Progressive Random Choice

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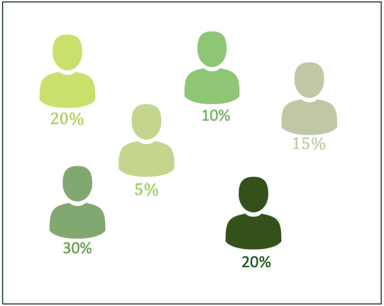
Random Choice

Think of a probabilistic choice coming from a heterogeneous population.

S - - →
Decision Problem

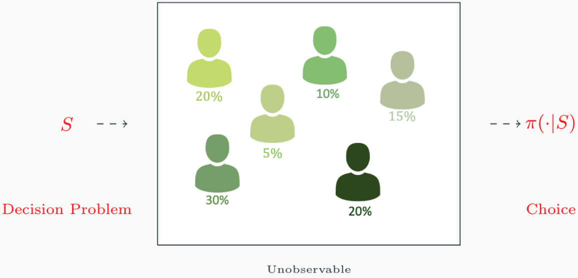


- - → $\pi(\cdot|S)$
Choice

Unobservable

$\pi(x|S)$ = frequency of types choosing x from S

Random Utility Model (RUM)

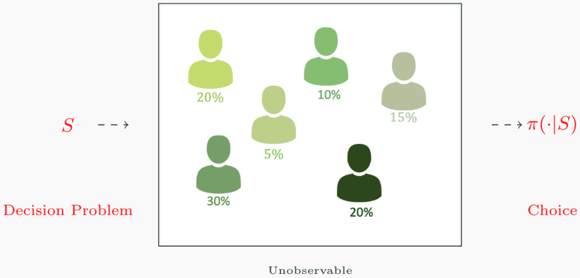


RUM

- each type is a utility maximizer
- μ: probability distribution over all preference relations

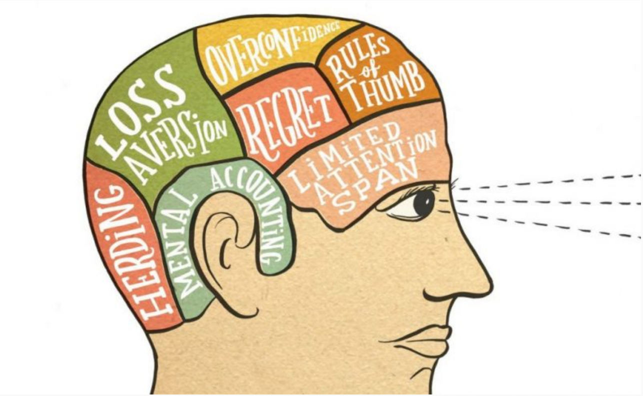
$$\pi(x|S) = \sum_{x \text{ is } \succ-\text{best in } S} \mu(\succ)$$

Limitations of RUM



- Each type must be "rational"
- No room for bounded rationality
- Distribution of types are not unique
- Complicated axioms (Block-Marschak Polynomials)

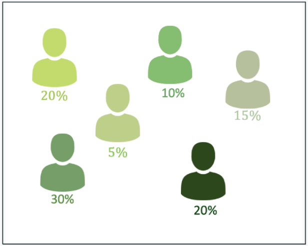
Bounded Rationality



Some papers....

Ambrus and Rozen [2015], Apesteguia and Ballester [2010], Apesteguia, Ballester, Masatlioglu [2014], Bordalo, Gennaioli, Shleifer [2013], Chambers and Yenmez [2017], Cherepanov, Feddersen, Sandroni [2013], de Clippel and Eliaz [2012], de Clippel and Rozen [2014], Dean, Kibris, Masatlioglu [2017], Dillenberger and Sadowski [2012], Dutta and Horan [2016], Eliaz and Spiegler [2011], Ellis and Masatlioglu [2019], Frick [2016], Horan [2020], Lleras, Masatlioglu, Nakajima, Ozbay [2017], Koszegi and Rabin [2006], Koszegi and Szeidl [2013], Manzini and Mariotti [2007], Manzini and Mariotti [2012a], Manzini and Mariotti [2012b], Manzini, Mariotti, Tyson [2013], Masatlioglu, Nakajima, Ozbay [2012], Masatlioglu and Nakajima [2013], Masatlioglu, Nakajima, Ozdenoren [2020] , Nishimura and Ok [2016], Nishimura [2018], Noor and Takeoka [2010], Ok, Ortoleva, Riella [2015], Papi [2012], Ravid and Steverson [2018], Rubinstein and Salant [2008], Simon [1955], Tserenjigmid [2015], Tyson [2013], Xu and Zhou [2007], and more...

Random Choice Model (RCM)

S - - → 

- - → $\pi(\cdot|S)$

Decision Problem — Choice

Unobservable

Random Choice Model

$$\pi(x|S) = \sum_{c(S)=x} \mu(c)$$

- "bounded rationality" is allowed
- μ: probability distribution over all choice functions

Ordered Domain

Random Choice Model is too general to learn about choice types.

We address this by imposing some meaningful structure on the domain

Consider domains where alternatives are sorted by an order:

- tax policies ordered by the total revenue (Roberts [1977])
- public goods ordered by the provision levels (Epple et al. [2001])
- insurances ordered by deductibles (Barseghyan et al. [2019])
- payments ordered by the present value (Manzini and Mariotti [2006])
- acts ordered by ambiguity level (Chew et al. [2017])
- food options ordered by temptation levels (Shiv and Fedorikhin [1999])
- uncertain payments ordered by probable maximum loss (Kremer [1990])
- products ordered by their carbon footprint (Rokeach [1970])

Ordered Types

$\triangleright$: the reference order

- policies ordered by being environmental friendly

$\{c_t\}$: Ordered types

- choice types ordered based on being environmentally conscious

less environmental ... more environmental

An Example

- Three policies: x, y, and z

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- Sorted by Eco-friendliness: $z \triangleright y \triangleright x$

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- The agent also cares her own selfish utility: u according to which x is the best and z is the worst policy.

An Example

- Three policies: x, y, and z
- Sorted by Eco-friendliness: $z \triangleright y \triangleright x$
- The agent also cares her own selfish utility: u according to which x is the best and z is the worst policy.
- Trade-off solved by the model of Dillenberger and Sadowski (2012): Ashamed to look non-eco-friendly with the shame parameter: s

$$c_s(T) = \underset{a \in T}{\operatorname{argmax}} \left\{ \underbrace{u(a)}_{\text{selfish utility}} - \underbrace{\left(\max_{b \in T} \psi(b) - \psi(a)\right)^s}_{\text{cost of shame to act non-ecofriendly}} \right\}$$

An Example

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- $u(x) = 4, u(y) = 3$ and $u(z) = 1$, and $\psi(z) = 6$, $\psi(y) = 4$ and $\psi(x) = 1$
- $s = 0$ (does not care being Eco-friendly)
- $c_{s=0}(\{x, y, z\}) = c_{s=0}(\{x, y\}) = c_{s=0}(\{x, z\}) = x$

An Example

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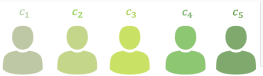
- $c_{s=0}(\{x, y, z\}) = c_{s=0}(\{x, y\}) = c_{s=0}(\{x, z\}) = x$
- There are five types with different levels of shame: $s \in \{0, 0.3, 0.6, 0.9, 1.2\}$

An Example



	c_1	c_2	c_3	c_4	c_5	
$\{x, y, z\}$	x	x	y	y	z	$x \lhd y \lhd z$
$\{x, y\}$	x	y	y	y	y	$x \lhd y$
$\{x, z\}$	x	x	x	z	z	$x \lhd z$
$\{y, z\}$	y	y	y	y	z	$y \lhd z$
s	0	0.3	0.6	0.9	1.2	

- $z \rhd y \rhd x$: reference order
- five types with different degree of consciousness: $\{c_1, ..., c_5\}$
- each choice function is becoming more inline with $\rhd$

Progressive

Progressive Collection

Definition

A collection of choice functions $\mathcal{C}$ is *progressive* with respect to $\triangleright$ if $\mathcal{C}$ can be sorted $\{c_1, c_2, \ldots, c_N\}$ such that $c_t(S) \trianglerighteq c_s(S)$ for all S and for any $t \geq s$.

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- inspired by single-crossing preferences (Mirrlees [1971], Roberts [1977], Grandmont[1978], Rothstein [1990], Milgrom and Shannon [1994], Gans and Smart [1996])

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- inspired by single-crossing preferences (Mirrlees [1971], Roberts [1977], Grandmont[1978], Rothstein [1990], Milgrom and Shannon [1994], Gans and Smart [1996])
- Apesteguia et al. [2017] studied it for RUM
- Model-free definition

Progressive Collection

(Class 1) $c_t(S) = \arg\max_{x \in S} \; u_t(x)$

Progressive Collection

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- Any utility maximization model

Progressive means: for any $x \triangleright y$ and $t > s$,

$$u_s(x) > u_s(y) \Rightarrow u_t(x) > u_t(y)$$

Progressive Collection

(Class 2) $c_t(S) = \arg\max_{x \in \Gamma_t(S)} \; u(x)$

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- Shortlisting of Manzini and Mariotti [2007],
- Preferred Personal Equilibrium of Kőszegi and Rabin [2006]
- Willpower of Masatlioglu et al. [2020]
- Rationalization of Cherepanov et al. [2013]
- Limited Attention of Masatlioglu et al. [2012]

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Progressive means: For any types $t > s$,

$$L_u(\Gamma_s(S)) \subseteq L_u(\Gamma_t(S))$$

Progressive Collection

(Class 3) $c_t(S) = \arg\max_{x \in S} \; u(x) - k_t(x, S)$

Progressive Collection

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- Temptation and Self Control by Gul and Pesendorfer [2001], Fudenberg and Levine [2006], Dekel et al. [2009], Noor and Takeoka [2010]
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Progressive means: For any types $t > s$ and alternatives $x \triangleright y$,

$$u(x) - k_s(x, S) > u(y) - k_s(y, S) \Rightarrow u(x) - k_t(x, S) > u(y) - k_t(y, S)$$

Progressive Random Choice

- Progressive Random Choice ($\text{PRC}_{\triangleright}$)
  - μ: probability distribution over all choice functions
  - $$\pi(x|S) = \sum_{c(S)=x} \mu(c)$$
  - the support of μ is progressive with respect to $\triangleright$

Main Result

Theorem

Let $\triangleright$ be a reference order. Then every probabilistic choice π has a PRC representation with respect to $\triangleright$. The representation is unique.

- High explanatory power (No prediction power)
- Identification unique
- Manzini and Mariotti [2006] data provides a unique opportunity to show that progressive structure indeed exists.

Proof

Constructive Proof

π	$\{x, y, z\}$	$\{x, y\}$	$\{x, z\}$	$\{y, z\}$
x	0.20	0.50	0.70	–
y	0.55	0.50	–	0.90
z	0.25	–	0.30	0.10

With the reference ranking $x \triangleright y \triangleright z$

Constructive Proof

π	$\{x, y, z\}$	$\{x, y\}$	$\{x, z\}$	$\{y, z\}$
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With the reference ranking $x \rhd y \rhd z$

Order the cumulative choice probabilities on lower contour sets:

$$\underbrace{\pi(z|yz)}_{0.10} < \underbrace{\pi(z|xyz)}_{0.25} < \underbrace{\pi(z|xz)}_{0.30} < \underbrace{\pi(y|xy)}_{0.50} < \underbrace{\pi(yz|xyz)}_{0.80}$$

Constructive Proof

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$\mu(c_t)$						
	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}						
{x,y}						
{x,z}						
{y,z}						

Constructive Proof

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	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z					
{x,y}	y					
{x,z}	z					
{y,z}	z					

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{x,y,z}	z					
{x,y}	y					
{x,z}	z					
{y,z}	z	y				

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{x,y,z}	z					
{x,y}	y					
{x,z}	z					
{y,z}	z	y	y	y	y	y

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{y,z}	z	y	y	y	y	y

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$$x \rhd y \rhd z$$

$$\underbrace{\underbrace{\pi(z|yz)}_{0.10} < \underbrace{\pi(z|xyz)}_{0.25}}_{0.15} < \underbrace{\pi(z|xz)}_{0.30} < \underbrace{\pi(y|xy)}_{0.50} < \underbrace{\pi(yz|xyz)}_{0.80}$$

$\mu(c_t)$	0.10	0.15				
	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y			
{x,y}	y	y				
{x,z}	z	z				
{y,z}	z	y	y	y	y	y

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$\mu(c_t)$	0.10	0.15	0.05			
	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y			
{x,y}	y	y	y			
{x,z}	z	z	z	x		
{y,z}	z	y	y	y	y	y

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{x,z}	z	z	z	x	x	x
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{x,y,z}	z	z	y	y		
{x,y}	y	y	y	y		
{x,z}	z	z	z	x	x	x
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$\mu(c_t)$	0.10	0.15	0.05	0.20		
	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y	y		
{x,y}	y	y	y	y	x	
{x,z}	z	z	z	x	x	x
{y,z}	z	y	y	y	y	y

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	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y	y	y	
{x,y}	y	y	y	y	x	x
{x,z}	z	z	z	x	x	x
{y,z}	z	y	y	y	y	y

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	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y	y	y	x
{x,y}	y	y	y	y	x	x
{x,z}	z	z	z	x	x	x
{y,z}	z	y	y	y	y	y

Constructive Proof

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$\mu(c_t)$	0.10	0.15	0.05	0.20	0.30	0.20
	c_1	c_2	c_3	c_4	c_5	c_6
{x,y,z}	z	z	y	y	y	x
{x,y}	y	y	y	y	x	x
{x,z}	z	z	z	x	x	x
{y,z}	z	y	y	y	y	y

First Result

Theorem 1

Let $\triangleright$ be a reference order. Then every probabilistic choice π has a PRC representation with respect to $\triangleright$. The representation is unique.

- Allows us to study phenomena outside of the utility maximization paradigm
- Any particular class of choice types of interest can be studied

An Application of Bounded Rationality:

"Less-is-More"

Less is More

Less is More

- reference order: $\triangleright$
- more likely to choose worse alternatives on larger choice sets
- choices are more in line with the order on smaller sets
- Iyengar and Lepper [2000], Chernev [2003], Iyengaret al. [2004], Caplin et al. [2009], Chernev et al [2015], ...

Less is More

Definition: Less-is-More

$\mathcal{C}$ satisfies *less-is-more* with respect to $\triangleright$ if for all t and for all $T \subset S$,

$$c_t(S) \in T \Rightarrow c_t(T) \trianglerighteq c_t(S)$$

Reference order: $(x \triangleright y \triangleright z)$

Progressive $\longrightarrow$

Less-is-More $\downarrow$

	Choice Types			
	c_1	c_2	c_3	c_4
$\{x, y, z\}$	y	y	y	x
$\{x, y\}$	y	y	x	x
$\{x, z\}$	z	x	x	x
$\{y, z\}$	y	y	y	y

Less-is-More

Less-is-more Progressive Random Choice

$$\pi(x|S) = \sum_{c(S)=x} \mu(c)$$

- the support of μ is progressive with respect to $\triangleright$
- the support of μ has less-is-more structure with respect to $\triangleright$

U-Regularity

Axiom: U-Regularity

For all $x \in T \subset S$ such that $\pi(x|S) \neq 0$

$$\pi(U_{\trianglerighteq}(x)|S) \leq \pi(U_{\trianglerighteq}(x)|T)$$

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- It resembles the standard regularity property:

$$\pi(x|S) \leq \pi(x|T) \text{ for all } x \in T \subset S$$

- but they are independent

Theorem: Characterization of Less-is-More

Let $\triangleright$ be a reference order. A probabilistic choice π satisfies U-regularity with respect to $\triangleright$ **if and only if** there is a unique $\text{PRC}_{\triangleright}$ representation of π in which each choice function satisfies the Less-is-More condition.

- Makes prediction
- Very simple axiom
- Representation is unique

Endogenous Reference Order

What happens if $\triangleright$ is unknown?

Revealed Order under Less-is-more

For any distinct x and y, define the following binary relation:

$$x \trianglerighteq_\pi y \quad \text{if} \quad (i)\ \pi(y|S) > \pi(y|\{x,y\}) \text{ for some } S \ni x,$$

$$(ii)\ \exists z\ s.t.\ \pi(y|\{x,y,z\}) > \pi(y|\{y,z\}) \text{ and } \pi(x|\{x,y,z\}) < \pi(x|\{x,y$$

$$(iii)\ \exists z\ s.t.\ \pi(z|\{x,z\}) > \pi(z|\{x,y,z\}) > \pi(z|\{y,z\}),$$

$$\pi(x|\{x,y,z\}) < \pi(x|\{x,y\}),\ \text{and } \pi(x|\{x,y,z\}) \neq 0$$

Revealed Order under Less-is-more

For any distinct x and y, define the following binary relation:

$x \trianglerighteq_\pi y$ if (i) $\pi(y|S) > \pi(y|\{x,y\})$ for some $S \ni x$,

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$\pi(x|\{x,y,z\}) < \pi(x|\{x,y\}),$ and $\pi(x|\{x,y,z\}) \neq 0$

Let $\triangleright_\pi^T$ be the transitive closure of $\triangleright_\pi$

Proposition:

If π has a $\text{PRC}_\triangleright$ representation satisfying the less-is-more property, then $\triangleright_\pi^T \subseteq \triangleright$.

Characterization Axioms

Axiom 2: Weak Binary Regularities

$$\forall x \in S, \quad \pi(x \mid S) \leq \max\{\pi(x|\{x,y\}), x \neq y \in S\}$$

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Axiom 3

$\forall S$ with $|S| \geq 3$, $\pi(x \mid S) > \pi(x \mid \{x,y\})$ for some $x, y \in S$

Note that the axioms do not require the knowledge of the reference order.

Axiom 2 allows binary regularity violations but it restricts the severity of them.

Axiom 3 is a natural one for a less-is-more representation because U-Monotonicity implies some regularity violations.

Identification of Reference Order

Theorem: Endogenizing $\trianglerighteq$

If a strict probabilistic choice function π satisfies Axioms 2 and 3, then $\triangleright_\pi^T$ is complete.

Identification of Reference Order

Theorem: Endogenizing $\trianglerighteq$

If a strict probabilistic choice function π satisfies Axioms 2 and 3, then $\triangleright_\pi^T$ is complete.

Note that this gives us the uniquely identified reference order for PRC satisfying less-is-more from the data.

- The theorem above provides a complete order as a candidate
- The previous Proposition established that it has to be the unique order representing the data

Summary

- Random Choice Model.
- This set up allows us to study behavior outside of utility maximization.
- Progressive structure implies unique representation.
- Progressive structure gives meaningful interpretation of heterogeneity of choice types and allows for comparative statics.
- Many ordered behavioral traits can be studied using our PRC algorithm
- Today:Less-is-More behavior is characterized.
- Revealed Preference Argument can be made when the reference order is not observable.
- Under certain conditions, the underlying order is uniquely identified.

The End

Backup Slides

Comparative Statics

$\forall \alpha \in (0,1]$, define $\mu_\alpha^{-1} := c_i \in \mathcal{C}$ such that

$\mu(c_1)+...+\mu(c_{i-1}) < \alpha \leq \mu(c_1)+...+\mu(c_i)$ for a given $\mathcal{C} = \{c_1, ..., c_T\}$ and μ

Hence, μ_α^{-1} identifies the choice function in the collection at which the cumulative distribution weakly exceeds α.

Definition: Higher

Probability distribution μ defined on $\mathcal{C}$ is **higher** than probability distribution η defined on $\mathcal{C}'$ if $\forall \alpha \in (0,1]$ and $\forall S \subset X$, $\mu_\alpha^{-1}(S) \trianglerighteq \eta_\alpha^{-1}(S)$.

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Theorem: Comparative Statics

Let π_μ and π_η be two PRC$_\triangleright$. π_μ first order stochastic dominates π_η **if and only if** μ is higher than η.

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