Limited Willpower

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Preferences and Choices

- Facing tempting alternatives, people sometimes make choices that are different from what they would have chosen according to their commitment preferences.
- Procrastination, impulse purchases, succumbing to the temptation of unhealthy foods are examples of such behavior.
- People do not always succumb to temptation and are sometimes able to overcome temptations by using cognitive resources.
- This ability is often called willpower.

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Willpower

Psychologists claim that Willpower is

- required to suppress and override our visceral urges,
- more than just a fairy tale or a metaphor,
- not unlimited resource,
- the same resource applies to different tasks,
  - If you perform a task requiring self-control, it is less likely/more difficult to exercise self-control in a different task. Baumeister et al (1994), Baumeister and Vohs (2003), Muraven (2011)

Psychology Experiments

- Stage 1: Experimental subjects are asked to perform a task of self-regulation (Do not eat cookies, Stroop Test, Do not look at subtitles). Control subjects do nothing. Willpower Depletion

- Stage 2: The "endurance" of all subjects is measured on an unrelated task (Working on insoluble puzzles, Squeezing hand exercisers, Refraining from impulse purchases). Less Endurance

- Experimental subjects exhibit MUCH less endurance on stage 2 tasks than the controls.

Related Work

- Ozdenoren, Salant, and Silverman (2011)
- Fudenberg and Levine (2006, 2012)
- Noor and Takeoka (2010)

Limited Willpower Model

- A choice theoretic foundation for the willpower as a limited cognitive resource model.
  - Provide a simple and tractable model,
  - Temptation modeled as a constraint,
  - Identification of one's willpower and visceral urge intensity,
  - Using a contracting example demonstrate unique implications

The Model

Three components:

$u(\cdot) \rightarrow$ utility

$v(\cdot) \rightarrow$ visceral urge intensity

$w \quad \rightarrow$ willpower

The Model

Choosing an alternative from set A:

$$c(A) = \arg\max_{x \in A} \quad u(x)$$

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$$\underbrace{\max_{y \in A} v(y)}_{\text{highest temptation in } A} \; - v(x) \leq w$$

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An illustration

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \quad \max_{y \in A} v(y) - v(x) \leq w$$

Example: Assume willpower stock, $w = 3$,

	u	v
going to gym	10	1
reading book	5	3

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⋆ Violation of WARP,

⋆ The middle option is chosen, "Compromise Effect"

Representation

$$c(A) = \underset{x \in A}{\text{argmax}} \quad u(x) \ \text{ subject to } \ \max_{y \in A} v(y) - v(x) \leq w$$

Two Extreme Cases

- $w = \infty$ (Standard) *NEVER* give in temptation
- $w = 0$ (Strotz) *ALWAYS* give in temptation

Setup

- X: a finite set of alternatives.
- Two pieces of information: $(\succsim, c)$
  - Preferences
  - Choices

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Question: What class of $(\succsim, c)$ can be explained by the Limited Willpower model?

Axioms

Axiom 1: $\succsim$ is complete and transitive.

Axiom 2: If $x \succ c(A \cup x)$ then $c(A) = c(A \cup x)$.

Axiom 3: $c(A) \succsim c(B) \Rightarrow c(A) \succsim c(A \cup B) \succsim c(B)$.

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Choice Betweenness vs. Set Betweenness

- Suppose $\succ_0$ is a preference over non-empty subsets of X.
- $\succ_0$ satisfies SB if $A \succ_0 B$ implies $A \succ_0 A \cup B \succ_0 B$.
- Consider commitment preferences and second-period choices implied by $\succ_0$.
- How are SB and CB related?

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SB but not CB

- $\succ_0$ has a *costly self-control representation* if represented by

$$V(A) = \max_{x \in A} \quad u(x) - \varphi(max_{y \in A} v(y) - v(x))$$

- Implied choices are:

$$c(A) = \operatorname*{argmax}_{x \in A} \quad u(x) - \varphi(max_{y \in A} v(y) - v(x)).$$

- If $\succ_0$ has a costly self-control representation then it satisfies SB (Noor and Takeoka (2010)) but implied choices violate CB when φ is concave.
- Suppose $\varphi(a) = a^{.5}$, $u(x) = 2$, $u(y) = 1$, $u(z) = 0$, and $v(x) = 0$, $v(y) = 1.5$, $v(z) = 3$. Then $x = c(x, z) = c(x, y, z) \succ y = c(x, y) \succ z = c(y, z)$.

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CB but not SB

- Suppose $\succ_0$ is represented by

$$W(A) = \max_{x \in A} u(x) - \left(\max_{y,z \in A, y \neq z} (v(y) + v(z)) - v(x) \right)$$

  and for singleton sets $W(\{x\}) = u(x)$

- Implied choices are:

$$c(A) = \operatorname*{argmax}_{x \in A} u(x) + v(x).$$

- $(\succ, c)$ (trivially) satisfies CB.
- To see that $\succ_0$ violates SB, let $X = \{x, y, z\}$, $u(x) = 7$, $u(y) = 3$, $u(z) = 2$, $v(x) = 0$, $v(y) = 1$ and $v(z) = 2$. Then, $\{x, y\} \succ_0 \{x, z\} \succ_0 \{x, y, z\}$.

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A Result

Theorem 0

$(\succsim, c)$ satisfies Axioms 1-3 if and only if it admits a generalized willpower representation:

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \max_{y \in A} v(y) - v(x) \leq w(x)$$

An Additional Axiom

When is $w(x) = w$?

Axiom 4 Suppose $y \succ c(y, z)$ and $c(t, z) = t$.
If $x \succ c(x, y)$ then $c(x, t) = t$.

- t is more tempting than y,
- x is not choosable over y.
- Then x is also not choosable t.

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Desired Result

Theorem 1

$(\succsim, c)$ satisfies Axioms 1-4 iff $(\succsim, c)$ admits a Limited Willpower representation.

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \max_{y \in A} v(y) - v(x) \leq w$$

Comparison with Costly Self Control

- When φ is linear (model of Gul and Pesendorfer), choices implied by the costly self control model satisfy WARP.
- As in our model, when φ not linear, there are WARP violations.
- When φ is concave, violates CB.
- When φ is convex, satisfies CB, hence special case of the generalized willpower model.
- In the convex case, consistency is violated.
  - provides a direct test to separate the two models based only on ex-ante preferences and ex-post choices and not on menu preferences.

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Non-Uniqueness

If preferences and choices coincide ($c(x, y) = x \succ y$), then

- No self-control problem
  - $0 < v(x) - v(y)$
- Self-control problem exists but enough willpower
  - $0 < v(y) - v(x) < w$

v is not even unique in ordinal sense !!!

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LOTTERIES

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LOTTERIES

Willpower with Lotteries

- X: the finite set of potentially available alternatives
- Δ: the set of all lotteries on X
- $\mathcal{X}$: the set of non-empty finite subsets of Δ
- $\succsim$: the preferences on X
- c: choices on $\mathcal{X}$

Linear Limited Willpower

$$c(A) = \underset{p \in A}{\operatorname{argmax}} \quad u(p)$$

subject to

$$\max_{q \in A} v(q) - v(p) \leq w$$

where

- u, v are linear functions
- w is a positive scalar.

New Axioms

Axiom A $\succsim$ admits an expected utility representation.

Axiom B Suppose $p_n \to p$ and $q_n \to q$ with $p_n \succ q_n$ for all n. If $c(p_n, q_n) = p_n$ then $p \in c(p, q)$.

New Axioms

- Independence axiom (adapted to choice correspondences) says that $y \in c(x, y)$ implies $y\alpha z \in c(x\alpha z, y\alpha z)$ where $\alpha \in [0, 1]$.
- Full independence is too strong for the limited willpower model.
  - Assume $u(x) = 1$ and $u(y) = 0$, $v(x) = 0$ and $v(y) = 3$, and $w = 2$.
  - $v(y) - v(x) = 3 > 2 = w$, so $c(x, y) = y$.
  - But $v(y) - v(x\frac{1}{2}y) = \frac{1}{2}v(y) - \frac{1}{2}v(x) = 1.5 < 2 = w$, and $c(x\frac{1}{2}y, y) = x\frac{1}{2}y$.

New Axioms

Axiom C (Temptation Independence) Let $p \succ q$ and $\alpha \in [0,1]$.

i) If $c(p,q) = p$, $c(p',q') = p'$ and $p' \succsim q'$, then $c(p\alpha p', q\alpha q') = p\alpha p'$

ii) If $c(p,q) = q$, $c(p',q') = q'$ and $p' \succ q'$ then $c(p\alpha p', q\alpha q') = q\alpha q'$

Axiom D (Invariance to Replacement) If $c(p\alpha r, q\alpha r) = p\alpha r$ then $c(p\alpha r', q\alpha r') = p\alpha r'$ for any r'.

New Axioms

Axiom E: (Conflict) There exist p and q such that $p \succ c(p, q)$.

Axiom F: (Limited Agreement) For all $p \succ q$, there exists $\alpha > 0$ such that $p\alpha q = c(p\alpha q, q)$.

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Characterization

Main Result

$(\succsim, c)$ satisfies the axioms iff $(\succsim, c)$ admits a linear Limited Willpower representation with $w > 0$.

Uniqueness: If (u, v, w) and (u', v', w') represent $(\succsim, c)$ then there exist scalars $\alpha > 0, \alpha' > 0, \beta, \beta'$ such that

$$u' = \alpha u + \beta, \quad v' = \alpha' v + \beta', \quad w' = \alpha' w$$

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Preferences from Choices

Can we reveal preferences from choices?

In the standard approach, preferences are revealed by choices.

$$x \succ y \quad \text{if} \quad x = c(x, y)$$

In the limited willpower, this is no longer true. It is possible that

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Preferences from Choices

Take two points x and y, and consider a mixture of them,

- If $u(x) > u(y)$ then $u(\alpha x + (1-\alpha)y) > u(y)$,
  - Order of utility does not change
- $v(y) - v(\alpha x + (1-\alpha)y) = \alpha(v(y) - v(x))$,
  - Self-control problem gets smaller

Preferences from Choices

Given c, we define revealed preference, $\succ^c$,

$x \succ^c y$ if one of the following is true

- $x = c(x, y)$ and no mixture can reverse the choice,
- $y = c(x, y)$ and some mixture can reverse the choice,

Preferences from Choices

Given c, define $\succ^c$

$x \succ^c y$ if one of the following is true

- $x = c(x, y)$ and $\nexists \alpha \in (0, 1)$ such that $y \in c(x\alpha y, y)$,
- $y = c(x, y)$ and $\exists \alpha \in (0, 1)$ such that $x\alpha y = c(x\alpha y, y)$.

Proposition

If $(\succsim, c)$ admits a linear willpower representation, then $\succsim = \succsim^c$.

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- Denote choices in the control vs. treatment group by c_{cont} and c_{treat}.
- Assume same commitment preference u.
- Subject gives into temptation in treatment but not in control: $c_{cont}(x,y) = x \succ c_{treat}(x,y) = y$.
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- How do we catch reversals in v?
- Suppose $p \succ q, q'$ and $c_{cont}(p, q) = p$ and $c_{cont}(p, q') = q'$.
  - q' is more tempting then p
- Suppose treatment is unable to choose p in either case.
- As β increases $p\beta q$ and $p\beta q'$. become less tempting, and former always less tempting for same v.
- Means treatment should never have $c_{treat}(p, p\beta q) = p\beta q$ and $c_{treat}(p, p\beta q') = p$

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Application

A monopolist facing a consumer with limited willpower

Examples...

- Buying a cell-phone plan,
- Buying a gym-membership,
- Checking in a hotel,
- Visiting a dealership or a restaurant,

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- Firm offers a set of services (contract),
  - p_s : the price of service s,
  - $c(s)$: the cost of producing service s,
- Firm's profit selling s is $p_s - c(s)$,
- Consumer can accept or reject it (outside option is 0),
- If accepted, both parties are committed to the contract,
- Consumer chooses a service from the contract.

Application

- Consumer has limited willpower.
- U and V are quasi-linear in price,

$$U(s, p_s) = u(s) - p_s, \quad V(s, p_s) = v(s) - p_s$$

- Higher price $\Rightarrow$ Less tempting,
- Consumer is NAIVE (incorrectly) believes that he has unlimited willpower

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- Call $v - u$ as Excess Temptation
- Let $y = argmin_{s \in X}(v(s) - u(s))$ be the service with lowest excess temptation and $Y = v(y) - u(y)$
- Let $z = argmax_{s \in X}(v(s) - u(s))$ be the service with highest excess temptation and $Z = v(z) - u(z)$
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EXAMPLE

There are four possible options: s_1, s_2, s_3, s_4.

	u	v	c
s_1	4	6	1
s_2	8	12	4
s_3	12	18	9
s_4	16	24	16

Commitment Contract

Suppose the consumer is standard (has unlimited willpower) or is able to commit.

- The firm offers only one option (Commitment Contract)

$$\max_{x,p} p - c(x) \quad \text{s.t. } u(x) - p \geq 0$$

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Commitment Contract

	u	v	c	$u-c$	
s_1	4	6	1	3	
s_2	8	12	4	4	$\Leftarrow x^u$
s_3	12	18	9	3	
s_4	16	24	16	0	

Profit: $u(x^u) - c(x^u) = 8 - 4 = 4$

No Willpower: $w = 0$

Now suppose the consumer has no willpower.
Is there a better contract for the firm?

• INDULGING CONTRACT: Attract the consumer with lowest excess temptation s_1 but actually sell s_3.

Consider $(s_1, 4; s_3, 16 - \epsilon)$

	u	v	c	p	$u-p$	$v-p$
s_1	4	6	1	4	0	2
s_3	12	18	9	$16-\epsilon$	$-4+\epsilon$	$2+\epsilon$

In period 1, the naive consumer believes that he will choose s_1,
In period 2, he ends up choosing s_3,
Profit: $7 - \epsilon$ (> 4)

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No Willpower

- Offer two services (Indulging Contract)

Firm's maximization problem (Attract consumer by y but make him buy x)

$$\max_{x,y,p(x),p(y)} p(x) - c(x)$$

subject to

Participation Constraint

$$u(y) - p(y) \geq 0$$

Make him buy x

$$v(x) - p(x) \geq v(y) - p(y)$$

Both of them are binding:

$$p(x) = v(x) - (v(y) - p(y)) = v(x) - (v(y) - u(y))$$

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No Willpower $w = 0$

The bottom line: The optimal contract is the INDULGING CONTRACT.

- Attract the consumer with lowest excess temptation y
- Actually sell $x_v = \arg\max(v - c)$
- Profit from indulging contract is $v(x^v) - c(x^v) - Y$

- Contracting with dynamically inconsistent naive agents,
- O'Donoghue and Rabin, 1999; Gilpatric, 2003; Sarafidis, 2004; DellaVigna and Malmendier, 2004; 2006; and especially Eliaz and Spiegler 2006,
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So far nothing new!!!

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Our Model $w > 0$

- Offer Indulging Contract

	u	v	c	p
s_1	4	6	1	4
s_2	8	12	4	
s_3	12	18	9	16
s_4	16	24	16	

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Consumer can resist some temptation,

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Consumer can resist some temptation,
Price of x^v must be lowered by w,
Hence, profit is lowered by w

Our Model $w = 2$

Is there a better contract for the firm?

Exploit the Compromise Effect and consider a contract with three services

	u	v	c	p
s_1	4	6	1	4
s_2	8	12	4	
s_3	12	18	9	16
s_4	16	24	16	20

In period 1, he believes that he will choose s_1,

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s_1	4	6	1	4	0	
s_2	8	12	4			
s_3	12	18	9	16	-4	
s_4	16	24	16	20	-5	

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Is there a better contract for the firm?
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	u	v	c	p	$u-p$	$v-p$
s_1	4	6	1	4	0	2
s_2	8	12	4			
s_3	12	18	9	16	-4	
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In period 1, he believes that he will choose s_1,
In period 2, s_4 is so tempting that he cannot choose s_1,
he ends up choosing s_3.
Profit: 7 (we recovered the same profit as if no willpower)

Lesson from this example

To exploit the consumer with some willpower, use the compromise effect.

Need to offer three choices in the menu:

- one with the lowest excess temptation (Decoy)
  - persuading the consumer to sign the contract
- one with the highest excess temptation (Temptation)
  - tempting the consumer not to choose decoy
- something middle (Target)

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- something middle (Target)

Maximization Problem

$$\max_{x,y,,z,p(x),p(y),p(z)} \quad p(x) - c(x)$$

subject to

Participation Constraint

$$u(y) - p(y) \geq 0$$

z makes y unchoosable

$$v(z) - p(z) \geq v(y) - p(y) + w$$

x is choosable

$$v(x) - p(x) \geq v(z) - p(z) - w$$

x is better than z

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Maximization Problem

- First two constraints binding:

$$p_y = u(y) \text{ and } p_z = v(z) - (v(y) - u(y)) - w$$

- Remaining two constraints become

$$p_x \leq v(x) - (v(y) - u(y))$$

$$p_x \leq u(x) - (v(y) - u(y)) + (v(z) - u(z)) - w.$$

- Constraints are $p_x \leq v(x) - Y$ and $p_x \leq u(x) - Y + Z - w$.

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Compromising Contract

- Compromising contract uses y as decoy and z as temptation.
- As target monopolist chooses x that maximizes:

$$min\{v(x) - c(x) - Y, u(x) - c(x) - Y + Z - w\}$$

- Compromising contract always better than indulging contract (which has profit $v(x) - c(x) - Y - w$.)
- To see this note $u(x) - c(x) - Y + Z - w \geq v(x) - c(x) - Y - w \iff Z \geq v(x) - u(x)$.
- If $w \leq Z - Y$ then compromising contract is best.
- If consumer's willpower exceeds this threshold, commitment contract is best.

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Example

- Let $X = [1,4]$ and $u(s) = 4s$, $v(s) = 6s$ and $c(s) = s^2$. Thus,

$$Y = 2, x^u = 2, x^v = 3, Z = 8$$

Example

- When $w < 2$, monopolist sells $x^v = 3$ and earns $v(x^v) - Y - c(x^v) = 7$ same as no willpower case.
- When $2 < w < 4$ monopolist sells $x = 4 - w/2$. As the willpower goes up, the actually sold service approaches the efficient level.
- When $4 < w < 6$, monopolist sells the efficient service $x^u = 2$, but exploits the consumer. Price goes down with more willpower.
- When $w > 6$, the monopolist sells the efficient service $x^u = 2$ at the price of $u(x^u) = 6$ without any exploitation.

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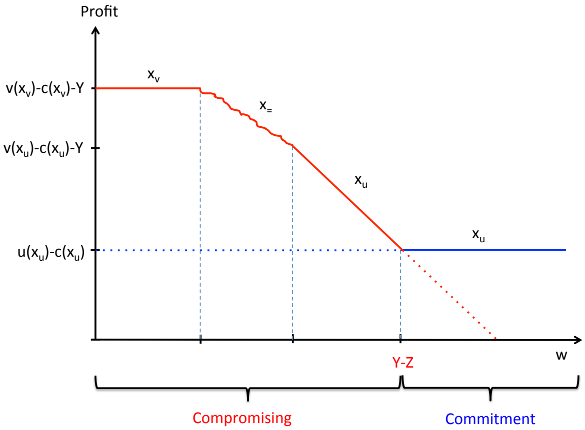
Optimal Contract



Comparative Statics w

- The monopolist sells a service somewhere between x_u and x_v.
- Profit is weakly decreasing in consumer's willpower.
- The consumer's welfare is weakly increasing in his willpower.
- When w is small, the monopolist can earn the same amount of the profit when the consumer has no willpower at all.
- When w is high, no exploitation.

Conclusion

- Provide a limited willpower model,
- Our characterization uses only choices,
- Temptation modeled as a constraint rather than a direct utility cost,
- Model is simple and tractable
  - A monopolist facing a consumer with limited willpower
  - Qualitatively different results (Strotz or Costly Self-control)
  - "Compromise Effect" as a market outcome
  - Unchosen alternatives play crucial role in actual choice

THANK YOU