text{ for every } i = 1,\ldots,k. \tag{A.8}$$

We now define the map $\mathbf{u} : X \to \mathbb{R}^k$ by $\mathbf{u}(\omega) := (u_1(\omega),...,u_k(\omega))$. Then, by definition of $\succcurlyeq$ and (A.8), we have

$$y \in \mathcal{Q}(x) \qquad \text{if and only if} \qquad \mathbf{u}(y) \geq \mathbf{u}(x). \tag{A.9}$$

Then, if $\mathbf{u}(y) = \mathbf{u}(x)$ were to hold for two distinct x and y in X, Claim 2.3 would have entailed that $U(y) > U(x)$, and hence, by (A.7), we would have found that $u_1(y) + \cdots + u_k(y) > u_1(x) + \cdots + u_k(x)$ which contradicts $\mathbf{u}(y)$ and $\mathbf{u}(x)$ being the same numbers. Therefore, $\mathbf{u}(y) = \mathbf{u}(x)$ can hold only if $x = y$, that is, $\mathbf{u}$ is an injection. Finally, combining (A.7) and (A.9) with (1) and (2) completes the proof of Theorem 3.

Independence of Axioms

Pick an arbitrary integer $k \geq 3$, and set $X := \{0,1,...,k\}^2$. First, consider the choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$ defined as

$$\mathbf{c}(S,\diamond) := \underset{(i,j)\in S}{\arg\max}\; i+j \qquad \text{and} \qquad \mathbf{c}(S,x) := \underset{(i,j)\in S \text{ and } (i,j)\geq x}{\arg\max}\; 2i+j,$$

for every non-empty subset S of X and $x \in S$. (Here $\geq$ is the coordinatewise ordering on X.) It is easily checked that this choice correspondence satisfies WARP, WSQB, and C, but not SQI. On the other hand, the choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$, defined as

$$\mathbf{c}(S,\diamond) := \begin{cases} \{(0,0)\}, & \text{if } S = X \\ \underset{(i,j)\in S}{\arg\max}\; i+j, & \text{otherwise} \end{cases} \qquad \text{and} \qquad \mathbf{c}(S,x) := \{x\}$$

for every non-empty subset S of X and $x \in S$, satisfies WSQB, SQI, and C, but not WARP. Next, consider the choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$, defined as

$$\mathbf{c}(S,\diamond) := \underset{(i,j)\in S}{\arg\max}\; i+j \qquad \text{and} \qquad \mathbf{c}(S,x) := \begin{cases} \{x\}, & \text{if } x \neq \mathbf{1} \\ \{\mathbf{0}\}, & \text{if } x = \mathbf{1} \text{ and } \mathbf{0} \in S \\ \{\mathbf{1}\}, & \text{otherwise.} \end{cases}$$

(Here $\mathbf{0} := (0,0)$ and $\mathbf{1} := (1,1)$.) This choice correspondence satisfies WARP, SQI, and C, but not WSQB. Finally, set $X := [0,1]^2$ and define the map $u : X \to \{0,1\}$ by $u(x) = 1$ iff $x = \mathbf{1}$. Then, the choice correspondence $\mathbf{c}$ on $\mathcal{C}(X)$, defined as $\mathbf{c}(S,x) := \arg\max u(S)$ for every non-empty subset S of X and $x \in S \cup \{\diamond\}$, satisfies WARP, WSQB, and SQI, but not C. Conclusion: the axioms used in Theorem 1 are logically independent.

As WSQB can be interchanged with SSQB in the previous paragraph, these examples also establish the logical independence of the axioms used in Theorems 2 and 3.

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