Journal of Monetary Economics 70 (2015) 84–99

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jme

Managing markets for toxic assets

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ARTICLE INFO

Article history:
Received 2 March 2012
Received in revised form
1 October 2014
Accepted 2 October 2014
Available online 14 October 2014

Keywords:
Liquidity
Adverse selection
TARP

ABSTRACT

A model in which banks trade toxic assets to raise funds for investment is analyzed. Toxic assets generate an adverse selection problem and, consequently, the interbank asset market provides insufficient liquidity. Investment is inefficiently low because acquiring funding requires banks to sell high-quality assets for less than their "fair" value. Equity injections reduce liquidity and may be counterproductive as a policy for increasing investment. Paradoxically, if it is directed to firms with the greatest liquidity needs, an equity injection will reduce investment further. Asset purchase programs, like the Public–Private Investment Program, often have favorable impacts on liquidity, investment and welfare.

1. Introduction

How can government agencies restore liquidity in financial markets during periods of financial distress? In 2008, the U.S. Treasury Department and the Federal Reserve were confronted with exactly this question. Banks and financial intermediaries with many "toxic assets" in their portfolios found it difficult to raise funds necessary for making loans. According to market observers and commentators in the popular press, these toxic assets could not be easily valued and were therefore illiquid. The Treasury and the Federal Reserve Bank responded with a combination of policy actions including asset purchases, loan guarantees and equity injections.

This paper uses a simple model to analyze the effects of policies in markets for toxic assets. We model the market for toxic assets with an adverse selection problem. Adverse selection seems like a natural modeling assumption for two reasons. First, many market observers emphasize the problem caused by having assets that buyers could not accurately value. Second, it has been well understood since Akerlof (1970) that adverse selection can cause significant market failures, and, in extreme cases, can cause markets to shut down completely.

In our model, banks possess both liquid and illiquid (toxic) assets. Banks trade the illiquid assets to finance investments and satisfy their needs for liquidity. In the absence of government interventions, the interbank market for toxic assets provides insufficient liquidity for banks to efficiently finance investment projects. Investment is inefficiently low because acquiring adequate funding requires banks to sell high quality assets at prices below their "fair" market value. This illiquidity is a direct consequence of the adverse selection problem in the secondary market for toxic assets.

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http://dx.doi.org/10.1016/j.jmoneco.2014.10.001
0304-3932/

We then consider whether government policies can increase liquidity in the secondary market and improve the allocation of investment. We focus our attention on equity injections and purchases of toxic assets. Equity injections do not increase liquidity in markets for toxic assets and, in some cases, may be counterproductive as a policy for increasing investment. In contrast, asset purchase programs can increase liquidity in the secondary market by driving up the price of toxic assets. In the model, under certain conditions, asset purchase programs may be more effective at increasing investment than policies that provide direct access to additional liquidity.

One counterintuitive disadvantage of equity injections is that they often allow financial institutions to fund investments directly without having to sell as many high-quality assets at unfavorable terms. While this is unambiguously good for a single financial institution at a given price, equity injections can have negative feedback effects in equilibrium. Because financial institutions can withhold high quality assets and instead rely on the new equity to fund investments, fewer of these high quality assets are traded. In such cases, equity injections have a *contamination effect* which causes the average quality (and thus the price) of toxic assets in the secondary market to drop. The reduction in price further reduces the incentive to trade, making toxic assets even less liquid. Paradoxically, if equity injections are directed to banks with the greatest needs for liquidity, the contamination effect can be so severe that investment falls in equilibrium.

Unlike equity injections, which often have a detrimental impact on the market for toxic assets, asset purchase programs – like the Public–Private Investment Program – often have favorable impacts on secondary markets, investment and welfare. Intuitively, while asset purchase plans can be designed to transfer the same amount of money to distressed financial institutions, the banks have to sell assets to obtain the funds. Asset purchases thus encourage trading in the secondary market, leading to greater liquidity and improved efficiency.

In the model, a key requirement of a successful asset purchase policy is that the government purchase assets at above-market prices. If the government buys assets at fair-market prices, the policy will have no effect. Because the government needs to purchase assets at above-market prices, successful asset purchases are costly in terms of the Federal Budget.

Modeling toxic assets with adverse selection obviously rules out other potentially important roles for such securities. For instance, in our model, the banks do not care directly about insolvency nor do they care about whether their balance sheet is sufficiently transparent for potential creditors. Other market frictions (bank runs, liquidity constraints, etc.) surely played important roles in shaping the crisis. Because available evidence does not point uniquely to adverse selection as the source of financial market failure, our results should be viewed somewhat narrowly. The paper presents one possible channel through which loan market failure may have arisen and is not meant to preclude other mechanisms.

2. Background

The Emergency Economic Stabilization Act of 2008 was signed on October 3, 2008 by President George W. Bush. The most prominent component of this legislation was the *Troubled Asset Relief Program* better known by its acronym TARP. At the time of the bill's passage, the conventional understanding was that the TARP would be used to purchase distressed assets in order to restore trading in interbank markets that were essentially frozen. Congress authorized the Treasury to purchase up to $700 billion of troubled assets.

The preamble of the bill laid out the intentions of the legislation as follows: "(t)he Secretary is authorized to establish the Troubled Asset Relief Program to purchase […] troubled assets from any financial institution, on such terms and conditions as are determined by the Secretary." The term "troubled assets" is defined by the law to mean "residential or commercial mortgages and any securities, obligations, or other instruments that are based on or related to such mortgages […] the purchase of which the Secretary determines promotes financial market stability."[1]

The purpose of the TARP was to restore liquidity to asset markets that had essentially ceased to function. In a formal press release, Secretary Henry Paulson described the problem in the asset markets and his proposed solution as follows:

> When the financial system works as it should, money and capital flow to and from households and businesses to pay for home loans, school loans and investments that create jobs. As illiquid mortgage assets block the system, the clogging of our financial markets has the potential to have significant effects on our financial system and our economy. […] The federal government must implement a program to remove these illiquid assets that are weighing down our financial institutions and threatening our economy. This troubled asset relief program must be properly designed and sufficiently large to have maximum impact, while including features that protect the taxpayer to the maximum extent possible.[2]

Paulson's description of the problem is exactly what we want to capture in our model.

The original design of the TARP was not greeted with unanimous support from academic economists, many of whom argued that capital injections would work better to stabilize conditions in the markets (see Stiglitz, 2008; Krugman, 2009; Diamond et al., 2008; Kashyap and Stein, 2008.)[3] In addition, two other issues ultimately led to a substantial re-design of the

[1] The Emergency Economic Stabilization Act is Division A of Public Law 110–343. The terms of the TARP are included in Title I of this Act.

[2] Statement by Secretary Henry Paulson on Comprehensive Approach to Market Developments, hp-1149, September 19, 2008.

[3] Not all economists viewed the original TARP unfavorably. Quoted by *The Wall Street Journal*, Anna Schwartz said that the problem was "exotic securities that the market does not know how to value. They are toxic because you cannot sell them, you do not know what they are worth, your balance sheet is not credible and the whole market freezes up." In her view, purchasing such assets would be a "step in the right direction." Also quoted in *The Wall*

TARP. First, it became clear that calculating the "correct" price of troubled assets would be difficult. This suggested that injecting equity might be easier than purchasing illiquid assets. Second, direct capital injections were tried with some apparent success in the U.K. As a result, the original TARP plan was abandoned in favor of equity injections.

In early 2009 a second plan to purchase toxic assets – the Public–Private Investment Program (P-PIP) – was suggested. Like the original TARP, the P-PIP aimed to purchase toxic assets from banks. The price for the assets would be determined by private auctions. The purchase would be made with a combination of funds from the TARP, the Federal Reserve and private investors. It is worth noting that, according to the original description of the P-PIP, banks would determine for themselves which assets they would put up for auction.

3. Model

We consider a static model of trade in illiquid assets. The main actors in the model are financial intermediaries that fund investment opportunities by either drawing on cash reserves or selling assets. There is a continuum of such intermediaries. We refer to the financial intermediaries simply as "banks." The cash reserves could be either actual cash or could be other liquid assets like Treasury bills. The illiquid assets are the source of the adverse selection problem. These toxic assets are referred to as mortgage backed securities (MBS) though in theory they could be any asset that buyers find difficult to value. The secondary market is referred to as the "interbank market."

Each bank has a portfolio of mortgage backed securities (measure one) and a measure m of liquid assets. The MBS differ in their default probabilities and the current holders of the securities have superior knowledge about the true quality of the assets compared to potential buyers. To keep matters simple, we assume that a given MBS either pays back fully or defaults. Solvent mortgages repay R dollars. If a mortgage defaults, the owner gets zero. The distribution of default probabilities is a common knowledge. Some assets are perfectly solvent; some have moderate default risk and some are sure to default. Let δ be the default rate on an individual mortgage asset. We assume that default rates are distributed according to the distribution function $G(\delta)$ with density $g(\delta)$. This distribution characterizes the portfolio of every bank. At the beginning of the period, banks know the default rate of each asset they own (they know which ones are underperforming and which ones are sound).

We do not take a strong stand on the exact source of this superior information. It could arise because the owners know details of the origination process such as how diligent they were in screening the borrowers. They could also have played a direct role in structuring the assets themselves. In addition, they could have direct observations on how well the underlying assets are doing. This is a feature of actual toxic assets, many of which are simply pools of mortgages that periodically pay off as the borrowers make mortgage payments. A bank that possesses a given MBS can observe the rate at which the mortgages pay off and infer whether the mortgages are good or whether they are likely to underperform.

Banks differ in their needs for liquidity. To motivate the demand for liquidity, we assume that each bank receives a different idiosyncratic investment opportunity. The maximum scale of the investment opportunity is i . The investment projects are perfectly divisible so a bank can choose to undertake a fraction of its investment opportunity if it wants. While i is the same for all investments, each investment opportunity has a different expected return z. The idiosyncratic return z is distributed across banks according to the distribution function $F(z)$ with density $f(z)$ and mean μ_z. To make the solution non-trivial, there are at least some projects for which $z > 1$. Banks in need of liquidity (banks with $z > 1$) face a trade-off between selling assets at a discount or foregoing profitable investment opportunities.

While we refer to the z shocks as investment projects, they could alternately be interpreted as liquidity needs for banks. Under this interpretation, banks with high z's have unexpectedly high needs for cash. This could be because the bank has to satisfy margin requirements or pay depositors (as in Uhlig, 2010a,b). The upper bound i would be the total margin requirement and the z could reflect alternate uses for the cash. Either interpretation works in the context of the model though, under the second interpretation, social welfare corresponds to the welfare of the banking system. For expositional purposes, we refer to the z's as investment projects.

In addition to the banks, the model also include a second set of agents that trade securities. This second group does not fund any investment but instead simply supplies a set of illiquid assets inelastically. These agents are referred to as traders, There is a measure T of such agents each with one toxic asset. The traders can be interpreted as bankrupt firms that are forced to liquidate irrespective of the price. Let the average default rate of the traders be δ^T, which may be greater or less than the default rate of the bank assets. The traders primarily play a technical role in the model.

Banks are risk neutral and seek to maximize expected profit. Banks with good projects (i.e., banks with high z) will attempt to raise liquid funds to invest. Banks' liquid assets are insufficient to fully fund their investment projects ($m < i$) and banks cannot share projects with each other. As a result, the idiosyncratic investment opportunities create a motive for trade in the secondary market for Mortgage Backed Securities to fund investment projects or satisfy liquidity needs. If the market price of the MBS is low, then few investment projects are funded. Our focus is on the equilibrium behavior of this secondary market and government policies that influence the equilibrium.

(*footnote continued*)
Street Journal, Martin Feldstein said "You cannot solve the overall economic problem if you cannot get the financial institutions lending again. And you cannot do that as long as they do not know what the value [of their troubled assets] is, particularly the residential mortgage-backed securities and derivatives based on them."

There are several key features of the model environment that deserve emphasis because they are crucial for obtaining many of the results. First, the projects require cash. This is essentially saying that a project's inputs must be purchased before the project pays off. Regardless of how productive a project is, it cannot self-finance its operation. Second, to confine attention to the liquidity role of toxic assets, other potential sources of funding are explicitly ruled out.[4] Project owners cannot approach other banks to obtain funds, banks cannot issue debt contracts and neither projects nor banks can issue equity contracts. Third, even though the financial intermediaries are referred to as "banks," they cannot freely borrow funds from the discount window of the central bank. Thus, perhaps a more reasonable interpretation of the financial intermediaries in the model is as members of the so-called "shadow banking sector" – institutions like money market funds and investment banks that play an important role in the allocation of investment funds but that are not member banks of the Federal Reserve System.

3.1. Equilibrium without government policy

We begin by analyzing the equilibrium without government intervention. The effects of government interventions are analyzed in the next section.

Optimization Problem: The bank's decision problem is to decide which mortgage backed securities to sell and whether to invest in its project. Let $q \geq 0$ be purchases of assets on the secondary market. Let x be investment in a bank's project. Investment can be at most i (the scale of the project) and cannot be negative. Investment is further constrained by the amount of liquidity that the bank has. A bank's liquidity includes its initial holding of liquid assets m plus proceeds from net sales of MBS.

Let p be the price for MBS in the secondary market. When a bank sells an asset, it knows the asset's default rate. When they buy, however, banks do not know the default rates of the assets they purchase. Let Δ be the average default probability of the MBS traded. The average default rate Δ is endogenously determined by the equilibrium distribution of asset sales across banks and traders.

To state the bank's optimization problem, notice that if it is optimal for a bank to sell assets of type δ, it must be optimal to sell all assets with $\delta' > \delta$. As a result, attention can be restricted to the choice of an optimal cutoff $\hat{\delta}$. The bank sells all assets with $\delta > \hat{\delta}$. Put differently, $\hat{\delta}$ is the worst asset that the bank chooses to retain. Taking z and Δ as given, a typical bank chooses a cutoff default rate $\hat{\delta}$, investment level x and asset purchases q to maximize

$$R\int_0^{\hat{\delta}} (1-\delta)g(\delta)\, d\delta + R(1-\Delta)q + zx + \left(m + p\left[1-G(\hat{\delta})\right] - x - pq\right) \tag{1}$$

subject to

$$m + p\left[1-G(\hat{\delta})\right] - x - pq \geq 0 \tag{2}$$

$$0 \leq x \leq i \quad \text{and} \quad 0 \leq q. \tag{3}$$

The first term in the objective (1) reflects the expected payoff of the MBS retained by the bank. These assets are the ones with the lowest default rates. The second term reflects the expected payoff from MBS purchased by the bank. By definition, assets sold on the secondary market have an average default rate Δ thus each MBS purchased provides the bank with an expected payoff of $R(1-\Delta)$. The third term is the payoff from funding the bank's idiosyncratic investment opportunity. The last term is simply the bank's remaining cash: its initial liquidity m, plus the proceeds from sales of MBS, less the money spent funding investment, less purchases of MBS. Naturally, these choices will depend on the idiosyncratic return z that the bank has. Denote the solution to the bank's optimization problem with the functions $q(z)$, $x(z)$ and $\hat{\delta}(z)$.

Market Clearing: The model requires the following market clearing conditions in equilibrium. First, the supply of toxic assets sold must be equal to the demand for purchases of toxic assets. Each bank with investment opportunity z supplies all assets with default rates greater than $\hat{\delta}(z)$. Asset demands are not differentiated by default rates since the purchasers cannot distinguish relatively risky assets from relatively safe assets. In addition to the banks' assets, the traders also supply T assets inelastically. Thus, integrating over all banks, the market clearing condition requires

$$T + \int \left[1-G(\hat{\delta})\right] f(z)\, dz = \int q(z) f(z)\, dz. \tag{4}$$

Second, aggregate investment can be no greater than total available liquidity,

$$\int x(z) f(z)\, dz \leq m. \tag{5}$$

Equilibrium: In equilibrium banks behave optimally and markets clear. Attention is confined to settings in which there is sufficient liquidity to achieve the socially optimal investment allocation. The only thing preventing the optimal level of investment is the adverse selection problem in the market for toxic assets.

[4] While we rule out other forms of financing, an alternative (and equivalent) interpretation of the model is one in which banks use their portfolio to write no-recourse repurchase agreements with the MBS serving as collateral.

Since the social opportunity cost of funding any given project is 1, the socially optimal investment allocation requires that all projects with $z>1$ are funded. Denote the socially optimal investment level as $I^* = i[1-F(1)]$. The following assumption ensures that there is sufficient liquidity in aggregate to achieve the social optimum and satisfy the liquidity needs of the traders but that any one bank is liquidity constrained.

Assumption 1. We assume that $i > m > I^* + RT$.

Assumption 1, together with optimal behavior on the part of the banks, has several implications. First, banks never over-invest in equilibrium. That is, no bank invests if $z<1$. Were a bank to undertake a low return project, it could increase its payoff simply by refraining from investing. Second, no bank ever simultaneously sells assets with $\delta < \Delta$ and purchases assets on the secondary market. If a bank were to do so, it could increase its payoff simply by keeping the relatively safe assets and purchasing fewer assets on the secondary market. Finally, if the price of toxic assets p were less than the diversified value of the assets in the secondary market $R(1-\Delta)$, then all banks with $z \le 1$ would strictly prefer to trade their cash for toxic assets. For the market to clear, banks with $z>1$ would have to hold more money than required for their investments (since $m > I^*$). This would not happen in equilibrium, however. Banks with $z>1$ could still invest i while selling fewer high-quality assets. Similarly, if the price of toxic assets were to exceed the diversified value, then no bank would willingly purchase these securities. Thus, if some assets are purchased in equilibrium, $p = R(1-\Delta)$. These observations are summarized in the following Lemma. Proofs are in Appendix A.

Lemma 1. *The following conditions hold in any equilibrium with trade*:

1. *No socially inefficient project is undertaken, i.e.,* $x(z)=0$ *whenever* $z<1$.
2. *Banks that buy assets do not sell assets with below average default rates, i.e.,* $q(z)>0 \Rightarrow \hat{\delta}(z) = \Delta$.
3. *The price of mortgage backed securities is actuarially fair, i.e.,* $p = R(1-\Delta)$.

With Lemma 1, we can treat $p=R(1-\Delta)$. Thus, Δ is sufficient to describe both the price of the mortgage backed securities and the return on these investments. We can now solve the banks' optimization problem without reference to the price p. The following proposition describes the optimal policy for an individual bank taking the endogenous default rate Δ, and its investment opportunity z as given.

Proposition 1. *Taking* Δ *as given, the optimal policy is described by a cutoff rule* $\hat{\delta}(z)$ *in* $[0,\Delta]$, *a purchase function* $q(z)$ *and an investment function* $x(z) \in [0,i]$.

1. *For* $z<1$, $\hat{\delta}(z)=\Delta$, $q(z) \in \left[0, m/(R(1-\Delta))\right]$ *and* $x(z)=0$.
2. *For* $1 \le z \le \bar{z}$, $\hat{\delta}(z) = 1 - z(1-\Delta)$, $q(z)=0$ *and* $x(z) = m + R(1-\Delta)(1-G(\hat{\delta}(z)))$.
3. *For* $\bar{z} \le z$, $\hat{\delta}(z)=\bar{\delta}$, $q(z)=0$ *and* $x(z) = \min\{i, m+R(1-\Delta)\}$.

Here

$$\bar{\delta} = G^{-1}\left(1 - \min\left\{\frac{i-m}{R(1-\Delta)}, 1\right\}\right)$$

and

$$\bar{z} = \frac{1-\bar{\delta}(\Delta)}{1-\Delta}, \quad \bar{z} \in \left[1, \frac{1}{1-\Delta}\right].$$

Only banks with $z<1$ *purchase assets on the secondary market. They are indifferent to the level of purchases.*

Proposition 1 says that banks fall into three categories. First, there are banks without profitable investment projects (i.e., $z<1$). These banks do not need to raise additional liquidity. Instead, they simply sell all of their toxic assets with $\delta \ge \hat{\delta}(z) = \Delta$ on the secondary market. The price they receive for these securities more than compensates them for the sale. These banks are willing to purchase assets sold on the secondary market at the actuarially fair price $p = R[1-\Delta]$.

Second, there are banks with profitable investment projects (i.e., $z>1$) but not so profitable that they are willing to part with their best assets. For these banks, there is a critical default rate $\hat{\delta}(z) = 1 - z(1-\Delta)$ at which the return on holding the marginal asset is equal to the marginal benefit of increasing investment. This cutoff depends negatively on the quality of the investment opportunity. Since $z>1$, the critical default rate is lower than the average default rate, so banks in this group sell above average quality assets to finance their investments. As z increases, banks part with higher-quality assets to increase investment.

Last, there are banks with projects so profitable (or liquidity needs so great) that they invest as much as they can. The critical z at which banks fully fund is $\bar{z}$. If the secondary market value of the toxic assets is sufficiently high, banks can fund their investments without selling all of their securities. This occurs if $R(1-\Delta) \ge i-m$. If the value of the toxic assets is less than $i-m$, then these banks sell all of their securities and fund as much as possible. Define $\hat{\Delta} = 1 - ((i-m)/R)$. For any $\Delta > \hat{\Delta}$, banks with $z > \bar{z}$ will be constrained (that is, they will have $\hat{\delta}(z) = 0$). The cutoff $\hat{\Delta}$ is important for characterizing several of the results below. Equilibria with $\Delta < \hat{\Delta}$ are referred to as *interior* equilibria since banks with $z > \bar{z}$ have $\hat{\delta}(z) = \bar{\delta} > 0$ and thus can fully fund their investments. Equilibria with $\Delta > \hat{\Delta}$ are referred to as *constrained* equilibria since banks with $z > \bar{z}$ have $\hat{\delta}(z) = 0$ and thus cannot fund their projects fully.

Fig. 1 depicts the cutoffs and investment policies as functions of z for a given Δ. The figure is drawn for a default rate for which $\Delta < \hat{\Delta}$ so it is possible for banks to fully fund their investments. For low z, banks simply sell their underperforming assets and do not invest. When $z=1$, banks begin to undertake their investment opportunities. Investment is discontinuous at $z=1$. Specifically, investment jumps from zero to $x(1) = m + R(1-\Delta)(1-G(\Delta))$ since banks are willing to exchange the full value of their liquidity to finance investment at this point. As z increases, the cutoff $\hat{\delta}(z)$ falls and investment rises until $x(z) = i$. To save space, the figure for a constrained equilibrium is omitted. In a constrained equilibrium, $\bar{\delta} = 0$ so $\bar{z}$ would occur where the $\hat{\delta}(z)$ cutoff intersects with the horizontal axis. Moreover, for banks with $z > \bar{z}$, investment is $x(z) = m + R(1-\Delta) < i$. As a consequence, rather than being confined to z's between 1 and $\bar{z}$, there is deadweight loss associated with every z greater than 1.

Given a cutoff function $\hat{\delta}(z)$, define A and B as follows. Let

$$A \equiv \int_0^\infty \left[\int_{\hat{\delta}(z)}^1 \delta g(\delta)\, d\delta \right] f(z)\, dz + T\delta^T \tag{6}$$

be the total number of assets sold on the secondary market that default and let

$$B \equiv \int_0^\infty \left[1 - G(\hat{\delta}(z)) \right] f(z)\, dz + T \tag{7}$$

be total sales of such securities. Recall that the traders supply T assets inelastically with an average default rate of δ^T. We can now define an equilibrium for our model.

Definition 1. A competitive equilibrium without government policy consists of an aggregate default rate Δ and functions $q(z)$, $x(z)$ and $\hat{\delta}(z)$ such that:

1. Taking Δ as given and $p = R(1-\Delta)$, the functions $q(z)$, $x(z)$, and $\hat{\delta}(z)$ maximize (1) subject to the constraints (2) and (3).
2. The functions $q(z)$, $x(z)$ and $\hat{\delta}(z)$ satisfy (4) and (5).
3. The policy functions $q(z)$, $x(z)$ and $\hat{\delta}(z)$ imply Δ; that is, $\Delta = A/B$ where A is given by (6) and B is given by (7).

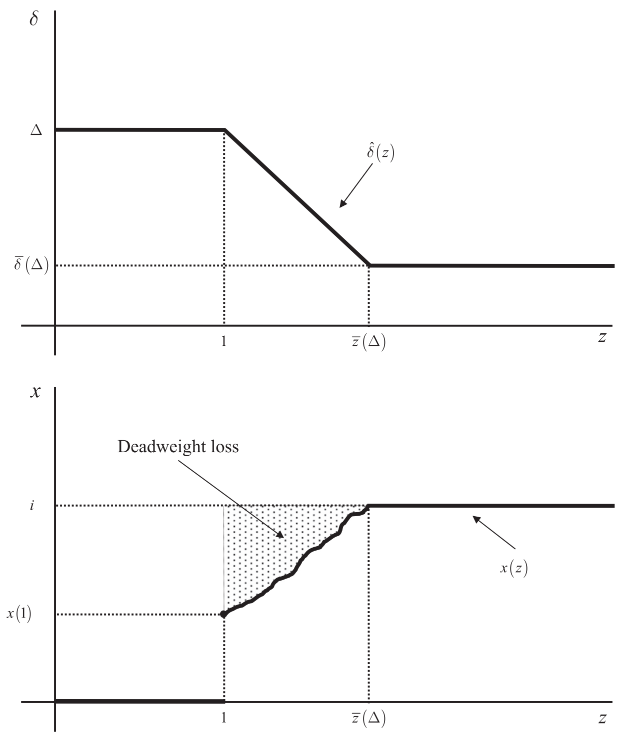

Fig. 1. Cutoff rules and investment decisions.

To prove that an equilibrium exists, we now define a fixed-point mapping in the default rate Δ. Let the mapping be $\Gamma:[0,1]\to[0,1]$. Given $\Delta\in[0,1]$, we have the optimal policies $\hat{\delta}(z)$ given in Proposition 1. Let $A(\Delta)$ and $B(\Delta)$ be given by Eqs. (6) and (7). Notice that A and B are written as functions of the default rate Δ since the optimal cutoff function $\hat{\delta}(z)$ depends on Δ. The fixed point mapping is then

$$\Gamma(\Delta)=\frac{A(\Delta)}{B(\Delta)}. \tag{8}$$

Since $A(\Delta)\le B(\Delta)$, $\Gamma(\Delta)\le 1$.

A fixed point Δ^* of Γ is an equilibrium default rate. Since the distributions F and G have density functions, Γ is continuous on $[0,1]$ provided that $T>0$. If there is a positive measure of traders in the market (i.e., if $T>0$) then Γ is continuous on the closed interval $[0,1]$. However, if $T=0$, the mapping in (8) is not well defined at $\Delta=1$. As $\Delta\to1$, fewer and fewer banks sell good assets and fund projects. One can show that if $T=0$, then $\lim_{\Delta\to1}\Gamma(\Delta)=1$. If $\Delta=1$, then the only assets "traded" on the secondary market would be those with $\delta=1$ and banks would be indifferent to such trades. To accommodate this limiting case, we assume that $\Gamma(1)=1$ when $T=0$. This trivially guarantees that a shutdown equilibrium always exists when there are no traders in the market.

If there are multiple equilibria, they are Pareto-ranked. All banks are better off with lower Δ. Banks sell toxic assets for which $\delta\ge\hat{\delta}(z)$ and the payoff from these sales is decreasing in Δ. Banks that purchase MBS are indifferent to the value of the equilibrium Δ since the price they pay is actuarially fair. That is, $p=R(1-\Delta)$. Thus all banks must be strictly better off with lower Δ.

The following proposition summarizes the existence of equilibria in the model.

Proposition 2. *Given any distributions F and G with densities f and g,*

1. *there exists at least one equilibrium* Δ^*;
2. *if there are multiple equilibria, lower* Δ^* *Pareto-dominate higher* Δ^*;
3. *if* $T=0$ *then* $\lim_{\Delta\to1}\Gamma(\Delta)=1$ *and* $\Delta=1$ *is an equilibrium.*

While it is straight-forward to prove existence of the equilibrium, it is not guaranteed that there is a unique equilibrium. Multiple equilibria may arise because of strategic complementarity in selling good assets. Strategic complementarity is reflected in whether Γ is increasing or decreasing. If a bank anticipates that the average quality of assets on the secondary market is high, the bank is more likely to sell high-quality securities. If banks sell high-quality securities, the average quality on the secondary market will be high. This positive feedback may cause Γ to slope up and may be strong enough to create multiple equilibria.

Consider an interior equilibrium in which banks with $z>\bar{z}$ can fully fund their projects (i.e., for which $\Delta<\hat{\Delta}$). Differentiating $\Gamma(\Delta)$ with respect to the default rate Δ gives

$$\Gamma_\Delta=\frac{1}{B}\left[\int_1^{\bar{z}}\left[\Delta-\hat{\delta}(z)\right]g\left(\hat{\delta}(z)\right)z\,dF(z)-\frac{i-m}{R(1-\Delta)}(\bar{z}-1)\left[1-F\left(\bar{z}(\Delta)\right)\right]\right]. \tag{9}$$

The expression for Γ_Δ has natural meaning in terms of the cutoffs $\hat{\delta}(z)$ shown in the top panel in Fig. 1. When Δ rises, banks with inefficient projects sell fewer assets but the marginal assets they withhold have the average default rate Δ (i.e., $\hat{\delta}(z)=\Delta$ for $z<1$) and so their behavior does not influence the implied default rate. Banks with intermediate z's (between 1 and $\bar{z}$) also respond by withholding marginal assets. These assets are higher quality than the average asset sold and thus their behavior causes the implied default rate to rise. The banks with the best projects (i.e., banks with $z>\bar{z}$) still find it in their interest to fully fund the project. As a result, they willingly sell more good assets (assets with the lowest default rates). This effect works to reduce the implied default rate on the secondary market. Whether Δ rises or not depends on the balance of the high z's (who sell more) and the intermediate z's (who sell less). These two effects correspond to the two terms in (9) above.

In a constrained equilibrium ($\Delta>\hat{\Delta}$), banks with $z>\bar{z}$ cannot fully fund their projects and instead sell all of their liquid assets and fund as much as possible. In this case, the increase in Δ does not make them sell more (since they have no more to sell). The derivative Γ_Δ is therefore missing the negative second term in (9) and thus in a constrained equilibrium, $\Gamma_\Delta>0$ and increases in Δ unambiguously make the pool worse.

The derivative Γ_Δ evaluated at an equilibrium Δ^* governs the stability of the equilibrium. Equilibria in which $\Gamma_\Delta<1$ are referred to as *stable equilibria* and equilibria with $\Gamma_\Delta\ge1$ are *unstable equilibria*. For there to be multiple equilibria, at least one equilibrium must be unstable. One way to eliminate multiple equilibria is to bound the maximum amount of investors on the margin and limit the minimum number of investors in the market. The following assumption provides conditions under which multiple equilibria cannot occur.

Assumption 2. The density g satisfies $g(\delta)<\bar{g}=T/((1-F(1))\sigma^2_{z|z\ge1})$.

The following proposition shows that Assumption 2 is sufficient to guarantee a unique equilibrium.

Proposition 3. *If Assumption 2 holds then the equilibrium is unique and stable.*

Intuitively, the reason multiple equilibria arise is due to the possibility that there may be many banks with intermediate projects that also have an abundance of assets with default rates near the cutoff $\hat{\delta}$. For these banks, when Δ increases, they reduce sales of good assets. If there are many such good assets at the margin, the implied Δ may rise considerably (as seen in the first term in (9)). Assumption 2 places a limit on the number of assets that there could be at the margin.

Comparative Statics: Before turning to the equilibrium with government interventions, we consider some comparative statics. We focus on the equilibrium changes in the default rate Δ in a neighborhood of a stable equilibrium as various underlying parameters change. We consider the effect of changes in internal funds m, the scale of investments i, the solvent repayment R and changes in the distributions F and G. Later, we consider the effect of policy changes on the equilibrium level of investment and welfare.

Consider first how an increase in internal funds m influences the equilibrium. With more internal funds, banks do not need to sell as many illiquid assets. As a result, banks with $z>1$ can afford to sell fewer high-quality assets. Since banks sell fewer assets with relatively low δ's, the average default rate on the secondary market rises.

When i increases, banks need more resources to fully fund any given project. Banks with profitable investment opportunities that wish to fully fund their projects must now sell more high-quality assets on the secondary market. (Banks with profitable investment projects that only partially fund their projects are not affected by an increase in the scale of the project.) As a result, the average quality in the pool increases and the equilibrium default rate falls.

When R increases, the value of MBS increases and thus banks get more money when they sell assets. As a result, banks do not need to sell as many high-quality assets. With fewer high-quality assets on the market, the average quality in the secondary market falls and the equilibrium default rate increases. The following proposition summarizes this discussion.

Proposition 4. *Let the distributions F and G be given and let Δ^* be a stable equilibrium (i.e., $\Gamma_\Delta(\Delta^*)<1$). Then, in the neighborhood of Δ^*, the equilibrium average default rate of assets sold in the market is increasing when there is*

1. *an increase in internal funds m,*
2. *a decrease in the scale of investment projects i, or*
3. *an increase in the mortgage rate R.*

We also consider the effects of changes in the distribution of investment projects F and the distribution of default rates G. Consider distributions that are ranked according to first-order stochastic dominance (Rothschild and Stiglitz, 1976).[5] Suppose F' first-order stochastically dominates F. More banks have profitable investment opportunities under F' compared to F and thus supply more high quality assets in the secondary market. As a result, the equilibrium default rate must be lower under F' than under F.

Suppose G' first-order stochastically dominates G. In this case, there are relatively more mortgages with high default probabilities. As a result, there are more low-quality assets in the secondary market and the equilibrium default rate is higher under G' than under G. The following proposition summarizes this discussion.

Proposition 5. *Let Δ^* be a stable equilibrium given F and G and suppose F' first-order stochastically dominates F and G' first-order stochastically dominates G. Then, in the neighborhood of Δ^*,*

1. *holding G fixed, the equilibrium default rate is lower under F' than under F and,*
2. *holding F fixed, the equilibrium default rate is higher under G' than under G.*

Proposition 5 shows that the market for toxic assets becomes less liquid when there is an exogenous reduction in the profitability of investment projects across banks. Since banks have less reason to sell high quality assets, the average quality of assets falls and the market becomes less liquid. This is an additional mechanism for generating financial crises in markets for toxic assets.

Aggregate Investment and Welfare: We close this section with a brief discussion of aggregate investment and social welfare. Given the default rate, aggregate investment is simply the sum of investment across banks, that is,

$$I(\Delta)=\int_0^\infty x(z)f(z)\,dz. \tag{10}$$

[5] Recall that F' first-order stochastically dominates F iff $F'(x)\le F(x)\forall x$. A simple example of first-order stochastic dominance would be a rightward shift of the distribution.

Social welfare is directly related to aggregate investment. Since the social opportunity cost of any investment x is $1 \times x$, aggregate welfare is simply

$$W(\Delta) = \int_0^{\infty} (z-1)x(z)f(z)\,dz. \tag{11}$$

Maximizing welfare requires setting $x(z) = i$ whenever $z > 1$. The allocation of investment is inefficient because banks with intermediate values of z do not fully fund their projects even though it is socially optimal to do so. Specifically, banks with z between 1 and $\overline{z}$ choose $x(z) < i$ whenever $\Delta > 0$. Thus, $I(\Delta) < I^*$ and $W(\Delta) < W^*$. The shaded area in Fig. 1 is the deadweight loss in equilibrium.

The welfare criterion in (11) deserves some additional comment. Adverse selection models like the one here are often constrained efficient (see Bigelow, 1990 for a precise analysis). Whether allocations can be improved by policy interventions is therefore a delicate matter. To avoid these complications, we imagine either that (a) the banks are owned by a representative agent who cannot transfer funds across banks directly or (b) prior to the realization of the idiosyncratic z's, the banks each pay a lump sum tax to the government for the purpose of the policy intervention. Under either interpretation (a) or (b) the (ex ante) welfare criterion in (11) is appropriate.

4. Policy

We now consider the effects of policies in our model. We focus on two types of policies: equity injections and asset purchases. These were the two most prominent policy options considered during the financial crisis of 2007–2009. Equity injections are modelled as an increase in m while (as demonstrated below) asset purchases are treated as a decrease in Δ.

Presumably both types of policies will be costly for the government. Asset purchases will be costly if the government pays more for assets than their "fundamental value." Equity injections will be costly if the government pays too much for the value of the equity.

We imagine that the government has potentially two separate objectives. First, the government might be concerned with the level and allocation of investment. Second, the government might be concerned with the functioning of the interbank market itself.

4.1. Equity injections

We model equity injections as a direct increase in m. This could be due to a forced equity injection, a subsidized government loan, or a direct transfer of funds from a government agency. Equity injections have two separate effects on allocations. First, by providing banks with additional funds, there is more investment for any fixed Δ. This direct effect increases both aggregate investment and welfare. Second, equity injections may have an indirect effect that arises due to changes in the equilibrium default rate Δ. Unlike the direct effect, this indirect effect reduces investment and welfare. As in Proposition 4, increase in m causes Δ to rise because banks rely less on the proceeds of their asset sales. This further contaminates the secondary market and reduces the exchange of toxic assets.

To see these effects, write aggregate investment I as an implicit function of internal funds,

$$I(m, \Delta(m)) = \int_1^{\infty} x(z, \Delta(m), m)f(z)\,dz. \tag{12}$$

Now consider an incremental increase in internal funds $dm > 0$. We analyze interior equilibria ($\Delta < \hat{\Delta}$) and constrained equilibria ($\Delta > \hat{\Delta}$) separately.

In the first case ($\Delta < \hat{\Delta}$), differentiating (12) with respect to m gives,

$$\frac{dI(m, \Delta(m))}{dm} = [F(\overline{z}) - F(1)] + I_{\Delta}\Delta_m \tag{13}$$

where $I_{\Delta} \equiv \partial I(m, \Delta)/\partial \Delta$ and $\Delta_m \equiv \partial \Delta / \partial m$.

Taking Δ as given, banks with low z ($z < 1$) do not change their behavior at all in response to the increase in m. They do not need the extra funds so the new liquidity remains idle and there is no effect on the interbank market or investment arising from these banks. Banks with intermediate projects ($1 \le z \le \overline{z}$) do not change their asset sales ($\hat{\delta}(z)$ is constant) but instead use the additional funds to expand investment. For these banks, each dollar of additional equity directly results in an extra dollar of investment. The increase in investment is directly proportional to the number of projects in this range, $F(\overline{z}) - F(1)$. This effect is captured by the first term in (13).

Banks with $z > \overline{z}$ make no change in investment but instead change their asset sales. Prior to the equity injection, they were selling high-quality assets sufficient to fund their investments. After the equity injection, these banks still finance their projects but now sell fewer high quality assets. This second effect is captured by the term $I_{\Delta}\Delta_m$. By Proposition 4, this effect must be negative. We refer to this side-effect of equity injections as the *contamination effect*. The contamination effect has a detrimental impact on the interbank market whenever $\Delta < \hat{\Delta}$ (i.e., whenever $\overline{\delta} > 0$) and implies that the increase in investment is less than the direct effect $F(\overline{z}) - F(1)$.

Fig. 2 shows the effects of an equity injection on the credit markets (the top panel) and on investment (the bottom panel). The upward shift in the investment curve (from $I(m_1, \Delta)$ to $I(m_2, \Delta)$) in the lower panel is the direct effect of having more funds. The movement along the $I(m_2, \Delta)$ curve from Δ_1 to Δ_2 is the contamination effect.

In the constrained case ($\Delta > \hat{\Delta}$), differentiating (12) with respect to m gives

$$\frac{dI(m, \Delta(m))}{dm} = [1 - F(1)]. \tag{14}$$

As before, given Δ, banks with low z's do not change their behavior. Unlike the first case however, all banks with projects $z \geq 1$ use the additional funds to expand investment dollar-for-dollar. Since all banks with $z \geq 1$ are liquidity constrained, the increase in investment is proportional to $1 - F(1)$. Unlike the case with $\Delta < \hat{\Delta}$, there is no contamination effect. The contamination effect arises when banks with $z > \overline{z}$ continue to finance their projects but sell fewer good assets. If $\Delta > \hat{\Delta}$, however, the banks with $z > \overline{z}$ are also constrained (they are already selling all of their assets). These banks continue to sell all of their assets after the equity injection. Thus, there is no contamination effect and the equity injection neither improves nor hinders the interbank market. The following proposition summarizes the discussion above.

Proposition 6. *Let Δ^* be a stable equilibrium (i.e., $\Gamma_\Delta(\Delta^*) < 1$) and consider an increase in internal funds $dm > 0$. In the neighborhood of Δ^*, the new equilibrium always features (weakly) greater toxicity in the secondary market (the new equilibrium always has a weakly higher default rate Δ). Also,*

1. *if the initial $\Delta^* > \hat{\Delta}$, the change in investment is $[1 - F(1)]\, dm$ and there is no improvement in the interbank market for toxic assets (the new equilibrium is $\Delta = \Delta^*$).*
2. *if $\Delta^* < \hat{\Delta}$ then the change in investment is less than $[F(\overline{z}(\Delta^*)) - F(1)]$ and the interbank market for toxic assets becomes less liquid (the new equilibrium is $\Delta > \Delta^*$).*

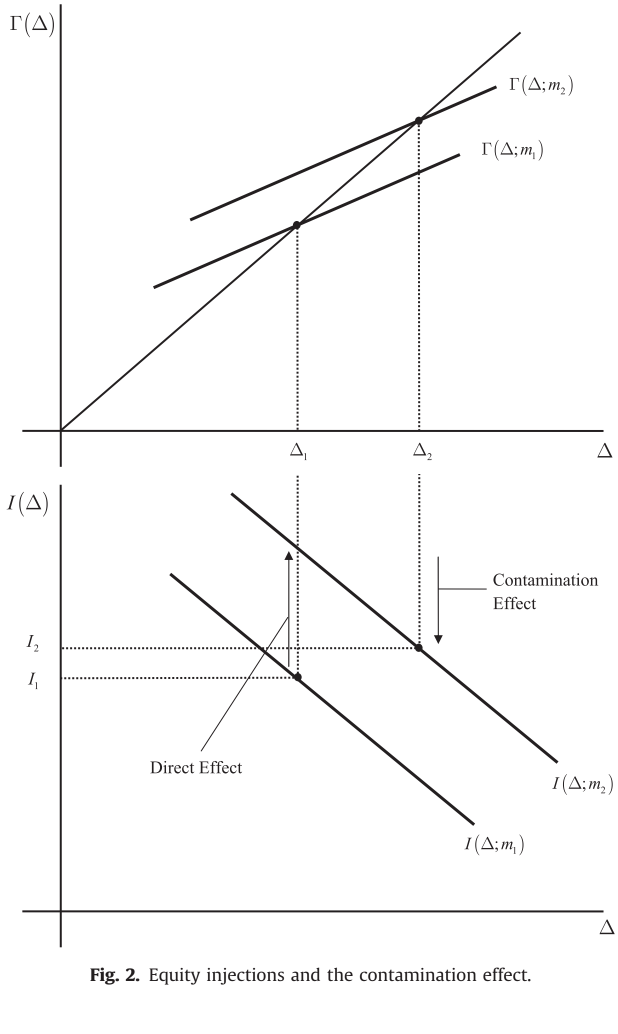

Fig. 2. Equity injections and the contamination effect.

The equity injection considered above provided an equal amount of new cash assets to all banks regardless of their liquidity needs. One might think that an equity injection targeted to banks with the best projects would be more effective than a policy which increases equity indiscriminately. In fact, in an interior equilibrium, an equity injection targeted toward the banks with the best projects is worse than an indiscriminate equity injection to all banks. Eq. (13) shows that investment only rises due to the increased investment of the banks with intermediate projects (those with $1<z<\overline{z}$). If the new equity were to go only to banks with $z>\overline{z}$, only the second term (the contamination effect) in (13) would be present and investment, welfare and liquidity would all decrease. In a constrained equilibrium, banks with the highest z's are also constrained and thus targeting these banks would be desirable. (Notice that directing funds to banks with high z's does not mean that funds are necessarily being directed to banks with the highest marginal value of additional investment. In the unconstrained equilibrium, banks with $z<\overline{z}$ have high marginal values of investment while the marginal value of investment for banks with $z>\overline{z}$ is zero.)

The Cost of Equity Injections: One might think that the cost to the government of a cash injection might be equal to the dollar value of the injection itself. This is incorrect. Cash injections like the ones undertaken in response to the financial crisis do not entail one-for-one costs. That is, an increase in liquidity of dm in our model might only cost tax payers $\xi\, dm$ where the per dollar cost of equity injections is given by the parameter $\xi \in [0,1]$.[6]

The actual equity injections undertaken by the TARP funds were initially seen as moderately costly. A 2009 CBO report estimated that the average subsidy associated with the equity purchases through the Capital Purchase Program (CPP) was roughly 18 percent (see Congressional Budget Office, 2009a,b). Additional equity purchases (outside the CPP) of AIG and Citigroup were estimated to be at prices that exceeded the fair value of the equity by 53 percent and 26 percent respectively. Veronesi and Zingales (2010) calculate that the cost of the 2008 U.S. equity injection was between 20 cents and 35 cents per dollar.

4.2. Asset purchases

The original design of the TARP called for the government to purchase toxic assets and take them off of the banks' balance sheets. The idea was that such an asset purchase would restore liquidity to the inter-bank market provided that it was (according to Secretary Paulson) "properly designed and sufficiently large." In this section we consider the effects of an asset purchase policy in our model.

Before turning to the formal analysis, notice that there is some reason to anticipate that an asset purchase program might work well in the adverse selection environment. Like the equity injection, an asset purchase program transfers funds to banks to the extent that the government overpays for the assets. Unlike an equity injection, however, banks must sell assets to get the transfer. This increased incentive to trade might well be expected to have beneficial effects. The implicit transfer from an asset purchase program would automatically be concentrated on banks that sold the most assets – presumably the banks with the greatest liquidity needs.

Suppose the government takes actions that lower Δ by an amount $d\Delta<0$. How costly is this policy? There are two ways to look at this. One way to think about such a policy is to imagine that the government provides a subsidy for sales of toxic assets (a negative "Tobin Tax").[7] The cost of such a policy would be the magnitude of the subsidy times the number of assets sold in equilibrium. A second way of thinking about such a policy is that the government purchases assets in sufficient amounts to increase the price. The costs to the government are the same in both cases though the initial out-of-pocket costs to the government are greater in the second case since the government needs to purchase effectively all of the assets sold on the market. We proceed with the second interpretation since it more accurately describes the policy interventions actually considered during the financial crisis.

If the asset purchase program succeeds, there will be a difference between the actual quality of the loan pool and the price of securities. In particular, $p>R(1-\Gamma(\Delta))$. This was exactly the concern voiced by many academic economists when the TARP and the P-PIP were unveiled. In our model, for the asset purchase program to work, the price *must* exceed the fair market value of the securities. Because the price will typically be higher than what would be justified by the underlying value of the securities, no private banks will purchase assets – only the government will be on the demand side of the market.

The fact that $p>R(1-\Gamma(\Delta))$ presents a slight technical problem because Δ can no longer be used to simultaneously describe both the average quality in the loan pool and the price in the secondary market.[8] We proceed by using Δ to describe the effective price to sellers. That is, if a bank sells an asset in the secondary market, it receives a price equal to $R(1-\Delta)$. We make this choice so we can continue to use the solutions in Proposition 1. The actual quality on the secondary market will be

6 If an equity injection could be achieved with zero budget cost ($\xi=0$) then a simple way of ensuring optimal investment would simply be to transfer i to the banks. Such a strategy would maximize the contamination effect. The only reason a bank would sell a toxic asset would be to get rid of assets with below average quality. In equilibrium, the secondary market for toxic securities would shut down completely.

7 Policies that subsidize trade are often beneficial in adverse selection environments. See House and Zhang (2012) for examples of how hiring taxes and subsidies can be used in labor markets with adverse selection. More recently, Guerrieri and Shimer (2014) consider a dynamic adverse selection model and suggest a direct trade subsidy as welfare improving policy.

8 Among other things it requires a modification to the definition of an equilibrium. The necessary modifications are natural. This modified definition is omitted to save space.

given by $\Gamma(\Delta) \neq \Delta$. It is natural to expect $\Gamma(\Delta) > \Delta$ since the government will want to increase the price of securities in the secondary market.

While we could analyze the program by starting with the size of the purchase, we instead analyze the program as if the government were choosing Δ directly. That is, we act as though the government chooses the price on the secondary market and then derive the amount of purchases that would imply that price.

Investment and Welfare: The effects of an asset purchase can be seen by differentiating (10) and (11) with respect to Δ in a neighborhood of an equilibrium $\Delta^* = \Gamma(\Delta^*)$. The effects naturally depend on whether the market is constrained or not. If $\Delta < \hat{\Delta}$ so that the banks with the highest z's can afford to fully fund their investment projects, an incremental change in Δ has the following effects on investment and welfare:

$$I_\Delta = -\int_1^{\bar{z}} R\left[(1-G(\hat{\delta}(z)))+(1-\Delta)g(\hat{\delta}(z))z\right]f(z)\,dz, \tag{15}$$

and

$$W_\Delta = -\int_1^{\bar{z}} (z-1)R\left[(1-G(\hat{\delta}(z)))+(1-\Delta)g(\hat{\delta}(z))z\right]f(z)\,dz. \tag{16}$$

The change in Δ influences only the banks with intermediate investment projects (the other banks are infra-marginal). For these banks, the change in Δ has two separate impacts on investment. First, as Δ increases, the value of their existing asset sales falls. This is captured by the first terms in the integrals in the equations above. Second, as Δ increases, these banks substitute away from selling high-quality MBS. There are $g(\hat{\delta}(z))$ such securities at the margin. Thus, in addition to the declining value of their sales, banks also choose to sell fewer assets. Both effects work to reduce investment and welfare as Δ increases.

If $\Delta > \hat{\Delta}$ the effects on investment and welfare are given by

$$I_\Delta = -R\left[\int_1^{\bar{z}} \left[(1-G(\hat{\delta}(z)))+(1-\Delta)g(\hat{\delta}(z))z\right]f(z)\,dz + \int_{\bar{z}}^{\infty} f(z)\,dz\right] \tag{17}$$

and

$$W_\Delta = -R\left[\int_1^{\bar{z}} (z-1)[(1-G(\hat{\delta}(z)))+(1-\Delta)g(\hat{\delta}(z))z]f(z)\,dz + \int_{\bar{z}}^{\infty} (z-1)f(z)\,dz\right]. \tag{18}$$

As before, the change in Δ influences investment and welfare by reducing the value of sales and the number of sales for the intermediate banks. Unlike the previous case however, the change in Δ now affects the infra-marginal banks as well (banks with $z > \bar{z}$). While these banks do not change the number of assets being sold (they are infra-marginal), an increase in Δ does reduce the value of their sales. As before, these effects work to unambiguously reduce investment and welfare.

Typically, asset purchases would be expected to increase the market price of MBS and reduce Δ. If the asset purchase succeeded in increasing the price, investment and welfare would both increase according to either (15) and (16) if $\Delta < \hat{\Delta}$ or (17) and (18) if $\Delta > \hat{\Delta}$.

Proposition 7. *A decrease in Δ unambiguously increases welfare and investment.*

The Cost of Asset Purchases: If the government purchases all toxic assets sold on the secondary market, its outlays are $RB(1-\Delta)$. The true value of the purchase is $RB(1-\Gamma(\Delta))$ so the net cost of the policy is $RB(\Gamma(\Delta)-\Delta)$.

At an equilibrium ($\Gamma(\Delta) = \Delta$) the net impact on the government budget is zero. However, if the subsidy causes $\Gamma(\Delta)$ to deviate from Δ (which must occur if $\Gamma_\Delta \neq 1$) then the costs are positive or negative depending on whether $\Gamma_\Delta < 1$ (as in a stable equilibrium) or $\Gamma_\Delta > 1$ (as in an unstable equilibrium). For our analysis, we assume that the equilibrium is stable.[9]

The budget cost of causing a change in Δ (by an amount $d\Delta$) is $RB[\Gamma_\Delta - 1]d\Delta$. The term in square brackets is negative provided that the equilibrium is stable. Presumably, the government will seek to reduce Δ, thus $d\Delta < 0$ and the overall cost of the asset purchase program is positive. As the positive feedback becomes stronger (as $\Gamma_\Delta \to 1$), the budget impact of the asset purchase is smaller.

An Alternate Asset Purchase Specification: From a modelling perspective, our treatment of asset purchases has some drawbacks. Perhaps the most glaring drawback is that, for the policy to be effective, the government needs to "take over" the entire demand side of the market. This feature can be eliminated by modifying the model to include two periods.[10] In the modified model, period 1 is exactly the same as the model analyzed in Section 3 without policy. In period 2, there are no investment projects, but the government purchases *some* assets. Specifically, for each asset, there is a probability θ that the government makes a purchase offer. The government offers to purchase assets at the same price p that prevails in period 1.

[9] If the equilibrium is unstable (i.e., if $\Gamma_\Delta > 1$) then the government can effectively raise revenue by purchasing toxic assets. In this case, asset purchases have such a strong positive feedback effect that the implied default rate falls by more than the reduction in Δ caused by the asset purchase.

[10] See Appendix B for a detailed discussion of this modification.

Any bank in possession of a MBS in period 2 know its type when it chooses whether to sell the asset. (Banks learn the type of assets purchased in period 1 before trade in period 2 occurs.) Banks that possess underperforming assets may be able to unload these assets in the second period. This option increases the value of the toxic assets and encourages trade in period 1. The policy does not require that the government take over the demand side of the model to be effective.

Intuitively, as the purchase probability θ increases, the number of assets purchased in period 2 rises and the price in period 1 increases. As a result, as it did in the one period model, the asset purchase program bids up the price and increases investment and welfare. Unlike the one period model, in the two period model the volume of trade in period 1 also increases as the government purchases more assets. Naturally, the cost to the government from the asset purchase program is also increasing in θ. One can show that in a neighborhood of $\theta = 0$ the two modeling specifications (the two-period model and the original one-period model) imply identical effects on investment and welfare for a given cost to the government.

5. Discussion and related literature

Numerical Illustrations: Using a numerical version of the model in Section 3 allows us to make simple direct comparisons between liquidity injection policies with asset purchase programs. The numerical model also allows us to present examples in which the solution switches from an interior equilibrium to a constrained equilibrium. Parameter values and functional forms are chosen for illustrative purposes only. Assume that F is a lognormal distribution and G is a truncated lognormal distribution (G receives no weight outside $[0, 1]$). The mean default rate (μ_δ) is set to 0.2 (comparable to actual default rates of collateralized debt obligations during the crisis) and the standard deviation of the default rates (σ_δ) is 0.1.[11] At these parameter values, the equilibrium default rate is roughly 31 percent – substantially greater than the average default rate in the portfolio (20 percent).

To compare asset purchases to equity injections, we calculate the critical cost of funds ξ^* at which an equity injection increases investment by exactly the same amount as an *equally costly* asset purchase program.[12] If $\xi > \xi^*$ then asset purchases are a more effective policy tool than liquidity injections. (Obviously if $\xi = 0$ then equity injections can achieve the first best allocation at no cost.) For the baseline parameter values, this critical cost of funds is 0.04. This is quite low in comparison to the 0.20–0.35 estimates by Veronesi and Zingales (2010).

Fig. 3 plots the equilibrium default rate (Δ), volume of trade (B), investment (I) and the critical cost of equity (ξ^*) as the level of internal funds and the standard deviation of default rates are varied. The shaded region corresponds to parameter combinations at which the equilibrium is constrained. While the critical cost of equity injections is fairly low overall, it rises substantially in some cases. In particular, ξ^* tends to be higher in constrained equilibria. This is not surprising since the contamination effect is not operative in the constrained case. One interpretation of this finding is that more consideration should be given to equity injections when the interbank market is most constrained. While asset purchases are a surprisingly viable policy option overall, they have their greatest benefits in cases where the market is not completely constrained.

Solvency versus Liquidity: The most important real-world feature that our model abstracts from is solvency. By eliminating solvency concerns we have removed what is arguably the most pressing rationale for increasing bank equity. Solvency may also interact with the lack of “mark-to-market” valuation in financial markets. In reality, if a bank were to sell a toxic asset, the low realized value from the sale could make the bank insolvent. Because these assets were listed at face-value on the bank's financial statements, banks had a strong incentive not to sell toxic assets. Thus, adding solvency concerns together with the lack of mark-to-market accounting might introduce a separate channel through which equity injections could increase liquidity.[13]

Solvency also may have played an important role in adding to the liquidity needs of participants in the interbank market. During the crisis (particularly after the failure of Lehman Brothers in September 2008), many financial institutions experienced sudden large withdrawals that many observers compared to traditional bank runs by creditors trying to withdraw funds before a possible bankruptcy. Even though our model cannot directly analyze bank run behavior, the model does point to potentially important channels through which runs might be affected. For instance, shocks that reduce liquidity and asset prices in the interbank market may increase the likelihood of runs on other financial institutions holding toxic assets in their portfolio. That is, a reduction in liquidity in the secondary market can contribute to an insolvency problem by any institution holding these assets as investments. On the other hand, a run that applies directly to the financial institutions in our model would (either by reducing m or increasing the average z drawn by the banks) actually increase

[11] The remaining parameter values are set as follows: $m = 0.6$, $R = i = 1$, mean z (μ_z) is 1, the standard deviation of ln z ($\sigma_{\ln z}$) is 0.3. See Appendix C for more details on the numerical model and its solution.

[12] This cutoff can be calculated as

$$\xi^* = \frac{dI}{dm}\left(\frac{\partial I}{\partial \Delta}\right)^{-1} \frac{RB[\Gamma_\Delta - 1]}{1+T}.$$

Here dI/dm is the total derivative (including the contamination effect and the direct effect). All derivatives are evaluated at the equilibrium. A similar cutoff can be calculated for welfare rather than investment. The two cutoffs are typically very close to each other. To save space, we report only the investment cutoff.

[13] We thank Richard Froyen for emphasizing this point.

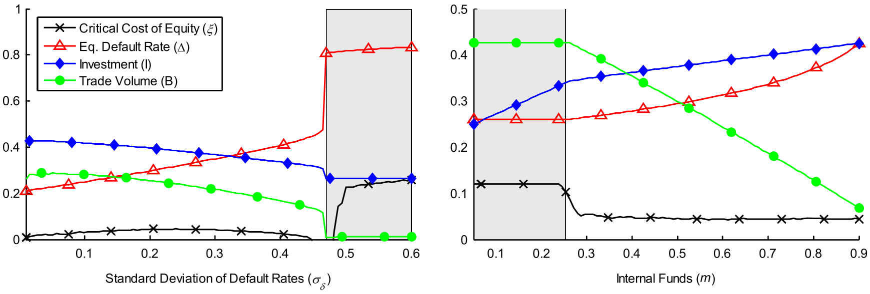

Fig. 3. Effects of parameter variations (default distribution G).

liquidity in equilibrium by encouraging firms to trade toxic assets of relatively higher quality. This increased liquidity is perhaps a beneficial side effect of what is otherwise a bad shock.[14]

Causes of Financial Crisis: There are many possible causes of financial crises that can be accommodated in our model. The deterioration of financial markets in the 2007 crisis was precipitated by sharp losses in subprime MBS. Moreover, housing loans, and derivative securities written on them, had sharply differing expected payouts. These effects could be accommodated in our model by a change in the distribution G. Prior to the crisis, MBS were highly liquid and banks perceived the default risk for these securities to be relatively low.[15] This could be represented by a distribution placing most (or all) mass on default rates near zero. As the crisis unfolded, MBS became illiquid and banks perceived the default risk to be substantially higher. This could be represented by a distribution that was stretched to the right relative to the initial distribution. According to Proposition 5, trade and liquidity in the interbank market would both fall. The market price of MBS and investment would also fall relative to pre-crisis levels.

Another potential factor contributing to the financial crisis is a reduction in the number of profitable investment opportunities. This could also be accommodated by the model. Specifically, a reduction in profitable investment projects could be represented by a leftward shift in the distribution F. Again, according to Proposition 5, this shift would result in reduced liquidity and lower asset prices.

Related Literature: Our paper adds both to the literature on adverse selection in financial markets and to the literature on the financial accelerator. Both literatures are very large, so we cannot provide a full summary here.[16] Instead, we briefly mention some recent papers that make contributions closely related to those in our paper.

Several papers emphasize the amplifying effects of adverse selection in macroeconomic contexts. Eisfeldt (2004) considers an optimal consumption allocation model in which financial assets are illiquid due to adverse selection. When productivity rises, agents want to invest more and thus sell more high-quality assets. This improves quality in the asset market and amplifies the productivity shock. Kurlat (2013) also emphasizes the amplifying role of adverse selection. In his model, investors sell old projects to finance new ones. Adverse selection in existing projects thus impedes investment. In Kurlat's model, learning about an assets' types slows down in recessions making adverse selection distortions worse. More recently, Bigio (2011) uses a numerical model of adverse selection in asset markets to quantitatively analyze the effects of illiquidity and the real effects of financial crises. In his model, losses to financial institutions make adverse selection problems worse. As a result, the recovery from a financial crisis takes a long time as these institutions slowly reaccumulate their net worth.

Philippon and Skreta (2012) focus on optimal policy in markets where the value of collateral is private information. Building on Myers and Majluf (1984), they consider a mechanism design setting in which banks finance investments by issuing claims on future payoffs including the future value of their toxic assets. Thus, in their setting, collateral is toxic. Their main focus is on the stigma attached to a bank that chooses to participate in a government program. In their model, markets conclude that participating financial institutions are bad risks. This negative stigma gives banks an incentive to refuse the government program as a way of signaling that they are a high type. Tirole (2012) also uses mechanism design to analyze market failure due to adverse selection. In his model, the optimal mechanism is designed to remove the worst assets from the market and leave only the best ones. Like our model, the government must overpay for toxic assets for the policy to work. Chari et al. (2011) study an adverse selection model in which banks may retain assets to signal quality to the market. Like the environment we study, if the government in the Chari et al. model is willing to incur losses on its trades, asset purchases can improve allocations.

Uhlig (2010a, 2010b) extends the Diamond and Dybvig (1983) model to analyze runs on "core" financial institutions. In his model, if banks are more distressed then the adverse selection problem is reduced because more high-quality assets are

[14] See Diamond and Dybvig (1983) for the original analysis of equilibrium bank runs. See Cooper and Ross (1998) and Ennis and Keister (2006) for recent updated versions of the bank run model.

[15] See Bachmann and House (2014) for more discussion. Much of the information on default rates for CDO's comes from Beltran et al. (2013) and Cordell et al. (2012). See also Barnett-Hart (2009). Similarly, Adrian and Shin (2009, 2010) emphasize the role that changes in balance sheets have on the ability of market participants to provide liquidity.

[16] For the literature on adverse selection in financial markets, see Stiglitz and Weiss (1981), De Meza and Webb (1987) and Mankiw (1986) (and obviously Akerlof, 1970). For the literature on the financial accelerator, see Bernanke and Gertler (1989), Bernanke et al. (1999) and the references therein.

sold. As in our model, asset purchases at above-market prices improve efficiency. Malherbe (2014) argues that financial intermediaries may strategically hoard liquidity prior to trade if they anticipate severe adverse selection. The channel through which this occurs is again closely tied to the contamination effect. As firms hoard more liquidity, they reduce their reliance on the secondary asset market and exacerbate the adverse selection problem – a potential unintended consequence of cavalierly allowing financial institutions easy access to liquid funds.

Using a search-theoretic framework, Camargo and Lester (2014) study the rate at which trades occur in markets for toxic assets and also whether policies can reduce the amount of time such markets remain "frozen." In their model, buyers wait for the most desperate sellers to accept low offers and clear out of the market. As a result, the quality of assets available for sale in the market rises over time. Eventually, such frozen markets will "thaw." However, if adverse selection is strong, it may take many periods for the market to clear. They then consider the effects of government assisted asset purchases in their environment. In their program (which is designed to be similar to the actual P-PIP), the government accepts some of the downside risk of asset purchases. This program has complex effects on the market. On one hand, the program increases the willingness to purchase toxic assets. On the other hand, it may also increase the incentive to hold-out for a high price.

Bolton et al. (2011) consider a model of the interbank market for liquidity in which banks gradually learn about the quality of assets that they originated. As the banks learn, there is an accumulation of asymmetric information. The key issue in the Bolton, Santos and Scheinkman paper is the decision of when to sell an asset to acquire liquidity. In a related paper, Camargo et al. (2014) show that government interventions similar to the P-PIP encourage traders to acquire information about the assets they are buying. The optimal amount of implicit government insurance offered through the P-PIP balances the gains from acquiring information with the costs that arise from moral hazard.

6. Conclusion

When the value of mortgage backed securities dropped sharply in 2007, banks and financial intermediaries faced a pronounced funding shortage. Many banks with MBS could not find willing buyers at fair prices and as a result could not raise the funds needed to finance investments. We have presented a model of an interbank market for toxic assets. In the model, the funding shortage arises from an adverse selection problem. Assets traded on the market are of the lowest quality, which creates a severe liquidity and funding crisis and reduces investment and welfare.

The model is used to analyze two types of government interventions: equity injections and asset purchases. While equity injections typically increase investment, they also have a negative side-effect. Allowing banks to have direct access to cash in the form of equity or liquidity injections endogenously reduces liquidity in the interbank market. Because banks have greater access to internal funds, they sell fewer high-quality assets. By reducing the number of high-quality assets traded, equity injections further contaminate the interbank market, reducing prices and liquidity. This contamination effect limits the effectiveness of direct equity or liquidity injections. In an extreme case, if equity injections are directed to banks with the greatest liquidity needs, the contamination effect causes investment and welfare to fall.

In contrast, by increasing the price of toxic assets, asset purchase programs increase liquidity in the interbank market and also increase investment. By bidding up the price of toxic assets, asset purchase plans effectively transfer more funds to the banks that need them the most. At the same time, to get the transfer, banks have to continue to sell high quality assets.

The conclusions of our analysis depend directly on adverse selection as the source of the financial friction. Obviously, there were many other factors operating in 2008 so our analysis cannot be viewed as a complete evaluation of the policies considered by the Treasury and the Federal Reserve. While it is not comprehensive, our analysis has identified an important channel that should be part of the evaluation of policies designed to improve liquidity in interbank lending markets.

Acknowledgments

We are indebted to Huberto Ennis and to an anonymous referee for their constructive input. We also thank George Akerlof, Robert Barsky, John Bryant, Anusha Chari, Richard Froyen, Laurien Gilbert, Lutz Hendricks, Lutz Kilian, Uday Rajan, Collin Raymond, Matthew Shapiro and seminar participants at Boston College, The University of California at Berkeley, The University of Michigan, The University of North Carolina at Chapel Hill, Southern Methodist University, the Federal Reserve and the IMF for insightful comments and suggestions.

Appendix A. Supplementary material

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2014.10.001.

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