A Theory of Reference Point Formation

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Abstract

We introduce a model of reference-dependent choice where the reference point is endogenously determined through maximization of a conspicuity ranking. This subjective ranking captures how eye-catching the alternatives are in relation to each other. The most conspicuous alternative in a choice set serves as its reference point and in turn, determines the reference-dependent utility the decision-maker will maximize to make a choice. We show that this *conspicuity based endogenous reference model* (CER) is characterized by an intuitive and simple behavioral postulate, called Single Reversal, and we discuss how choice data can be used to reveal information about CER's parameters. We additionally analyze special cases where a reference-free utility function, combined with psychological constraints, is used to make reference-dependent choices.

Keywords: Conspicuity, Reference Point Formation, Reference Dependence, Psychological Constraints, Revealed Preference, Choice Reversal

JEL Codes: D03, D83

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1 Introduction

Starting with the seminal works of Markowitz (1952) and Kahneman and Tversky (1979), the idea of reference-dependence has played a very significant role in economics. Numerous empirical and experimental studies have documented that choices are often reference dependent With this motivation, researchers have developed a variety of theoretical models in which an exogenously given reference point affects choice behavior. However, with the exception of a few studies, this literature remains silent on how the reference point is determined. This has been recognized as a major drawback (*e.g.* see Markowitz (1952), Tversky and Kahneman (1991), Levy (1992), Wakker (2010), Barberis (2013)). For example, Wakker (2010, p. 245) argues that *"If too much liberty is left concerning the choice of reference points, then the theory becomes too general and is almost impossible to refute empirically. It does not then yield valuable predictions."* In other words, a full-blown theory of reference-dependence necessitates a theory of reference point formation.

Many studies informally relate determination of a reference point to some notion of conspicuity (or equivalently, salience) and argue that in a choice set the "most conspicuous alternative" becomes the reference point (Brickman, Coates, and Janoff-Bulman (1978), Samuelson and Zeckhauser (1988), Pratkanis (2007), DellaVigna (2009), Larrick and Wu (2012), Bhatia and Golman (2015), Bhatia (2017)). To quote Bhatia and Golman (2015), *"reference points are merely options that are especially salient to the decision maker."* For example, when purchasing an airline ticket, most consumers sort alternatives according a criterion important to them (say, price), and then use the top of that list (such as the cheapest ticket) as a reference point when evaluating others.[1] Similarly, in online platforms like Amazon, the best reviewed or the most purchased alternative might serve as a reference point. Our main objective is to formalize this intuition to offer a theory of endogenous reference point formation, and analyze its behavioral implications.

In our model, alternatives are ranked according to how conspicuous (equivalently, salient) they are in relation to others and *the most conspicuous alternative serves as the reference point*. The conspicuity ranking captures how eye-catching the alternatives are in relation to each other.[2] In the above example, a cheaper product is more conspicuous for a price-

[1]Marketing literature establishes price to be a particularly important criterion for conspicuity (*e.g.* see Winer (1986), Kalyanaram and Winer (1995), Erdem, Mayhew, and Sun (2001)).

[2]Empirical findings suggest that conspicuous alternatives are more likely to attract attention and affect decision-making (*e.g.* see Lohse (1997), Milosavljevic, Navalpakkam, Koch, and Rangel (2012), Navalpakkam, Kumar, Li, and Sivakumar (2012)).

conscious customer. However, in general, conspicuity of a product might depend on features potentially irrelevant for its valuation such as the size and/or color of its package (*e.g.* see Milosavljevic, Navalpakkam, Koch, and Rangel (2012)). Furthermore, individuals can differ in their criteria for conspicuity, and typically, what a person finds conspicous is not directly observable from outside.[3] This (subjective) conspicuity ranking is the first component of our model.[4]

The second component of our model is a set of reference-dependent utility functions $\{U_\rho\}_{\rho \in X}$. We do not impose any particular functional form on them. Generality in the utility component allows our model to encompass a wide range of reference-dependent utility functions used in the literature, including those of Tversky and Kahneman (1991), Munro and Sugden (2003), Masatlioglu and Ok (2005, 2014), Sagi (2006), Kőszegi and Rabin (2006), and Bordalo, Gennaioli, and Shleifer (2013). This enables us to compare our model to the existing literature, as discussed below.

We are now ready to define the choice procedure of our agent. Given a choice problem S, the most conspicuous alternative according to the conspicuity ranking, denoted by $r(S)$, serves as the reference point. Next, the reference point $r(S)$ induces a utility function $U_{r(S)}$. The agent finalizes her choice by maximizing this utility function $U_{r(S)}$ on S. This model, summarized in Figure 1, is called the *Conspicuity based Endogenous Reference model* (hereafter, **CER**). In what follows, we study the basic properties of CER as well as its economic implications, and discuss to what extent its ingredients can be inferred from choice data.

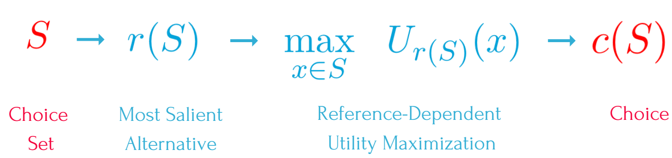

Figure 1: Conspicuity based Endogenous Reference model (CER)

The first contribution of our paper is the concept of a *conspicuity ranking* through which

[3]Subjectivity of the conspicuity ranking is in line with recent evidence which suggests that different individuals facing similar decision environments might end up with distinct reference points. For example, Terzi, Koedijk, Noussair, and Pownall (2016) present experimental data in which there is heterogeneity among individuals in the reference points that they employ.

[4]A related yet different notion is discussed in Bordalo, Gennaioli, and Shleifer (2013). In their framework, each product has different attributes and depending on the context and the reference point, one of the attributes becomes salient and receives a higher weight on the final evaluation. Thus, in their model, it is the reference point that determines salience (and of an attribute). Conversely, in our model, conspicuity determines the reference point. Hence, the two approaches are conceptually different.

reference point formation is endogenized. To highlight the significance of this innovation, consider the constant loss aversion model of Tversky and Kahneman (1991). Due to its tractable form, this highly celebrated reference-dependent model is widely used in applications. Yet it has also been criticized on the basis that it cannot accommodate well-known behavioral patterns such as the attraction and compromise effects. However, if the reference point is endogenously formed through a conspicuity ranking, as in our model, the constant loss aversion model can accommodate both. Figure 1 (left) presents an example with three alternatives, where the conspicuity ranking is $A \gg B \gg C$ and the constant loss aversion parameter is $\lambda = 2$, as commonly used in the literature. In the figure, B is chosen over C when only these two alternatives are available. However, the choice switches from B to C when (and only when) a third alternative A is added to the colored regions. Note that these areas are

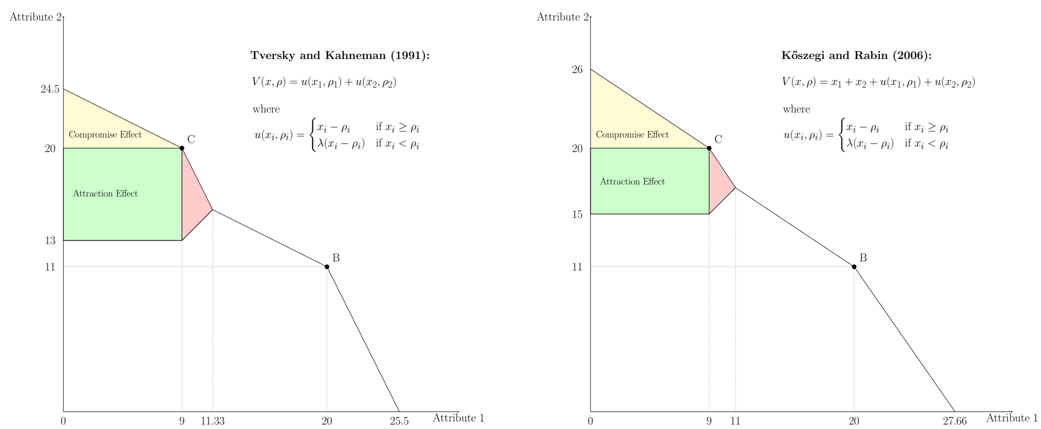

Figure 2: Attraction and compromise effects when conspicuity based reference-point formation is combined with two standard reference-dependent utility functions. The conspicuity ranking is $A \gg B \gg C$ and the loss aversion parameter is $\lambda = 2$.

predominantly consistent with the underlying motivation of the attraction and compromise effects, as detailed in Section 4. Particularly, a choice reversal does not occur when A is added to a region dominated by B, or to a region that turns B into a "compromise".

We should also point out that this improvement is a result of the conspicuity ranking and not just endogenization of the reference point. For example, if we utilize the reference-dependent utilities introduced by Kőszegi and Rabin (2006) as our underlying reference-dependent model, conspicuity based reference-point formation is capable of explaining the compromise and attraction effect (see Figure 1 (right panel)). On the other hand, if reference-point formation was based on their own preferred personal equilibrium concept, the implied behavior would be identical to the classical model (see Proposition 3 in Kőszegi and Rabin (2006)). This highlights the importance of reference-point formation for a given underlying reference-dependent choice. Overall, choice of the reference-point formation process has the potential to significantly improve the performance of existing models by allowing them to accommodate additional (seemingly anomalous) choice patterns.

Our second contribution is that we allow the conspicuity ranking to be *subjective* (that is, to depend on the decision maker's individual characteristics) and we show how this subjective ranking can be *inferred from choice data*. This inference relies on an important feature of our model concerning choice reversals. A *choice reversal* is said to occur when the elimination of an unchosen alternative affects the choice.[5] In our model such reversals can only be induced by the elimination of the most conspicuous alternative in a choice set. This feature allows us to infer the conspicuity ranking from observed choices.

To better understand CER, we explore its behavioral implications. It turns out that one intuitive and simple behavioral postulate, that we call the *Single Reversal Axiom*, fully characterizes CER. This axiom is motivated by the aforementioned observation on how CER regulates choice reversals. The Single Reversal Axiom requires that if there is a choice problem where an alternative x causes choice reversal when y is available, there cannot be a choice problem where y causes choice reversal when x is available.[6] This implies that for a given choice problem, we can observe at most one choice reversal. Since WARP does not allow any choice reversals, the Single Reversal Axiom can be thought of as the minimal deviation from it. Overall, CER enjoys an intuitive and simple axiomatic foundation that provides a clear picture of what type of choice behavior CER can address, and which enables the design of simple experiments to test its validity.

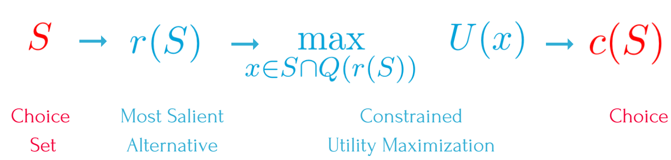

Figure 3: Psychologically Constrained CER Model (PC-CER)

One important criticism of reference-dependent choice models is that each reference point

[5] Formally, the statement "x induces a choice reversal in S" can be stated as $x \neq c(S) \neq c(S \setminus x)$ where $c(S)$ is the choice from S.

[6] When stated in terms of "revealed conspicuity," Single Reversal means that if x is revealed to be more conspicuous than y, then y cannot be revealed to be more conspicuous than x.

induces a new utility function as if there is a new self and that, this makes welfare comparisons across different reference points problematic. To deal with this criticism, Masatlioglu and Ok (2014) proposes a model where there is a single utility function applied under all reference points, but each reference point in turn induces a "psychological constraint" which eliminates certain alternatives. In Section 6, we analyze a special case of CER where choices are made by the procedure of Masatlioglu and Ok (2014). This special case, called the *Psychologically-Constrained Conspicuity based Endogenous Reference model* (PC-CER), is summarized in Figure 3 (where U is the reference-free utility and $Q(r(S))$ is the psychological constraint imposed by the reference point $r(S)$).

We analyze the behavioral implications of PC-CER as well. It turns out that a *Consistency Axiom*, together with Single Reversal, fully characterizes PC-CER. Consistency simply states that the revealed (reference-free) preference of this model has no cycles. This characterization also helps us to compare our study with the previous literature, including Manzini and Mariotti (2007), Masatlioglu, Nakajima, and Ozbay (2012), Ok, Ortoleva, and Riella (2015), Masatlioglu, Nakajima, and Ozdenoren (2017).

To place our paper in the literature, we provide a brief discussion of existing reference-dependent models, a classification of which is presented in Figure 4.[7] Columns in Figure 4 follow an approximate historical order to classify existing models in terms of how they treat reference-point formation. Rows, on the other hand, classify models in terms of how choice is made once the reference point is determined.

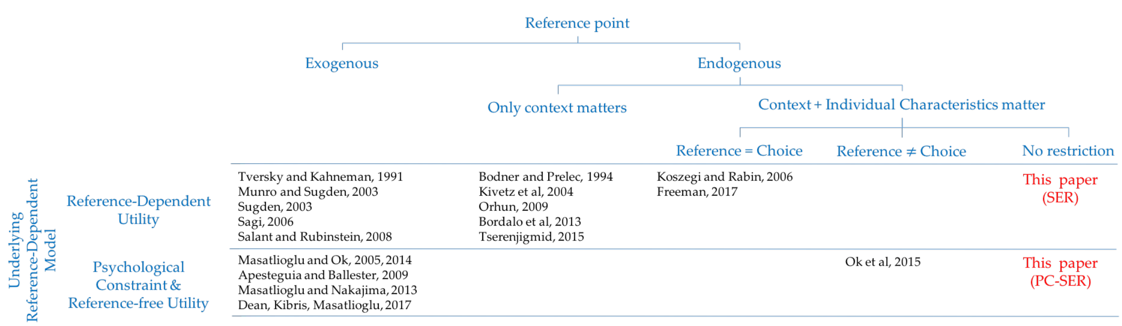

Figure 4: Reference-Dependent Models

Studies represented in the first row of Figure 4 all employ reference-dependent utility functions, but differ in terms of reference-point formation. The earliest strand of literature on this specification treats the reference point as exogenous (*e.g.* Tversky and Kahneman

[7]For a more detailed discussion, see Section 7.

(1991), Munro and Sugden (2003), Sugden (2003), Sagi (2006), Salant and Rubinstein (2008)). Later studies (columns 2 to 5) endogenize reference-point formation. In models of Bodner and Prelec (1994), Kivetz, Netzer, and Srinivasan (2004), Orhun (2009), Bordalo, Gennaioli, and Shleifer (2013), and Tserenjigmid (2015) (column 2), the reference point depends on the structure of the choice set, but is independent of individual characteristics. Thus, these models analyze environments where all decision makers facing the same choice problem necessarily have the same reference point. Alternatively, Kőszegi and Rabin (2006) and Freeman (2017) (see column 3) analyze models where the endogenous reference point can depend on individual characteristics. But in these models, the final choice always coincides with the reference point. CER is related to these earlier studies since it endogenizes reference-point formation, allows it to depend on individual characteristics, and does not restrict choice to coincide with the reference point.

The second row of Figure 4 represents another strand of literature that replaces reference dependent utilities with a reference-free utility function combined with psychological constraints. This special case of reference-dependent choice is important for welfare comparisons, as mentioned earlier (and further discussed in Section 6). Most studies in this strand of the literature treat the reference point as exogenous (*e.g.* Masatlioglu and Ok (2005, 2014), Apesteguia and Ballester (2009), Masatlioglu and Nakajima (2013), Dean, Kıbrıs, and Masatlioglu (2017)).[8] One exception is Ok, Ortoleva, and Riella (2015) where the reference point is determined endogenously but it is required to be distinct from actual choice. PC-CER is closely related to this strand of literature: it endogenizes reference-point formation, employs a reference-free utility and psychological constraints, and does not restrict choice to be distinct from the reference point.

As will be discussed in Section 4, our model is consistent with three well-known behavioral patterns frequently observed in empirical studies, namely, Compromise Effect, Attraction Effect, and Cyclical Choice. None of the other studies listed above can accommodate all three. Our study is also unique in the sense that it characterizes the distinction between the two types of models represented in rows 1 and 2 of Figure 4. To elaborate, a comparison of theorems 1 and 2 shows that, (in the confines of our framework) this distinction can be

[8] Maltz (2017) presents a hybrid model which combines an exogenous reference point (the endowment) with endogenous reference-point formation. In this model, alternatives are partitioned into categories and, given the endowment, the most-preferred feasible alternative in its category serves as the reference point. As far as we know, this is the only model that combines an exogenous reference point with endogenous reference-point formation.

characterized by the Consistency axiom, which is closely related to the revealed (reference-free) preference of PC-CER.

The paper is organized as follows. In Section 2, we present and discuss our model. In Section 3, we introduce the Single Reversal Axiom and show that it characterizes CER. In Section 4, we discuss three important behavioral patterns. In Section 5, we show how information is revealed from choice data consistent with CER. In Section 6, we discuss a special case of CER where psychological constraints introduce extra structure on the reference-dependent utility functions. In Section 7, we discuss the related literature. We conclude in Section 8. The Appendix contains all the proofs.

2 Conspicuity and Endogenous Reference Dependence

Let X denote a finite set of alternatives and let $\mathcal{X}$ be the set of all nonempty subsets of X. A **choice problem** is a set of alternatives $S \in \mathcal{X}$ from which the decision maker needs to make a choice. A **choice function** $c : \mathcal{X} \to X$ maps every choice problem $S \in \mathcal{X}$ to an alternative $c(S) \in S$.[9] We assume that c represents data on the choice behavior of a decision maker (hereafter, **DM**).

Our model has two components: (i) a family $\mathcal{U} = \{U_\rho\}_{\rho \in X}$ of **(reference-dependent) utility functions**, each associated with a potential reference point, and (ii) a **conspicuity ranking** $\gg$. In our interpretation, $\gg$ reflects the DM's perception of how prominent or eye-catching the alternatives are in relation to each other. We assume that $\gg$ is a *strict linear order*.[10] We theorize that the reference point in a choice set is the most conspicuous alternative in it. Formally, given $\gg$, the **endogenous reference function** $r : \mathcal{X} \to X$ maps each choice set S to the endogenous reference point $r(S) \in S$, defined as

$$r(S) = \text{argmax}(\gg, S).$$

Given the reference point $r(S)$ for a choice problems S, the DM uses the induced reference-dependent utility function, $U_{r(S)} : X \to \mathbb{R}$ to evaluate alternatives in S. The maximizer of $U_{r(S)}$ in S is the chosen alternative.[11] This process is formally stated in the next definition.

[9]While we work with choice functions, our results can be extended to choice correspondences, that is, to environments where the decision maker chooses more than one alternative. An extension of our main theorem that allows choice correspondences is available upon request.

[10]A binary relation R on X is a strict linear order if it is weakly connected, irreflexive and transitive.

[11]Without loss of generality, we can impose a widely accepted property from the reference dependence

Definition 1. *A choice function c admits a conspicuity based endogenous reference (CER) representation if there is a family of (reference-dependent) utility functions $\mathcal{U} = \{U_\rho\}_{\rho \in X}$ and a conspicuity ranking $\gg$ such that for each $S \in \mathcal{X}$,*

$$c(S) = \arg\max_{x \in S} \; U_{r(S)}(x) \qquad \textit{where} \quad r(S) = \text{argmax}(\gg, S).$$

The standard rational choice model is a special case: any CER where all reference-dependent utility functions are identical (*i.e.* $U_\rho = U$ for all $\rho \in X$) behaves similar to the rational choice model with the utility function U.

An important difference of our model from the standard model of rational choice is that removal of an unchosen alternative from a choice set can in turn affect the chosen alternative. This pattern, called a *choice reversal*, can only be observed if the removed alternative is the reference point, and thus, is the most conspicuous alternative. For demonstration, imagine that we observe the choice reversal $y \neq c(S) \neq c(S \setminus y)$. If $r(S) = r(S \setminus y)$ was true, then we would have $c(S) = c(S \setminus y)$ as $c(S)$ belongs to $S \setminus y$. Therefore, the reference point must have changed when we remove y, that is, $r(S) \neq r(S \setminus y)$. Since r maximizes the conspicuity ranking, it must be that $r(S) = y$. How to use choice behavior to reveal such information will be discussed in detail in the coming sections.

3 Representation Theorem

As discussed in the previous paragraph, removal of an alternative from a choice set can only induce a choice reversal when the removed alternative is its reference point. This observation motivates our first axiom, *Single (choice) Reversal*, which states that in every choice set there is at most one alternative that can induce choice reversal.

Single Reversal Axiom: For each $S, T \in \mathcal{X}$ and distinct $x, y \in X$ with $\{x, y\} \subseteq S \cap T$, if $x \neq c(S) \neq c(S \setminus x)$, then either $c(T) = y$ or $c(T \setminus y) = c(T)$.

In the above statement, $x \neq c(S) \neq c(S \setminus x)$ means that x causes a choice reversal in S. Then, the axiom states that no other alternative in S can cause a choice reversal in the presence of x.

Single Reversal is a necessary condition of CER. Once x causes a choice reversal, CER

literature about how two reference dependent utility functions are related: $U_y(x) > U_y(y)$ implies $U_x(x) > U_x(y)$. This property states that if a person is willing to abandon her reference point y for an alternative x, then she will not abandon x for y when x is itself the reference point.

infers that x is the most conspicuous alternative in S. Hence, any alternative in $S \setminus x$ is less conspicuous than x and cannot induce a choice reversal in a set that contains x. Our main result thus states that all CER satisfy Single Reversal. But more importantly, it also establishes that any choice data that satisfies this axiom can be represented with a CER.

Theorem 1. *A choice function c admits a CER representation if and only if it satisfies Single Reversal.*

For the detailed proof, we refer the interested reader to the Appendix. Here, we provide a sketch of the sufficiency argument. To construct the conspicuity ranking, we first define a binary relation R as follows: xRy if there is a choice set that contains x and y and in which x causes a choice reversal. The binary relation R need not be complete. But using the Single Reversal axiom, we show that R is transitive. A completion of R serves as our conspicuity ranking. Then, we define the endogenous reference function r as picking the maximizer of the conspicuity ranking.

For each reference point ρ, we next define an associated binary relation P_ρ as follows: $xP_\rho y$ if there is a choice set with reference point ρ such that in this set, x is chosen even though y was also available: formally, $\rho = r(S)$, $x = c(S)$, and $y \in S$. We then show that each such P_ρ is transitive and take U_ρ to represent a completion of P_ρ. Finally, we show that the conspicuity ranking and the set of reference-dependent utility functions we constructed induce choice behavior identical to the original choice function c.

4 Behavioral Patterns

In addition to providing a full-blown theory of reference dependence, CER is capable of exhibiting several interesting behavioral patterns. Among them, we discuss three particularly important ones, namely, Compromise Effect, Attraction Effect, and Cyclical Choices. While such "seemingly anomalous" choices are frequently observed in empirical studies, it is difficult to reconcile them in a single model. This point will be further discussed in Section 7 to distinguish CER from the existing literature.

THE COMPROMISE EFFECT refers to a phenomenon where an individual tends to choose the middle option of a selection set rather than the extreme ones (*e.g.* see Simonson (1989), Simonson and Tversky (1992)). To illustrate, consider Figure 5 (left) where four alternatives are described in terms of two attributes (such as price and quality). Here, the compromise effect requires that B is chosen more frequently in $\{A, B, C\}$ than in $\{A, B\}$. This means for some individuals we must observe $c(A, B) = A$ together with $c(A, B, C) = B$.[12] Similarly, we must observe $c(B, C) = B$ and $c(B, C, D) = C$.

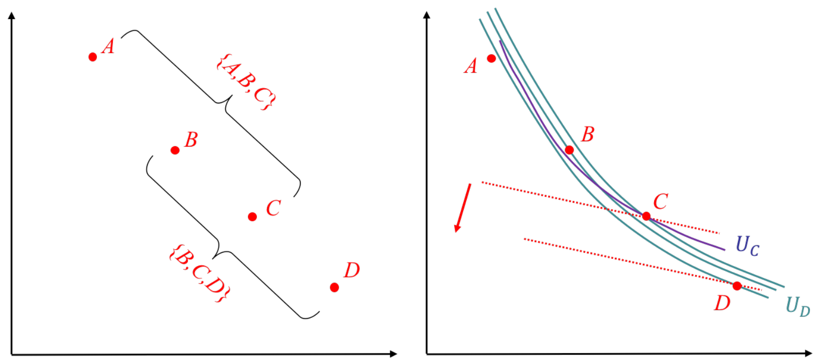

Figure 5: Compromise Effect with Four Alternatives

We next explain how our model accommodates such choice patterns by introducing a DM, Comyn, whose choices are consistent with CER.

Example 1. *For Comyn, whose choice behavior is in accordance with our model, D is more conspicuous than C, C than B, and B than A (as represented by the dotted indifference lines in the right panel of Figure 5). Also assume that Comyn's reference dependent preferences are such that under reference point D we have $U_D(C) > U_D(B) > U_D(D) > U_D(A)$, under reference point C we have $U_C(B) > U_C(C) > U_C(A)$, and under reference point B we have $U_B(A) > U_B(B)$.*

If Comyn faces only A and B, the most conspicuous option is B, which implies that A must be chosen since it yields higher utility in U_B. When C is introduced, however, it becomes the most conspicuous alternative. And since U_C attains its maximum at B, Comyn chooses B from $\{A, B, C\}$. (As the reader will note, the added alternative causes the compromise effect by changing the reference point.) In addition, when Comyn faces $\{B, C, D\}$, the most conspicuous option becomes D. Given that the reference point is D, alternative C which maximizes U_D is chosen. Overall, Comyn exhibits the compromise effect choice patterns.

In case of three alternatives, there is only one instance of the compromise effect. Having four alternatives, however, creates more opportunities to observe this type of choice patterns. Such cases are harder to replicate in theoretical models, and hence, present a stricter test of

[12] Throughout the paper, we will abuse the notation and write $c(x, y, \dots)$ instead of $c(\{x, y, \dots\})$. Similarly, we omit braces and write $S \cup x$ instead of $S \cup \{x\}$.

whether a model displays the compromise effect. For example, B and C are the compromises in $\{A, B, D\}$ and $\{A, C, D\}$, respectively. Hence the compromise effect predicts that the DM should choose B from $\{A, B, D\}$ and C from $\{A, C, D\}$. This is exactly what Comyn does in our example.

THE ATTRACTION EFFECT refers to an inferior product's ability to increase the attractiveness of a superior one, when added to a choice set (*e.g.* see Huber, Payne, and Puto (1982), Ratneshwar, Shocker, and Stewart (1987)). To illustrate, consider Figure 6 (left) where the two-dimensional product space is as in the previous example and alternatives A and B are not comparable, as before. Adding to $\{A, B\}$ a third product a which is dominated in both attributes by A, but is not comparable to B makes A more attractive. The attraction effect then requires that the addition of a increases the likelihood of A being chosen. Similarly, adding a product b dominated by B makes it more attractive and increases the likelihood that B will be chosen. This means we must observe $c(A, B, a) = A$ and $c(A, B, b) = B$.

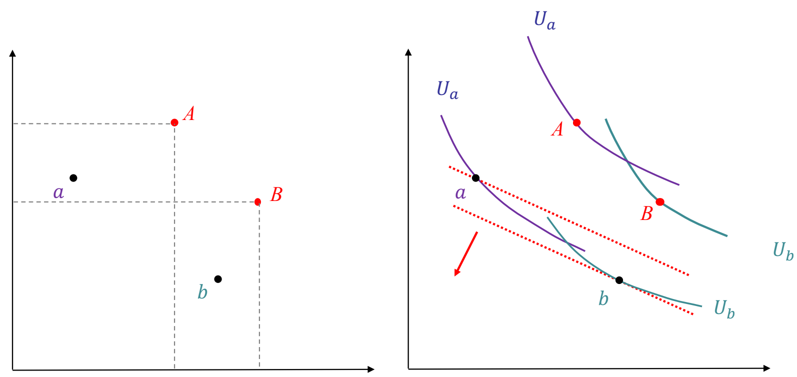

Figure 6: Attraction Effect with Four Alternatives

We next introduce Attila to explain how CER accommodates the attraction effect.

Example 2. *Attila's conspicuity ranking, presented in Figure 6 (right), is similar to Comyn's (i.e. Figure 5): he ranks b above a, a above B, and B above A in terms of their conspicuity. Attila's reference dependent preferences are so that the two top alternatives are A and B but, under reference point a we have $U_a(A) > U_a(B)$ and under reference points b and B we have $U_b(B) > U_b(A)$ and $U_B(B) > U_B(A)$.*

If Attila faces only A and B, the most conspicuous option is B, and it also maximizes U_B. When a is introduced, however, a becomes the most conspicuous alternative. And since U_a attains its maximum at A, Attila chooses A from $\{A, B, a\}$. Furthermore, when b is introduced *to $\{A, B, a\}$, it becomes the most conspicuous alternative, and acting as the reference point, switches choice back to B. Addition of a to $\{A, B\}$ reverses choice from B to A and further addition of b reverses choice back to B, in line with the attraction effect.*

Cyclical Choice refers to instances where choices from binary sets violate transitivity. The first experiment which illustrated that individuals can exhibit cyclical choice patterns has been provided by May (1954). Since then intransitivity of preferences has been observed in many different choice environments (*e.g.* Tversky (1969), Loomes, Starmer, and Sugden (1991), Manzini and Mariotti (2009), Mandler, Manzini, and Mariotti (2012)). This choice pattern can be summarized as

$$c(x, y) = x, \quad c(y, z) = y, \quad \text{and} \quad c(x, z) = z.$$

Even though x is chosen over y and y is chosen over z, the choice from $\{x, z\}$ is not x.

We next introduce Cyler to explain how CER accommodates cyclical choices.

Example 3. *Cyler ranks z to be more conspicuous than x and x to be more conspicuous than y. His reference-dependent preferences are so that under reference point z we have $U_z(y) > U_z(z) > U_z(x)$, and under reference points x we have $U_x(x) > U_x(y)$.*

Under this specification, z is the endogenous reference point for binary problems $\{x, z\}$ and $\{y, z\}$ (as well as the trinary problem $\{x, y, z\}$). Hence, the choices from these two sets are obtained by maximization of U_z. As a result, z is chosen from $\{x, z\}$ and y is chosen from $\{y, z\}$. On the other hand, the endogenous reference point for $\{x, y\}$ is x and choice from this set is made by maximization of U_x. As a result, x is chosen from $\{x, y\}$. This creates a binary choice cycle.

Hence, as in the previous examples, the change in the endogenous reference point is what induces cyclical binary choices for Cyler.

5 Revealed Information

Our model has two components: the reference-dependent utility functions and the conspicuity ranking. We now discuss how to infer information about them from observed choices.

First note that, due to the finite domain assumption, our utility functions are ordinal. That is, any monotonic transformation of a utility function continues to represent the same underlying preference relation. Since choice data only let us infer about binary comparisons, any two such utility functions are observationally equivalent. Hence for simplicity, in this section we will notate a utility function with the preference relation it represents. That is, we will use the $(\{\succ_\rho\}_{\rho\in X}, \gg)$ representation instead of $(\mathcal{U}, \gg)$ where $U_\rho(x) > U_\rho(y)$ if and only if $x \succ_\rho y$.

The following example demonstrates that there can be multiple CER representations of the same choice data.

Example 4. *Consider the following choice data on $X = \{x, y, z\}$.*

$$c(xyz) = y, \quad c(xy) = x, \quad c(yz) = y, \text{ and } c(xz) = z.$$

As discussed at the end of Section 2, the choice reversal $z \neq c(x, y, z) \neq c(x, y)$ reveals that z is more conspicuous than x and y (and thus, serves as the reference point of $\{x, y, z\}$, as well as $\{x, z\}$ and $\{y, z\}$). Thus, any CER $(\{\succ_\rho\}_{\rho\in X}, \gg)$ consistent with this choice data needs to exhibit $z \gg x$ and $z \gg y$. Furthermore, $c(x, y, z) = y$ and $c(x, z) = z$ imply that any CER $(\{\succ_\rho\}_{\rho\in X}, \gg)$ that represents c also needs to satisfy $y \succ_z z \succ_z x$. However, the conspicuity ranking between x and y, as well as the preferences $\succ_x$ and $\succ_y$ are not identified.

Since there might be multiple CER representations of the same choice data, we need to formally define what revealed preference and revealed conspicuity mean.

Definition 2. *Assume that c admits k different CER representations, $(\{\succ_\rho\}^i_{\rho\in X}, \gg^i)_{i\in\{1,\ldots,k\}}$. Then*

- *x is revealed to be preferred to y under reference point ρ if $x \succ^i_\rho y$ for all $i \in \{1, \ldots, k\}$,*
- *x is revealed to be more conspicuous than y if $x \gg^i y$ for all $i \in \{1, \ldots, k\}$.*

This definition is very conservative. For example, we say x is revealed to be preferred to y under the reference point ρ (or x is more conspicuous than y) only when *all* possible representations agree on it. This conservative approach, proposed by Masatlioglu, Nakajima, and Ozbay (2012), guarantees that we do not make any claims that are not fully implied by the data.

The above definition raises a potential difficulty. For example, if one wants to know whether x is more conspicuous than y, it appears necessary to check the consistency of every $(\{\succ_\rho\}^i_{\rho\in X}, \gg^i)_{i\in\{1,\ldots,k\}}$ with this claim. However, this is not practical, especially when there are too many alternatives. Instead we shall now provide a method to obtain revealed conspicuity and revealed preference.

We first discuss revealed conspicuity. As argued in Example 4, if x causes choice reversal when y is in the choice set, then we can conclude that x is more conspicuous than y in every CER representation of c. A natural question to ask is whether the converse is also true, that is, whether such choice reversals fully characterize revealed conspicuity. Formally, for any $x \neq y$, we define a binary relation R as follows:

$$xRy \text{ if there is } S \supseteq \{x, y\} \text{ such that } x \neq c(S) \neq c(S \setminus x).$$

The binary relation R is transitive, as established in the proof of Theorem 1. The following claim additionally establishes that R fully characterizes revealed conspicuity.

Remark 1. *(Revealed Conspicuity) Suppose c admits a CER representation. Then x is revealed to be more conspicuous than y if and only if xRy.*

We next discuss revealed preference. Example 4 shows that, under a fixed reference point, choices from bigger sets are revealed preferred to choices from smaller sets. Using this idea, we characterize revealed preference under a reference point z. For any x, y, z such that $x \neq y$, we define

$$xP_zy \text{ if there is } S \supseteq T \supseteq \{x, y, z\} \text{ such that } \begin{array}{ll} (i) & z \neq c(S) \neq c(S \setminus z), \textit{ and} \\ (ii) & x = c(T). \end{array}$$

As shown in the proof of Theorem 1, P_z is transitive for each z. Our second remark establishes that P_z is indeed the revealed preference relation under reference point z.

Remark 2. *(Revealed Preference) Suppose c admits a CER representation. Then x is revealed to be preferred to y under reference point z if and only if xP_zy.*

6 Reference-Dependent Choice with Psychological Constraints

The model we considered so far assumes that choices are represented by maximization of a reference-dependent utility function U_ρ which is potentially distinct for different reference points. That is,

$$c(S|\rho) := \arg\max_{x \in S} \; U_\rho(x)$$

One disadvantage of this model is that, since the utility function changes with the reference point, there is no overall welfare criterion. To overcome this critique, Masatlioglu and Ok (2014) proposes an alternative model of reference-dependent choice. Their model views choice as arising from a "psychologically constrained utility maximization" where the constraints are induced by one's initial endowment.[13] This model allows construction of a (reference-independent) ranking of alternatives that can be used to carry out meaningful welfare analyses.

Reference-dependent choice in Masatlioglu and Ok (2014) involves a psychological constraint function, formally defined as follows.

Definition 3. *A psychological constraint function Q maps each alternative $\rho \in X$ to its psychological constraint set $Q(\rho) \in \mathcal{X}$ such that $\rho \in Q(\rho)$.*

Namely, $Q(\rho)$ denotes the set of alternatives "acceptable" to the DM when her point of reference is ρ. The only restriction imposed on Q is $\rho \in Q(\rho)$, that is, a reference point does not exclude itself from consideration.

Overall, if the choice set is S and the reference point is $\rho \in S$, the DM only considers alternatives in $Q(\rho) \cap S$. She makes her choice by maximizing a reference-free utility function U on this set. Formally,

$$c_{MO}(S|\rho) = \arg \max_{x \in S \cap Q(\rho)} U(x).$$

This reference-dependent choice procedure is a special case of our general model. To see this, start with a reference-free utility function and psychological constraints, and note that if an alternative does not belong to the psychological constraint set of the reference point, it should not be chosen in the presence of this reference point. Hence, when constructing the associated reference-dependent utility function, such alternatives can be assigned any utility level lower than the (reference-free) utility level of the reference point. Otherwise, we can assign the original (reference-free) utility level. Formally, given U and Q, and letting $m < \min_{x \in X} U(x)$, define the reference-dependent utility functions U_ρ as follows:

$$U_\rho(x) := \begin{cases} U(x) & \text{if } x \in Q(\rho), \\ m & \text{if } x \notin Q(\rho). \end{cases}$$

[13] This idea was first introduced by Samuelson and Zeckhauser (1988), who state that "Assuming that he or she understands his or her current plan, a reasonable strategy would be to undertake a comparative analysis including only some subset of competing plans (ignoring the others altogether)".

We now define a special case of CER where the underlying reference-dependent choice is based on Masatlioglu and Ok (2014). The rest of the choice procedure is the same.

Definition 4. *A choice function c admits a Psychologically-Constrained CER (PC-CER) representation if there is a (reference-independent) utility function U, a conspicuity ranking $\gg$, and a psychological constraint function Q such that for each $S \in \mathcal{X}$,*

$$c(S) = \arg \max_{x \in Q(r(S)) \cap S} U(x) \qquad \textit{where} \quad r(S) = \text{argmax}(\gg, S).$$

The standard rational choice model is still a special case. When $Q(\rho) = X$ for each $\rho \in X$, the DM behaves identical to a rational-choice agent whose utility function is U. Similarly, when $Q(\rho)$ is a singleton for each $\rho \in X$, the model behaves identical to a rational choice agent whose utility function has the same ranking as $\gg$. For choice data that satisfies WARP, it is impossible to distinguish these two special cases.

PC-CER exhibits both endogenous reference-point formation (as in CER) and a reference-free utility function (as in the rational choice model). Being a special case of CER, PC-CER satisfies Single Reversal and exhibits choice reversals only due to a change in the underlying reference point. Due to the existence of a reference-free utility function, PC-CER also satisfies an additional consistency property, which is closely related to the following property of the rational choice model:

For each $S \in \mathcal{X}$, there is $x \in S$ such that if $x \in T$ then either $c(T) = x$ or $c(T) \notin S$.

The above statement says that there is a best element x in S and if x and some other member, say y of S were to be considered together in some other set T, it cannot be that y is chosen from T as the best alternative: either x or another alternative outside S must be chosen from T. This simple consistency requirement, however, does not take into account the fact that, in case of reference effects, x might be considered as an alternative in S but might be ruled out by in some other set T by its reference point's psychological constraint. To exclude this possibility, we revise the above statement to additionally require that the reference point z of T does not rule out x from consideration. In sum, to capture PC-CER (which is more general than the standard rational choice model), we relax the above statement as follows.

Consistency: For each $S \in \mathcal{X}$, there is $x \in S$ such that if $\{x, z\} \subseteq T \subseteq T'$, $z \neq c(T') \neq c(T' \setminus z)$ and $x = c(x, z)$, then either $c(T) = x$ or $c(T) \notin S$.

In the above statement, the choice reversal $z \neq c(T') \neq c(T' \setminus z)$ tells us that z is the reference point of T' and thus, of its subsets T and $\{x, z\}$. The statement $x = c(x, z)$ additionally informs us that the reference point z does not rule out x from consideration. Since x is considered in T as an alternative, it is then not possible that another member of S is chosen instead of x.

The more general CER violates Consistency.[14] Hence, Consistency is a formulation of the behavioral difference between CER and PC-CER. This behavioral difference arises from a replacement of the reference-dependent utilities in CER with a combination of reference-free utility and psychological constraints in PC-CER. Consistency is thus expected to be related to the revealed (reference-free) preference of PC-CER. Indeed, it guarantees that this revealed preference has no cycles, as will be discussed later in this section. For choice data that is consistent with CER but not with PC-CER (such as Comyn's behavior), such reference-free preferences cannot even be constructed.

The following theorem states that any PC-CER satisfies both Single Reversal and Consistency. Furthermore, any choice data that satisfies these two axioms can be rationalized by a PC-CER.

Theorem 2. *A choice function c admits a PC-CER representation if and only if it satisfies Single Reversal and Consistency.*

For the detailed proof, we refer the interested reader to the Appendix (where we also establish the independence of our axioms). Here, we provide a sketch of the sufficiency argument.[15] To construct the conspicuity ranking, we first define a binary relation R as in the proof of Theorem 1. As before, Single Reversal guarantees that R is transitive. However, unlike in Theorem 1, we cannot take any arbitrary completion of R to be our conspicuity ranking. Instead, the conspicuity ranking $\gg$ is constructed as follows: $x \gg y$ if either xRy or x and y cannot be compared by R and $x = c(x, y)$. The binary relation $\gg$ is complete. We additionally show that $\gg$ is transitive.

[14] To see this, let us revisit Example 1 where Comyn, who follows CER, exhibits the Compromise effect. To see that Comyn violates Consistency, let $S = \{B, C\}$. First, let $x = z = C$ and $T = T' = \{A, B, C\}$. Note that $C \neq c(A, B, C) \neq c(A, B)$ and $C = c(C)$, but $C \neq c(A, B, C) = B \in S$. So C cannot be the "best element" in S. Alternatively, let $x = B$, $z = D$, $T = \{B, C, D\}$, and $T' = \{A, B, C, D\}$. Note that $D \neq c(A, B, C, D) \neq c(A, B, C)$, $B = c(B, D)$, but $B \neq c(B, C, D) = C \in S$. So B can also not be the "best element" in S.

[15] In general, given a choice function that satisfies Single Reversal and Consistency, there are multiple possible constructions of $(U, \gg, Q)$ that can explain the choice behavior. The construction that is used in the proof works with the minimal psychological constraint function in the sense that if there is another representation of the same choice behavior given by $(U', \gg', Q')$, then $Q(x) \subseteq Q'(x)$ for all $x \in X$. This point will be useful when we later discuss revealed psychological constraints.

Our next step is to construct the psychological constraint function Q. We say $y \in Q(x)$ if and only if $x \gg y$ and $y = c(x, y)$. As discussed in Footnote 15, our definition of $\gg$ guarantees that Q is the minimal psychological constraint function that can be used.

The third step is to define the preferences. To this end, we define a binary relation P as follows: xPy if there is $z \in X$ such that $\{x, y\} \subseteq Q(z)$ and $c(x, y, z) = x$. Consistency guarantees that P defined as such is acyclic. Using this, we then let $\succ$ be a completion of P. Since the domain is finite, we can find a utility function representing $\succ$. Finally, we show that the CER with the triple $(U, \gg, Q)$ as created above induces choice behavior identical to the original choice function c.

Revealed Information

In this section, we illustrate how to infer the DM's utility function/preference ranking from her observed choices, given that the choice data is consistent with PC-CER.[16] For revealed conspicuity and revealed psychological constraint, please see the Appendix.

The following example demonstrates how choice data can be used to infer the components of PC-CER.

Example 5. *Consider the following choice data on* $X = \{x, y, z\}$*:*

$$c(x, y, z) = x, \;\; c(x, y) = y, \quad c(y, z) = y, \quad \textit{and} \;\; c(x, z) = x.$$

The choice reversal $z \neq c(x, y, z) \neq c(x, y)$ *reveals that* z *is more conspicuous than* x *and* y*. We also observe that* $c(x, z) = x$ *and* $c(y, z) = y$*. Since* x *and* y *are chosen over* z *despite the fact that they are less conspicuous than* z*, we infer that* x *and* y *must be both more preferred than* z*. We also learn that the psychological constraint under* z *does not rule out either* x *or* y*. In particular,* y *is considered in the choice set* $\{x, y, z\}$*. This, together with the observation that* $c(x, y, z) = x$*, leads us to conclude that* x *is more preferred to* y*. Finally, since* x *is preferred to* y *but* $y = c(x, y)$*, it must be that* y *is more conspicuous than* x*.*

Overall, we uniquely identify preference and conspicuity as $x \succ y \succ z$ *and* $z \gg y \gg x$*. The psychological constraint is identified as* $Q(z) = \{x, y, z\}$*,* $\{y\} \subseteq Q(y) \subseteq \{y, z\}$*, and* $\{x\} \subseteq Q(x)$*.*

[16] Similar to Section 5, we focus on the preference relations rather than their utility representations. Hence, we write $(\succ, \gg, Q)$ instead of $(U, \gg, Q)$.

As demonstrated in Example 5, under a fixed reference point choices from bigger sets are revealed preferred to choices from smaller sets. Using this idea, we construct the following binary relation. For any x, y, z such that $x \neq y$, we define

$$xPy \text{ if } \exists S, T \text{ with } \{x,y,z\} \subseteq T \subseteq S \text{ such that } \begin{array}{ll} (i) & z \neq c(S) \neq c(S \setminus z), \\ (ii) & c(y,z) = y, \textit{ and} \\ (iii) & x = c(T). \end{array} \tag{1}$$

Condition (i) implies that z is the most conspicuous alternative and the reference point in S as well as in T and $\{y, z\}$. Since y is chosen from $\{y, z\}$, we can infer that y is in the psychological constraint set of z. Then observing $x = c(T)$ reveals that y is considered and x is chosen. Hence the DM prefers x over y.

Since the binary relation P is not necessarily transitive (but all PC-CER that represent c have transitive preferences), let P^T be the transitive closure of P. The next remark establishes that P^T fully characterizes revealed preference.

Remark 3. *(Revealed Preference) Suppose c admits a PC-CER representation. Then x is revealed to be preferred to y if and only if xP^Ty.*

The same construction that is used in the proof of Theorem 2 can be used to prove Remark 3, as well as the remarks on revealed conspicuity and revealed psychological constraint, presented in the Appendix.

6.1 Reference-Dependent Choice with Ordered Psychological Constraints

In this section, we consider a special case of PC-CER where the psychological constraint is "ordered with respect to the conspicuity relation". Intuitively, this means that a more conspicuous alternative induces a harsher psychological constraint than a less conspicuous one.

Definition 5. *A psychological constraint function Q is ordered with respect to $\gg$ if*

$$x \gg y \textit{ implies } Q(x) \subseteq Q(y).$$

The ordering assumption on Q is natural under a wide range of circumstances. For example, consider a consumer whose reference point is the cheapest alternative in the menu and her psychological constraint rules out alternatives that are too expensive relative to the reference point. More specifically, if the consumer's reference alternative costs p dollars, she is willing to spend at most $p+m$ dollars on a purchase. Formally, $Q(x) = \{z \in X | p_z \leq p_x + m\}$. In this example, cheaper alternatives are more conspicuous, that is, $x \gg y$ is equivalent to $p_x < p_y$. This in turn implies $Q(x) \subseteq Q(y)$. Hence Q is ordered with respect to $\gg$.

As a second example, consider the willpower model of Masatlioglu, Nakajima, and Ozdenoren (2017). The conspicuity ranking captures the amount of temptation each alternative creates. Under this interpretation, the most tempting alternative in a choice set becomes its reference point. Let $v(x)$ denote the temptation value of an alternative x. And assume that the DM has a willpower stock w which she can use to resist temptation. That is, a DM with a reference point x is able to consider an alternative y only if its temptation value is not less than $v(x) - w$, that is, $v(y) \geq v(x) - w$. Then it is easy to see that the psychological constraint function Q defined as $Q(x) = \{y \in X | \, v(y) \geq v(x) - w\}$ is ordered with respect to $\gg$.

For such examples, the following restriction of the PC-CER is appropriate.

Definition 6. *A choice function c admits an ordered PC-CER representation if there exists a PC-CER representation of c such that the psychological constraint function is ordered with respect to the conspicuity ranking.*

The additional ordering assumption on the psychological constraint allows us to learn more about the DM's preferences. To illustrate this, we revisit Example 4:

$$c(xyz) = y, \quad c(xy) = x, \quad c(yz) = y, \text{ and } c(xz) = z.$$

As discussed before, $z \neq c(x, y, z) = y \neq c(x, y)$ tells us that $z \gg x$, $z \gg y$, $y \in Q(z)$, and $y \succ z$. Without an ordering assumption on Q this is all we can learn. With an ordered Q and $z \gg x$, we must have $y \in Q(z) \subseteq Q(x)$. We now show that $x \succ y$. We need to consider two cases: either (i) y is more conspicuous than x, or (ii) x is more conspicuous than y. If the former hold, then $c(x, y) = x$ implies $x \succ y$. If the latter holds, $y \in Q(x)$ and $c(x, y) = x$ imply $x \succ y$. Hence, with the ordering assumption on Q we also learn that $x \succ y$. The revealed preference thus becomes $x \succ y \succ z$.[17]

[17]The ordering assumption on Q, however, does not help us to identify the conspicuity ranking between x and y. It is possible that $x \gg y$ and $Q(x) = \{x, y, z\}$ or $y \gg x$ and $Q(y) = \{x, y, z\}$.

As illustrated above, we now have two types of preference revelation. The first one is the binary relation P defined in Equation (1). The second type of preference revelation is new. (It is related to the "Less is More" concept in the earlier literature, which states that in case of choice reversals, the alternative chosen from the smaller set is revealed preferred to the one chosen in the larger set.) So if x is chosen over y in some choice set T and y is chosen over x in some superset S of T, then x is revealed to be preferred to y. To see why, notice that since y is chosen over x in S it must belong to the psychological constraint set induced by the most conspicuous alternative in that set. Due to the ordered nature of the psychological constraint function, however, y must also belong to the psychological constraint set induced by the most conspicuous alternative in T. Since x is chosen over y in T, x must be more preferred than y. Formally, for any $x \neq y$,

$$xP'y \text{ if } \exists S \supset T \supseteq \{x, y\} \text{ such that } c(T) = x \text{ and } c(S) = y.$$

It turns out that a necessary condition for a choice function c to have an ordered PC-CER representation is acyclicity of the union of these two revealed preferences: $P_o = P \cup P'$. The following axiom imposes this requirement. In that sense, it is very similar to the Strong Axiom of Revealed Preference, commonly used in the literature.

Acyclicity: P_o is acyclic.

The main result of this section states that the ordered PC-CER satisfies both the Acyclicity and Single Reversal axioms. Furthermore, any choice function that satisfies these two axioms admits an ordered PC-CER representation.

Theorem 3. *A choice function c admits an ordered PC-CER representation if and only if it satisfies Single Reversal and Acyclicity.*

The proof of the Theorem also provides the characterization of the revealed preference. Let P_o^T denote the transitive closure of P_o. The next remark says that P_o^T is indeed the revealed preference ranking.

Remark 4. *(Revealed Preference) Suppose c admits an ordered-CER representation. Then x is revealed to be preferred to y if and only if xP_o^Ty.*

The ordered PC-CER has some interesting links to the earlier literature, as will be discussed in the next section.

7 Related Literature

In this section, we discuss two strands of literature that is related to our work, namely, (i) models of reference dependence and (ii) other choice procedures involving choice reversals.

Reference Dependence: Among the studies listed in Figure 4, Kőszegi and Rabin (2006), Bordalo, Gennaioli, and Shleifer (2013), Tserenjigmid (2015), and Ok, Ortoleva, and Riella (2015) are most closely related to ours. Below, we discuss them in more detail.

Kőszegi and Rabin (2006) extend the loss aversion model of Tversky and Kahneman (1991) by proposing that "a person's reference point is the probabilistic beliefs she held in the recent past about outcomes." Due to the assumption that expectations are rational, the reference point in Kőszegi and Rabin (2006) coincides with the actual choice. Since this model does not allow any alternative other than the actual choice to act as the reference point, it cannot accommodate Comyn's and Attila's choice patterns (Examples 1 and 2).

Mainly motivated by attraction-effect type phenomena, Ok, Ortoleva, and Riella (2015) present an axiomatic study of a reference-dependent model. As opposed to Kőszegi and Rabin (2006), in this model the reference point can never coincide with actual choice. Another key assumption in Ok, Ortoleva, and Riella (2015) is that the DM does not exhibit reference dependence in binary choice problems. This rules out cyclical choice behavior (see Example 3). This assumption allows choices from binary problems to fully reveal the DM's (reference-independent) preferences. In contrast, CER, PC-CER and ordered PC-CER allow reference dependence in binary choice problems and do not assume reference-independent preferences to be observable from binary choice problems. In line with its main motivation, the Ok, Ortoleva, and Riella (2015) model can accommodate Attila's behavior (Example 2). However, it does not accommodate Comyn's and Cyler's behavior (Examples 1 and 3).[18]

Bordalo, Gennaioli, and Shleifer (2013) model a commodity as a vector of K attributes and identify the reference point as the commodity with *average attributes* (see also Bodner and Prelec (1994), Kivetz, Netzer, and Srinivasan (2004)). Unlike in other studies, the reference point in this model need not be in the choice set or in the commodity space. Under certain parametrizations this model can explain versions of attraction and compromise effects

[18] Ok, Ortoleva, and Riella (2015) rule out cyclical binary choice by assumption and thus, cannot accommodate Cyler's behavior. We next show that it does not accommodate Comyn's behavior as well. Using their terminology, we say that z "helps" x against y and z "hurts" y against x if $y \in c(x, y)$ but $x = c(x, y, z)$. One of the key assumptions in Ok, Ortoleva, and Riella (2015) is that if z "helps" x against one alternative, it can never "hurt" x against any other alternative. In contrast, Example 1 exhibits $c(A, B) = A$ and $c(A, B, D) = B$ (*i.e.* D "helps" B against A) as well as $c(B, C) = B$ and $c(B, C, D) = C$ (*i.e.* D "hurts" B against C).

(see Proposition 4 in Bordalo, Gennaioli, and Shleifer (2013)), but not the more elaborate behavioral patterns of Comyn and Attila. This model is capable of accommodating cyclical binary choices by utilizing violations of monotonicity. For a behavioral foundation, see Ellis and Masatlioglu (2017).

Related to Bordalo, Gennaioli, and Shleifer (2013), Tserenjigmid (2015) presents an axiomatic study of a reference-dependent model in which the endogenous reference point has the worst attributes in all dimensions. As in Bordalo, Gennaioli, and Shleifer (2013), in this model the commodities' attributes are observable, the reference point need not be a member of the choice set and it is independent of individual characteristics. Similar to Ok, Ortoleva, and Riella (2015), binary choices are assumed not to have cycles. Hence, Tserenjigmid (2015) cannot capture Cyler's behavior. However, this model can accommodate the choice patterns of Comyn and Attila.

Other Choice Procedures: Our model is also related to the following papers, though to a lesser extent. While these models have separate psychological motivations than ours (and from each other), similar to our study they all generate choice reversals.

In the limited attention model of Masatlioglu, Nakajima, and Ozbay (2012), a DM only compares (and chooses from) a subset of all available options (called "the consideration set"). This consideration set is assumed not to be affected from elimination of an "unconsidered" alternative. It turns out that our model is a subset of theirs.[19]

In the rational shortlisting model of Manzini and Mariotti (2007), the DM iteratively applies two asymmetric binary relations to make a choice. CER and PC-CER are not logically related to this model.[20] But surprisingly, the ordered PC-CER turns out to be a rational shortlist method in which the first binary relation is an interval order and the second binary relation is complete and transitive.[21]

Masatlioglu, Nakajima, and Ozdenoren (2017) propose a model of temptation where individuals have imperfect control over their immediate urges and they are able to overcome temptation by exerting mental effort. In their model this ability, called willpower, is a limited

[19]The "difficult choice" patterns, i.e., $c(x, y, z) = x,\ c(x, y) = y$, and $c(x, z) = z$, can be captured by their model but not by ours.

[20]The models of Manzini and Mariotti (2012a), Cherepanov, Feddersen, and Sandroni (2013), Lleras, Masatlioglu, Nakajima, and Ozbay (2017) are generalizations of Manzini and Mariotti (2007). All of them satisfy Weak-WARP, which is logically independent of our Single Reversal axiom.

[21]Some of the related papers contributing to this literature are Houy (2008), Au and Kawai (2011), Manzini and Mariotti (2012a,b), Apesteguia and Ballester (2013), Matsuki and Tadenuma (2013), Tyson (2013), Dutta and Horan (2015), Horan (2016).

resource.[22] We uncover a surprising relationship between our work and two nested models considered in Masatlioglu, Nakajima, and Ozdenoren (2017). It turns out that both of their models are special cases of PC-CER. Furthermore, a choice function c admits an ordered PC-CER representation if and only if it admits a generalized limited willpower representation.

Noor and Takeoka (2010) extends the costly self-control model of Gul and Pesendorfer (2001). In their model, deviation from the most tempting alternative in the menu imposes a cost. The decision maker maximizes the welfare utility minus the cost of deviation from the most tempting alternative in the menu. For the self-control cost, they consider a general form which depends on both the choice and the maximum temptation value in the menu. Even though the domain of this paper is ex-ante menu preferences, one can focus on the implied ex-post choices to make a comparison. Indeed, the ex-post choices in this model exhibit choice-reversals as in ours. Furthermore, these ex-post choices satisfy our Single Reversal axiom. Recently, Ravid and Steverson (2018) focus on the ex-post choices and shows that the cost function of Noor and Takeoka (2010) can be written as a function of the difference between the realized utility from choice and the minimum utility possible in the menu. The authors call this the "bad temptation" model since the temptation value is the opposite of welfare utility. Given this equivalence result, the bad temptation model also satisfies Single Reversal. As opposed to Noor and Takeoka (2010), Ravid and Steverson (2018) provide a characterization of the bad temptation model in ex-post choice. They prove that their model is characterized by the Axiom of Revealed Temptation which states that in any choice set there exists an alternative such that if we consider any subsets of the given choice set which includes that alternative, then choices satisfy WARP. CER satisfies their axiom. Hence Single Reversal and their axiom generate equivalent behavioral patterns.[23] These models are thus indistinguishable from each other on the basis of choice data alone even though they capture very different positive models of behavior.

[22]As opposed to ours, Masatlioglu, Nakajima, and Ozdenoren (2017) take a preference-choice function pair $(\succeq, c)$ as their data and analyze conditions under which the limited willpower model can represent the pair $(\succeq, c)$.

[23]The advantage of the Single Reversal axiom is that it can be falsified using only two observations. On the other hand, the Axiom of Revealed Temptation is an existence axiom that requires many observations to falsify and hence, harder to test empirically.

8 Conclusion

We provide a simple model of reference-dependent choice (CER) in which the reference point is determined endogenously. The main component of CER is a *conspicuity ranking*, whose maximization in a choice set determines its reference point. The reference point in turn determines the (reference-dependent) preferences. We show that CER can be characterized by a simple and easily testable Single Reversal axiom on observed choices. We also demonstrate how one can reveal the conspicuity ranking and reference-dependent preferences of a decision maker. Imposing an additional Consistency axiom on choice behavior results in a more specialized model (PC-CER) where the reference point affects choices through a psychological constraint and the DM has reference-free preferences that can be used to make welfare evaluations. We also analyze the implications of an order structure on the psychological constraints (ordered PC-CER).

We would like to point out that this paper studies just one particular theory of reference point formation. Alternative and equally valuable theories of reference points such as expectations, aspirations, minimal and average alternatives as references have already been explored in the literature. More research is needed to determine which theory of reference point formation is the most appropriate in different applications. We hope that this paper will contribute to this line of research.

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Appendix

Proof of Theorem 1

Here we show that Single Reversal is sufficient for a CER representation. To construct the conspicuity ranking, we first define the following binary relation. For each $x, y \in X$ such that $x \neq y$, let xRy if there exists $S \in \mathcal{X}$ such that $\{x, y\} \subset S$ and $x \neq c(S) \neq c(S \setminus x)$.

Claim 1. *R is asymmetric.*

Proof. Directly follows from the statement of Single Reversal Axiom. □

Claim 2. *If $c(\bigcup_{i=1}^n S_i) \in \bigcap_{i=1}^n S_i$, then $c(\bigcup_{i=1}^n S_i) = c(S_i)$ for some $i \in \{1, ..., n\}$.*

Proof. Let $S = S_1$ and $T = \bigcup_{i=2}^n S_i$.

Suppose $c(S \cup T) \in S \cap T$, but $c(S \cup T) \notin \{c(S), c(T)\}$. Then $T \setminus S$ and $S \setminus T$ are both nonempty. Enumerate $T \setminus S = \{t_1, \ldots, t_k\}$ and $S \setminus T = \{s_1, \ldots s_l\}$. By our supposition, there are $k' \in \{1, ..., k\}$ and $l' \in \{1, ..., l\}$ such that $c(S \cup T) = c(T \cup \{s_1, ..., s_{l'}\}) \neq c(T \cup \{s_1, ..., s_{l'-1}\})$ and $c(S \cup T) = c(S \cup \{t_1, ..., t_{k'}\}) \neq c(S \cup \{t_1, ..., t_{k'-1}\})$. Also, since $c(S \cup T) \in S \cap T$, we have $c(S \cup T) \notin \{s_{l'}, t_{k'}\}$. But then $t_{k'} \neq c(S \cup \{t_1, ..., t_{k'}\}) \neq c(S \cup \{t_1, ..., t_{k'-1}\})$ implies $t_{k'} R\ s_{l'}$. Similarly, $s_{l'} \neq c(T \cup \{s_1, ..., s_{l'}\}) \neq c(T \cup \{s_1, ..., s_{l'-1}\})$ implies $s_{l'} R\ t_{k'}$. This contradicts asymmetry of R, which was established in Claim 1.

The previous paragraph guarantees that the claim holds when $n = 2$. Next, assume that the claim holds for all $n' < n$. We will prove it for n. By the previous paragraph, either $c(S) = c(\bigcup_{i=1}^n S_i)$ or $c(T) = c(\bigcup_{i=1}^n S_i)$. If the former is true, we are done. Otherwise, since $T = \bigcup_{i=2}^n S_i$, by the induction hypothesis, for some $i \in \{2, ..., n\}$ we have $c(S_i) = c(T) = c(\bigcup_{i=1}^n S_i)$. □

Claim 3. *Assume $x \neq c(S) \neq c(S \setminus x)$ and $y \neq c(T) \neq c(T \setminus y)$. Then for any $S' \in \mathcal{X}$ such that $\{x, y\} \subseteq S' \subseteq S \cup T$ and $c(S \cup T) \in S'$, we have $c(S') = c(S \cup T)$.*

Proof. Suppose not. Let $\{t_1, \ldots t_n\} = (S \cup T) \setminus S'$. For each $i \in \{1, ..., n\}$, define $T'_i = S' \cup \{t_i\}$. Since $\bigcup_{i=1}^n T'_i = S \cup T$ and $\bigcap_{i=1}^n T'_i = S'$, we have $c(\bigcup_{i=1}^n T'_i) \in \bigcap_{i=1}^n T'_i$. This, by Claim 2, implies $c(T'_i) = c(S \cup T)$ for some $i \in \{1, ..., n\}$. Since $c(S \cup T) \in S'$, we have $t_i \neq c(T'_i)$. Since $c(S') \neq c(S \cup T)$, we have $c(T'_i) \neq c(S')$. Since $\{x, y\} \subseteq S'$, we then have $t_i R x$ and $t_i R y$. But if $t_i \in S$, by our assumption $x R t_i$, contradicting asymmetry of R established in Claim 1. Alternatively if $t_i \in T$, we have a similar contradiction due to $y R t_i$. □

Claim 4. *R is transitive.*

Proof. Assume $xRyRz$. Then there are $S \supset \{x, y\}$ and $T \supset \{y, z\}$ such that $x \neq c(S) \neq c(S \setminus x)$ and $y \neq c(T) \neq c(T \setminus y)$. Note that, by asymmetry of R we have $x \notin T$. If $z \in S$, then xRz and we are done. Alternatively assume $z \notin S$. Then there are two cases to consider.
Case 1: $c(S \cup T) \in S$.
Let $\bar{S} = S \cup \{z\}$. Since $\{x, y\} \subset S \subset \bar{S} \subseteq S \cup T$, Claim 3 implies $c(S) = c(\bar{S}) = c(S \cup T)$. Thus, $x \neq c(\bar{S})$. First, assume $c(\bar{S}) = c(\bar{S} \setminus x)$. But, then $c(S) = c(\bar{S}) = c(\bar{S} \setminus x) = c(S \setminus x)$, in contradiction to the original hypothesis. Hence, $c(\bar{S}) \neq c(\bar{S} \setminus x)$ and xRz follows.
Case 2: $c(S \cup T) \in T \setminus S$.
Let $\bar{T} = T \cup \{x\}$. Since $\{x, y\} \subset \bar{T} \subseteq S \cup T$, Claim 3 implies $c(\bar{T}) = c(S \cup T)$. Since $c(S \cup T) \in T \setminus S$, $c(\bar{T}) \notin \{x, y\}$. Hence, if $c(\bar{T}) \neq c(T)$, we have xRz, the desired result. Alternatively assume $c(\bar{T}) = c(T)$. Since $y \neq c(\bar{T})$, if $c(\bar{T}) \neq c(\bar{T} \setminus y)$, we have yRx, contradicting asymmetry of R. Thus, $c(T) = c(\bar{T}) = c(\bar{T} \setminus y)$. If $c(\bar{T} \setminus y) = c(T \setminus y)$, the previous equality implies $c(T) = c(T \setminus y)$, contradicting the original hypothesis. Thus, $c(\bar{T} \setminus y) \neq c(T \setminus y)$. Since $x \neq c(\bar{T}) = c(\bar{T} \setminus y)$, this implies xRz, the desired conclusion. □

Let $\gg$ be any completion of R. Using $\gg$, we next define the reference function. For each $S \in \mathcal{X}$, let

$$r(S) = \text{argmax}(\gg, S).$$

Claim 5. *If $x \in S$ is such that $x \notin \{r(S), c(S)\}$, then $c(S) = c(S \setminus x)$.*

Proof. Suppose we have $x \neq r(S)$ and $x \neq c(S) \neq c(S \setminus x)$. By definition of R, this implies $xRr(S)$, contradicting the definition of r. □

Now for each $\rho \in X$, we define the following binary relation. Let $xP_\rho y$ if there exists a choice set such that $r(S) = \rho$, $c(S) = x$ and $y \in S$.

Claim 6. *For each $\rho \in X$, P_ρ is a transitive binary relation.*

Proof. Suppose $xP_\rho y$ and $yP_\rho z$. Then there exists S and T with $y \in S$ and $z \in T$ such that $r(S) = r(T) = \rho$, $c(S) = x$ and $c(T) = y$. Now consider $S \cup T$. Since $r(S) = r(T) = \rho$, we must have $r(S \cup T) = \rho$. We claim that $c(S \cup T) = x$. By the way of contradiction, suppose $c(S \cup T) = t \neq x$. Since $t \in S \cup T$, either $t \in S$ or $t \in T$. If $t \in S$, then by repeated application of the previous claim we get that $c(S) = t$, which is a contradiction as $t \neq x$. Suppose $t \in T \setminus S$. By repeated application of the previous claim, we get that $c(T) = t$ so that $t = y$. But this is a contradiction as $y \in S$. Hence, we conclude that $c(S \cup T) = x$. Since $r(S \cup T) = \rho$ and $z \in S \cup T$, by definition, $xP_\rho z$. □

For each $r \in X$, let $\succ_\rho$ be any completion of P_ρ and let U_ρ be the utility representation of $\succ_\rho$, i.e., $x \succ_\rho y$ if and only if $U_\rho(x) > U_\rho(y)$. Now by definition, $x = c(S)$ if and only if $U_{r(S)}(x) > U_{r(S)}(y)$ for all $y \in S \setminus x$. This completes the proof of the theorem.

Proof of Theorem 2

Let R be as in the proof of Theorem 1. Note that then, R is asymmetric and transitive. We next define a second binary relation, which we will later use together with R to construct the conspicuity ranking. For each $x, y \in X$ such that $x \neq y$, let $xR'y$ if $\neg(xRy)$, $\neg(yRx)$ and $c(x, y) = x$.[24]

Claim 7. *If $xR'yR'z$, then $\neg(zRx)$.*

Proof. First note that, x, y, z are distinct alternatives. Also, by definition $c(x, y) = x$ and $c(y, z) = y$. Now, if $c(x, y, z) = y$, we have $z \neq c(x, y, z) \neq c(x, y)$ implying zRy. This contradicts $yR'z$. Similarly, if $c(x, y, z) = z$, then $x \neq c(x, y, z) \neq c(y, z)$ implying xRy. This contradicts $xR'y$. Therefore, $c(x, y, z) = x$.

Suppose zRx. Then there exists $S \supset \{x, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$. Note that since $yR'z$, $y \notin S$. Let $\bar{S} = S \cup \{y\}$. There are two cases to consider.
Case 1: $c(\bar{S}) \neq y$.
Since $\neg(yRx)$, we have $c(S) = c(\bar{S})$ and thus, $z \neq c(\bar{S})$. Since $\neg(zRy)$, we then have $c(\bar{S}) = c(\bar{S} \setminus z)$. But then, $y \neq c(\bar{S} \setminus z)$. This, and $\neg(yRx)$ imply $c(\bar{S} \setminus z) = c(S \setminus z)$. But combining these equalities, we get $c(S) = c(S \setminus z)$, a contradiction.
Case 2: $c(\bar{S}) = y$.
Since $c(\{x, y, z\}) = x$, the set $\bar{S} \setminus \{x, y, z\} = \{t_1, ..., t_n\}$ is nonempty. For each $i \in \{1, ..., n\}$, let $T_i = \{x, y, z, t_i\}$. Since $c(\bigcup_{i=1}^n T_i) = c(\bar{S}) \in \bigcap_{i=1}^n T_i$, by Claim 2 $c(\bar{S}) = c(T_i)$ for some $i \in \{1, ..., n\}$. But then $t_i \neq y = c(x, y, z, t_i) \neq c(x, y, z) = x$, implies t_iRz. This contradicts zRt_i due to Claim 1.

Since both cases lead to a contradiction we cannot have zRx. □

We define the conspicuity ranking as $\gg = R \cup R'$. The next claim establishes that $\gg$ is a strict linear order.

Claim 8. *The conspicuity ranking $\gg$ is a strict linear order.*

Proof. For any $x \neq y$, either x and y are compared by R or by R'. Hence, $\gg$ is weakly connected. By definition, it is also irreflexive. To establish that $\gg$ is a strict linear order, we show that $\gg$ is transitive. Assume $x \gg y \gg z$. We have a few cases to consider.
Case 1: $xRyRz$.
We then have xRz by Claim 4.
Case 2: $xR'yR'z$.
By Claim 7, $\neg(zRx)$. If xRz, we are done. Alternatively assume $\neg(xRz)$. Note that by definition, $c(x, y) = x$ and $c(y, z) = y$. Now, if $c(x, y, z) = y$, we have $z \neq c(x, y, z) \neq c(x, y)$ implying zRy. This contradicts $yR'z$. Similarly, if $c(x, y, z) = z$, then $x \neq c(x, y, z) \neq c(y, z)$ implying xRy. This contradicts $\neg(xRy)$. Therefore, $c(x, y, z) = x$. If $c(x, z) = z$, then $y \neq c(x, y, z) \neq c(x, z)$, contradicting $yR'z$. Therefore, $c(x, z) = x$. This implies $xR'z$.
Case 3: $xRyR'z$.
Note that if zRx, by Claim 4 we have zRy. This contradicts $yR'z$. Alternatively if $zR'x$, by Claim 7 we have $\neg(xRy)$, again a contradiction. Therefore, either xRz or $xR'z$.

The case where $xR'yRz$ is similar to Case 3. □

[24] The notation $\neg(xRy)$ denotes "not xRy".

Using $\gg$, we next define the reference function. For each $S \in \mathcal{X}$, let

$$r(S) = \text{argmax}(\gg, S).$$

Claim 5 implies that if $x \in S$ is such that $x \notin \{r(S), c(S)\}$, then $c(S) = c(S \setminus x)$.

Given two alternatives x, y which are dominated by z according to binary relation R, it is useful to understand under which conditions we can find a choice set which includes $\{x, y, z\}$ and z causes choice reversal in that set. The next claim provides a sufficient condition which will be useful in the proof of final claim.

Claim 9. *Assume zRx, zRy and $c(x, y) = y$. Then either xRy or there is $S \supseteq \{x, y, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$.*

Proof. Since zRx and zRy, there are $T_1, T_2 \in \mathcal{X}$ such that $x \in T_1$, $y \in T_2$, $z \neq c(T_1) \neq c(T_1 \setminus z)$ and $z \neq c(T_2) \neq c(T_2 \setminus z)$.

Let $\bar{T}_2 = T_2 \cup \{x\}$. If $T_2 = \bar{T}_2$, letting $S = T_2$ concludes the proof. Alternatively, assume $T_2 \neq \bar{T}_2$. If $c(\bar{T}_2) = z$, then $x \neq c(\bar{T}_2) \neq c(T_2)$, implying xRz. This contradicts asymmetry of R. So $c(\bar{T}_2) \neq z$. We have two cases.

Case 1: $c(\bar{T}_2) = x$.

Since $z \neq c(T_2) \neq c(T_2 \setminus z)$, Single Reversal then implies $c(x, y, z) = x$. But then $z \neq c(x, y, z) \neq c(x, y)$ and letting $S = \{x, y, z\}$ concludes the proof.

Case 2: $c(\bar{T}_2) \neq x$.

Then, by asymmetry of R, $c(T_2) = c(\bar{T}_2)$. Also, if $c(\bar{T}_2) \neq c(\bar{T}_2 \setminus z)$, letting $S = \bar{T}_2$ concludes the proof. So assume $c(\bar{T}_2) = c(\bar{T}_2 \setminus z)$. Then $x \neq c(\bar{T}_2 \setminus z)$. If $c(\bar{T}_2 \setminus z) = c(T_2 \setminus z)$, then $c(T_2) = c(\bar{T}_2) = c(\bar{T}_2 \setminus z) = c(T_2 \setminus z)$, which is a contradiction to our original hypothesis. So $c(\bar{T}_2 \setminus z) \neq c(T_2 \setminus z)$. But then xRy, concluding the proof. □

We next define the psychological constraint function $Q : X \to \mathcal{X}$ as

$$Q(x) = \{y \in X \mid r(x, y) = x \text{ and } c(x, y) = y\}.$$

Our next objective is to define the preference ranking $\succ$. For each $x, y \in X$ such that $x \neq y$, let xPy if there are $z \in X \setminus x$ and $T, S \in \mathcal{X}$ such that $\{x, y, z\} \subseteq T \subseteq S$, $z \neq c(S) \neq c(S \setminus z)$, $c(T) = x$, and $c(y, z) = y$.

Claim 10. *P is acyclic.*

Proof. Suppose $x_1 P x_2 P \cdots P x_n P x_{n+1}$ where $x_1 = x_{n+1}$. Then there are $\{(z_i, T_i, S_i)\}_{i=1}^{n}$ such that for each $i \in \{1, ..., n\}$, we have $\{x_i, x_{i+1}, z_i\} \subseteq T_i \subseteq S_i$, $z_i \neq c(S_i) \neq c(S_i \setminus z_i)$, $c(x_{i+1}, z_i) = x_{i+1}$, and $c(T_i) = x_i$. Now consider the set $S^* = \{x_2, \ldots, x_{n+1}\}$. For any $x_i \in S^*$, there is z_{i-1} such that $z_{i-1} \neq c(S_{i-1}) \neq c(S_{i-1} \setminus z_{i-1})$, $c(x_i, z_{i-1}) = x_i$, $\{x_i, z_{i-1}\} \subseteq T_{i-1} \subseteq S_{i-1}$ and $c(T_{i-1}) \in S^*$, but $c(T_{i-1}) \neq x_i$, contradicting Consistency. □

Finally, we define the preference ranking $\succ$ to be any completion of P. The next claim shows that $c(S)$ is $\succ$-maximal in $Q(r(S)) \cap S$, thus concluding the proof.

Claim 11. *$c(S)$ maximizes $\succ$ in $Q(r(S)) \cap S$.*

Proof. First, if $c(S) = r(S)$, then by Claim 5, $c(r(S), x) = r(S)$ for all $x \in S$. Hence, $Q(r(S)) \cap S = \{r(S)\}$ and the claim trivially holds.

Alternatively assume $c(S) \neq r(S)$. By Claim 5, $c(r(S), c(S)) = c(S)$ which implies $\neg(r(S)R'c(S))$. But then by Claim 8, $r(S)Rc(S)$. Hence there is $T \supset \{r(S), c(S)\}$ such that $r(S) \neq c(T) \neq c(T \setminus r(S))$. Then by definition of P, $c(S)Pr(S)$. Thus, $c(S) \succ r(S)$.

Notice that $c(r(S), c(S)) = c(S)$ implies $c(S) \in Q(r(S))$. Now let $x \in Q(r(S)) \cap S$ be such that $x \notin \{c(S), r(S)\}$. By definition, $c(x, r(S)) = x$. We need to show that $c(S)Px$. There are two possible cases.

Case 1: $c(x, c(S)) = x$.

By Claim 5, $c(x, r(S), c(S)) = c(S)$. Hence, $r(S) \neq c(x, r(S), c(S)) \neq c(x, c(S))$. Since $x = c(x, r(S))$, by definition of P, $c(S)Px$.

Case 2: $c(x, c(S)) = c(S)$.

Since $c(r(S), c(S)) = c(S)$ and $c(x, r(S)) = x$, we have $\neg(r(S)R'x)$ and $\neg(r(S)R'c(S))$. Therefore, by Claim 7, it must be the case that $r(S)Rx$ and $r(S)Rc(S)$. Then by Claim 9, either there is $T \supseteq \{x, r(S), c(S)\}$ such that $r(S) \neq c(T) \neq c(T \setminus r(S))$ or $xRc(S)$. In both cases $c(S)Px$ follows. □

Independence of Axioms

To see that Single Reversal does not imply Consistency, recall Comyn from Example 1. His choices satisfy Single Reversal but not Consistency. To see that Consistency does not imply Single Reversal, let $c(x, y, z) = x$, $c(x, y) = c(y, z) = y$ and $c(x, z) = z$. Then removing either y or z from $\{x, y, z\}$ causes choice reversal. However these choices satisfy Consistency.

Proof of Theorem 3

Let $\gg$ and $Q : X \to \mathcal{X}$ be as in the proof of Theorem 2. We define $Q' : X \to \mathcal{X}$ by

$$Q'(x) = \{y \in X |\, y \in Q(z) \text{ where } z \gg x \text{ or } z = x\}$$

Notice that Q' is ordered with respect to $\gg$. For any $x \neq y$, we define xP_oy if

- there exist z and $S \supseteq \{x, y, z\}$ such that $z \neq c(S) \neq c(S \setminus z)$, $c(y, z) = y$, and $c(x, y, z) = x$; or,
- there exist $T \subset S$ containing $\{x, y\}$ such that $c(T) = x$ and $c(S) = y$.

Acyclicity axiom states that P_o is acyclic. Let $\succ$ be any completion of P_o.

Claim 12. *$c(S)$ maximizes $\succ$ in $Q'(r(S)) \cap S$.*

Proof. Firstly, suppose $c(S) = r(S)$. By Single Reversal, $c(x, r(S)) = r(S)$ for all $x \in S$. Hence, $\{r(S)\} = Q(r(S)) \cap S$. This implies that if $x \in Q'(r(S)) \cap (S \setminus r(S))$, then there exists $y \gg r(S)$ such that $c(x, y) = x$. Then x, y, and $r(S)$ are distinct as $y \gg r(S) \gg x$. There are three cases to consider.

Case 1: $c(x, y, r(S)) = y$.

Notice that $r(S) \neq c(x, y, r(S)) \neq c(x, y)$. This implies $r(S) \gg y$, a contradiction.

Case 2: $c(x, y, r(S)) = x$.

Since $c(x, r(S)) = r(S)$, by definition, $r(S)P_ox$. Thus $r(S) \succ x$.

Case 3: $c(x, y, r(S)) = r(S)$.
Since $y \gg x$, we should have $c(y, r(S)) = r(S)$. Since we also have $c(x, y) = x$, it cannot be the case that $yR'r(S)$ or $yR'x$ where R' is defined as in the proof of Theorem 2. By definition of $\gg$, yRx and $yRr(S)$. Also notice that $c(x, r(S)) = r(S)$. By Claim 9, either there exists $T \supseteq \{x, y, r(S)\}$ such that $y \neq c(T) \neq c(T \setminus y)$ or $xRr(S)$. Since $xRr(S)$ is not possible, the former must be true. Then, by definition, $c(x, y, r(S)) = r(S)$ and $c(x, y) = x$ imply $r(S)P_o x$, and hence $r(S) \succ x$.

Now suppose $c(S) \neq r(S)$. Let $x \in Q'(r(S)) \cap (S \setminus c(S))$ be given. In the proof of Theorem 2, we already showed that $c(S) \succ x$ for all $x \in Q(r(S)) \cap (S \setminus c(S))$. Hence, we can assume $x \notin Q(r(S))$. Then there exists $y \gg r(S)$ such that $c(x, y) = x$. Since $r(x, c(S), r(S)) = r(S)$, we have $c(x, c(S), r(S)) = c(c(S), r(S)) = c(S)$. There are four cases to consider.
Case 1: $c(x, y, c(S), r(S)) = y$.
Since $r(x, y, c(S), r(S)) = y$, we have $c(x, y) = y$, a contradiction.
Case 2: $c(x, y, c(S), r(S)) = x$.
Since $c(x, c(S), r(S)) = c(S)$, by definition, $c(S)P_o x$ and we are done.
Case 3: $c(x, y, c(S), r(S)) = r(S)$.
Since $c(x, c(S), r(S)) = c(S)$, we have $y \neq c(x, y, c(S), r(S)) \neq c(x, c(S), r(S))$. Single Reversal implies $c(x, y, r(S)) = r(S)$. This together with $c(x, y) = x$ imply $r(S)P_o x$. Furthermore, $c(r(S), c(S)) = c(S)$ and $c(x, y, c(S), r(S)) = r(S)$ imply $c(S)P_o r(S)$. Hence, $c(S) \succ x$.
Case 4: $c(x, y, c(S), r(S)) = c(S)$.
Since $y \gg r(S)$, we have $c(x, y, c(S)) = c(S)$. If $c(x, c(S)) = x$, then $c(x, y, c(S)) = c(S) \neq c(x, c(S))$ and $c(x, y) = x$ imply $c(S)P_o x$ and we are done. Suppose $c(x, c(S)) = c(S)$. Since $y \gg x$, we have $c(y, c(S)) = c(S)$. Then it cannot be the case that $yR'c(S)$ or $yR'x$. Hence, by definition, $yRc(S)$ and yRx. By Claim 9, either there exists $T \supseteq \{x, y, c(S)\}$ such that $y \neq c(T) \neq c(T \setminus y)$ or $xRc(S)$. In both cases, $c(S)P_o x$ follows. □

Revealed Information Under PC-CER

Continuing the revealed preference discussion on Section 6, we illustrate here how to infer the DM's conspicuity ranking and psychological constraint from her observed choices, given that the choice data is consistent with PC-CER.

First, we extend Definition 2 to state what revealed psychological constraint means under PC-CER. Assume that c admits k different PC-CER representations, $(\succ_i, \gg_i, Q_i)_{i \in \{1,\ldots,k\}}$. Then x is revealed to be in the psychological constraint of ρ if $x \in Q_i(\rho)$ for all i. Similarly, x is revealed to be outside the psychological constraint of ρ if $x \notin Q_i(\rho)$ for all i.

Recall that the construction in Theorem 2 uses the minimal possible Q. This guarantees that if the preference between x and y is not identified by P^T, then we can pick either $x \succ y$ or $y \succ x$ without affecting choice behavior.

We next use the same idea to provide a characterization of revealed psychological constraint. If we know that x is the reference point of a set T, the fact that y is chosen from T informs us that y belongs to the psychological constraint set of x. Given that the reference point is endogenously determined, we can learn that x is the reference point of T if it induces a choice reversal in T or a superset of it. Formally,

$$Q_M(x) = \{y \in X \mid \text{there is } S \supseteq T \supseteq \{x, y\} \text{ such that } x \neq c(S) \neq c(S \setminus x) \text{ and } y = c(T)\}.$$

As will be discussed later, observing choice reversals is not necessary for revealed conspicuity in PC-CER. In other words, x may be revealed to be more conspicuous than y even when x never causes a choice reversal in the existence of y. Since the definition above only uses choice reversals, one may wonder if it does fully capture revealed consideration. It turns out that the answer is yes. Indeed, if x and y cannot be compared by R, which is the binary relation capturing choice reversals, then whenever $c(x, y) = y$ we can construct a PC-CER representation of c with $y \gg x$. This is the construction used in the proof of Theorem 2. Hence, if x is revealed to be more conspicuous than y in the absence of a choice reversal, then $c(x, y) = x$ and $c(T) \neq y$ for any T with $r(T) = x$ must be true.

Choice data can also inform us whether an alternative lies outside the psychological constraint of another. Consider the observations that x is revealed preferred to y and $y = c(x, y)$. Then it must be the case that y is more conspicuous than x and x is outside the psychological constraint set of y. The opposite is also true. If x is not revealed preferred to y or $c(x, y) = x$, then we cannot reveal that x is outside the psychological constraint set of y. To see the first point, notice that if x is not revealed preferred to y, then there exists a representation $(\succ, \gg, Q)$ with $y \succ x$ and $x \in Q(y)$. To illustrate the second point, if $c(x, y) = x$, then in any representation $(\succ, \gg, Q)$, either x is more conspicuous than y in which case it is without loss to assume $x \in Q(y)$ or y is more conspicuous than x, which in turn implies $x \in Q(y)$. The following remark summarizes these points.

Remark 5. *(Psychological Constraint) Suppose c admits a PC-CER representation. Then (i) x is revealed to be in the psychological constraint set of y if and only if $x \in Q_M(y)$, (ii) x is revealed to be outside the psychological constraint set of y if and only if xP^Ty and $c(x, y) = y$.*

Finally, we discuss revealed conspicuity. As noted earlier, if x causes choice reversal when y is in the choice set, then we can conclude that x is more conspicuous than y. However, Example 5 suggests that more information about conspicuity can be revealed: the fact that x is revealed preferred to y and $y = c(x, y)$ informed us that y is more conspicuous than x. Hence, we need to modify the revealed conspicuity relation to accommodate this additional revelation: For any $x \neq y$

$$
\begin{aligned}
x\bar{R}y \text{ if } &(i) \quad \exists S \supseteq \{x, y\} \text{ such that } x \neq c(S) \neq c(S \setminus x), \text{ or} \\
&(ii) \quad yP^Tx \text{ and } x = c(x, y).
\end{aligned}
$$

Let $\bar{R}^T$ stand for the transitive closure of $\bar{R}$.

Remark 6. *(Revealed Conspicuity) Suppose c admits a PC-CER representation. If $x\bar{R}^Ty$ then x is revealed to be more conspicuous than y.*

A natural question to ask is whether $\bar{R}^T$ characterizes revealed conspicuity. The following example shows that the answer is no.

Example 6. *Consider the following choice data on $X = \{x, y, z, t\}$.*

S	$xyzt$	xyz	xyt	xzt	yzt	xy	xz	xt	yz	yt	zt
$c(S)$	x	x	y	x	y	y	x	x	y	y	z

Since $z \neq c(x, y, z, t) \neq c(x, y, t)$ we have $z\bar{R}x$, $z\bar{R}y$, and $z\bar{R}t$. Furthermore, $c(x, y, z) = x$ and $c(y, z) = y$ imply $xPyPz$. Now xPy and $c(x, y) = y$ imply $y\bar{R}x$. Hence, we have $z\bar{R}y\bar{R}x$

and $z\bar{R}t$. Now suppose we take the following completion of $\bar{R}$: $z \gg t \gg y \gg x$. Even though $\gg$ includes $\bar{R}$ there is no PC-CER representation of c with this $\gg$. First notice that given $\gg$, any psychological constraint function rationalizing this data must satisfy $Q(z) \supseteq \{x, y, z\}$ and $Q(t) \supseteq \{x, y, t\}$. Furthermore, since the preference ranking must include P, we have $x \succ y$. But then $r(x, y, t) = t$, $Q(t) \cap \{x, y, t\} = \{x, y, t\}$ and $c(x, y, t) \neq x$. Hence, no PC-CER representation of this data with this $\gg$ exists.

The following table summarizes all admissible preference-conspicuity combinations in Example 6.

	$t \succ x \succ y \succ z$	$x \succ t \succ y \succ z$	$x \succ y \succ t \succ z$	$x \succ y \succ z \succ t$
$z \gg y \gg x \gg t$	✓	✓	✓	✓
$z \gg y \gg t \gg x$	✗	✓	✓	✓
$z \gg t \gg y \gg x$	✗	✗	✗	✗

Notice that even though $y\bar{R}t$ does not hold, in all possible PC-CER representations we have $y \gg t$. To put it differently, there is no preference ranking that contains the revealed preference and is also compatible with the conspicuity ranking $z \gg t \gg y \gg x$. In this case, we say that $z \gg t \gg y \gg x$ is not consistent with revealed preference. Our next definition generalizes the intuition from Example 6.

Definition 7. *A conspicuity ranking $\gg$ is consistent with revealed preference if there is a completion of revealed preference P^T denoted by $\succ$ and a psychological constraint function Q defined by $Q(x) = \{y \in X | \, x \gg y$ and $c(x, y) = y\}$ such that*

$$c(S) = \text{argmax}(\succ, Q(\text{argmax}(\gg, S)) \cap S))$$

for all $S \in \mathcal{X}$.

Notice that if $(\succ, \gg, Q)$ is a CER representation of a choice function c, then $\succ$ must include revealed preference P^T, and, given $\gg$, Q must include the minimal possible psychological constraint. But then, by definition, $\gg$ is consistent with revealed preference. It then follows from definitions that for x to be revealed to be more conspicuous than y, it must be ranked higher in all conspicuity rankings consistent with revealed preference and vice versa.