A Behavioral Foundation for Endogenous Salience

Andrew Ellis
LSE

Yusufcan Masatlioglu
Maryland

Goethe University and Frankfurt School

Motivation

- Many models of salience/attention (List A)
  - Gabaix and Laibson [2006]
  - Koszegi and Szeidl [2013]
  - Bhatia and Golman [2013]
  - Bordalo, Gennaioli, and Shleifer [2013]
  - Cunningham [2013]
  - Gabaix [2014]
  - Schwartzstein [2014]
  - Bushong, Rabin and Schwartzstein et al. [2015]

Motivation

- Bordalo, Gennaioli, and Shleifer [2013] (BGS)

Taylor and Thompson [1982] says

> *"[S]alience refers to the phenomenon that when one's attention is differentially directed to one portion on the environment rather than to others, the information contained in that portion will receive disproportionate weighing in subsequent judgments."*

Motivation

- Understand the basic intuition behind BGS
- Introduce a stripped-down version of BGS
- Regional Preference Model (RPM)
- Uncover surprising relationships between existing models

An Overview

- Bordalo, Gennaioli, and Shleifer [2013] (BGS)

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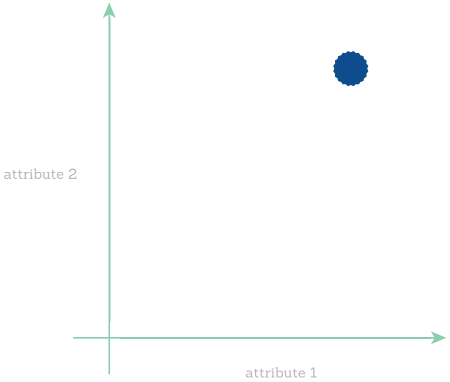
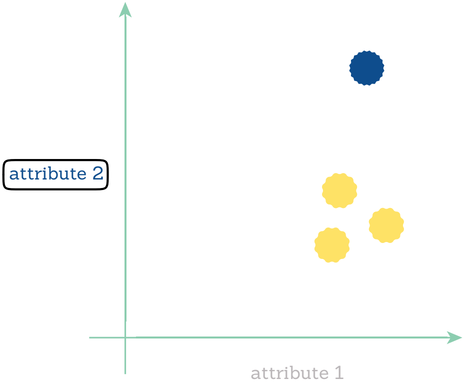


Bordalo, Gennaioli, Shleifer [2013]



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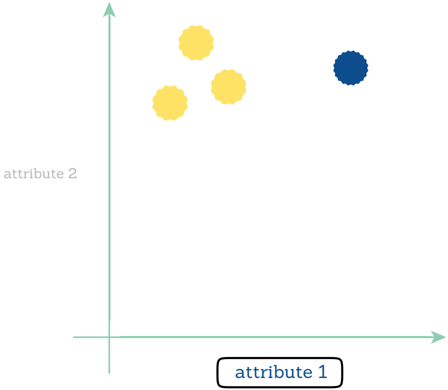

Salience Function

- Salience function $\sigma(a, b)$
  - measures the salience of a compared to b
  - independent of both products and attributes
- The average level is the reference point
  - Salience level of (x_1, x_2) in attribute 1: $\sigma(x_1, \bar{x}_1)$
  - Salience level of (x_1, x_2) in attribute 2: $\sigma(x_2, \bar{x}_2)$

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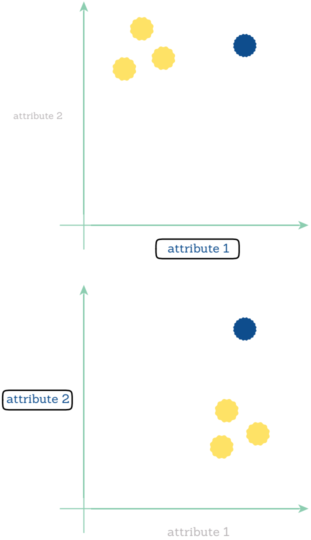
Salience

Attribute 1 is salient for good x if $\sigma(x_1, \bar{x}_1) > \sigma(x_2, \bar{x}_2)$

Attribute 2 is salient for good x if $\sigma(x_1, \bar{x}_1) < \sigma(x_2, \bar{x}_2)$



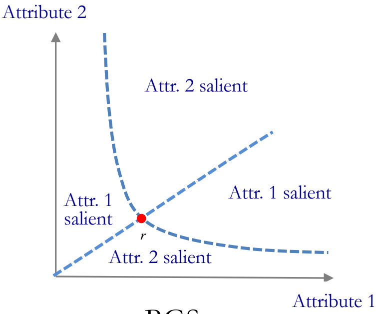
Salience Function

- σ is symmetric and continuous
- Three additional properties
  - Ordering: Salience increases in contrast: For $\epsilon, \epsilon' \geq 0$ with $\epsilon + \epsilon' > 0$
    If $x > \bar{x}$, then $\sigma(x + \epsilon, \bar{x} - \epsilon') > \sigma(x, \bar{x})$
    If $x < \bar{x}$, then $\sigma(x - \epsilon, \bar{x} + \epsilon') > \sigma(x, \bar{x})$
  - Diminishing Sensitivity: Salience decreases as the value of an attribute uniformly increases for all goods: For $\epsilon > 0$,
    $$\sigma(x + \epsilon, \bar{x} + \epsilon) < \sigma(x, \bar{x})$$
  - Homogeneity: For all $\alpha > 0$,
    $$\sigma(\alpha x, \alpha \bar{x}) = \sigma(x, \bar{x})$$

Bordalo, Gennaioli, Shleifer [2013]

To illustrate this model, consider the salience function proposed by BGS,

$$\sigma(a,b) = \frac{|a-b|}{a+b}$$



BGS

Utility Function

Attribute 1 of option x attracts more attention than attribute 2 and receives greater "decision weight" when attribute 1 "stands out"

$$V_{BGS}(x|r) := \begin{cases} wx_1 + (1-w)x_2 & \text{if attribute 1 is salient} \\ (1-w)x_1 + wx_2 & \text{if attribute 2 is salient} \end{cases}$$

where $w \in (1/2, 1)$

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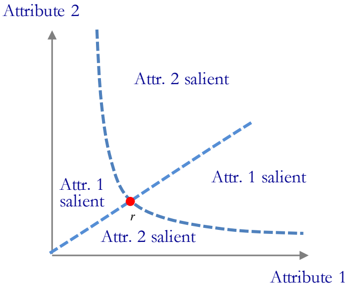

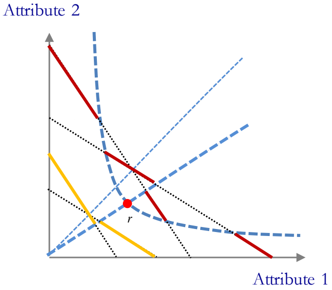

Regions of BGS

- $r = (r_1, r_2)$ the reference good
- $R_k(r)$ be the set of products that are k-salient for the reference point r
- $R_k(r)$ as the k-salient region.

Some Questions

- How do these pictures change as the underlying salience function changes?
- Given a salience function σ, what are the properties of regions, $R_1(r)$ and $R_2(r)$, it generates?

Regions of BGS

- S0 no bundle is both 1-salient and 2-salient, and almost every bundle is either one or the other.
- S1 making a bundle's less salient attribute closer to the reference point does not change the salience of the bundle.
- S2 if every attribute of a good differs from the reference point by the same percentage, then none of the attributes stands out.
- S3 there is no bundle completely surrounded by k-salient bundles that is not an k-salient bundle itself.

Regions of BGS

S0 (Basic) For any $r \in X$: $R_1(r) \bigcap R_2(r) = \emptyset$ and $R_1(r) \bigcup R_2(r)$ is dense in X.

S1 (Moderation) For any $\lambda \in (0, 1]$ and $r \in X$:
if $x \in R_k(r)$, $y_k = x_k$, and $y_{-k} = \lambda x_{-k} + (1 - \lambda) r_{-k}$, then $y \in R_k(r)$.

S2 (Equal Salience) For any $x, r \in X$: if $\frac{x_1}{r_1} = \frac{x_2}{r_2}$ or $\frac{x_1}{r_1} = \frac{r_2}{x_2}$, then $x \notin R_k(r)$ for $k = 1, 2$.

S3 (Regular regions) For all $r \in X$ and $k = 1, 2$: $R_k(r)$ is a regular open set.

Regions of BGS

Theorem

The following are equivalent:
(i) The functions R_1 and R_2 satisfy S0-S3
(ii) There exists a salience function σ s.t.

$$x \in R_k(r) \iff \sigma(x_k, r_k) > \sigma(x_{-k}, r_{-k})$$

(iii) For any salience function σ,

$$x \in R_k(r) \iff \sigma(x_k, r_k) > \sigma(x_{-k}, r_{-k})$$

- no need for the functional form assumptions
- all salience functions lead to the same regions
- our figure independent of the salience function

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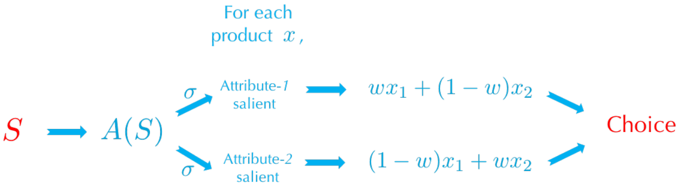
BGS

what are behavioral implications of the salient thinking model?



properties of choices $(c(S))$?

Axioms for BGS

- First axiom is a version of WARP
- Consider two budget sets S^1 and S^2
- $x^1 \in c(S^1)$ and $x^2 \in S^1$
- $x^2 \in c(S^2)$ and $x^1 \in S^2$
- Then WARP implies $x^1 \in c(S^2)$

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- The salience of products does not change when the menu changes from S^1 to S^2
  - E.g., x^1 is 1-salient in both sets and x^2 is 2-salient in both sets
- Then $x^1 \in c(S^2)$.

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  - $x^i \in R_k(A(S^1)) \cap R_k(A(S^2))$ for some k
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Axioms for BGS

Axiom (Salience-SARP)

For any finite sequences of pairs $(x^i, S^i)_{i=1}^n$ *such that* $x^i \in c(S^i)$, $x^{i+1} \in S^i$, *and* $x^{i+1} \in R_k(A(S^i)) \cap R_k(A(S^{i+1}))$ *for some* $k \in \{1, 2\}$ *for every* $i = 1, \ldots, n-1$*:*
if $x^n \in c(S^n)$, $x^1 \in S^n$, *and* $x^1 \in R_k(A(S^1)) \bigcap R_k(A(S^n))$ *for some* k, *then* $x^1 \in c(S^n)$.

Axioms for BGS

Axiom (Regional Linearity)

For $\alpha \in (0,1]$, take S and y such that $S \subset R_k(A(S))$ and $\alpha S + (1-\alpha)\{y\} \subset R_k(A(\alpha S + (1-\alpha)\{y\}))$ for some k. Then, $x \in c(S)$ if and only if $\alpha x + (1-\alpha) y \in c(\alpha S + (1-\alpha)\{y\})$.

▷ Provided all alternatives have the same salient attribute in both menus, choice obeys the usual linearity axiom.

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Axioms for BGS

Axiom (Regional Monotonicity)

For any $x, y \in S$ with $x \neq y$, if $x \geq y$ and $x, y \in R_k(A(S))$ for some k, then $y \notin c(S)$.

▷ Choices respect the usual monotonicity axioms for alternatives within the same salient region

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Axioms for BGS

- The indifference curves in Region 1 should be steeper than in Region 2

Axiom (Salient Dimension Overvalued (SDO))

For $x, y \in S \bigcap S'$ with $x_k > y_k$ and $y_{-k} > x_{-k}$, if $x, y \in R_k(A(S))$, $x, y \in R_{-k}(A(S'))$, and $y \in c(S)$, then $x \notin c(S')$.

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Axioms for BGS

▷ both salience and preference treat attributes symmetrically, permuting the attributes of all objects in the same way does not change rankings.

Axiom (Reflection)

For any $S \in \mathcal{X}$, if $(a, b) \in c(S)$ and T is the reflection of S, then $(b, a) \in c(T)$.

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Axioms for BGS

Axiom (Regional Continuity)

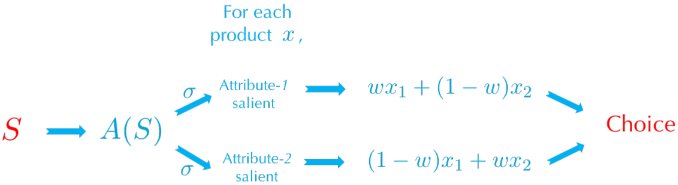
Let $y^n \to y$, $x^n \to x$, and $x, y \notin S$. Then
i) if $x_n \in c(S \cup \{x_n\})$ for all n, then $x \in c(S \cup \{x\})$, and
ii) if $z \in c(S \cup \{y_n\})$ for all n, then $z \in c(S \cup \{y\})$

Characterization of BGS

Theorem

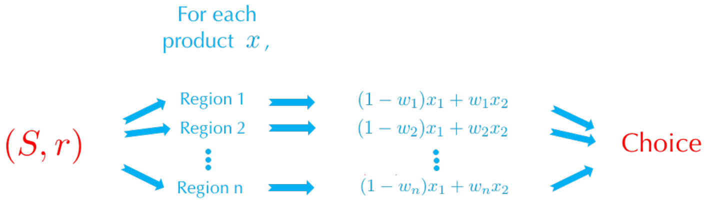
The choice correspondence $c(\cdot)$ satisfies Axioms 1-6 if and only if it has a salient thinking representation.

BGS


For each product x,
S
$A(S)$
σ
Attribute-1 salient
$wx_1 + (1-w)x_2$
σ
Attribute-2 salient
$(1-w)x_1 + wx_2$
Choice

Regional Preference Model (RPM)

Consider a strip down version of BGS



Motivating Questions

- What are the behavioral implications of RPM?
- Which models belong to this class? (List A?)

Setup

- **Domain** $X = I_1 \times I_2$ for open intervals I_1 and I_2
  - $x = (x_1, x_2)$
  - Each attribute **desirable**
  - $x\alpha y$ denotes the coordinate-by-coordinate mixture of x and y, i.e. $[x\alpha y]_i = \alpha x_i + (1-\alpha) y_i$ for all i

Setup

Regions:

- Every region is "well behaved" and non-empty
  - If x in region, so are all points close enough to x
  - A path between points in a region
- All regions surround the reference point
- Almost everything is in a region
- Nothing is in two regions
- As the frame changes, regions change smoothly

Setup

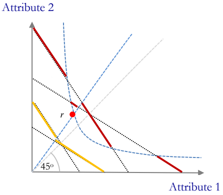
Definition

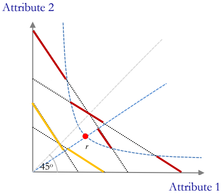
A vector-valued function $\mathcal{R} = (R_1, R_2, \ldots, R_n)$ is a **regional function** if each $R_i : X \to 2^X$ satisfies the following properties:

1. $R_i(r)$ is a non-empty open set and $R_i(r) \bigcup \{r\}$ is connected
2. $r \in \bigcap_{i=1}^{n} bd(R_i(r))$
3. $\bigcup_{i=1}^{n} R_i(r)$ is dense
4. $R_i(r) \bigcap R_j(r) = \emptyset$ for all i, j
5. $R_i(\cdot)$ is continuous.

Regional Preference Model

- Within a given region:
  - Indifference curves are straight lines
  - The relative ranking of two alternatives are the same as long as both of the alternatives lie in the same region
  - Affine functions $u_1(\cdot|r), u_2(\cdot|r), \cdots, u_n(\cdot|r)$
  - $u_i(\cdot|r)$ is a positive affine transformation of $u_i(\cdot|r')$





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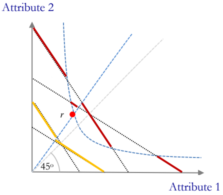

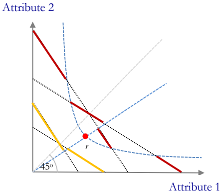

Regional Preference Model

Definition

$\{\succsim_r\}_{r \in X}$ is a RPM under $\mathcal{R}$ if

- $\exists$ positive, strictly increasing, affine functions $u_i(\cdot|r)$
- $u_i(\cdot|r)$ is a positive affine transformation of $u_i(\cdot|r')$

such that for all $x \in R_i(r)$ and $y \in R_j(r)$,

$$x \succsim_r y \;\text{ iff }\; u_i(x|r) \geq u_j(y|r).$$

List B

- Classical Theory
- Tversky and Kahneman [1991] (TK)
- Koszegi and Rabin [2006] (KR) (without expectation)
- Masatlioglu and Ok [2005] (MO)
- Bordalo, Gennaioli, and Shleifer [2013] (BGS)

Tversky and Kahneman [1991]

- Extends Prospect Theory to the case of riskless consumption bundles
- The workhorse of modeling behavior in risk-less environment
- Loss Aversion
- 5339 Google citations (as of Oct 2017)
- Exogenous reference point

Tversky and Kahneman [1991]

$$V_{TK}(x|r) = \begin{cases} (x_1 - r_1) + (x_2 - r_2) & \text{if gain-gain} \\ \lambda_1(x_1 - r_1) + (x_2 - r_2) & \text{if loss-gain} \\ (x_1 - r_1) + \lambda_2(x_2 - r_2) & \text{if gain-loss} \\ \lambda_1(x_1 - r_1) + \lambda_2(x_2 - r_2) & \text{if loss-loss} \end{cases}$$

- Losses hurt: λ_1 and λ_2 are greater than 1
- Constant Loss Aversion

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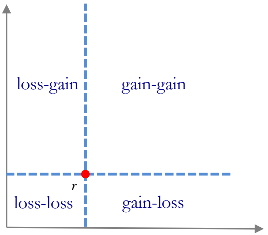

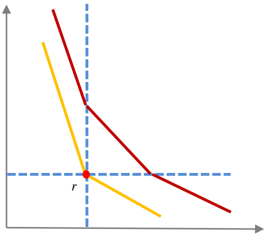

Masatlioglu and Ok [2005,2014]

- Status Quo Bias
- Psychologically constrained utility maximization
- The status quo imposes a psychological constrain on decision makers ($\mathcal{Q}(r)$)

Masatlioglu and Ok [2005,2014]

$$V_{MO}(x|r) := \begin{cases} x_1 + x_2 & \text{if } x \in \mathcal{Q}(r) \\ x_1 + x_2 - c(r) & \text{otherwise} \end{cases}$$

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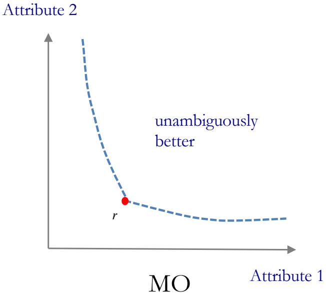
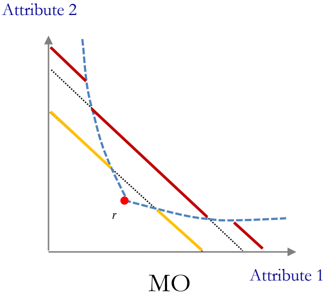
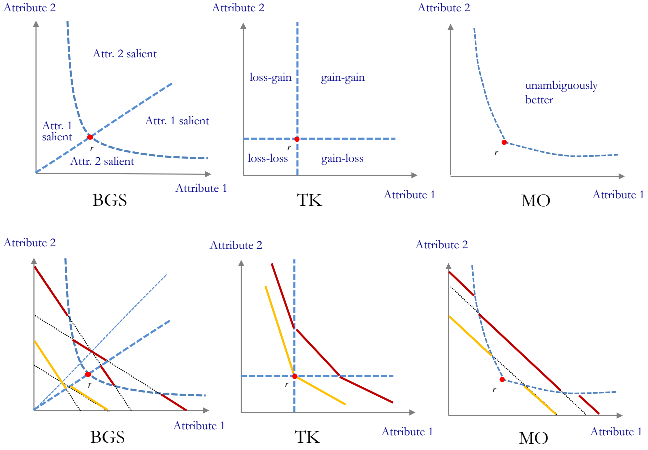




Comparing Models



Comparing Models

- Consider three properties
  - Strong Frame Independence
    - Reference point does not affect ranking, only the regions do
  - Monotonicity
    - More is better
  - Cancellation

Comparing Models

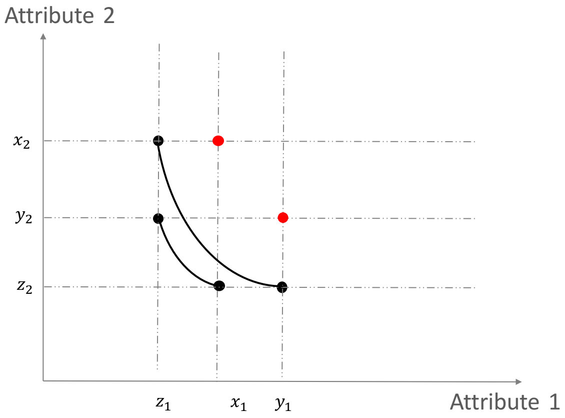
- Strong Frame Independence
  - Reference point does not affect ranking, only the regions
  - If $x \in R_i(r) \bigcap R_i(r')$ and $y \in R_j(r) \bigcap R_j(r')$

$$x \succsim_r y \text{ iff } x \succsim_{r'} y$$

- Monotonicity
  - More is better
  - If $y \geq x$, then $y \succsim_r x$, strictly whenever $y \neq x$.
- Cancellation

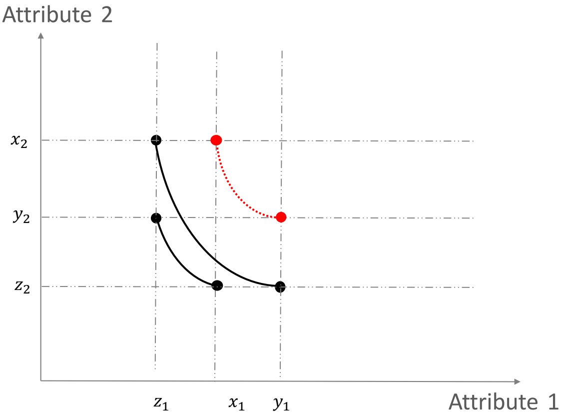
Comparing Models

- Cancellation



Comparing Models

- Cancellation

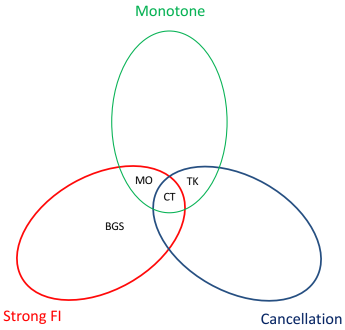


Comparing Models

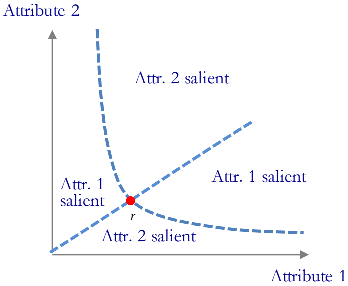
	?	?	?	?
Monotonicity	✓	✗	✓	✓
Strong FI	✓	✓	✗	✓
Cancellation	✓	✗	✓	✗

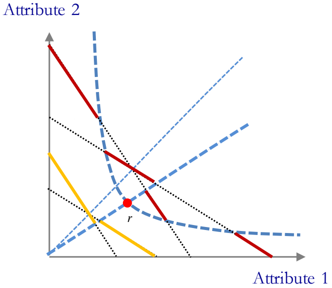
Comparing Models

	Classical	BGS	TK	MO
Monotonicity	✓	✗	✓	✓
Strong FI	✓	✓	✗	✓
Cancellation	✓	✗	✓	✗



BGS and Monotonicity





- What causes BGS to violate monotonicity?
- Is there a model “close” to BGS that satisfies it?

RPM and Monotonicity

- A version of RPM that
  - satisfies SFI,
  - satisfies Monotonicity
  - permits “salience” to affect preference
- Salience reweighs utilities through regions
- No region uniformly better or worse
- An RPM has **salience utilities** if different slopes in different regions

RPM and Monotonicity

Proposition

Suppose there exists some x such that $(x,x) \in X$.
If $\{\succsim_r\}_{r \in X}$ is a RPM under $\mathcal{R}$ with at least two regions, has salience utilities and satisfies SFI, then $\succsim_r$ ***violates Monotonicity*** *for some r.*

- No way to specify regions and weights that prevents violations of Monotonicity

Literature Review

Reference point

Reference –dependent choice	Exogenous	Endogenous	
		Semi	Full
$\{U_r\}$	Tversky and Kahneman, 1991 Munro and Sugden, 2003 Sugden, 2003 Sagi, 2006 Salant and Rubinstein, 2008 …	Bodner and Prelec, 1994 Kivetz et al, 2004 Orhun, 2009 Bordalo et al, 2013 Tserenjigmid, 2015 Ellis and Masatlioglu, 2018	Koszegi and Rabin, 2006 Freeman, 2017 Kıbrıs, Masatlioglu, Suleymanov, 2018 Lim, Xi Zhi (RC), 2019
$(U, Q(r))$	Masatlioglu and Ok, 2005, 2014 Apesteguia and Ballester, 2009 Ortoleva, 2010, Masatlioglu and Nakajima, 2013 Dean, Kıbrıs, Masatlioglu, 2017 …		Ok et al, 2015 Kıbrıs, Masatlioglu, Suleymanov, 2018

Wrapping up

- BGS enjoys a nice characterization
- RPM generalizes the main idea behind BGS
- Other salience/attention models are outside of RPM (List A)
- $\Rightarrow$ BGS offers vey distinct way of modeling salience
- Uncover surprising relationships (List B)
- Tradeoffs between SFI and Monotonicity

Wrapping up

- Stark contrast between
  - attention to attributes (RPM, BGS)
  - attention to alternatives (Masatlioglu et al. [2012])
  - attention to information (Ellis [2013])

THANK YOU

Axioms for RPM

- Simply restrict usual axioms to within regions
  - Regional Frame Independence
  - Regional Monotonicity
  - Regional Continuity
  - Regional Linearity

Axioms for RPM

A1 (Regional Frame Independence) If $x, y \in R_i(r) \cap R_i(r')$, then $x \succsim_r y \iff x \succsim_{r'} y$.

A2 (Regional Monotonicity) For any $x, y \in R_i(r)$, if $y \geq x$, then $y \succsim_r x$, strictly whenever $y \neq x$.

A3 (Regional Continuity) For any x and region $R_i(r)$, the sets $\{y \in R_i(r) : y \succ_r x\}$ and $\{y \in R_i(r) : x \succ_r y\}$ are open.

A4 (Regional Linearity) For any $x, y, a, b \in X$, $r \in X$ and $\alpha \in (0, 1]$ such that $x, x\alpha y, y \in R_i(r)$ and $a, a\alpha b, b \in R_j(r)$: if $x \succsim_r a$ and $y \succsim_r b$, then $x\alpha y \succsim_r a\alpha b$, strictly if $x \succ_r a$

Representation Theorem

Theorem

Let $\mathcal{R}$ be a regional function. Then $(\{\succsim_r\}_{r\in X}, \mathcal{R})$ satisfies A1-A4 if and only if $\{\succsim_r\}_{r\in X}$ is RPM under $\mathcal{R}$.

Proof outline:

- Each region contains a mixture space
- Use the mixture space to get a "within representation"
- Stitch together the within representations using Regional Linearity and Transitivity; show still affine
  - tougher, especially if two regions have no indifference points
- Regional Frame Independence and continuity of the regional function allow us to show indifference curves have same slope across frames

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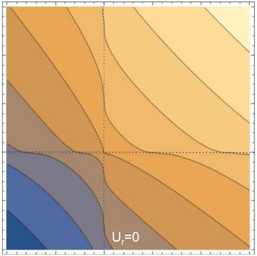
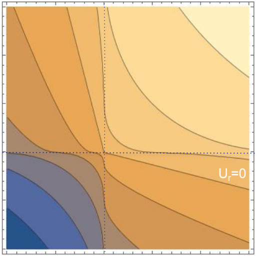
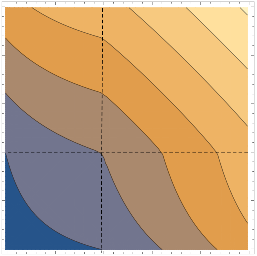
Continuous Salience



KR



TK



Continuous BGS