BOOK REVIEWS

DYNAMIC NONLINEAR ECONOMETRIC MODELS—ASYMPTOTIC THEORY
by Benedikt M. Pötscher and Ingmar R. Prucha
Springer-Verlag, 1997

REVIEWED BY
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1. THE AUTHORS

Benedikt Pötscher and Ingmar Prucha are two exceptional econometricians who combine an extraordinary knowledge of the statistics and econometrics literature with great analytical skills. Both are excellent mathematicians, and the comment that can be heard among mathematicians that econometricians are "self-made mathematicians" definitely does not apply to them. Therefore, given the task of writing an overview of asymptotic theory for minimization estimators for dependent processes, one could hardly imagine a better choice of writers for such a task. The result is a remarkable book that deserves to receive attention from an audience that is broader than experts in the field only.

2. CONTENTS AND SCOPE OF THE BOOK

The book deals with the asymptotic theory of minimization estimators in the presence of weakly dependent data. Consistency, asymptotic normality, and covariance matrix estimation are covered by the book, and the results that are derived for general minimization estimators are specialized in its last chapter to the normal full information maximum likelihood estimator of a dynamic nonlinear simultaneous equation system. The book is reminiscent of Gallant and White's book *A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models* that some readers may be familiar with. The topics of both books are largely overlapping. However, although the approaches in both books are comparable, they use different terminology, and results in both books are often comparable, but conditions and proofs are never identical. Also, Pötscher and Prucha's book seeks to be a complete treatment of the subject at this point in time; the authors try

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to prove every result under as minimal conditions as possible, point out other approaches and overlapping or similar results, and discuss their results in great detail.

The book consists of 14 chapters, and all proofs are deferred to appendices (that in total take up 111 pages). Chapters 2 through 7 deal with the consistency proof, Chapters 8 through 11 deal with the asymptotic normality proof, and Chapters 12 and 13 discuss covariance matrix estimation using heteroskedasticity and autocorrelation consistent covariance matrix estimators. Chapter 14 contains the application that was mentioned earlier.

The book is based on two large articles by Pötscher and Prucha that appeared in 1991 issues of *Econometric Reviews.* The first part of the book corresponds to the first article and aims to develop as weak as possible regularity conditions for the consistency proof. To this end, various results from the literature are combined and improved, and one of its achievements is the introduction of a new concept of weak dependence (L_p-approximability) that encompasses the nustability or stochastic stability concept used by Bierens and the concept of near epoch dependence that was used by authors such as White, Gallant, and Andrews. All three concepts of weak dependence are modern implementations of ideas that are more than 30 years old; for example, a weak dependence concept that is similar to the three concepts mentioned previously can be found in Billingsley's 1967 book *Convergence of Probability Measures*, in which we can find a section entitled "Functions of Mixing Processes." This first part of Pötscher and Prucha's book I liked best because this part seems to come close to a definitive discussion of the issue in the chosen setting. The second part of the book discusses asymptotic normality. The usual techniques are used for obtaining the asymptotic normality result, and results are provided for both the case where the errors are martingale differences (corresponding to the correct specification of the time series model) and for general weakly dependent errors. The third part of the book discusses estimation of autocorrelation and heteroskedasticity consistent covariance matrices using kernel-type estimators. The results require that the bandwidth parameter that is used is $o(n^{1/3})$, and the discussion of the literature on this subject is excellent. The reference list of the book contains no less than 240 references, illustrating the authors' vast knowledge of the field. Especially strong are those paragraphs where the authors discuss results, place results in context, and point out minor slips that were left by previous authors. Numerous times one reads sentences such as "The proof by . . . seems to be in error here" or "for a corrected version of this lemma see the Appendix."

3. READERSHIP

This book is aimed at a readership of professional econometricians and perhaps ambitious econometrics graduate students. I do not think that it is suitable for graduate teaching, except perhaps at one of the world's best universities. It is not at all straightforward to list the prerequisites for reading this book. In principle,

the material of the book is largely self-contained, and the mathematics that is used is not very hard by nature. However, to fully appreciate the book, some prior familiarity with the consistency proof for minimization estimators, sigma-fields, the uniform law of large numbers and its proof, and mixing concepts for weak dependence will be very useful. This makes the book less suitable as an introduction to this type of asymptotic theory. The book is intended to take the reader all the way to the frontiers of knowledge in this field, and the reader should be prepared for the trip.

4. THE BOOK OR THE ARTICLES?

Given the fact that the book is based on two articles by Pötscher and Prucha, a natural question that arises is whether this book adds much to the article version. One difference is the addition of Chapter 14, the last chapter, which deals with "Quasi Maximum Likelihood Estimation of Dynamic Nonlinear Simultaneous Systems." This applies the "minimization estimator" theory as presented in the first 13 chapters. More essential I thought was the addition of extra references and discussion; however, the structure and results of the *Econometric Reviews* papers remain intact. As I mentioned earlier, the particular strength of the book is its discussion and the references, and I think that any reader would want the "full and last version" of these. Also, the book is beautifully typeset using LaTex; this is not only pleasing for the eye but helps very much in reading the mathematics. Therefore, if I would not have been so lucky as to have received a free review copy of Pötscher and Prucha's book, I definitely would have bought a copy of this book, in spite of the presence of copies of Pötscher and Prucha's original articles in my paper archive.

5. LIMITATIONS

Although one of the accomplishments of the book is the introduction of the weak dependence concept of "L_p-approximability" that replaces the nu-stability concept by Bierens and the near epoch dependence concept, the asymptotic normality and covariance matrix estimation parts of the book assume near epoch dependence. The problem that the authors faced here was that no central limit theorems for L_p-approximable processes are available at this point. Because of the average form that the L_p-approximability concept is formulated in, it will be difficult to formulate elegant central limit theorems for this dependence concept, and it will also be hard to deal with trending objective functions. The switch from L_p-approximability to near epoch dependence provides a discontinuity in the treatment of results that however seemed impossible to avoid. Also, when writing a book that intends to take the reader to the edge of science, it is unavoidable that elsewhere in the literature overlapping or better results are reported. The treatment of heteroskedasticity and autocorrelation consistent covariance matrix estimators, for example, overlaps with results by

Andrews, Hansen, and Davidson and De Jong. However, the authors did an extremely good job making their book as definitive as possible, and I do not expect that within 10 years from now, their book will be outdated in any sense, especially its first part.

6. FUTURE OF THE SUBJECT

As mentioned before, it will be very difficult to improve upon the results that the authors provide in the first part of their book. One possible route to extension of their results could be the law of large numbers and the uniform law of large numbers that are used. The basis for these results in the book is a result first published by Andrews, and in scenarios where we have trending random variables this result will not apply. However, because trending random variables cannot be stationary, such a contribution to the theory will be limited. The second part of the book has more opportunities for improvement. For example, there is the problem of the connection between the near epoch dependence and the approximability concept when central limit theorems are proven. Also, the results for kernel estimation of covariance matrices suggest some room for improvement.

The book does not include discussion of minimization estimators in the presence of long memory processes, integrated processes, or cointegrating relationships. The issues for such topics are very different from the ones that will come up in the analysis of weakly dependent processes. Whereas these topics will fall outside the scope of the techniques that were used in this book, they will probably become important research topics of the future.

7. CONCLUSION

Pötscher and Prucha have written an excellent book. Their willingness to share their vast knowledge of the econometrics and statistics literature with the reader means that even experts in this field will find a lot of interesting material in the book. This is a book that will be a standard reference in the future, and I recommend it to any reader with an interest in asymptotic theory for weakly dependent processes.