

B. M. Pötscher
I. R. Prucha

Dynamic Nonlinear Econometric Models

Asymptotic
Theory



Springer

Preface

Many relationships in economics, and also in other fields, are both dynamic and nonlinear. A major advance in econometrics over the last fifteen years has been the development of a theory of estimation and inference for dynamic nonlinear models. This advance was accompanied by improvements in computer technology that facilitate the practical implementation of such estimation methods.

In two articles in *Econometric Reviews*, i.e., Pötscher and Prucha (1991a,b), we provided an expository discussion of the basic structure of the asymptotic theory of M-estimators in dynamic nonlinear models and a review of the literature up to the beginning of this decade. Among others, the class of M-estimators contains least mean distance estimators (including maximum likelihood estimators) and generalized method of moment estimators. The present book expands and revises the discussion in those articles. It is geared towards the professional econometrician or statistician.

Besides reviewing the literature we also presented in the above mentioned articles a number of then new results. One example is a consistency result for the case where the identifiable uniqueness condition fails. Another of these contributions was the introduction of the concept of L_p -approximability of a stochastic process by some (mixing) basis process. This approximation concept encompasses the concept of stochastic stability and the concept of near epoch dependence. Both of the latter approximation concepts had been used in the literature on the estimation of dynamic nonlinear econometric models, but the implications of the differences in these concepts were unclear at that time. Based on the encompassing approximation concept it was then possible to gain a better understanding of the differences and common grounds between the two approximation concepts. The encompassing framework made it, furthermore, possible to derive new results for the consistency and asymptotic normality of M-estimators of dynamic nonlinear models. Other contributions in the two review papers included improved consistency results for heteroskedasticity and autocorrelation robust variance-covariance matrix estimators in case of near epoch dependent data.

The theory presented in Pötscher and Prucha (1991a,b) and in the literature reviewed therein maintains catalogues of assumptions that are kept at a quite general and abstract level. As a consequence those catalogues of assumptions cover a wide range of applications. However this also means

that for a specific estimation problem it is typically necessary to still expend considerable effort to verify if those assumptions are satisfied for the problem at hand. One of the features of this book is that we apply the general theory to an important more specific estimation problem. In particular, we analyze the full information maximum likelihood estimator of a dynamic nonlinear equation system. Apart from illustrating the applicability of the general theory, this analysis also provides new catalogues for the consistency and asymptotic normality of the nonlinear full information maximum likelihood estimator. We consider both the case of a correctly specified model and that of a misspecified model. An important question that seems natural when dealing with dynamic nonlinear systems is under which conditions the output process of such a system is L_p -approximable or near epoch dependent, given the input process has this property. In this book we provide several new results in this regard. Those results cover not only first order but also higher order dynamic systems.

As usual we would like to express our thanks to all who have contributed to the preparation of this monograph over the years. In particular we would like to thank Donald W.K. Andrews, Herman Bierens, Michael Binder, Immanuel M. Bomze, A. Ronald Gallant, David Pollard, and Halbert White for their helpful comments. Special thanks are due to Manfred Deistler and Harry H. Kelejian for their ongoing support and gracious advice on this as well as other research projects. We would also like to express our gratitude to Christian Cener for expert advice on TeX-issues and to Birgit Ewald for helping with the preparation of the TeX-version of the manuscript. Finally we thank Michael Kumhof for his help in proof-reading the manuscript, and the editors of Springer-Verlag for their support and patience.

B. M. PÖTSCHER
I. R. PRUCHA

December 1996

Contents

PREFACE	vii
1 INTRODUCTION	1
2 MODELS, DATA GENERATING PROCESSES, AND ESTIMATORS	9
3 BASIC STRUCTURE OF THE CLASSICAL CONSISTENCY PROOF	15
4 FURTHER COMMENTS ON CONSISTENCY PROOFS	23
4.1 Transforming the Objective Function	23
4.2 Weakening the Uniform Convergence Assumption	24
4.3 Uniform Convergence and Compactness	26
4.4 Approximate M-Estimators	29
4.5 Limitations: An Illustrative Example	29
4.6 Identifiable Uniqueness	31
5 UNIFORM LAWS OF LARGE NUMBERS	37
5.1 ULLNs for Dependent and Heterogeneous Processes	38
5.2 Further Remarks on ULLNs	43
6 APPROXIMATION CONCEPTS AND LIMIT THEOREMS	45
6.1 Dynamic Models and Mixing Processes	46
6.2 Approximation Concepts	48
6.3 Laws of Large Numbers for L_p -Approximable and Near Epoch Dependent Processes	52
6.4 Preservation of Approximation Concepts under Transformation	56
6.5 Illustrations of Local Laws of Large Numbers	72
6.6 Comparison of ULLNs for L_p -Approximable and Near Epoch Dependent Processes	76
7 CONSISTENCY: CATALOGUES OF ASSUMPTIONS	79

8	BASIC STRUCTURE OF THE ASYMPTOTIC NORMALITY PROOF	83
9	ASYMPTOTIC NORMALITY UNDER NONSTANDARD CONDITIONS	95
10	CENTRAL LIMIT THEOREMS	99
	10.1 A Central Limit Theorem for Martingale Differences	100
	10.2 A Central Limit Theorem for Functions of Mixing Processes	101
11	ASYMPTOTIC NORMALITY: CATALOGUES OF ASSUMPTIONS	105
	11.1 Asymptotic Normality of Least Mean Distance Estimators	107
	11.2 Asymptotic Normality of Generalized Method of Moments Estimators	110
	11.3 Further Discussion and Comparison of Assumptions	116
12	HETEROSKEDASTICITY AND AUTOCORRELATION ROBUST ESTIMATION OF VARIANCE COVARIANCE MATRICES	121
	12.1 An Outline of the Variance Covariance Matrix Estimation Problem	121
	12.2 Sufficient Conditions for Consistency	124
	12.3 Further Remarks	134
13	CONSISTENT VARIANCE COVARIANCE MATRIX ESTIMATION: CATALOGUES OF ASSUMPTIONS	137
	13.1 Estimation of the Variance Covariance Matrix of Least Mean Distance Estimators	137
	13.2 Estimation of the Variance Covariance Matrix of Generalized Method of Moments Estimators	141
14	QUASI MAXIMUM LIKELIHOOD ESTIMATION OF DYNAMIC NONLINEAR SIMULTANEOUS SYSTEMS	145
	14.1 A General Consistency Result for Quasi-NFIML Estimators	147
	14.2 Asymptotic Results for the Quasi-NFIML Estimator in Case of a Correctly Specified System	150
	14.2.1 Sufficient Conditions for L_p -Approximability of the Data Generating Process	150
	14.2.2 Consistency	154
	14.2.3 Asymptotic Normality and Variance Covariance Matrix Estimation	159
15	CONCLUDING REMARKS	171

A PROOFS FOR CHAPTER 3	175
B PROOFS FOR CHAPTER 4	183
C PROOFS FOR CHAPTER 5	187
D PROOFS FOR CHAPTER 6	193
E PROOFS FOR CHAPTER 7	225
F PROOFS FOR CHAPTER 8	227
G PROOFS FOR CHAPTER 10	235
H PROOFS FOR CHAPTER 11	239
I PROOFS FOR CHAPTER 12	249
J PROOFS FOR CHAPTER 13	259
K PROOFS FOR CHAPTER 14	263
REFERENCES	287
INDEX	305

Pötscher · Prucha
Dynamic Nonlinear Econometric Models

The book provides an extensive discussion of asymptotic theory of M-estimators in the context of dynamic nonlinear models. The class of M-estimators contains least mean distance estimators (including maximum likelihood estimators) and generalized method of moments estimators. In addition to establishing the asymptotic properties of such estimators, the book provides a detailed discussion of the statistical and probabilistic tools necessary for such an analysis. In particular, the appropriate weak dependence concepts and limit theorems – (uniform) laws of large numbers, central limit theorems – and their relationship to dynamic nonlinear models are investigated in depth. This discussion includes sufficient conditions for a dynamic nonlinear system to generate an output process that is weakly dependent in an appropriate form. The book also gives a careful treatment of estimators of asymptotic variance covariance matrices for dependent processes. Furthermore, the book applies the general asymptotic theory for M-estimators developed therein to derive consistency and asymptotic normality results for the normal full information maximum likelihood estimator for the parameters of an implicit dynamic nonlinear simultaneous equation system.

ISB N 3-540-62857-6



9 783540 628576