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Journal of Economic Theory 121 (2005) 1–29

JOURNAL OF  
**Economic  
Theory**

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# Rational choice with status quo bias<sup>☆</sup>

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Received 15 January 2003; final version received 17 March 2004

Available online 3 September 2004

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## Abstract

Motivated by the empirical findings concerning the importance of one's current situation on her choice behavior, the main objective of this paper is to propose a rational choice theory that allows for the presence of a status quo bias, and that incorporates the standard choice theory as a special case. We follow a revealed preference approach, and obtain two nested models of rational choice that allow phenomena like the status quo bias and the endowment effect, and that are applicable in any choice situation to which the standard (static) choice model applies.

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*JEL classification:* D11; D81

*Keywords:* Revealed preferences; Incomplete preferences; Status quo bias; Endowment effect; Expected utility; Overpricing; Preference reversal phenomenon

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## 1. Introduction

In the last two decades, a sizable amount of experimental data on the individual choice behavior has been obtained, and a number of startling regularities have been noted.<sup>1</sup> Among such regularities is the observation that, relative to other alternatives, a current choice or a default option is often evaluated markedly positively by the individuals. This

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<sup>☆</sup> Various conversations with Jean-Pierre Benoit, Faruk Gul, and Jacob Sagi have helped to the development of this paper to a significant degree; the second author gratefully acknowledges his intellectual debt to them. A careful referee of this journal contributed substantially to the exposition of the paper; we are grateful to him/her. We are also thankful for the support of the C.V. Starr Center for Applied Economics at New York University.

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<sup>1</sup> See [6,25] for careful surveys.

phenomenon is termed the *status quo bias* [27], and is documented not only by experimental studies but also by empirical work in the case of actual markets.<sup>2</sup> Motivated by these findings, the main objective of this paper is to propose a rational choice theory that allows for the presence of a status quo bias, and that incorporates the standard choice theory as a special case. Our approach is axiomatic, and yields an individual choice model that is general enough to be applicable in any situation in which the standard (static) choice model is applicable.

We think of a *choice problem* in this paper either as a feasible set  $S$  of alternatives, or as a feasible set  $S$  and a point  $x$  in  $S$ , which is interpreted as the default option of the individual.<sup>3</sup> As usual, a *choice correspondence* is then defined as assigning to any given choice problem a subset of the feasible set of the problem. We introduce to the model a status quo bias by requiring that if an alternative is chosen when it is *not* a status quo, it should be chosen *uniquely* when it is itself a status quo, other things being equal. In addition to this property, we consider four other rationality requirements in the first part of the paper. Two of these are straightforward reflections of the standard axioms of revealed preference theory, and the other two are new properties that link the choice behavior of the decision maker across problems with and without a status quo.

These five axioms characterize a decision-making model which is quite reminiscent of some earlier suggestions present in the literature. According to this model, the agent has an incomplete preference relation that distinguishes between choice alternatives on the basis of various criteria (several complete preference relations), and in the absence of a status quo choice, she solves her choice problems by maximizing an aggregation of these criteria. This can, of course, be viewed as a particular instance of the individual choice model of the classical revealed preference theory. When, however, there is a status quo  $x$  in the problem, then the incompleteness of agent's preferences becomes operational. In this case, the agent sticks with  $x$ , unless there are some feasible alternatives that dominate the status quo choice  $x$  with respect to her incomplete preference relation (that is, in terms of *all* decision criteria she deems relevant to the problem). Put differently, having an initial entitlement allows the agent to get "confused/indecisive", when comparing the other alternatives with her current holdings, and she always resolves this situation in favor of her status quo; hence the *status quo bias*. If no such confusion arises (because some alternatives are unambiguously superior to the status quo), then the initial position of the agent becomes irrelevant, and the agent settles her decision problem by aggregating her decision criteria as if there is no status quo in the problem. Fig. 1 provides a geometric illustration of this choice procedure which is, as we shall discuss below, closely linked to the choice procedures suggested by Simon [30] and Bewley [4].

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<sup>2</sup> Samuelson and Zeckhauser [27] identified this effect experimentally by an extensive study concerning portfolio choices. Most of the experimental studies that find gaps between buying and selling prices provide support for the status quo bias as we understand the term here; see, for instance, [15–17]. [11,13,20] report substantial amounts of status quo bias in the field settings (where individual choices concern reliability levels of residential electrical services, car insurance, and participation in 401(k) plans, respectively).

<sup>3</sup> This latter model of choice problems was also considered by Bossert and Sprumont [5] and Zhou [35]; more on the relation between the present work and these papers shortly.

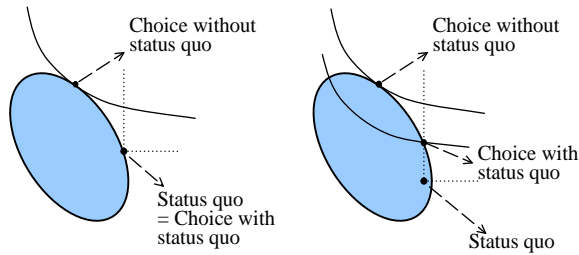


Fig. 1.

By strengthening the axioms that connect how the problems with and without status quo are solved, one can provide sharper characterizations of the nature of the “indecisiveness” of the agent in the presence of a status quo. Our second characterization theorem is a case in point. This result provides a model which is apparently suitable for capturing the famous *endowment effect*. In this model, again, the agent solves the standard problems by maximizing a utility function  $u(\cdot)$ , but when there is a status quo  $x$  (which, in view of the strengthened set of axioms, is best interpreted as an alternative that the agent is *endowed* with), she gives up  $x$  if, and only if, another choice item provides her higher utility than  $u(x)$  plus a “utility pump” of  $\varphi(x) > 0$ , which may perhaps be thought of as a *psychological switching cost*. Put differently, if the “value” of an object  $x$  is some number  $u(x)$  when the object is not owned, its value is  $u(x) + \varphi(x)$ , when it is owned; hence the *endowment effect*.<sup>4</sup> What is more, this decision-making model is easily extended to the risky choice situations (via positing the independence axiom on choice correspondences).

A few remarks on the relationship of the present work to the literature are in order. The motivation of the papers by Zhou [35] and Bossert and Sprumont [5] are very close to this paper, and there are some similarities between the models that we consider. In particular, both of these papers adopt the revealed preference approach and consider the choice problems with a status quo as we define them here. A major difference between these papers and the present work is that we allow here problems *without* a status quo as well in our domain of choice problems. Not only that this allows us to produce a theory that admits the standard rational choice model as a special case, but it also gives rise to a rich setup in which one can consider properties regarding how choices are made in problems with and without status quo in a consistent manner. The implications of this are surprisingly far reaching.

In particular, our choice model is in stark contrast with that of Zhou [35] who works with choice *functions*. While the only axiom considered by that paper (menu-independence) is ill-defined for choice correspondences in general, even the single-valued choice correspondences of the form characterized here (such as the Simon–Bewley choice procedures) need not satisfy this axiom. On the other hand, in the case of abstract choice problems (the case we consider here), [5] characterize those choice correspondences that choose in a problem

<sup>4</sup> While this representation has certain attractive features for applications, the issue of “uniqueness” of  $u$  and  $\varphi$  naturally arises at this junction. Suffices it to say that our theory is close to being ordinal (in the riskless case), but given the nonstandard nature of the representations that we propose, this issue will have to be carefully addressed. We will do so in the body of the paper.

a set *only if* every element of this set is superior to the status quo (according to a linear order on the universal set of alternatives). While quite interesting, this approach gives only little information about how to model the status quo bias in concrete situations. Indeed, the axioms adopted by Bossert and Sprumont [5] are too weak to produce a well-structured choice procedure in problems with status quo.<sup>5</sup> Certainly all choice procedures we consider here satisfy the Bossert–Sprumont axioms, but more importantly, they enjoy relatively sharp representations that may be used in applications in a straightforward way. To reiterate, we do this by studying in conjunction the choice problems with and without status quo, and examining certain consistency properties that tie such problems together.

Our work is also related to the literature on *reference dependent* preferences. In particular, Tversky and Kahneman [32] suggest a deterministic utility theory over a finite-dimensional commodity space in which an alternative is preferred to another alternative according to a preference relation that depends on a reference state, which can be interpreted in our context as the status quo point. Yet, Tversky and Kahneman *assume* a particular choice behavior generated by such reference dependent preferences (that is, impose a model that parallels the choice behavior that we derive axiomatically here), and do not discuss the structure of representation for such preferences. In the case of risky prospects, however, this situation is remedied by Sagi [26] who provides an axiomatic foundation for the reference dependent decision model of [32].<sup>6</sup>

A major difference of the present work from these studies is that we take here the choices as the starting point, and *derive* the (reference dependent) preferences thereof, as opposed to following the opposite direction. While, under certain assumptions, the two approaches are dual to each other, the appeal and strength of rationality axioms differ across these models. A second major difference is our insistence of developing a model that allows for reference independence (the absence of a status quo), and exploiting this in order to link the standard theory to reference-based choice. As noted earlier, this approach yields a decision theory which is somewhat more comprehensive than the one in which agents maximize a given (reference dependent or otherwise) preference relation. (See Theorem 1 and the discussion that follows.) We also note that this formulation contrasts with (and should be viewed complementary to) the axiomatization provided by Sagi [26] in that the analysis here is *nonlinear* (hence *ordinal*) for the most part, as in the classical revealed preference theory. As such it is arguably more appropriate for models of riskless choice, even though, as we shall show in Section 2.5, it is not difficult to extend the proposed choice theory to risky choice situations through the classical independence axiom.

The rest of the paper is organized as follows. In Section 2 we introduce a set of axioms that seem particularly suitable for a rational choice theory that allows for status quo bias, and characterize the choice correspondences that satisfy these properties. We then add further postulates to the model in order to capture the so-called endowment effect, identify exactly

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<sup>5</sup> These axioms are not even strong enough to yield a complete characterization of how an agent makes her choices on the basis of this rationalizing linear order. Indeed, given this order  $R$ , these axioms cannot distinguish between the rules “choose only the status quo” and “choose everything that is better than the status quo in terms of  $R$ .” In this sense, it is not clear how to use the model of [5] in applications.

<sup>6</sup> We view this as a much needed remedy. For, notions like loss aversion and diminishing sensitivity are *cardinal* in nature, and are thus arguably ill-founded in the context of riskless choice.

which choice correspondences satisfy these axioms, and finally, extend the model to the context of risky alternatives. Section 3 contains two applications. First we show how our model of the endowment effect predicts a discrepancy between buying and selling prices of goods, and second, we use our risky-choice model to offer a new explanation for the (in)famous preference reversal phenomenon. Concluding remarks and the proofs are given in Sections 4 and 5, respectively.

## 2. An axiomatic model of status quo bias

### 2.1. Basic axioms

We consider an arbitrary compact metric space  $X$ , and interpret each element of  $X$  as a potential choice alternative (or prize).<sup>7</sup> The set  $X$  is thus viewed as the grand alternative space. For reasons that will become clear shortly, we designate the symbol  $\diamond$  to denote an object that does not belong to  $X$ . Also let  $\mathcal{X}$  denote the set of all nonempty closed subsets of  $X$ . By a *choice problem* in this paper, we mean a list  $(S, x)$  where  $S \in \mathcal{X}$  and either  $x \in S$  or  $x = \diamond$ . The set of all choice problems is denoted as  $\mathcal{C}(X)$ .

If  $x \in S \in \mathcal{X}$ , then the choice problem  $(S, x)$  is referred to as a *choice problem with a status quo*. The interpretation is that the individual is confronted with the problem of choosing an alternative from the feasible set  $S$  while either she is currently endowed with the alternative  $x$  or her default option is  $x$ . Viewed this way, choosing an alternative  $y \in S \setminus \{x\}$  means that the individual gives up her status quo  $x$  and switches to  $y$ . We denote by  $\mathcal{C}_{sq}(X)$  the set of all choice problems with a status quo.

On the other hand, many real-life choice situations do not have a natural status quo alternative. Within the formalism of this paper, the choice problems of the form  $(S, \diamond)$  model such situations. Formally, then, we define a *choice problem without a status quo* as the list  $(S, \diamond)$  for any set  $S$  in  $\mathcal{X}$ . (While the use of the symbol  $\diamond$  is clearly redundant here, it will prove quite convenient in the foregoing analysis.)

Just to fix ideas, and help illustrate some of what follows, we shall carry with us a concrete (yet hypothetical) example which is rather close to home. We have in mind the choice problem of an economist, call her Prof.  $\sigma$ , who is currently employed at Cornell and is pondering over two new job offers, one from NYU and one from UCSD. In the terminology of this paper, then, the choice problem of Prof.  $\sigma$  is one with a status quo, where the status quo point is to stay at Cornell, and the feasible set is the jobs she might have in all three of the schools. If we change the scenario a little bit, and instead say that Prof.  $\sigma$  is about to graduate from UPenn, and after a successful job market experience, has now three offers from Cornell, NYU and UCSD, then it would make sense to model her choice problem as one without a status quo.

By a *choice correspondence* in the present setup, we mean a map  $c : \mathcal{C}(X) \rightarrow \mathcal{X}$  such that

$$c(S, x) \subseteq S \quad \text{for all } (S, x) \in \mathcal{C}(X).$$

<sup>7</sup> Throughout this paper we adopt the innocuous convention of assuming that every metric space is nonempty.  $cl(\cdot)$ ,  $int(\cdot)$ , and  $N_\varepsilon(\cdot)$  (with  $\varepsilon > 0$ ) denote the closure, interior, and  $\varepsilon$ -neighborhood operators, respectively.

(Notice that a choice correspondence must be nonempty-valued by definition.) We shall next consider some basic properties for choice correspondences. The first two of these are straightforward reflections of the classical theory of revealed preference; they allow one to identify when a “choice” can be viewed as an outcome of a utility maximization exercise. While in the standard theory such axioms regulate the alteration of choices in response to the alteration of feasible sets of alternatives, our properties here condition things also with regard to the changes of status quo points across choice problems.

**Property  $\alpha$ .** For any  $(S, x), (T, x) \in \mathcal{C}(X)$ , if  $y \in T \subseteq S$  and  $y \in c(S, x)$ , then  $y \in c(T, x)$ .

**Property  $\beta$ .** For any  $(S, x) \in \mathcal{C}(X)$ , if  $z, y \in c(S, x)$ ,  $S \subseteq T$ , and  $z \in c(T, x)$ , then  $y \in c(T, x)$ .

There is little need to motivate these properties; they are none other than the obvious extensions of the classical axioms of revealed preference theory. (See, for instance, [18, pp. 11–15] for an expository account.) Suffices it to say that these properties are jointly equivalent to the statement that, for any  $x \in X \cup \{\diamond\}$ , the correspondence  $c(\cdot, x) : \mathcal{X} \rightarrow \mathcal{X}$  satisfies Houthakker’s *weak axiom of revealed preference*, or Arrow’s *choice axiom* [1].

**Axiom D (Dominance).** For any  $(S, x) \in \mathcal{C}(X)$ , if  $\{y\} = c(S, x)$  for some  $S \subseteq T$ , and  $y \in c(T, \diamond)$ , then  $y \in c(T, x)$ .

Recall that if  $y \in c(T, \diamond)$ , we understand that  $y$  is one of the most preferred alternatives in the set  $T$  *in the absence of a status quo*. So the only reason why  $y$  would not be chosen from  $(T, x)$  is because  $x$  may defeat  $y$  when it is endowed with the additional strength of being the status quo. But if  $\{y\} = c(S, x)$  for some  $S \subseteq T$ , then it is clear that this cannot be the case, because  $y$  is then revealed to be (strictly) preferred to  $x$  even when  $x$  is designated as the status quo point. Thus, it seems that  $y$  (weakly) dominates everything feasible in the choice problem  $(T, x)$  along with the status quo point  $x$ , and hence, so Axiom D posits, it should be a potential choice from  $(T, x)$  as well.

To illustrate, suppose that Prof.  $\sigma$ , who is currently employed at Cornell, would accept a job offer from NYU. We somehow also know that, upon getting her degree (and thus not having a status quo), she would have taken the NYU offer over the offers of Cornell and UCSD. Assuming that her tastes have not changed through time, what would one expect Prof.  $\sigma$  to do, when she gets the NYU and UCSD offers simultaneously while she is at Cornell? We contend that most people would not be surprised to see her at NYU the following year, for our knowledge about her preferences indicates that NYU dominates the offers of both Cornell and UCSD, and this regardless of her being currently employed at Cornell. Axiom D is based precisely on this sort of a reasoning.

**Axiom SQI (Status-quo irrelevance).** For any  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ , if  $y \in c(S, x)$  and there does not exist any nonempty  $T \subseteq S$  with  $T \neq \{x\}$  and  $x \in c(T, x)$ , then  $y \in c(S, \diamond)$ .

To understand the intuitive appeal of this property, assume that  $x$  is never chosen from any subset of  $S$  despite the fact that it is the status quo. Thus  $x$  cannot be thought of as playing a significant role in the choice situation  $(S, x)$ ; it is completely irrelevant for the problem at hand. This means that there is practically no difference between the choice problem without

a status quo  $(S, \diamond)$  and the choice problem  $(S, x)$  in the eyes of the decision maker. So, if  $y$  is chosen from the latter problem, it should also be chosen from the former.

To illustrate more concretely, suppose again that Prof.  $\sigma$ , who is currently tenured at Cornell and who has two offers, one from NYU and one from UCSD, has indicated that she would take the NYU offer. Moreover, it is somehow known that both of the offers dominate staying at Cornell. What would one expect Prof.  $\sigma$  to do when confronted with the offers of Cornell, NYU and UCSD, upon getting her degree (and thus not having a status quo)? The information at hand indicates that she deems being employed or not being employed at Cornell as irrelevant with regard to her choice problem at hand—her problem is really to choose between the offers of NYU and UCSD.<sup>8</sup> But we already know that she likes the former at least as much as the latter, so it appears rational that she take the NYU offer; this is precisely what Axiom SQI posits.

The final property that we will consider here for choice correspondences is central to the development of this paper. It specifies a distinctive role for the status quo point in choice problems with a status quo.

**Axiom SQB** (Status-quo bias). For any  $(S, x) \in \mathcal{C}(X)$ , if  $y \in c(S, x)$  then  $\{y\} = c(S, y)$ .

This axiom is based on the idea that if the decision maker reveals that  $y$  is no worse than any other alternative in a feasible set  $S$ , including the status quo point  $x$  (if there is such a point), then, when  $y$  is itself the status quo, its position can only be stronger relative to the alternatives in  $S$ . The axiom posits that in this case  $y$  must be the *only* choice from the alternative set  $S$ , thereby requiring a choice correspondence to exhibit some bias towards the status quo. If it is somehow revealed elsewhere that the status quo is at least as desirable as all other feasible alternatives, then “why move?”; SQB requires in this case the individual to keep the status quo point. This property is the least normative among the axioms considered so far, being motivated instead by empirical research that established the importance of the presence of a status quo in individual decision-making. It seems quite appealing for a choice theory that envisages a status quo bias, and yet is otherwise rational.

## 2.2. Basic choice correspondences

Each of the first four properties considered above corresponds to a particularly appealing rationality trait in the case of choice problems that may possess a status quo point. The fifth property, on the other hand, introduces to the model a form of inertia towards the status quo, but it does this in a very conservative manner. Consequently, the choice correspondences that satisfy all five of these properties appear to be focal for a rational choice theory that would allow for status quo bias. This prompts the following definition.

**Definition.** Let  $X$  be a compact metric space. We say that a choice correspondence  $c$  on  $\mathcal{C}(X)$  is *basic* if it satisfies properties  $\alpha$  and  $\beta$ , and Axioms D, SQI and SQB.

<sup>8</sup> It is important to note that the irrelevance of the Cornell offer is *relative* to the offers from NYU and UCSD. Provided that the preferences of Prof.  $\sigma$  have not changed, the only conclusion that one may draw from this is that at the time she chose Cornell (making it her status quo), the offers from NYU and/or UCSD were not available.



The characterization of the structure of basic choice correspondences is the primary aim of this paper. In this section, we shall investigate this issue under the assumption of finiteness of the prize space. Let us first introduce the order-theoretic nomenclature we adopt in this paper. A binary relation on a nonempty set is called a *preorder* if it is reflexive and transitive. An antisymmetric preorder is called a *partial order*, and a complete partial order is called a *linear order*. If  $\succsim$  is some preorder on  $X$ , we say that  $\succsim^*$  is a *completion* of  $\succsim$  if  $\succsim^*$  is a complete preorder with  $\sim \subseteq \sim^*$  and  $\succ \subseteq \succ^*$ , where  $\succ$  is the strict (asymmetric) part of  $\succsim$  and  $\sim$  is the weak (symmetric) part of  $\succsim$ , and similarly for  $\sim^*$  and  $\succ^*$ . For any nonempty subset  $S$  of  $X$ , by  $M(S, \succsim)$  we mean the set of all maximal elements in  $S$  with respect to  $\succsim$ , that is,  $M(S, \succsim) := \{x \in S : y \succ x \text{ for no } y \in S\}$  where  $\succ$  denotes the strict part of  $\succsim$ . Finally, for any  $x \in X$ , by  $U_{\succ}(x)$ , we denote the strict upper contour set of  $x$  with respect to  $\succsim$ , that is,  $U_{\succ}(x) := \{y \in X : y \succ x\}$ .

We are now ready to exhibit what sort of structure one would expect a basic choice correspondence to possess in general.

**Lemma 1.** *Let  $X$  be a nonempty set. If the choice correspondence  $c$  on  $\mathcal{C}(X)$  is basic, then there exists a partial order  $\succsim$  and a completion  $\succsim^*$  of this partial order such that  $c(\cdot, \diamond) = M(\cdot, \succsim^*)$  and*

$$c(S, x) = \begin{cases} \{x\} & \text{if } x \in M(S, \succsim), \\ M(S \cap U_{\succ}(x), \succsim^*) & \text{otherwise} \end{cases}$$

for all  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ .

Unfortunately, absent any continuity requirements on the choice correspondence, one cannot hope to prove the converse of this fact since one cannot then guarantee in general the existence of a maximal element in a feasible set. However, when  $X$  is finite, then we can not only escape this technical problem, but also get a multi-dimensional utility representation for the partial order found in Lemma 1. Before we discuss the choice structure found in this lemma, therefore, we make note of the ensuing situation in this case. As a final bit of notation, we note that, for any positive integer  $n$  and any  $S \in \mathcal{X}$ , we denote the upper contour set of any  $x \in X$  with respect to a function  $\mathbf{u} : X \rightarrow \mathbb{R}^n$  as  $U_{\mathbf{u}}(S, x)$ , that is,

$$U_{\mathbf{u}}(S, x) := \{y \in S : \mathbf{u}(y) > \mathbf{u}(x)\}.$$

The following characterization theorem identifies all basic choice correspondences in the case where the grand alternative space  $X$  is finite.

**Theorem 1.** *Let  $X$  be a nonempty finite set. A choice correspondence  $c$  on  $\mathcal{C}(X)$  is basic if, and only if, there exists a positive integer  $n$ , an injective function  $\mathbf{u} : X \rightarrow \mathbb{R}^n$  and a strictly increasing map  $f : \mathbf{u}(X) \rightarrow \mathbb{R}$  such that*

$$c(S, \diamond) = \arg \max_{x \in S} f(\mathbf{u}(x)) \quad \text{for all } S \in \mathcal{X} \tag{1}$$



and

$$c(S, x) = \begin{cases} \{x\} & \text{if } U_{\mathbf{u}}(S, x) = \emptyset, \\ \arg \max_{y \in U_{\mathbf{u}}(S, x)} f(\mathbf{u}(y)) & \text{otherwise} \end{cases} \quad (2)$$

for all  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ .

Theorem 1 shows that a basic choice correspondence models a surprisingly well-structured choice behavior, at least when  $X$  is finite. An agent whose choice behavior is characterized by such a choice correspondence evaluates all alternatives by means of a vector-valued utility function  $\mathbf{u}$ . We may interpret this as the evaluation of the alternatives on the basis of various distinct criteria; in this interpretation, the  $i$ th component of  $\mathbf{u}$  can be thought of as representing the agent's (complete) ranking of the alternatives with respect to the  $i$ th criterion.

If the agent is dealing with a choice problem without a status quo, breaking down her preferences in this way is not essential. For, in this case, she has a particular way of aggregating all criteria (i.e. the components of  $\mathbf{u}$ ) by means of a map  $f$ . A moment's reflection shows that the standard maximizing choice paradigm is but a special case of this setting.

When, however, there is a status quo  $x$  in the problem, the multidimensional way in which the agent makes a first pass at evaluating the alternatives is detrimental. First of all, the agent compares the status quo point  $x$  with all other feasible alternatives with respect to *all* criteria that she deems relevant.<sup>9</sup> If none of the alternatives weakly dominates  $x$  in each criterion, and strictly in at least one criterion, then the agent sticks with her status quo, thereby depicting a pronounced status quo bias. If at least one alternative passes this test, then the decision maker decides to leave her status quo, and considers choosing an alternative among all those that "beat" her status quo in each criterion. The final choice among such alternatives is made on the basis of maximizing her aggregate utility, where to aggregate the various criteria she uses the same aggregator  $f$  (hence the same trade-offs between these criteria) that she uses in choice problems without a status quo.

Consider the now familiar example of Prof.  $\sigma$  who has job offers from Cornell, NYU and UCSD, and whose choice correspondence is basic. If she does not have a status quo job (say, because she is fresh out of the graduate school), then her choice problem is completely standard. She distinguishes between the offers on the basis of a number of criteria that she deems relevant, for instance, salary, location, preferences of the spouse and reputation of the school (or some linear combinations of these criteria). But, given that she does not have a status quo at present, there is no benchmark for her to compare these dimensions against, and hence she somehow aggregates the potential of each offer across the relevant criteria, thereby assigning an aggregate utility to each offer, and then chooses the one that yields her the maximum aggregate utility. If, on the other hand, Prof.  $\sigma$  is already tenured at Cornell, so her Cornell offer is really her status quo, then she compares the promises of NYU and UCSD with those of Cornell with respect to every criteria, and unless at least one of her outside offers dominates Cornell in every dimension, she stays at Cornell. If only NYU

<sup>9</sup> It is worth noting here that there are some axiomatic studies about individual preferences that admit such a multi-utility representation. See, for instance, [2,8,22,29].

passes this test, then she moves to NYU. If, finally, both NYU and UCSD offers dominate Cornell with respect to each of her criteria, then the problem becomes choosing between NYU and UCSD, and Prof.  $\sigma$  settles this problem as if her choice set consists only of these two alternatives and there is no status quo.

It is interesting that the behavioral choice procedures stipulated by both Simon [30] and Bewley [3] are closely related to basic choice correspondences (at least when  $X$  is finite). Simon [30] suggests a choice procedure in which an agent tries “to implement a number of values that do not have a common denominator—e.g., he compares two jobs in terms of salary, climate, pleasantness of work, etc.”, and then searches for the set of feasible alternatives which is “satisfactory” in terms of all these values. Simon is a bit imprecise about what “satisfactory” means in this context, but it is clear that he has in mind some sort of a dominance (in terms of *all* values/criteria) over what is guaranteed to the agent, presumably at his status quo. In the same spirit the inertia assumption of Bewley [4, p. 1] “asserts that in some circumstances one can define a *status quo*, which the decision maker abandons in favor of an alternative only if doing so leads to an improvement”.

The formalization of this procedure, which we shall refer to as the *Simon–Bewley choice procedure*, is straightforward in the present setup. If  $\mathbf{u}$  denotes the multidimensional evaluation criteria, given a choice problem with a status quo  $(S, x)$ , the agent’s aspiration levels in each of these criteria is given by the vector  $\mathbf{u}(x)$ . So the Simon–Bewley procedure chooses the status quo  $x$  if no  $y \in S$  dominates  $x$  in all these criteria (i.e., if  $\mathbf{u}(y) > \mathbf{u}(x)$  for no  $y \in S$ ), and choose *all* dominating alternatives if there is such an alternative  $y$ , that is, choose the set  $U_{\mathbf{u}}(S, x)$  whenever this set is nonempty.

To illustrate, let  $X := \{x, y, z, w\}$ , and assume that the decision maker uses exactly two criteria (so  $\mathbf{u}(X) \subset \mathbb{R}^2$ ), and we have  $\mathbf{u}(x) := (0, 0)$ ,  $\mathbf{u}(y) := (1, 4)$ ,  $\mathbf{u}(z) := (2, 1)$ ,  $\mathbf{u}(w) := (-1, 4)$ . Then the Simon–Bewley procedure chooses precisely the set  $\{y, z\}$  in the case of the problem  $(X, x)$ . By contrast, a basic choice correspondence can be more refined than this. It certainly agrees with the Simon–Bewley procedure, in that it chooses a subset of  $\{y, z\}$ , but the final decision requires comparing  $y$  and  $z$  as if there is no status quo in the problem. (After all,  $x$  did its job; it eliminated  $w$  from consideration, and in turn, it is itself eliminated by both  $y$  and  $z$ .) If the individual aggregates the components of  $\mathbf{u}$  by means of a function  $f : \mathbf{u}(X) \rightarrow \mathbb{R}$  in the absence of a status quo point, then the agent’s unique choice from  $(S, x)$  is  $y$  whenever  $f(\mathbf{u}(y)) > f(\mathbf{u}(z))$ . This situation is depicted in Fig. 2, where  $f$  is represented by the indifference curves drawn in the criteria space.<sup>10</sup>

Note that a basic choice correspondence makes use of a status quo points in two ways: (i) to eliminate those alternatives that do not dominate the status quo in all evaluation criteria; and (ii) to act as the unique choice if all alternatives are eliminated by the test of (i). To make this point clear, let  $X := \{x, y, z\}$ , and consider the basic choice correspondence defined as in Theorem 1 with  $\mathbf{u}(x) := (2, 2)$ ,  $\mathbf{u}(y) := (3, 3)$ ,  $\mathbf{u}(z) := (1, 10)$ , and with  $f : \mathbf{u}(X) \rightarrow \mathbb{R}$  defined by  $f(\mathbf{a}) := a_1 a_2$ . This correspondence chooses  $z$  from the problem

<sup>10</sup> It is worth noting that a basic choice correspondence collapses to (our formulation) of the Simon–Bewley choice procedure in sequential decision problems where  $(S, x)$  is such that  $S = \{x, y\}$  with  $y$  standing for a new alternative offered to the agent against  $x$ . In this sense, and informally speaking, our characterization in Theorem 1 can be thought of as providing an axiomatic support for the *inertia assumption* of Bewley’s theory of Knightian uncertainty which is often criticized for being ad hoc.

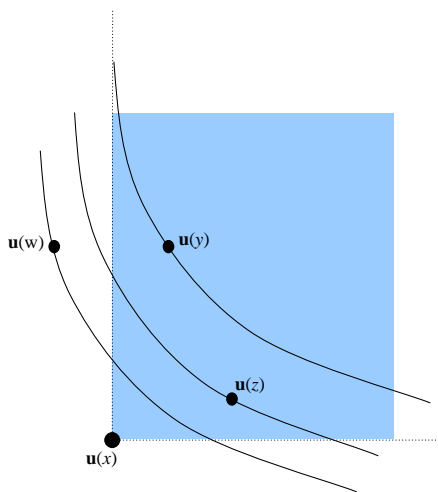


Fig. 2.

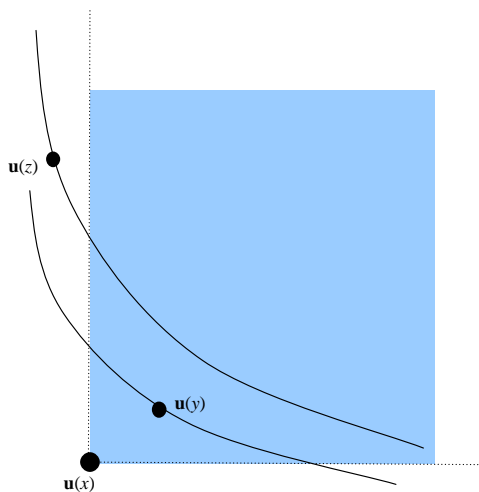


Fig. 3.

$(X, \diamond)$  while it chooses  $y$  from the problem  $(X, x)$ ; see Fig. 3. The upshot is that, a basic choice correspondence maintains that, a status quo point may alter one's choices even if it will itself not be chosen! If Prof.  $\sigma$  has such a choice correspondence, then she may choose to go to UCSD over NYU when she is not employed anywhere (a problem without a status quo), but may as well choose instead NYU over UCSD when she is employed at Cornell.

This observation reveals that status quo-dependent choices are not really irrational, at least insofar as one would agree that there is a good deal of rationality contained in a basic choice correspondence. In an intuitive sense too there is no need to view the choice behavior of Prof.  $\sigma$  in this example as irrational. For instance, suppose that UCSD is indeed her best choice (with other alternatives being Cornell and NYU) when she does not have a status quo job. Now consider the case where she is employed at Cornell, and an offer from UCSD came through. If UCSD does not dominate Cornell in all criteria that Prof.  $\sigma$  deems relevant, and she indeed possesses a status quo bias the way modeled by a basic choice correspondence, then it makes good sense that she will stay at Cornell. If after she had turned down the UCSD offer, comes along an NYU offer, and if the NYU offer dominates Cornell according to all her criteria, she will move to New York. After the dust settles, then, there is hardly anything surprising about seeing Prof.  $\sigma$  employed at NYU. Notice that the apparent intransitivity of her choices does not make her subject to a “money pump” argument, for when employed at NYU, Prof.  $\sigma$  will not move to UCSD, precisely because according to the scenario at hand, UCSD offer cannot dominate that of NYU. (See [21] for more on this issue.)

We conclude this section with a few relatively technical remarks.

**Remark 1.** Theorem 1 is sharp in the sense that the axioms used in this result constitute a logically independent set. For brevity, we do not provide the easy proof of this claim here.

**Remark 2.** To clarify why we need  $X$  to be a finite set in Theorem 1, let us note that under substantially more general conditions (namely, when  $X$  is a compact metric space and  $c$  satisfies the upper hemicontinuity condition that is stated below), one can guarantee that a basic choice correspondence  $c$  must be of the form depicted in Lemma 1. (For future reference, let us agree to say in such a situation that  $(\succ, \succ^*)$  represents  $c$ .) Once this result is obtained, Theorem 1 becomes self-evident, because any partial order on a finite set can be written as the intersection of some linear orders each of which admits a utility representation. When  $X$  is infinite, the latter fact is no longer valid, and the additional conditions (such as those given in [22]) that may reinstate it are difficult to sustain in the present setup where we “derive” the preference relations from choice correspondences. This is the main reason why we could not avoid assuming  $|X| < \infty$  in Theorem 1.

**Remark 3.** A natural question concerns the uniqueness of the representation of basic choice correspondences given in Theorem 1. We note that this representation is *ordinal* in the sense that if  $\succ_i$  is a partial order on  $X$ , and  $\succ_i^*$  is a completion of  $\succ_i$  such that  $(\succ_i, \succ_i^*)$  represents  $c$  (as described in Remark 2),  $i = 1, 2$ , then  $\succ_1 = \succ_2$  and  $\succ_1^* = \succ_2^*$ .<sup>11</sup>

### 2.3. The endowment effect

Another important anomaly that has made frequent appearance in experimental studies of individual choice is the so-called *endowment effect*; that is, the tendency of an individual

<sup>11</sup> An immediate implication of this is that if,  $m \in \mathbb{N}$ ,  $\mathbf{v} : X \rightarrow \mathbb{R}^m$  is an injection, and  $g : \mathbf{v}(X) \rightarrow \mathbb{R}$  is a strictly increasing function such that (1) and (2) hold, then there must exist a strictly increasing  $H : \mathbf{u}(X) \rightarrow \mathbb{R}^m$  and a strictly increasing  $F : f(\mathbf{u}(X)) \rightarrow \mathbb{R}$  such that  $\mathbf{v} = H \circ \mathbf{u}$  and  $g = F \circ f \circ H^{-1}$ .

to value an object more when she is the owner of it [14,15]. This phenomenon is often envisioned as if an agent gets a “utility boost” out of possessing an object, thereby creating a wedge between willingness to pay and willingness to buy. It is noted in the literature that this has important implications; in particular, it launches an unexpected attack on the famous Coase theorem. (More on this in Section 3.2.)

While this sort of a phenomenon may at first seem quite distinct from the choice behavior we have characterized in Theorem 1, it is in fact a special case of such behavior. More precisely, every choice correspondence that envisages the endowment effect (in a way that is formalized below) is in fact a basic choice correspondence. The main objective of this section is in fact to characterize precisely this subclass of basic choice correspondences by means of strengthening the set of axioms we utilized when proving Theorem 1.

The first property we need states simply that if two alternatives are chosen in the absence of a status quo, then in the presence of a status quo that is distinct from these alternatives, either they should be chosen together or neither of them should be chosen.

**Axiom SQI\*** (Status-quo independence). For any  $(S, x) \in \mathcal{C}_{sq}(X)$ , if  $y, z \in c(S, \diamond) \setminus \{x\}$  and  $z \in c(S, x)$ , then  $y \in c(S, x)$ .

This assumption cuts one of the channels through which choices might depend on the status quo. As discussed by means of two examples above (recall Fig. 3), a basic choice correspondence need not satisfy this assumption. But if one is really attempting to model the endowment effect, then SQI\* is an apparently reasonable rationality requirement. If  $y$  and  $z$  are equally good when an individual does not own an object  $x$  (distinct from  $y$  and  $z$ ), they should also be equally good (so it is not the case that one is chosen but the other is not) when the individual owns  $x$ .

The second property we will consider is a straightforward strengthening of the SQB axiom.

**Axiom SQB\*** (Strong status-quo bias). For any  $(S, x) \in \mathcal{C}(X)$ , the following hold:

- (i) If  $y \in c(S, x)$ , then  $\{y\} = c(S, y)$ ;
- (ii) If  $y \in c(S, x) \setminus \{x\}$ , then  $y \in c(S, \diamond)$ ;
- (iii) For any  $x \in X$ , then there exists an  $\varepsilon > 0$  such that  $x \in c(cI(N_\varepsilon(x)), x)$ .

While the statement (i) here is a restatement of SQB, (ii) says that if the decision maker qualifies  $y$  no worse than any other alternative in a feasible set  $S$ , including the status quo, then, when there is no status quo, its position should not deteriorate relative to the alternatives in  $S$ . While it need not be satisfied by a basic choice correspondence, it is clear that this property sits well with the intuitive understanding of the endowment effect. Finally, (iii) is a nontriviality requirement that says that the power of being the status quo makes any alternative  $x$  be the choice from a set that consists only of alternatives the nature of which are arbitrarily close to  $x$ . This requirement is trivially satisfied when  $X$  is finite (and is thus endowed with the discrete metric).

Axioms SQI\* and SQB\* are enough to transform Theorem 1 into a characterization of choice correspondences that are arguably suitable for the modeling of the endowment effect. With the help of the following standard continuity assumption, however, we can in fact state our new characterization in a more general framework that allows for infinitely

many alternatives. Of course, we need to introduce a metric on  $\mathcal{X}$  for this purpose, and as usual, we adopt the Hausdorff metric towards this end.<sup>12</sup>

**Axiom UHC** (Upper hemicontinuity). For every  $x \in X \cup \{\diamond\}$  and  $S, S_m \in \mathcal{X}$ ,  $m = 1, 2, \dots$ , if  $S_m \rightarrow S$  and  $x \neq y_m \in c(S_m, x)$  for each  $m$ , and  $y_m \rightarrow y$ , then  $y \in c(S, x)$ .

The following theorem characterizes those basic choice correspondences that satisfy the above three properties. Due to some redundancies in the set of axioms we posited so far, we can state this result by using only five properties.

**Theorem 2.** *Let  $X$  be a compact metric space. A choice correspondence  $c$  on  $\mathcal{C}(X)$  satisfies properties  $\alpha$  and  $\beta$ , and Axioms  $SQI^*$ ,  $SQB^*$  and  $UHC$  if, and only if, there exist a continuous map  $U : X \rightarrow \mathbb{R}$  and a function  $\varphi : X \rightarrow \mathbb{R}_{++}$  such that*

$$c(S, \diamond) = \arg \max_{x \in S} U(x) \quad \text{for all } S \in \mathcal{X} \quad (3)$$

and

$$c(S, x) = \begin{cases} \{x\} & \text{if } U(x) + \varphi(x) > U(y) \text{ for all } y \in S, \\ \arg \max_{y \in S} U(y) & \text{otherwise} \end{cases} \quad (4)$$

for all  $(S, x) \in \mathcal{C}_{sq}(X)$ .

The interpretation of the choice behavior identified by this result is quite straightforward. In the absence of a status quo point, an agent with such a choice correspondence solves her choice problems by maximizing a utility function  $U$  (as in the standard theory). But if there is a status quo  $x$ , then we interpret  $x$  as the object that the agent “owns”. While the utility of  $x$  for the agent is  $U(x)$ , out of owning the object, the agent gets a “utility pump”  $\varphi(x) > 0$  in the sense that to move away from her status quo (that is, to exchange  $x$  for some other alternative) she needs to be compensated by  $\varphi(x)$  in addition to  $U(x)$ , that is, she must be “paid” at least as much as  $U(x) + \varphi(x)$ ; hence the phenomenon of the *endowment effect*.

An arguably good way of interpreting  $\varphi(x)$  is thus to view it as a *psychological switching cost*. If the value of no alternative in her feasible set exceeds this psychological cost plus the value of the status quo, the agent sticks with her status quo (i.e. endowment). If, on the other hand, some alternatives are materially more desirable than  $x$  despite the endowment effect, (i.e.  $U(y) \geq U(x) + \varphi(x)$  for some feasible  $y$ ), then the agent chooses the alternative with the highest utility. This model seems to correspond well to the experimental observation that there is in general a discrepancy between the willingness to buy and willingness to sell. If the agent deems the worth of an alternative  $U(x)$  when she does not possess it, she values it at  $U(x) + \varphi(x)$  when she owns it.

<sup>12</sup> The metric structure postulated here is not essential to the analysis. As far as Theorem 2 is concerned,  $X$  can actually be taken as any Hausdorff topological space, provided that we topologize  $\mathcal{X}$  by means of the Vietoris topology.

**Remark 4.** One way to see that the set of axioms of Theorem 2 is stronger than that of Theorem 1 (when  $|X| < \infty$ ) is to note that if a choice correspondence satisfies the axioms of Theorem 2, then it can be represented by  $(\succ, \succ^*)$ , where  $\succ$  is an *interval order* on  $X$ , and  $\succ^*$  is a completion of  $\succ$  (in the sense of Remark 2), but not conversely.<sup>13</sup> Consequently, a choice model that conforms with the one given in Theorem 2 can always be written in a form that conforms with the one given in Theorem 1 (although there is no obvious formula for obtaining the tuple  $(\mathbf{u}, f)$  from the tuple  $(U, \varphi)$ ). Easy examples would show that the converse is not true in general.<sup>14</sup>

**Remark 5.** It is worth noting that, when  $X$  is a connected metric space, then the knowledge of the utility function  $U$  determines the psychological switching cost function  $\varphi$  essentially uniquely. More precisely,  $(U, \varphi)$  and  $(U, \phi)$  represent a given choice correspondence  $c$  on  $\mathcal{C}(X)$  as in Theorem 2 if, and only if,  $\varphi|_{X_c} = \phi|_{X_c}$ , where  $X_c := \{x \in X : x \notin c(S, x) \text{ for some } S \in \mathcal{X} \text{ with } x \in S\}$ .<sup>15</sup>

**Remark 6.** In the case where  $X$  has additional structure, one can give sharper characterizations than the one given in Theorem 2. For instance, if  $X$  is a compact subset of a Euclidean space  $\mathbb{R}^d$  (interpreted perhaps as a commodity space), and if we further postulate the natural rationality property that  $c(S, \diamond) \subseteq \{y \in S : x > y \text{ for no } x \in S\}$ , then we guarantee that  $U$  found in Theorem 2 is *strictly increasing* in addition to being continuous. (This follows from the argument sketched for the proof of Theorem 2 in Section 5, and Theorem 1 of Ok and Zhou [23].) If we further assume that the decision maker has a preference for compromises in convex choice problems without a status quo, and formalize this by requiring that  $|c(S, \diamond)| \neq 2$  for any convex  $S$  in  $\mathcal{X}$ , then  $U$  turns out to be a continuous, strictly increasing and *quasiconcave* function. (This follows from the previous observation and Lemma 3 of [23].)<sup>16</sup>

<sup>13</sup> A binary relation  $R$  on a nonempty set  $A$  is an *interval order* if it is reflexive, antisymmetric, and  $xRa$  and  $yRb$  imply either  $xRb$  or  $yRa$ , for all  $x, y, a, b \in A$ . Real functional representations of interval orders are studied extensively within order theory (cf. Chapter 7 of [9]). These theorems, however, make use of algebraic separability conditions which do not sit well with the revealed preference approach. The proof of Theorem 2 is, in fact, not based on a standard interval order representation theorem.

<sup>14</sup> For instance, let  $X := \{x, y, z\}$ , and consider the basic choice correspondence defined as in Theorem 1 with  $\mathbf{u}(x) := (2, 2)$ ,  $\mathbf{u}(y) := (3, 3)$ ,  $\mathbf{u}(z) := (1, 10)$ , and with  $f : \mathbf{u}(X) \rightarrow \mathbb{R}$  defined by  $f(\mathbf{a}) := a_1 a_2$ . (Recall Fig. 3.) This correspondence violates part (ii) of Axiom SQB\*, and hence does not conform with the model given in Theorem 2.

<sup>15</sup> *Proof.* Suppose that there is an  $x \in X_c$  such that  $\varphi(x) > \phi(x)$ . Then there exists a  $y \in X$  with  $\{y\} = c(\{x, y\}, x)$ , and hence  $U(y) > U(x) + \varphi(x) > U(x) + \phi(x) > U(x)$ . Since the continuous image of a connected set is connected, we must have  $[U(x), U(y)] \subseteq U(X)$ . Thus there exists a  $z \in X \setminus \{x\}$  such that  $U(x) + \varphi(x) > U(z) > U(x) + \phi(x)$ . But then  $\{x\} = c(\{x, z\}, z)$  and  $\{z\} = c(\{x, z\}, x)$  contradicting the hypothesis that  $(U, \varphi)$  and  $(U, \phi)$  represent the same choice correspondence. Reversing the roles  $\varphi$  and  $\phi$  in this argument completes the proof.

<sup>16</sup> If we assume that the choice correspondences under consideration are all defined only for *convex* problems, however, substantial changes in the theorems would be needed, for then the property  $\alpha$  loses much of its strength. In this case, a less appealing theory would obtain, where the properties  $\alpha$  and  $\beta$  are replaced with the strong axiom of revealed preference as analyzed by Peters and Wakker [24].



#### 2.4. Monotonicity of the endowment effect

Theorem 2 does not give any structure for the psychological switching cost function  $\varphi$  other than its strict positivity. Consequently, it does not answer the following question: Does the status quo bias increase or decrease with the value of the initial endowment? To make things precise, let us consider the following situation:  $y \in c(\{x, y\}, x)$  and  $x \in c(\{x, z\}, \diamond)$ . In words, the agent in question views  $y$  more valuable than  $x$  even if  $x$  is the status quo, while she values  $x$  (weakly) higher than  $z$  (free of any status quo bias). What would this agent choose from  $\{y, z\}$  if the status quo was  $z$ ?

According to the choice model of Theorem 2, the agent may choose either of the alternatives; both  $\{y\} = c(\{y, z\}, z)$  and  $\{z\} = c(\{y, z\}, z)$  are consistent with the axioms we have considered so far. However, the latter situation is possible only if the psychological switching cost from  $z$  is higher than that from  $x$ , even though  $x$  is deemed more valuable than  $z$  by the decision maker. Therefore, if one wishes to model the endowment effect as monotonically increasing in the valuation of the alternatives, then she would wish to see instead that  $\{y\} = c(\{y, z\}, z)$ . While its empirical plausibility is not self-evident, this sort of a requirement would give rise to a more refined model of choice. In particular, as we shall show formally in Section 3.1, it would entail a model which envisages that an agent would charge a higher price for the alternative that he values more in the absence of any endowment effect.

This discussion prompts the following hypothesis:

**Axiom MEE** (Monotonicity of the endowment effect). For any  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ , if  $\{y\} = c(S, x)$  and  $\{x\} = c(T, \diamond)$  for some  $T \in \mathcal{X}$ , then  $y \in c(S \cup \{z\}, z)$  for all  $z \in T$ .

In words, Axiom MEE says simply that if an agent prefers to move away from the status quo  $x$  in favor of an alternative  $y$ , he would also do so if her initial endowment was less valuable than  $x$ . The final result of this section shows the implication of this property for the choice model we have developed thus far.

**Theorem 3.** *Let  $X$  be a compact and connected metric space. A choice correspondence  $c$  on  $\mathcal{C}(X)$  satisfies properties  $\alpha$  and  $\beta$ , and Axioms  $SQI^*$ ,  $SQB^*$ ,  $UHC$ , and  $MEE$  if, and only if, there exist a continuous map  $U : X \rightarrow \mathbb{R}$  and a function  $\varphi : X \rightarrow \mathbb{R}_{++}$  such that*

- (i) (3) and (4) hold for all  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ ;
- (ii)  $U$  and  $U + \varphi$  are comonotonic.<sup>17</sup>

The interpretation of this result is identical to that of Theorem 2, except that the choice model of Theorem 3 conditions the structure of the endowment effect more sharply. In particular, this model says that if the agent finds  $x$  more valuable than  $y$  (i.e.  $U(x) \geq U(y)$ ), then her status quo bias when  $x$  is the status quo is larger than that when  $y$  is the status quo (i.e.  $U(x) + \varphi(x) \geq U(y) + \varphi(y)$ ). Once again, we note that we do not see a strong reason why this is a normatively more compelling theory; the final arbiter of the usefulness of Theorem 3 is the experimental testing of Axiom MEE.<sup>18</sup>

<sup>17</sup> That is,  $U(x) \geq U(y)$  implies  $U(x) + \varphi(x) \geq U(y) + \varphi(y)$  for any  $x, y \in X$ .

<sup>18</sup> One test of the model is through the classical experiment of the preference reversal phenomenon [10]. As we shall show in Section 3.2, this test would refute the model of Theorem 3, but not of Theorem 2.

2.5. The endowment effect under risk

In this section, we extend the main result of Section 2.3 to the case of choice sets that consist of lotteries. Our development parallels the classical von Neumann–Morgenstern expected utility theory. We designate an arbitrary compact metric space  $Z$  (that contains at least two elements) as the set of all (certain) prizes, and let  $\mathbf{C}(Z)$  denote the Banach space of all continuous real maps on  $X$  (under the sup-norm). By a *lottery*, we mean a Borel probability measure on  $Z$ , and denote the set of all lotteries by  $\mathcal{P}(Z)$ . Of course,  $Z$  is naturally embedded in  $\mathcal{P}(Z)$  by identifying any certain prize  $a \in Z$  with the (Dirac) probability measure that puts full mass on the set  $\{a\}$ . This identification allows us to write  $a \in \mathcal{P}(Z)$  with a slight abuse of notation.

For any  $p \in \mathcal{P}(Z)$  and any continuous real function  $f$  on  $Z$ , we denote the Lebesgue integral  $\int_Z f dp$  by  $\mathbb{E}_p(f)$ . We endow  $\mathcal{P}(Z)$  with some metric that induces the topology of weak convergence, so for any sequence  $(p_n)$  in  $\mathcal{P}(Z)$ ,  $p_n \rightarrow p$  means that  $p_n$  converges to  $p$  weakly, that is,  $\mathbb{E}_{p_n}(f) \rightarrow \mathbb{E}_p(f)$  for all  $f \in \mathbf{C}(Z)$ .<sup>19</sup> It is well-known that this renders  $\mathcal{P}(Z)$  a compact metric space.

The following axiom is a straightforward reflection of the classical independence axiom of expected utility theory, but note that it is stated only in terms of the choice problems without a status quo.

**Axiom I** (Independence). For any  $(S, \diamond) \in \mathcal{C}(\mathcal{P}(Z))$ ,  $\lambda \in (0, 1)$ , and  $r \in \mathcal{P}(Z)$ ,

$$p \in c(S, \diamond) \text{ implies } \lambda p + (1 - \lambda)r \in c(\lambda S + (1 - \lambda)r, \diamond),$$

where  $\lambda S + (1 - \lambda)r = \{\lambda q + (1 - \lambda)r : q \in S\}$ .

It turns out that adding this property to the set of assumptions employed in Theorem 2 yields easily a characterization of choice correspondences that exhibit the endowment effect in risky choice situations as well.

**Theorem 4.** Let  $Z$  be a compact metric space. A choice correspondence  $c$  on  $\mathcal{C}(\mathcal{P}(Z))$  satisfies properties  $\alpha$  and  $\beta$ , and Axioms  $SQI^*$ ,  $SQB^*$ ,  $UHC$ , and  $I$  if, and only if, there exist a continuous function  $u : Z \rightarrow \mathbb{R}$  and a function  $\varphi : \mathcal{P}(Z) \rightarrow \mathbb{R}_{++}$  such that

$$c(S, \diamond) = \arg \max_{p \in S} \mathbb{E}_p(u)$$

for any nonempty closed subset  $S$  of  $\mathcal{P}(Z)$ , and

$$c(S, q) = \begin{cases} \{q\} & \text{if } \mathbb{E}_q(u) + \varphi(q) > \mathbb{E}_p(u) \text{ for all } p \in S \\ \arg \max_{p \in S} \mathbb{E}_p(u) & \text{otherwise} \end{cases}$$

for all  $(S, q) \in \mathcal{C}_{sq}(\mathcal{P}(Z))$ .

The interpretation of this result is analogous to that of Theorem 2, so we do not discuss it further here. We should note, however, that adding Axiom MEE to Theorem 4 would ensure

<sup>19</sup> There are various distance functions that may be used for this purpose (e.g the Prohorov metric). For the present purposes, however, it is inconsequential which of these metrics is chosen.

in this result the comonotonicity of the maps  $p \mapsto \mathbb{E}_p(u)$  and  $p \mapsto \mathbb{E}_p(u) + \varphi$ . (Recall Theorem 3.)

In passing, we should note that an alternative model of risky choice with status quo bias is proposed recently by Sagi [26]. The choice correspondence  $C$  entailed by Sagi's model is defined on  $C_{\text{sq}}(\mathcal{P}(Z))$  as

$$C(S, q) := \arg \max_{p \in S} \left( \inf_{u \in \mathcal{U}} (\mathbb{E}_p(u) - \mathbb{E}_q(u)) \right),$$

where  $\mathcal{U}$  is a nonempty, closed and convex subset of  $\mathbf{C}(Z)$ . This is certainly an interesting choice model—we will refer to it as *Sagi's model*—the interpretation of which parallels to that of Theorem 1 (as a model of multi-criteria decision-making). A few remarks comparing this model to that obtained in Theorem 4 are in order.

Perhaps, the most important difference between the two models is that Sagi's model is defined *only* on  $C_{\text{sq}}(\mathcal{P}(Z))$ , that is, it does not apply to choice problems *without* status quo. This contrasts with the model advanced in Theorem 4, which builds a tight connection between choice situations with and without status quo outcomes (via the utility function  $u$ ). Secondly, it appears to us that Sagi's model is more suitable for modeling the phenomenon of status quo bias than the endowment effect. The latter phenomenon is arguably better modeled by means of the notion of a psychological switching cost which is captured in Theorem 4 by means of the function  $\varphi$ . This, for instance, allows one to perform easy comparative statics exercises such as in Section 2.4, and as will be seen in the next section, this formulation is particularly easy to adopt in applications. Finally, we note that there are differences in the implications of the two models as well. In particular, Sagi's model does not satisfy Axiom SQB, for it allows a status quo outcome to be chosen along with other outcomes. In turn, the model of Theorem 4 is less continuous than that of Sagi—it is not continuous with respect to the status quo point (and thus fails to satisfy Axiom 3 of [26]).

### 3. Applications

#### 3.1. The overpricing phenomenon

A major channel through which the endowment effect was discovered in experimental environments is the discrepancy found between buying and selling prices of commodities by the individuals, which is often referred to as the gap between *willingness to pay* and *willingness to buy* of a person. (See [15,6, pp. 665–670] for detailed surveys.) We will show below that the model of the endowment effect derived in the previous section would indeed predict this sort of a gap.

Since we wish to talk about buying and selling prices of commodities, we need to introduce slightly more structure to the present setup. Consequently, we let  $Y$  stand for a compact metric space, and in order to interpret this space as the set of all *non-monetary* choice alternatives (such as physical goods and/or lotteries), we assume  $Y \cap \mathbb{R} = \emptyset$ . On the other hand, take any  $M > 0$  and let  $[0, M]$  denote the set of all potential prices. The outcome space of the model is then obtained by putting  $[0, M]$  and  $Y$  together. Letting  $\uplus$  denote the disjoint-union operation, then, we let  $X := [0, M] \uplus Y$ , and make this set a compact metric space

in a natural way.<sup>20</sup> The choice correspondences in this context are thus defined on  $\mathcal{C}(X)$ . A particularly interesting subclass of these correspondences is the one that consists of the *monotonic* ones, that is, those  $c$  that satisfy

$$c(\{a, b\}, \diamond) = \{a\} \quad \text{whenever} \quad M \geq a > b \geq 0 \tag{5}$$

and

$$M \in c(\{M, y\}, y) \quad \text{and} \quad y \in c(\{0, y\}, 0) \quad \text{for all } y \in Y. \tag{6}$$

Property (5) is an obviously appealing condition that requires that more money is preferred to less in the absence of a status quo. On the other hand, the first part of (6) says that there is a price for every feasible commodity at which that commodity would be sold, and the second part says that all goods are “good”—any member of  $Y$  is better than holding \$0.

Let us consider an individual whose choice correspondence  $c$  on  $\mathcal{C}(X)$  is monotonic. We define the map  $S_c : Y \rightarrow [0, M]$  by

$$S_c(y) := \inf\{a \in [0, M] : c(\{y, a\}, y) \ni a\}$$

and the map  $B_c : Y \rightarrow [0, M]$  by

$$B_c(y) := \sup\{a \in [0, M] : c(\{y, a\}, a) \ni y\}.$$

(Observe that these functions are well-defined in view of (6).) In words,  $S_c(y)$  is the *minimum selling price* (or *willingness to sell*) for the nonmonetary alternative  $y$  according to the individual with the choice correspondence  $c$ . Similarly,  $B_c(y)$  is interpreted as the *maximum buying price* (or *willingness to pay*) for  $y$  in the eyes of this agent.

The so-called price gap between buying and selling prices translates, therefore, into the comparison of the maps  $B_c$  and  $S_c$ . The following result thus establishes that the present model predicts precisely the overpricing phenomenon found repeatedly in various experimental settings. It also shows that, provided that the agent abides by Axiom MEE, then her willingness to sell is monotonically decreasing in the value of the object.<sup>21</sup>

**Proposition 1** (The overpricing phenomenon). *Let  $M > 0$ ,  $Y$  a compact metric space and  $X := [0, M] \cup Y$ . If  $c$  is a monotonic choice correspondence on  $\mathcal{C}(X)$  that satisfies properties  $\alpha$  and  $\beta$ , and Axioms SQI\*, SQB\* and UHC, then*

$$S_c(y) > B_c(y) \quad \text{for all } y \in Y.$$

Moreover, if  $c$  also satisfies Axiom MEE, then

$$x \in c(\{x, y\}, \diamond) \quad \text{implies} \quad S_c(x) \geq S_c(y).$$

<sup>20</sup> Since  $[0, M] \cap Y = \emptyset$ , there is an obvious way of doing this. Let  $d$  stand for the metric of  $Y$  and let  $\theta > \max\{M, \text{diam}(Y)\}$ . We define  $D : X^2 \rightarrow \mathbb{R}_+$  by  $D(y, y') := d(y, y')$ ,  $D(a, a') := |a - a'|$ , and  $D(y, a) := D(a, y) := \theta$  for all  $y, y' \in Y$  and  $a, a' \in [0, M]$ . It is easy to check that  $D$  is a metric on  $X$  which makes  $X$  compact.

<sup>21</sup> Here by the “value” of the object we mean the utility of the object as obtained in the absence of status quo bias and/or endowment effect.

While our axiomatic model entails that the endowment effect is strictly positive (albeit possibly infinitesimal) even for monetary outcomes, one may wish to use the choice model found in Theorem 2 coupled with the assumption that  $\varphi(a) = 0$  for all  $a \in [0, M]$  (the psychological switching cost of money is nil) and  $\varphi(y) > 0$  for all  $y \in Y$ . We note that Proposition 1 applies also to this marginally more general model; in fact, the proof we have given for this result above does not use the strict positivity of  $\varphi$  on  $[0, M]$ .

### 3.2. The preference reversal phenomenon as an endowment effect

Among the many experimental observations that refute the basic premises of expected utility theory, a particularly striking one is the so-called *preference reversal phenomenon*. First noted by Slovic and Lichtenstein [31], this phenomenon has caused a great deal of theoretical and experimental debate among decision theorists, especially after the seminal contribution of Grether and Plott [10]. The basic experiment behind this phenomenon asks the decision maker to choose between two lotteries, one offering a high probability of winning a small amount of money, and the other offering a low probability of winning a large payoff. For concreteness, let us couch the discussion by means of a specific example: Let  $h$  stand for the lottery that pays \$10 with probability  $\frac{8}{9}$  and nothing otherwise, and let  $\ell$  stand for the lottery that pays \$85 with probability  $\frac{1}{9}$  and nothing otherwise. When confronted with such a choice problem, most individuals were observed to prefer  $h$  over  $\ell$  in the experiments. The subjects were then asked to state the minimum price at which they would be willing to sell  $h$  and  $\ell$  (had they owned these lotteries), and surprisingly, about half of them were found to charge a strictly higher price for  $\ell$  than for  $h$ ; hence the term *preference reversal (PR) phenomenon*.

While there have been a large number of experimental and theoretical studies concerning the explanations for this phenomenon,<sup>22</sup> to our knowledge no one has suggested that this phenomenon is but a particular instant of the endowment effect. This is quite surprising because the structure of the PR phenomenon is very reminiscent of the overpricing phenomenon (Section 3.1) which is often explained by this effect. At any rate, it may be worthwhile to note that the present choice model (as envisaged by Theorem 4) provides an immediate test of whether or not it is mainly the endowment effect that underlies the PR phenomenon.

Let  $Z := [0, z]$  for some  $z > 85$ , and assume that the choice correspondence  $c$  on  $\mathcal{C}(\mathcal{P}(Z))$  satisfies the axioms of Theorem 4 along with the following innocuous monotonicity property:  $c(\{\delta_a, \delta_b\}, \diamond) = \{\delta_a\}$  for all  $a, b \in Z$  with  $a > b$ . Just as in Section 3.1, we define the *minimum selling price* induced by  $c$  as the map  $S_c : \mathcal{P}(Z) \rightarrow Z$  given by

$$S_c(p) := \inf\{a \in Z : c(\{p, \delta_a\}, p) \ni \delta_a\}.$$

This formulation recognizes the fact that a seller prices a lottery *when she is in possession of it*, a potentially important aspect of the PR experiments. The data of the PR phenomenon is then the following:

$$\{h\} = c(\{h, \ell\}, \diamond) \quad \text{and} \quad S_c(h) < S_c(\ell).$$

<sup>22</sup> See, *inter alia*, [7,12,16,19,28,33].

In the language of Theorem 4, these statements are tantamount to the following two inequalities:

$$\mathbb{E}_h(u) > \mathbb{E}_\ell(u) \quad \text{and} \quad \mathbb{E}_\ell(u) + \varphi(\ell) > \mathbb{E}_h(u) + \varphi(h), \quad (7)$$

with  $u$  and  $\varphi$  as found in that theorem. Since one can easily choose  $u$  and  $\varphi$  in a way to satisfy these inequalities, we find that the PR phenomenon is consistent with the choice model of Theorem 4. Perhaps more importantly, the inequalities of (7) provide us with an immediate experimental test of the model (which will be undertaken in future research): Ask the subjects to choose from  $\{h, \ell\}$  when they are endowed with  $\ell$ . On the basis of (7), our model predicts that a substantial fraction of the agents with  $\{h\} = c(\{h, \ell\}, \diamond)$  will in fact change their choices to  $\ell$  when  $\ell$  is the status quo of the problem, i.e.,  $\{\ell\} = c(\{h, \ell\}, \ell)$ , for  $\mathbb{E}_\ell(u) + \varphi(\ell) > \mathbb{E}_h(u) + \varphi(h) > \mathbb{E}_h(u)$ . If the subjects keep choosing  $h$  from  $\{h, \ell\}$  even when endowed with  $\ell$ , then this would, in turn, refute the model envisaged by Theorem 4.

#### 4. Conclusion

We have sketched here a revealed preference theory that modifies the standard static choice theory by introducing the possibility that the decision maker may have an initial reference point which can be interpreted as a default option, current choice and/or an endowment. This expands the classical setup, and leads to some intuitive representations of choice behavior. In particular, the representations we provide here allow for phenomena like the status quo bias and the endowment effect, and notably, draw a connection between how problems with and without a status quo are settled.

There are, of course, several directions that need to be explored. For one thing, like other related papers mentioned in Section 1, our analysis applies to only *static* choice problems. While the status quo bias phenomenon is presumably more pressing in static problems, it is not known if and how dynamic choice procedures would induce static choice behavior that would exhibit a status quo bias. This sort of an analysis would provide a deeper model in which the status quo bias is endogenized. While the present work might provide useful in modeling such a phenomenon in the stage problems of a dynamic choice model, its exclusively static nature is of course a serious shortcoming.<sup>23</sup> Secondly, on the applied front, it will be interesting to see if and how the choice models introduced here might affect the conventional conclusions of the standard search and buyer–seller models, where a status quo bias and/or the endowment effect are likely to play important roles.

#### 5. Proofs

**Proof of Lemma 1.** Take any choice correspondence  $c$  that satisfies all five of the postulated properties, and define the binary relation  $\succsim$  on  $X$  by

$$y \succsim x \quad \text{if and only if} \quad y \in c(\{x, y\}, x).$$

<sup>23</sup> The only paper in this regard that we know is Vega-Redondo [34] who provides a dynamic decision model (with learning) the limit behavior of which covers decisions that depend on the status quo.

Since  $c$  is nonempty-valued,  $\succsim$  is reflexive. By SQB,  $y \in c(\{x, y\}, x)$  and  $x \in c(\{x, y\}, y)$  can hold simultaneously if and only if  $y = x$ . Thus  $\succsim$  is antisymmetric. To see that  $\succsim$  is also transitive, take any  $y, x, z \in X$  with  $y \succsim x \succsim z$ , that is,

$$y \in c(\{x, y\}, x) \quad \text{and} \quad x \in c(\{x, z\}, z).$$

We assume that  $y, x$  and  $z$  are distinct outcomes, otherwise the claim is trivial. The first expression above and SQB then jointly imply that  $x \notin c(\{x, y\}, x)$  so, by property  $\alpha$ ,  $x \notin c(\{x, y, z\}, x)$ . Then, by SQB,  $x \notin c(\{x, y, z\}, z)$ . But property  $\alpha$  and SQB imply that  $z \in c(\{x, y, z\}, z)$  is possible only if  $\{z\} = c(\{x, z\}, z)$  which contradicts  $x \succsim z$ . Thus we have  $y \in c(\{x, y, z\}, z)$ , and by property  $\alpha$ , it follows that  $y \in c(\{y, z\}, z)$ , that is,  $y \succsim z$ . Consequently, we conclude that  $\succsim$  is a partial order on  $X$ .  $\square$

**Claim 1.** For any choice problem with a status quo  $(S, x) \in \mathcal{C}_{sq}(X)$ ,

$$c(S, x) \subseteq \begin{cases} \{x\} & \text{if } U_{\succ}(x) \cap S = \emptyset, \\ U_{\succ}(x) & \text{otherwise.} \end{cases}$$

**Proof.** Take any  $(S, x) \in \mathcal{C}_{sq}(X)$ , and assume that  $U_{\succ}(x) \cap S = \emptyset$ . If  $x \neq y \in c(S, x)$ , then, by property  $\alpha$ ,  $y \in c(\{x, y\}, x)$ , which yields the contradiction  $y \in U_{\succ}(x) \cap S$ . Thus, in this case, we have  $c(S, x) = \{x\}$ . Assume next that  $U_{\succ}(x) \cap S \neq \emptyset$ , and pick any  $y \in c(S, x)$ . If  $y = x$ , then, by property  $\alpha$  and SQB,  $\{x\} = c(\{x, z\}, x)$  for all  $z \in S$ , and this yields  $U_{\succ}(x) \cap S = \emptyset$ , a contradiction. So  $y \neq x$ . Then, by property  $\alpha$ ,  $y \in c(\{y, x\}, x)$  so that  $y \in U_{\succ}(x)$ .  $\square$

**Claim 2.** For any choice problem with a status quo  $(S, x) \in \mathcal{C}_{sq}(X)$ , if  $U_{\succ}(x) \cap S \neq \emptyset$ , then

$$c(S, x) = c(U_{\succ}(x) \cap S, \diamond).$$

**Proof.** Take any  $(S, x) \in \mathcal{C}_{sq}(X)$ , and assume that  $U_{\succ}(x) \cap S \neq \emptyset$ . Let  $y \in c(S, x)$ . By Claim 1,  $y \in U_{\succ}(x) \cap S$ , so by property  $\alpha$ ,  $y \in c((U_{\succ}(x) \cap S) \cup \{x\}, x)$ . But by Claim 1,  $x \notin c(T, x)$  for any nonempty  $T \subseteq (U_{\succ}(x) \cap S) \cup \{x\}$  with  $T \neq \{x\}$ . Thus SQI gives  $y \in c((U_{\succ}(x) \cap S) \cup \{x\}, \diamond)$ , and so by property  $\alpha$ ,  $y \in c(U_{\succ}(x) \cap S, \diamond)$ .

To prove the converse containment, let  $y \in c(U_{\succ}(x) \cap S, \diamond)$ , and notice that if  $z \in c((U_{\succ}(x) \cap S) \cup \{x\}, \diamond)$ , then by property  $\alpha$ , we must have  $z \in c(U_{\succ}(x) \cap S, \diamond)$ , and hence, by property  $\beta$ , we obtain  $y \in c((U_{\succ}(x) \cap S) \cup \{x\}, \diamond)$ . Moreover, since  $y \in U_{\succ}(x)$ , we have  $\{y\} = c(\{x, y\}, x)$  by SQB, and therefore, we may apply Axiom D to conclude that  $y \in c((U_{\succ}(x) \cap S) \cup \{x\}, x)$ . Now take any  $z \in c(S, x)$ , and apply property  $\alpha$  to get  $z \in c((U_{\succ}(x) \cap S) \cup \{x\}, x)$ . It then follows from property  $\beta$  that  $y \in c(S, x)$ .  $\square$

Given that  $c$  satisfies the properties  $\alpha$  and  $\beta$ , by a standard result of choice theory, there must exist a complete preorder  $\succsim^*$  such that  $c(\cdot, \diamond) = M(\cdot, \succsim^*)$ . To complete the proof,



then, it is enough to show that  $\succ$  is contained in  $\succ^*$ . But for any distinct  $x, y \in X$  with  $y \in c(\{x, y\}, x)$ , SQB implies that  $x \in c(\{x, y\}, \diamond)$  cannot hold, so it follows that  $y \succ^* x$ .

**Proof of Theorem 1.** The “if” part of the claim is easily verified. To prove the “only if” part, take any choice correspondence  $c$  that satisfies all five of the postulated properties, and consider the binary relations  $\succcurlyeq$  and  $\succcurlyeq^*$  found in Lemma 1.

**Claim.** There exists a positive integer  $n$  and an injection  $\mathbf{u} : X \rightarrow \mathbb{R}^n$  such that

$$y \succcurlyeq x \text{ if and only if } \mathbf{u}(y) \geq \mathbf{u}(x) \text{ for all } x, y \in X.$$

**Proof.** Let  $e(\succcurlyeq)$  stand for the set of all linear orders such that  $\succ \subseteq R$ . Given that  $X$  is finite, it is obvious that  $e(\succcurlyeq)$  is a nonempty finite set. Let us enumerate this set as  $\{R_1, \dots, R_n\}$ . It is readily checked that  $\succcurlyeq = \bigcap_{i=1}^n R_i$ . Moreover, since  $X$  is finite, there exists a map  $u_i : X \rightarrow \mathbb{R}$  such that  $y R_i x$  iff  $u_i(y) \geq u_i(x)$  for all  $x, y \in X$ . Thus, defining  $\mathbf{u}(x) := (u_1(x), \dots, u_n(x))$ , we find that  $y \succcurlyeq x$  iff  $\mathbf{u}(y) \geq \mathbf{u}(x)$  for all  $x, y \in X$ . Since  $\succcurlyeq$  is antisymmetric,  $\mathbf{u}$  must be an injection.  $\square$

Now observe that  $c(\cdot, \diamond) : \mathcal{X} \rightarrow \mathcal{X}$  is a standard choice correspondence that satisfies the classical properties  $\alpha$  and  $\beta$ . Given that  $X$  is finite, it follows that there exists a map  $v : X \rightarrow \mathbb{R}$  such that

$$c(S, \diamond) = \arg \max_{z \in S} v(z) \text{ for all } S \in \mathcal{X}. \tag{8}$$

Consequently, by Lemma 1, we may conclude that

$$U_{\mathbf{u}}(S, x) \neq \emptyset \text{ implies } c(S, x) = \arg \max_{z \in U_{\mathbf{u}}(S, x)} v(z) \tag{9}$$

for any  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ . To complete the proof, we define  $f : \mathbf{u}(X) \rightarrow \mathbb{R}$  by  $f(\mathbf{a}) := v(\mathbf{u}^{-1}(\mathbf{a}))$ . Since  $\mathbf{u}$  is injective,  $f$  is well-defined. Moreover, if  $\mathbf{u}(y) = \mathbf{a} > \mathbf{b} = \mathbf{u}(x)$  for some  $x, y \in X$ , then  $\{y\} = c(\{x, y\}, x)$  by Lemma 1 and the claim proved above. But then  $x \in c(\{x, y\}, \diamond)$  cannot hold by SQB, and hence (8) yields  $f(\mathbf{a}) = v(y) > v(x) = f(\mathbf{b})$ . We conclude that  $f$  is strictly increasing. Finally, observe that, by Lemma 1 and the claim proved above, we have

$$U_{\mathbf{u}}(S, x) = \emptyset \text{ implies } c(S, x) = \{x\} \tag{10}$$

for any  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ . Combining (10), (9), and (8), and noting that  $v = f \circ \mathbf{u}$ , completes the proof.  $\square$

**Proof of Theorem 2.** The only nontrivial statements in the “if” part of the claim concern the verification of UHC and part (iii) of SQB\*. To establish these properties of a choice correspondence  $c$  which is of the form given in Theorem 2, fix any  $U$  and  $\varphi$  that satisfy the requirements of this result, and define the correspondence  $\Upsilon : \mathcal{X} \rightrightarrows \mathcal{X}$  by

$$\Upsilon(S) := \arg \max_{z \in S} U(z).$$

Since  $U$  is continuous, and  $\mathcal{X}$  is a compact metric space (under the Hausdorff metric), Berge’s Maximum Theorem implies that  $\Upsilon$  has closed graph. Now fix any  $x \in X \cup \{\diamond\}$  and  $S, S_m \in \mathcal{X}, m = 1, 2, \dots$  with  $S_m \rightarrow S$ . If  $x \neq y_m \in c(S_m, x)$  for each  $m$ , and  $y_m \rightarrow y$ , then the structure of  $c$  entails that  $y_m \in \Upsilon(S_m)$ . Thus, the closed graph property of  $\Upsilon$  yields  $y \in \Upsilon(S) = c(S, \diamond)$ . Thus if  $x = \diamond$ , we are done. If, on the other hand,  $x \neq \diamond$ , then we have  $U(y_m) \geq U(x) + \varphi(x)$  for each  $m$  by (4), so by continuity of  $U$ , we have  $U(y) \geq U(x) + \varphi(x)$ . But then  $x \notin c(S, x)$ , so we have  $y \in \Upsilon(S) = c(S, \diamond) = c(S, x)$ , thereby proving that  $c$  satisfies UHC. To establish that  $c$  also satisfies the part (iii) of SQB\*, fix any  $x \in X$ , and note that, by continuity of  $U$ , there must exist a  $\delta > 0$  such that

$$|U(x) - U(y)| < \varphi(x) \quad \text{for all } y \in N_\delta(x).$$

But then letting  $\varepsilon := \delta/2$ , we find

$$|U(x) - U(y)| < \varphi(x) \quad \text{for all } y \in clN_\varepsilon(x),$$

so it follows from (4) that  $x \in c(cl(N_\varepsilon(x)), x)$ .

To prove the “only if” part of the theorem, take any choice correspondence  $c$  that satisfies all five of the postulated properties, and define the binary relation  $\succsim$  on  $X$  by

$$y \succsim x \quad \text{if and only if} \quad y \in c(\{x, y\}, \diamond).$$

It is easily verified that  $\succsim$  is complete preorder on  $X$  by using properties  $\alpha$  and  $\beta$ . It is easy to see that UHC ensures the continuity of this preorder. Indeed, for any  $x \in X$ , and any sequence  $(y_m)$  in  $X$  such that  $y_m \succsim x$  and  $y_m \rightarrow y$  for some  $y \in X$ , we have then  $\{x, y_m\} \rightarrow \{x, y\}$ , so it follows from UHC that  $y \in c(\{x, y\}, \diamond)$ , that is,  $y \succsim x$ . Since  $x$  is arbitrary in  $X$ , this proves the upper semicontinuity of  $\succsim$ . Lower semicontinuity of  $\succsim$  is verified similarly.

Given that  $\succsim$  is upper semicontinuous, and any  $S$  in  $\mathcal{X}$  is compact, it follows that

$$\{y \in S : y \succsim x \text{ for all } x \in S\} \neq \emptyset \quad \text{for all } S \in \mathcal{X}.$$

Moreover, by using properties  $\alpha$  and  $\beta$ , one may easily verify that

$$c(S, \diamond) = \{y \in S : y \succsim x \text{ for all } x \in S\} \quad \text{for all } S \in \mathcal{X}.$$

But, given that  $\succsim$  is continuous, by the Debreu utility representation theorem, there exists a continuous real function  $U$  on  $X$  (which is compact, hence separable) such that  $y \succsim x$  iff  $U(y) \geq U(x)$  for all  $x, y \in X$ . Combining this fact with the previous observation, we obtain

$$c(S, \diamond) = \arg \max_{z \in S} U(z) \quad \text{for all } S \in \mathcal{X}. \tag{11}$$

Now define

$$I(x) := \{y \in X \setminus \{x\} : y \in c(\{x, y\}, x)\}, \quad x \in X$$

and

$$X_c := \{x \in X : x \notin c(S, x) \text{ for some } S \in \mathcal{X} \text{ with } x \in S\}.$$

**Claim 1.**  $I(x)$  is a nonempty compact subset of  $X$  for any  $x \in X_c$ .

**Proof.** Fix any  $x \in X_c$ . By definition, there exists an  $S \in \mathcal{X}$  with  $x \in S$  and  $x \notin c(S, x)$ . Let  $z \in c(S, x)$ . Then  $z \neq x$ , and  $z \in c(\{x, z\}, x)$  by property  $\alpha$ , that is,  $z \in I(x)$ , establishing that  $I(x) \neq \emptyset$ .

Given that  $X$  is compact, it is then enough to show that  $I(x)$  is a closed set. To this end, take any sequence  $(y_m)$  in  $I(x)$  with  $y_m \rightarrow y$  for some  $y \in X$ . Clearly,  $\{x, y_m\} \rightarrow \{x, y\}$  and  $x \neq y_m \in c(\{x, y_m\}, x)$  for each  $m$ , so by UHC,  $y \in c(\{x, y\}, x)$ . Moreover, by SQB\*, there exists an  $\varepsilon > 0$  such that  $\{x\} = c(cI(N_\varepsilon(x)))$ , so if  $y = x$ , then there exists an integer  $M$  such that  $y_m \in N_\varepsilon(x)$  for all  $m \geq M$ . But then, by property  $\alpha$ ,  $\{x\} = c(\{x, y_m\}, x)$  which gives  $x = y_m$  for each  $m \geq M$ , a contradiction. Thus,  $y \neq x$ , that is,  $y \in I(x)$ .  $\square$

Given Claim 1 and the fact that  $U$  is continuous, we may define  $\lambda : X_c \rightarrow \mathbb{R}$  by

$$\lambda(x) := \min_{z \in I(x)} U(z).$$

**Claim 2.**  $\lambda(x) > U(x)$  for all  $x \in X_c$ .

**Proof.** For any  $x \in X_c$  and  $z \in X \setminus \{x\}$ , if  $U(x) \geq U(z)$  holds, then  $x \in c(\{x, z\}, \diamond)$ , so by SQB,  $\{x\} = c(\{x, z\}, x)$ , that is,  $z \notin I(x)$ . Thus,  $U(z) > U(x)$  holds for all  $z \in I(x)$ , and hence the claim.  $\square$

**Claim 3.** For any  $(x, y) \in X_c \times X$ ,  $U(y) \geq \lambda(x)$  implies that  $y \in I(x)$ .

**Proof.** By hypothesis,  $U(y) \geq U(z)$  for some  $z \in I(x)$ . By Claim 2, therefore,  $y \in c(\{x, y, z\}, \diamond)$ . Now suppose that  $y \notin c(\{x, y, z\}, x)$ . Since  $z \neq x$ , SQB and property  $\alpha$  imply that  $\{z\} = c(\{x, y, z\}, x)$ , but this contradicts SQI\*. Thus  $y \in c(\{x, y, z\}, x)$ , and hence  $y \in c(\{x, y\}, x)$  by property  $\alpha$ . But, by Claim 2,  $U(y) \geq \lambda(x) > U(x)$ , so  $y \neq x$ . Thus  $y \in I(x)$ .  $\square$

**Claim 4.** For any  $x \in X_c$  and  $(S, x) \in \mathcal{C}_{sq}(X)$ , if  $U(y) \geq \lambda(x)$  for some  $y \in S$ , then

$$y \in c(S, x) \quad \text{if and only if} \quad U(y) = \max U(S).$$

**Proof.** Take any  $(S, x) \in \mathcal{C}_{sq}(X)$  with  $x \in X_c$  and

$$Y := \{y \in S : U(y) \geq \lambda(x)\} \neq \emptyset.$$

Suppose first that  $x \in c(S, x)$ . Take any  $y \in Y$ , and note that  $y \neq x$  by Claim 2. Then by property  $\alpha$  and SQB,  $y \notin c(\{x, y\}, x)$ , that is,  $y \notin I(x)$ , which contradicts Claim 3. Thus  $x \notin c(S, x)$ , so  $y \in c(S, x)$  implies that  $y \neq x$ . Then, by SQB\*,  $y \in c(S, x)$  holds only if  $y \in c(S, \diamond)$ , that is,  $U(y) = \max U(S)$  by (11). Conversely, if  $y \in \arg \max_{z \in S} U(z)$ , then  $y \in c(S, \diamond)$ . But if  $z \in c(S, x)$ , the previous argument yield that  $z \neq x$ , so  $z \in c(S, \diamond)$  by SQB\*. Then, by SQI\*, we get  $y \in c(S, x)$ .  $\square$

**Claim 5.** For any  $(S, x) \in \mathcal{C}_{sq}(X)$ , if  $\lambda(x) > U(y)$  for all  $y \in S$ , then  $c(S, x) = \{x\}$ .

**Proof.** By definition of  $\lambda$ ,  $\lambda(x) > U(y)$  implies that  $y \notin I(x)$ , so  $\{x\} = c(\{x, y\}, x)$  for all  $y \in S$ . Thus, by property  $\alpha$ , we have  $c(S, x) = \{x\}$ .  $\square$

To complete the proof, we define  $\varphi : X \rightarrow \mathbb{R}_{++}$  as

$$\varphi(x) := \begin{cases} \lambda(x) - U(x) & \text{if } x \in X_c, \\ \max U(X) - \min U(X) + 1 & \text{otherwise.} \end{cases}$$

Take any  $(S, x) \in \mathcal{C}_{sq}(X)$ , and suppose that  $U(x) + \varphi(x) > U(y)$  for all  $y \in S$ . If  $x \in X_c$ , then  $\lambda(x) > U(y)$  for all  $y \in S$ , so  $c(S, x) = \{x\}$  by Claim 5. If  $x \notin X_c$ , then  $x \in c(S, x)$  by definition of  $X_c$ , so by SQB, we have  $c(S, x) = \{x\}$ . Now suppose that  $U(x) + \varphi(x) \leq U(y)$  for some  $y \in S$ . If  $x \notin X_c$ , then  $\max U(X) - \min U(X) + 1 \leq U(y) - U(x)$  for some  $y \in X$ , which is impossible. Thus  $x \in X_c$ , and in this case, we have  $\lambda(x) \leq U(y)$  for some  $y \in S$ , and there follows  $c(S, x) = \arg \max_{z \in S} U(z)$  by Claim 4. The proof of Theorem 2 is now complete.  $\square$

**Proof of Theorem 3.** We only need to talk about the “only if” part. To this end, we define the set  $X_c$  and the maps  $I$ ,  $\lambda$ ,  $U$  and  $\varphi$  exactly as in the proof of Theorem 2, and note that, by Theorem 2, we only need to establish the comonotonicity of  $U$  and  $U + \varphi$ . We will use the following claim for this purpose.

**Claim 1.** For any  $x, y \in X_c$ ,  $U(x) \geq U(y)$  implies  $\lambda(x) \geq \lambda(y)$ .

**Proof.** Take any  $x, y \in X_c$  with  $U(x) \geq U(y)$ , and to derive a contradiction, assume  $\lambda(y) > \lambda(x)$ . Since  $I(y)$  and  $I(x)$  are nonempty compact sets (by Claim 1 of the proof of Theorem 2) and  $U$  is continuous, there exists a  $(z_x, z_y) \in I(x) \times I(y)$  such that  $U(z_x) = \lambda(x)$  and  $U(z_y) = \lambda(y)$ . Given that  $X$  is connected, we may then use the intermediate value theorem to find a  $z \in X$  such that  $\lambda(y) > U(z) > \lambda(x)$ . Now, by Claim 3 of the proof of Theorem 2,  $z \in I(x)$ , that is,  $z \in c(\{x, z\}, x)$ , so we have  $x \notin c(\{x, y, z\}, x)$  by Property  $\alpha$  and SQB. On the other hand, (4) and  $U(x) \geq U(y)$  imply that  $y \notin c(\{x, y, z\}, x)$ , so we must have  $\{z\} = c(\{x, y, z\}, x)$ . But again by (4) we have  $x \in c(\{x, y\}, \diamond)$ , so by Axiom MEE we get  $z \in c(\{x, y, z\}, y)$ . By property  $\alpha$ , this means that  $z \in c(\{y, z\}, y)$ , that is,  $z \in I(y)$ . But this is impossible, for  $U(z) < \lambda(y) = \min_{z \in I(y)} U(z)$ .  $\square$

Now take any  $x, y \in X$  with  $U(x) \geq U(y)$ . Consider first the case where  $x \notin X_c$ . In this case, if  $y \in X_c$ , then

$$\begin{aligned} U(x) + \varphi(x) &= \max U(X) + (U(x) - \min U(X)) + 1 \\ &> \max U(X) \\ &\geq \lambda(y) \\ &= U(y) + \varphi(y) \end{aligned}$$

and if  $y \notin X_c$ , then

$$\begin{aligned} U(x) + \varphi(x) &= U(x) + \max U(X) - \min U(X) + 1 \\ &\geq U(y) + \max U(X) - \min U(X) + 1 \\ &= U(y) + \varphi(y) \end{aligned}$$

as we sought. Now let  $x \in X_c$ . This implies that  $x \notin c(S, x)$  for some  $S \in \mathcal{X}$  with  $x \in S$ , so by properties  $\alpha$  and  $\beta$ , we have  $\{z\} = c(\{x, z\}, x)$  for some  $z \in X$ . Moreover,  $U(x) \geq U(y)$  implies  $x \in c(\{x, y\}, \diamond)$ , so it follows from Axiom MEE that  $z \in c(\{x, y, z\}, y)$ . Then by SQB,  $y \notin c(\{x, y, z\}, y)$ , that is,  $y \in X_c$ . By Claim 1, therefore,

$$U(x) + \varphi(x) = \lambda(x) \geq \lambda(y) = U(y) + \varphi(y),$$

and we are done.

**Proof of Theorem 4** (Sketch). We proceed exactly as in the proof of Theorem 2 (by setting  $X := \mathcal{P}(Z)$ ). To prove the “only if” part, then, we take any choice correspondence  $c$  that satisfies all six of the postulated properties, and define the complete and continuous preorder  $\succsim$  on  $\mathcal{P}(Z)$  by

$$p \succsim q \text{ if and only if } p \in c(\{p, q\}, \diamond).$$

Using Axiom I, it is readily verified that  $\succsim$  satisfies the classical independence axiom so that by the von Neumann–Morgenstern expected utility theorem (see [18]), there must exist a function  $u \in \mathbf{C}(Z)$  such that  $p \succsim q$  iff  $\mathbb{E}_p(u) \geq \mathbb{E}_q(u)$  for all  $p, q \in \mathcal{P}(Z)$ . But the properties  $\alpha$  and  $\beta$  imply

$$c(S, \diamond) = \{p \in S : p \succsim q \text{ for all } q \in S\}$$

so that  $c(S, \diamond) = \arg \max_{p \in S} \mathbb{E}_p(u)$  for all nonempty closed subsets  $S$  of  $\mathcal{P}(Z)$ . The rest of the proof is identical to that of Theorem 2 with  $p \mapsto \mathbb{E}_p(u)$  playing the role of  $p \mapsto U(p)$ .  $\square$

**Proof of Proposition 1.** By Theorem 2, there exist a function  $\varphi : X \rightarrow \mathbb{R}_{++}$  and continuous  $U : X \rightarrow \mathbb{R}$  such that (3) and (4) hold for all  $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ . Consequently,  $S_c(y) = \inf\{a \in [0, M] : U(a) \geq U(y) + \varphi(y)\}$  for any fixed  $y$ . But (3) and monotonicity of  $c$  imply that  $U$  is strictly increasing and  $U(M) \geq U(y) + \varphi(y) \geq U(0)$ . Since  $U$  is continuous, therefore, we find  $U(S_c(y)) = U(y) + \varphi(y)$ .

Now define  $\mathcal{B}_y := \{a \in [0, M] : U(y) \geq U(a) + \varphi(a)\}$  and apply Theorem 2 to conclude that  $B_c(y) = \sup \mathcal{B}_y$ . Since  $\varphi(a) \geq 0$ , for any  $a \in \mathcal{B}_y$  we have

$$U(S_c(y)) = U(y) + \varphi(y) \geq U(a) + \varphi(a) + \varphi(y) \geq U(a) + \varphi(y).$$

Thus by continuity of  $U$ , we find

$$U(S_c(y)) \geq \sup U(\mathcal{B}_y) + \varphi(y) = U(\sup \mathcal{B}_y) + \varphi(y) > U(B_c(y)).$$

Given that  $U$  is strictly increasing, this implies that  $S_c(y) > B_c(y)$  as we sought.

To see the second claim, observe that  $x \in c(\{x, y\}, \diamond)$  implies  $U(x) \geq U(y)$ , so since  $U$  and  $U + \varphi$  are comonotonic by Theorem 3, we get

$$U(S_c(x)) = U(x) + \varphi(x) \geq U(y) + \varphi(y) = U(S_c(y)).$$

Thus the claim follows from the monotonicity of  $U$ .  $\square$

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