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Endogenous capital utilization and productivity measurement in dynamic factor demand models Theory and an application to the U.S. electrical machinery industry

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Abstract

The literature on dynamic factor demand models has, until recently, largely overlooked the issue of capital utilization. In this paper we allow for variations in the rate of capital utilization within the context of a dynamic factor demand model by adopting a modeling framework within which the firm combines its beginning-of-period stocks with other inputs to produce its outputs as well as its end-of-period stocks. We also define measures of productivity and capacity utilization for the adopted framework. As a by-product, the framework also provides for a consistent decomposition of gross investment into replacement and expansion investment. As an illustration, the model is applied to U.S. electrical machinery data.

Key words: Dynamic factor demand; Productivity; Capital stock; Capital utilization; Capacity utilization; Depreciation
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1. Introduction

In the last two decades a large body of literature emerged regarding the utilization of capital.[1] Still, studies of the firm's demand for factor inputs often assume a constant rate of utilization of the inputs and ignore the fact that the firm can simultaneously choose the level and the rate of utilization of its inputs. In particular, the literature on dynamic factor demand models has, until recently, largely overlooked the issue of capital utilization, and/or did not distinguish carefully between the distinct concepts of capital and capacity utilization.

In this paper we allow for variations in the rate of capital utilization within the context of a dynamic factor demand model by adopting a modeling framework in which the firm combines its beginning-of-period stocks with other inputs to produce its outputs as well as its end-of-period stocks. Of course, in deciding on how much of its beginning-of-period stocks are left over at the end of the period, the firm effectively decides on the rate of depreciation of its stocks. The adopted modeling framework goes back to Hicks (1946), Malinvaud (1953), and Diewert (1977, 1980). In the literature on dynamic factor demand models this framework was first adopted by Epstein and Denny (1980), and more recently by Kollintzas and Choi (1985) and Bernstein and Nadiri (1987a, b).[2] Only the first two papers implement the model empirically. In contrast to Epstein and Denny (1980) we model and estimate not only the firm's demand for its variable factors, but also the demand for its quasi-fixed factors. Kollintzas and Choi's (1985) model differs from ours in that adjustment costs are modeled as external. In contrast to both studies we allow for more than one quasi-fixed factor. The quasi-fixed factors may become productive immediately or with a lag. (Apart from those general modeling differences, these studies also differ from the current one in terms of the actual empirical specification, implementation, and detail of the analysis of the empirical results.)

To facilitate a full interpretation of the empirical results it seems of interest to also report estimates of technical change. Consequently we define measures of technical change and capacity utilization within the adopted modeling

[1] Most of this literature focused on the firm's long-run decision. Excellent summaries (including important extensions) of this literature are given in Betancourt (1987), Betancourt and Clague (1981), and Winston (1982).

[2] Bischoff and Kokkelenberg (1987) adopt a related framework where the depreciation rate is modeled as a function of capacity utilization. Other contributions to the literature on dynamic factor demand models that allow for the firm to operate at different levels of utilization, but are based on alternative modeling frameworks, include papers by Nadiri and Rosen (1969), Abel (1981), Bernstein (1983), Kokkelenberg (1984), Honkapohija and Kanniainen (1985), and Shapiro (1986).

framework (which allows for temporary equilibrium and for the endogenous determination of the depreciation rate). The specified productivity measures also cover multiple-output technologies and generalize various productivity measures previously introduced in the literature. Furthermore, we give a decomposition of the traditional measure of total factor productivity growth into technical change, scale effect, adjustment cost effect, and the variable depreciation effect.

While the approach adopted in this paper allows in general for the depreciation rate to be variable over time it also permits a constant depreciation rate as a special case. The question whether the depreciation rate of the capital stock is constant or variable was the subject of considerable debate in the literature. The approach considered here can be used to formally test the hypothesis of a constant depreciation rate.

Given the depreciation rate is estimated, existing capital stock series cannot be employed for estimation. A consistent capital stock series must be generated during estimation of the model from gross investment data. Thus, as a by-product, we generate alternative capital stock series that can be contrasted with 'official' capital stock estimates. We also obtain a consistent decomposition of gross investment into replacement and expansion investment which is important from the vantage point of public policy analysis.

To illustrate the various features of the model we have estimated the model using data from the U.S. electrical machinery industry. We test the hypothesis of a constant depreciation rate and furthermore compare our estimates with those obtained by Nadiri and Prucha (1990a) from a model with an exogenously given capital depreciation rate. We find that for the U.S. electrical machinery industry we cannot reject the hypothesis of a constant depreciation rate. Of course, as mentioned above, in general our model allows for the depreciation rate to be variable over time and in other applications the data may lead us to reject the hypothesis of a constant depreciation rate.

The paper is organized as follows. The theoretical specification of the model is presented in Section 2. Both primal and dual measures of technical change for multi-product firms, and measures of capacity utilization are discussed in Section 3. In this section we also explore the traditional measure of total factor productivity growth as a measure of technical change in more detail and identify sources of (possible) bias. In Section 4 we give an empirical specification of the model and apply this model to U.S. electrical machinery data. We report on our test of the hypothesis of a constant depreciation rate, and present estimates of the model parameters, price and output elasticities, estimates of technical change and scale, as well as report on the internally generated capital stock series and the decomposition of gross investment into replacement and expansion investment. We also give a decomposition of the traditional measure of total factor productivity growth. Concluding remarks are given in Section 5. Most of the underlying mathematic derivations are relegated to several appendices.

2. Theoretical model specification[3]

We consider a firm that combines a set of variable inputs and a set of quasi-fixed inputs to produce a set of outputs for current sale, as well as a set of capital inputs for future production. More specifically, we consider a firm that can choose how much of the beginning-of-period stocks of some of the quasi-fixed capital inputs will be left over at the end of the period. I.e., we allow the firm to endogenously choose the rate of depreciation of some, but not necessarily all, of its quasi-fixed factors. To keep the theoretical specification general we also allow for some quasi-fixed factors to become immediately productive and for some to become productive with a lag.

In more detail, let M_t be some variable factor, then we assume that the firm's technology can be represented by the following factor requirement function:

$$M_t = M(Y_t, L_t, K_t^{\mathrm{o}}, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t), \tag{2.1}$$

with $\underline{K}_t = \phi^K K_t + (I - \phi^K)K_{t-1}$, $\underline{R}_t = \phi^R R_t + (I - \phi^R)R_{t-1}$, $\Delta K_t = K_t - K_{t-1}$, and $\Delta R_t = R_t - R_{t-1}$. Here Y_t denotes the vector of output goods for current sale, and L_t represents the vector of the variable inputs other than M_t. With K_t we denote the vector of the end-of-period stocks of the quasi-fixed capital inputs whose depreciation rates can be chosen endogenously by the firm. K_t^{o} denotes the vector of 'old' stocks left over at the end of period t from K_{t-1}. As mentioned, the firm is assumed to be able to choose the level of K_t^{o} by, e.g., choosing appropriate levels of maintenance. Of course, this is equivalent to choosing the rate of depreciation for respective stocks, since we can always write $K_t^{\mathrm{o}} = (1 - \delta_t^K)K_{t-1}$ and interpret δ_t^K as a diagonal matrix of depreciation rates. R_t is the vector of the end-of-period stocks of the quasi-fixed factors whose depreciation rates are exogenous to the firm. We allow for the possibility that all or part of the net investments become productive with a one-period lag. More specifically, with $\underline{K}_t$ and $\underline{R}_t$ we denote the vectors of productive stocks in period t, and ϕ^K and ϕ^R denote diagonal matrices (where the diagonal elements are assumed to lie between zero and unity). Observe that $\underline{K}_t = \phi^K \Delta K_t + K_{t-1}$ and $\underline{R}_t = \phi^R \Delta R_t + R_{t-1}$; hence the diagonal elements of ϕ^K and ϕ^R represent the fractions of the net investments that become immediately productive. If, in particular, a diagonal element is one, then the corresponding quasi-fixed factor becomes immediately productive; if a diagonal element is zero, then the corresponding quasi-fixed factor becomes productive with a one-period lag. The

[3] The subsequent discussion makes use of the following notational conventions (unless explicitly indicated otherwise): Let Z_t be some $k \times 1$ vector of goods in period t, then p_t^Z refers to the corresponding $k \times 1$ price vector; Z_{ti} and p_{ti}^Z denote the ith elements of Z_t and p_t^Z, respectively. Furthermore, in the following we often write $(p_t^Z)'Z_t$ for $\sum_{i=1}^{k} p_{ti}^Z Z_{ti}$ where the prime (′) stands for transpose.

vectors ΔK_t and ΔR_t in the factor requirement function represent internal adjustment costs in terms of foregone output due to changes in the quasi-fixed factors. The variable T_t represents an index of technology.

The stocks K_t and R_t accumulate according to the following equations:

$$K_t = I_t^K + K_t^\circ, \qquad R_t = I_t^R + (I - \delta_t^R)R_{t-1}, \tag{2.2}$$

where I_t^K and I_t^R denote the respective vectors of gross investment and δ_t^R denotes the diagonal matrix of exogenous depreciation rates (some of which may be zero).

The firm's cost in period τ, normalized by the price of the variable factor M_τ, is given by

$$M_\tau + (p_\tau^L)'L_\tau + (p_\tau^K)'K_\tau + (p_\tau^R)'R_\tau + (q_\tau^K)'I_\tau^K + (q_\tau^R)'I_\tau^R. \tag{2.3}$$

To keep the theoretical specification general we distinguish between the price (cost) associated with operating the stocks, p_τ^K and p_τ^R, and the price of new investment goods after taxes, q_τ^K and q_τ^R, possibly normalized by $1 - u_\tau$, where u_τ denotes the corporate tax rate.[4] (It is maintained that at least one price in each corresponding pair is positive.) We assume that the firm faces perfectly competitive markets with respect to its factor inputs.

Suppose the firm's objective is to minimize the expected present value of its future cost stream.[5] Substitution of (2.1) and (2.2) into (2.3) then yields the following expression for the firm's objective function:

$$\begin{aligned} \mathrm{E}_t \sum_{\tau=t}^{\infty} [&M(Y_\tau, L_\tau, K_\tau^\circ, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p_\tau^L)'L_\tau - (q_\tau^K)'K_\tau^\circ + (p_\tau^K)'K_\tau \\ &+ (p_\tau^R)'R_\tau + (q_\tau^K)'K_\tau + (q_\tau^R)'[R_\tau - (I - \delta_\tau^R)R_{\tau-1}]] \prod_{s=t}^{\tau} (1 + r_s)^{-1}, \end{aligned} \tag{2.4}$$

where E_t denotes the expectations operator conditional on the set of information available in period t and r denotes the real discount rate (which may possibly also incorporate variations in the corporate tax rate).

Suppose the firm follows a stochastic closed loop feedback control policy in minimizing the expected present value of its future cost stream (2.4). Then, in

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[4] As an illustration, suppose R is a scalar and corresponds to the number of nonproduction workers; then p^R may represent total compensation per nonproduction worker with q^R equal to zero. As a further illustration, suppose K is a scalar and corresponds to the stock of a certain capital good; then p^K may represent the insurance cost and q^K may equal $[1 - c - u(1 - mc)B]p^{IK}/(1 - u)$, where p^{IK} denotes the price of new investment goods, u denotes the corporate tax rate, c is the rate of the investment tax credit, m is the portion of the investment tax credit which reduces the depreciable base for tax purposes, and B is the present value of the depreciation allowances.

[5] We note that the subsequent theoretical discussion can be readily modified to also apply to the case of a profit-maximizing firm.

period t the firm will choose optimal values for its current inputs L_t, K_t, R_t, and for K_t^o. At the same time the firm will choose a contingency plan for setting L_τ, K_τ, R_τ, and K_τ^o in periods $\tau = t+1, t+2, \ldots$ optimally, depending on observed realizations of the exogenous variables and past choices for the quasi-fixed factors. Of course, for given optimal values for L_τ, K_τ, R_τ, and K_τ^o the optimal values for M_τ are implied by (2.1). Prices, output, and the discount rate are assumed to be exogenous to the firm's optimization problem.

Since L_τ and K_τ^o can be changed without adjustment costs the stochastic closed loop feedback control solution can be found conveniently in two steps. In the first step we minimize the total (normalized) cost in each period τ with respect to L_τ and K_τ^o for given values of the quasi-fixed factors and the exogenous variables. Substitution of the minimized expressions into (2.4) then leads in the second step to an optimal control problem that only involves the quasi-fixed factors K_τ and R_τ.

The part of total cost that actually depends on L_τ and K_τ^o is given by $M(Y_\tau, L_\tau, K_\tau^o, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p_\tau^L)' L_\tau - (q_\tau^K)' K_\tau^o$, i.e., variable cost minus the value of the 'old' stocks left over at the end of the period from the beginning-of-period stocks. Assuming that $M(\,.\,)$ is differentiable and that a unique interior minimum of the above expression exists, the first-order conditions for that minimum are given by

$$\partial M_\tau / \partial L_\tau + p_\tau^L = 0, \qquad \partial M_\tau / \partial K_\tau^o - q_\tau^K = 0. \tag{2.5}$$

Let $\hat{L}_\tau$ and $\hat{K}_\tau^o$ denote the minimizing vectors, then the minimum of the variable cost minus the value of the 'old' stocks is given by

$$G_\tau = G(p_\tau^L, q_\tau^K, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) = \hat{M}_\tau + (p_\tau^L)' \hat{L}_\tau - (q_\tau^K)' \hat{K}_\tau^o, \tag{2.6}$$

with $\hat{M}_\tau = M(Y_\tau, \hat{L}_\tau, \hat{K}_\tau^o, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau)$. The function $G(\,.\,)$ has the interpretation of a normalized variable cost function net of the value of the 'old' stocks left over at the end of the period from the beginning-of-period stocks. Technically it can be viewed as the negative of a normalized restricted profit function. For duality results between factor requirement functions and normalized variable profit functions see, e.g., Diewert (1982) and Lau (1976). We assume that the function $G(\,.\,)$ is twice continuously differentiable in all its arguments, homogeneous of degree zero in p^L and q^K, nondecreasing in Y, $|\Delta K|$, $|\Delta R|$, and p^L, nonincreasing in $\underline{K}$, $\underline{R}$, and q^K, concave in p^L and q^K, and convex in $\underline{K}$, $\underline{R}$, ΔK, and ΔR.

As indicated above, the stochastic closed loop optimal control solution for the quasi-fixed factors can now be found by replacing $M_\tau + (p_\tau^L)' L_\tau - (q_\tau^K)' K_\tau^o$ in (2.4) by $G(p_\tau^L, q_\tau^K, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau)$ defined in (2.6), and then by minimizing

$$\begin{aligned} \mathrm{E}_t \sum_{\tau=t}^{\infty} \big[& G(p_\tau^L, q_\tau^K, Y_\tau, \underline{K}_\tau, \underline{R}_\tau, \Delta K_\tau, \Delta R_\tau, T_\tau) + (p_\tau^K)' K_\tau + (p_\tau^R)' R_\tau \\ & + (q_\tau^K)' K_\tau + (q_\tau^R)' [R_\tau - (I - \delta_\tau^R) R_{\tau-1}] \big] \prod_{s=t}^{\tau} (1 + r_s)^{-1} \end{aligned} \tag{2.7}$$

with respect to the quasi-fixed factors $\{K_\tau, R_\tau\}_{\tau=t}^{\infty}$ only. Standard control theory implies that the stochastic closed loop feedback control solution that minimizes (2.7), say $\{\hat{K}_\tau, \hat{R}_\tau\}_{\tau=t}^{\infty}$, must satisfy the following set of stochastic Euler equations ($\tau = t, t+1, \dots$):[6]

$$- \phi^K \frac{\partial G_\tau}{\partial \underline{K}_\tau} - (I - \phi^K)\,\mathrm{E}_\tau \frac{\partial G_{\tau+1}}{\partial \underline{K}_{\tau+1}} \Big/ (1 + r_{\tau+1})$$

$$= p_\tau^K + q_\tau^K + \left\{ \frac{\partial G_\tau}{\partial \Delta K_\tau} - \mathrm{E}_\tau \frac{\partial G_{\tau+1}}{\partial \Delta K_{\tau+1}} \Big/ (1 + r_{\tau+1}) \right\}, \tag{2.8}$$

$$- \phi^R \frac{\partial G_\tau}{\partial \underline{R}_\tau} - (I - \phi^R)\,\mathrm{E}_\tau \frac{\partial G_{\tau+1}}{\partial \underline{R}_{\tau+1}} \Big/ (1 + r_{\tau+1})$$

$$= p_\tau^R + c_\tau^R + \left\{ \frac{\partial G_\tau}{\partial \Delta R_\tau} - \mathrm{E}_\tau \frac{\partial G_{\tau+1}}{\partial \Delta R_{\tau+1}} \Big/ (1 + r_{\tau+1}) \right\}, \tag{2.9}$$

where $c_\tau^R = \mathrm{E}_\tau[q_\tau^R(1 + r_{\tau+1}) - (I - \delta_\tau^R)q_{\tau+1}^R]/(1 + r_{\tau+1})$ can be viewed as a vector of rental prices. The stochastic Euler equations (2.8) and (2.9) have the following economic interpretation: The optimizing firm invests in the quasi-fixed factors K and R until, at the margin (and properly discounted), the reduction in the variable cost plus the increase in the value of the 'old' stocks K^o equals the price (cost) of operating the quasi-fixed factor plus the acquisition price, plus current-period adjustment costs, minus the expected adjustment cost that would have occurred if the investment would be undertaken in the next period (rather than the current one). The firm's optimization decisions with respect to L_τ and K_τ^o are incorporated in the stochastic Euler equations via G_τ. [Recall from (2.6) that G_τ gives the minimal value of the variable cost net of the value of the 'old' stocks for given values of the quasi-fixed factors and exogenous variables.] The optimal values for L_τ and K_τ^o can be found by differentiating G_τ with respect to p_τ^L and q_τ^K and then making use of (2.5), i.e., via Shephard's and Hotelling's lemma:[7]

$$\hat{L}_\tau = \partial G_\tau / \partial p_\tau^L, \qquad \hat{K}_\tau^o = -\partial G_\tau / \partial q_\tau^K. \tag{2.10}$$

The derivatives on the r.h.s. of the above equations need to be evaluated at the optimal control solution for the quasi-fixed factors.

The formulation of a stochastic closed loop control policy generally requires knowledge of the entire distribution of the exogenous variables. Alternatively

[6] Compare, e.g., Stokey, Lucas, and Prescott (1989, Ch. 9) for a more detailed list of assumptions and a careful exposition of stochastic control theory, as well as for a discussion of the transversality condition. An explicit solution and a more detailed list of assumptions for the case where $G(.)$ is linear quadratic will be given in Section 4.

[7] In case of a profit maximizing model we have furthermore the following condition for the output vector: $\partial G_\tau/\partial Y_\tau = p_\tau^Y + [\partial p_\tau^Y/\partial Y_\tau]Y_\tau$.

one may postulate – as will be the case in the empirical application – that the firm formulates a certainty equivalence feedback control policy, which only requires knowledge of the first moment (mean) of the exogenous variables. In that case the firms objective function is given by (2.4) or (2.7) with the expectations operator moved next to each of the exogenous variables. The firm would now devise in each period t an optimal plan for its inputs in periods $t, t+1, \ldots$ such that its objective function in period t is optimized, and then choose its inputs in period t accordingly. In each future period the firm will revise its expectations and optimal plan for its inputs based on new information. In case of a certainty equivalence feedback control policy the first-order conditions for the optimal plan in period t for the quasi-fixed factors would be given by (2.8) and (2.9) with all exogenous variables replaced by their expected values (conditional on information available at time t and the expectations operator in front of the respective derivatives suppressed). Eqs. (2.10) remain the same. If $G(.)$ is linear-quadratic, then the well-known certainty equivalence principle implies that the stochastic closed loop and the certainty equivalence feedback control policy are identical.

3. Generalized measures of technological characteristics

The traditional measure of total factor productivity based on the Divisia index formula assumes, in particular: (1) that producers are in long-run equilibrium, (2) that the technology exhibits constant returns to scale, (3) that output and input markets are perfectly competitive, and (4) that factors are utilized at a constant rate. If any one of those assumptions are violated, the traditional measure of total factor productivity will in general yield biased estimates of technical change. The puzzle of the observed slowdown of productivity growth during the 1970's has initiated a critical methodological review of the traditional measures of productivity.[8]

The model considered here relaxes all of the above listed assumptions that correspond to the traditional measure of productivity. In the following we define, within the context of our model, appropriate measures of technical change. We also define a measure of capacity utilization. Furthermore we decompose the traditional measure of productivity into technical change and sources of potential bias. The measures of technical change and capacity utilization and the decomposition of the traditional measure of productivity discussed in this section will be used to evaluate the empirical results presented in Section 4.

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[8] Cp., e.g, Berndt and Fuss (1981, 1986, 1989), Bernstein and Mohnen (1988), Caves, Christensen, and Swanson (1980, 1981), Denny, Fuss, and Waverman (1981a), Griliches (1988), Hulten (1986), Mohnen, Nadiri, and Prucha (1983), Morrison (1985a, b, 1986, 1989), Nadiri and Prucha (1984, 1990a, b), and Nadiri and Schankerman (1981a, b).

3.1. Primal and dual measures of technical change

In discussing appropriate measures of technical change we first define those measures, in order to avoid ambiguities, in terms of a transformation function, say,

$$F(Y_t, V_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t) = M(Y_t, L_t, K_t^o, \underline{K}_t, \underline{R}_t, \Delta K_t, \Delta R_t, T_t) - M_t \equiv 0, \tag{3.1}$$

where $V_t = [M_t, L_t']'$ denotes the vector of all variable factors. We then show how those measures can be evaluated from the normalized variable cost function net of the value of the 'old' stocks, G. We note that in the original version of this paper, Prucha and Nadiri (1990), we also discuss corresponding index number formulae. The measures defined for the model considered in this paper generalize corresponding measures discussed in Berndt and Fuss (1981, 1986, 1989), Morrison (1985a, b, 1986), and Nadiri and Prucha (1990b) in that the model considered here not only allows for the firm to choose its factor inputs but also its rate of factor utilization optimally. For ease of notation we drop in the following time subscripts whenever those subscripts are obvious from the context.

Assume that the technology index T shifts by, say, Δ. Let $a = a(\Delta, Y, V, K^o, \underline{K}, \underline{R}, \Delta K, \Delta R, T)$ be the proportionality factor by which all outputs Y can be increased, and let $b = b(\Delta, Y, V, K^o, \underline{K}, \underline{R}, \Delta K, \Delta R, T)$ be the proportionality factor by which all inputs and K^o can be decreased corresponding to this shift in technology such that the firm remains on its production surface, i.e., $F(aY, V, K^o, \underline{K}, \underline{R}, \Delta K, \Delta R, T + \Delta) = 0$ and $F(Y, bV, bK^o, b\underline{K}, b\underline{R}, b\Delta K, b\Delta R, T + \Delta) = 0$. Furthermore let $c = c(\kappa, Y, V, K^o, \underline{K}, \underline{R}, \Delta K, \Delta R, T)$ be the proportionality factor by which all outputs Y can be increased corresponding to an increase in all inputs and K^o by a factor κ such that the firm remains on its production surface, i.e., $F(cY, \kappa V, \kappa K^o, \kappa\underline{K}, \kappa\underline{R}, \kappa\Delta K, \kappa\Delta R, T) = 0$. We can now give the following two definitions of technical change, λ_Y and λ_X, and returns to scale, ρ:

$$\lambda_Y = \frac{\partial a}{\partial \Delta}\bigg|_{\Delta=0} = -\frac{\partial F}{\partial T}\bigg/\left[\sum_i (\partial F/\partial Y_i) Y_i\right], \tag{3.2}$$

$$\begin{aligned}\lambda_X = -\frac{\partial b}{\partial \Delta}\bigg|_{\Delta=0} = \frac{\partial F}{\partial T}\bigg/\Bigg[&\sum_j (\partial F/\partial V_j) V_j + \sum_k (\partial F/\partial \underline{K}_k)\underline{K}_k \\ &+ \sum_l (\partial F/\partial \underline{R}_l)\underline{R}_l + \sum_k (\partial F/\partial K_k^o) K_k^o \\ &+ \sum_k (\partial F/\partial \Delta K_k)\Delta K_k + \sum_l (\partial F/\partial \Delta R_l)\Delta R_l\Bigg],\end{aligned}$$

$$\rho = \frac{\partial c}{\partial \kappa}\bigg|_{\kappa=1} = \lambda_Y/\lambda_X.$$

We refer to λ_Y and λ_X as the rates of, respectively, output- and input-based technical change or productivity growth. The definitions given above coincide with those given in Caves, Christensen, and Swanson (1981) and Caves, Christensen, and Diewert (1982a, b) for the case of technologies without explicit adjustment costs and constant factor utilization rates.

We next show how the above measures can be evaluated from the cost side. Observe from (3.1) that $\partial F/\partial M = -1$ and $\partial F/\partial Z = \partial M/\partial Z$ for $Z = Y, L, \underline{K}, \underline{R}, K^o, \Delta K, \Delta R, T$. Hence it follows from (2.5) that $\partial F/\partial L = -p^L$ and $\partial F/\partial K^o = q^K$. Observe furthermore from (2.5) and (2.6) that $\partial G/\partial Z = \partial M/\partial Z$ and hence $\partial F/\partial Z = \partial G/\partial Z$ for $Z = Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T$. Therefore we can write the above expressions for technical change and returns to scale alternatively in terms of the normalized variable cost function net of the 'old' stocks, G, as

$$\lambda_Y = -[\partial G/\partial T] \Big/ \left[\sum_i (\partial G/\partial Y_i) Y_i \right], \tag{3.3}$$

$$\lambda_X = -[\partial G/\partial T] \Big/ \left[G - \sum_k (\partial G/\partial \underline{K}_k)\underline{K}_k - \sum_l (\partial G/\partial \underline{R}_l)\underline{R}_l - \sum_k (\partial G/\partial \Delta K_k)\Delta K_k - \sum_l (\partial G/\partial \Delta R_l)\Delta R_l \right],$$

$$\rho = \lambda_Y/\lambda_X = \left[G - \sum_k (\partial G/\partial \underline{K}_k)\underline{K}_k - \sum_l (\partial G/\partial \underline{R}_l)\underline{R}_l - \sum_k (\partial G/\partial \Delta K_k)\Delta K_k - \sum_l (\partial G/\partial \Delta R_l)\Delta R_l \right] \Big/ \left[\sum_i (\partial G/\partial Y_i) Y_i \right].$$

The above expressions for output-based and input-based technical change and returns to scale generalize, in particular, those previously given in Nadiri and Prucha (1984, 1990a, b) for single-output technologies with adjustment costs, but constant factor depreciation rates.[9] Due to space constraints we do not present index number formulae corresponding to (3.3) in this paper; however, those formulae are given in Prucha and Nadiri (1990).

3.2. Measures of capacity utilization

For ease of notation, we assume in this and the subsequent subsection that $L, K^o, \underline{K}$, and $\underline{R}$ are scalars (and that all quasi-fixed factors exhibit positive

[9] Nadiri and Prucha (1990b) also provide expressions for multiple-output technologies. We note, furthermore, that the algebra adopted here is analogous to that used by Caves, Christensen, and Swanson (1981) for multiple-output technologies without explicit adjustment costs and constant factor utilization rates.

growth).[10] Furthermore, to simplify the discussion we assume that all quasi-fixed factors only become productive with a lag, i.e., $\phi_K = 0$ and $\phi_R = 0$ and $\underline{K} = K_{-1}$ and $\underline{R} = R_{-1}$. We also assume that $p^K = 0$ and $p^R = 0$ and that all price expectations are static.

For further interpretation of our input-based and output-based technical change measures observe that the total shadow cost (normalized by the price of the variable factor M) is defined as

$$C = G(p^L, q^K, Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T) - (\partial G/\partial \underline{K})\underline{K} - (\partial G/\partial \underline{R})\underline{R}$$
$$- (\partial G/\partial \Delta K)\Delta K - (\partial G/\partial \Delta R)\Delta R, \tag{3.4}$$

where $-\partial G/\partial \underline{K}$, $-\partial G/\partial \underline{R}$, $-\partial G/\partial \Delta K$, and $-\partial G/\partial \Delta R$ denote the respective shadow prices. Furthermore, consider the following measure of total cost (normalized by the price of the variable factor M):

$$C^+ = M + p^L L + \underline{c}^K \underline{K} + \underline{c}^R \underline{R}$$
$$= G(p^L, q^K, Y, \underline{K}, \underline{R}, \Delta K, \Delta R, T) + (1+r) q^K \underline{K} + c^R \underline{R}, \tag{3.5}$$

where $c^K = q^K(r + \delta^K)$ and $c^R = q^R(r + \delta^R)$ denote respective rental prices; the second equality follows given M, L, and K^o are chosen optimally and observing that $c^K \underline{K} = (1+r) q^K \underline{K} - q^K K^o$. Now suppose we attempt to measure technical change in terms of the total cost function C^+ by $\lambda_X^+ = -(\partial C^+/\partial T)/C^+$. Observing that $\partial C^+/\partial T = \partial G/\partial T$ it follows immediately from (3.3) and (3.4) that

$$\lambda_Y = \rho \lambda_X^+ (C^+/C), \qquad \lambda_X = \lambda_X^+ (C^+/C). \tag{3.6}$$

Clearly, in long-run equilibrium C^+ equals C and hence λ_X^+ equals λ_X. In general, however, λ_X^+ differs from λ_X and λ_Y. We note that the above formulae generalize analogous formulae given, e.g., in Morrison (1986) for adjustment cost technologies in case of a single-output good and exogenously given factor depreciation rates. Analogously to Berndt, Fuss, Hulten, and Morrison we can interpret

$$CU = C/C^+ \tag{3.7}$$

as a measure of capacity utilization and we can therefore interpret our input- and output-based measures for technical change as being derived from λ_X^+ via an

[10] The results generalize trivially to the case where L, K^o, $\underline{K}$, and $\underline{R}$ are vectors.

adjustment in terms of a capacity utilization measure to account for temporary equilibrium; cp. also Nadiri and Prucha (1990b).

3.3. Sources of bias in the traditional measure of TFP growth

As indicated by the above discussion the traditional measure of total factor productivity growth, say $T\dot{F}P$, is only a proper measure for technical change under the assumption that returns to scale equal unity, that producers are in long-run equilibrium, that output and input markets are perfectly competitive and factors are utilized at a constant rate. In the following we provide – under the less restrictive assumptions maintained here – a decomposition of the $T\dot{F}P$ measure into technical change and sources of potential bias. We provide this decomposition for the case of a firm that produces a single-output good and $T_t = t$.

Consider the following typical Törnquist approximation for the traditional measure of total factor productivity growth:

$$\Delta TFP_t = \Delta \ln Y_t - \Delta \ln N_t, \tag{3.8a}$$

where $\Delta \ln Y_t$ denotes the growth rate of output and $\Delta \ln N_t$ denotes the growth rate of a cost-share-weighted index of aggregate inputs. The index of aggregate inputs, N, is defined by

$$\Delta \ln N_t = \tfrac{1}{2}[\Delta \ln N_t^t + \Delta \ln N_t^{t-1}], \tag{3.8b}$$

$$\Delta \ln N_t^\tau = \bar{s}^M(\tau)\Delta \ln M_t + \bar{s}^L(\tau)\Delta \ln L_t + \bar{s}^K(\tau)\Delta \ln \underline{K}_t + \bar{s}^R(\tau)\Delta \ln \underline{R}_t,$$

$$\bar{s}^M(\tau) = M_\tau/C_\tau^+, \qquad \bar{s}^L(\tau) = p_\tau^L L_\tau/C_\tau^+,$$

$$\bar{s}^K(\tau) = c_\tau^K \underline{K}_\tau/C_\tau^+, \quad \bar{s}^R(\tau) = c_\tau^R \underline{R}_\tau/C_\tau^+,$$

where total cost C^+ is defined in (3.5) and the $\bar{s}^Z$ denote the respective cost shares ($Z = M, L, K, R$). In Appendix A we show that ΔTFP can essentially be decomposed as follows:[11]

$$\Delta TFP_t = \Delta TFP_t^1 + \Delta TFP_t^2 + \Delta TFP_t^3 + \Delta TFP_t^4 + \Delta TFP_t^5, \tag{3.9}$$

where

$$\Delta TFP_t^1 = \tfrac{1}{2}[\lambda_X(t) + \lambda_X(t-1)],$$

$$\Delta TFP_t^2 = (1 - 1/\rho(t))\Delta \ln Y_t,$$

[11] For ease of notation we give in (3.3) the decomposition under the assumption that $\rho(t) = \rho(t-1)$. Appendix A covers the general case $\rho(t) \neq \rho(t-1)$.

$$\Delta TFP_t^3 = \tfrac{1}{2} \sum_{\tau=t,t-1} [[-\partial G_\tau/\partial \underline{K}_\tau - (1+r)q_\tau^K]\underline{K}_\tau/C_\tau^+][\Delta \ln \underline{K}_t - \Delta \ln N_t^\tau]$$

$$+ \tfrac{1}{2} \sum_{\tau=t,t-1} [[-\partial G_\tau/\partial \underline{R}_\tau - \underline{c}_\tau^R]\underline{R}_\tau/C_\tau^+][\Delta \ln \underline{R}_t - \Delta \ln N_t^\tau],$$

$$\Delta TFP_t^4 = \tfrac{1}{2} \sum_{\tau=t,t-1} [[-\partial G_\tau/\partial \Delta K_\tau]\Delta K_\tau/C_\tau^+][\Delta \ln \Delta K_t - \Delta \ln N_t^\tau]$$

$$+ \tfrac{1}{2} \sum_{\tau=t,t-1} [[-\partial G_\tau/\partial \Delta R_\tau]\Delta R_\tau/C_\tau^+][\Delta \ln \Delta R_t - \Delta \ln N_t^\tau],$$

$$\Delta TFP_t^5 = \tfrac{1}{2} \sum_{\tau=t,t-1} [-q_\tau^K K_\tau^o/C_\tau^+][\Delta \ln K_t^o - \Delta \ln K_{t-1}].$$

The first term in the above decomposition of ΔTFP corresponds to actual technical change. The remaining terms decompose the difference between ΔTFP and technical change, i.e., they reflect sources of potential bias of ΔTFP as a measure of technical change. More specifically, the second term reflects scale effects. We note that under increasing returns to scale and positive output growth ΔTFP will overestimate technical change. The third term reflects the difference in the marginal conditions for the quasi-fixed factors between short- and long-run equilibrium due to adjustment cost, i.e., the difference between the shadow price and (long-run) rental price. Suppose the shadow price for a particular quasi-fixed factor exceeds the long-run price used in the computation of ΔTFP. In this case ΔTFP will, ceteris paribus, overestimate the technical change effects given the growth rate of the quasi-fixed input exceeds that of the aggregate input index. The fourth term reflects the direct effect of adjustment costs in the sense that due to the presence of ΔK_t and ΔR_t in the transformation function the growth rates of those terms also enter the decomposition of the output growth rate. The fifth term stems from the fact that the firm can choose the depreciation rate for some of its quasi-fixed factors endogenously. Clearly, in case of a constant depreciation rate K^o and K_{-1} will grow at the same rate and this latter term will be zero.

4. Empirical application

In the following we apply the above described approach to analyze the production structure, factor demand, productivity growth, capacity utilization, and the rate of capital depreciation in the U.S. electrical machinery industry. In Nadiri and Prucha (1990a) we have estimated the production structure and factor demand for that industry based on capital stock data from the Office of Business Analysis (OBA). We use this previous study as a benchmark. The model specified below coincides with that of our previous study, except that we now estimate the depreciation rate of the capital stock econometrically and generate the capital stock series internally. While the above described approach allows in general for the depreciation rate to be variable over time, it also permits a constant depreciation rate as a special case. The question whether the depreciation rate of the capital stock is constant or variable was the subject of considerable debate in the literature. The approach considered here can be used to test the hypothesis of a constant depreciation rate. We note that here even in case of a constant depreciation rate this rate is estimated econometrically and the corresponding capital stock series is generated internally.

4.1. Empirical specification and estimation procedure

Following Nadiri and Prucha (1990a) we specialize the model for the empirical analysis to two variable inputs, two quasi-fixed factors, and one output good. More specifically, in the following L_t and M_t denote, respectively, labor input and material input, and K_t and R_t denote, respectively, the end of period stocks of physical capital and R&D, and Y_t denotes gross output. The specification allows for the firm to determine the depreciation rate of capital endogenously, while the depreciation rate of R&D is fixed. p_t^L now denotes the price of labor, q_t^K and q_t^R denote the after-tax acquisition price for capital and R&D normalized by the price of material goods, respectively, and $p_t^K = p_t^R = 0$; cp. footnote 5. The real discount rate is taken to be constant over time at 5 percent.[12]

To model the technology we specify (dropping subscripts t) the following functional form for the normalized variable cost function net of the value of the 'old' stocks as

$$\begin{aligned} &G(p^L, q^K, K_{-1}, R_{-1}, \Delta K, \Delta R, Y, T) \qquad (4.1)\\ &= Y^{1/\rho}\{\alpha_0 + \alpha_L p^L + \alpha_{LT} p^L T + \tfrac{1}{2}\alpha_{K^\circ K^\circ}(q^K)^2 + \alpha_{LK^\circ} p^L q^K + \tfrac{1}{2}\alpha_{LL}(p^L)^2\}\\ &\quad + \alpha_K K_{-1} + \alpha_R R_{-1} + \alpha_{KL} K_{-1} p^L + \alpha_{KK^\circ} K_{-1} q^K\\ &\quad + \alpha_{RL} R_{-1} p^L + \alpha_{RK^\circ} R_{-1} q^K + \alpha_{KT} K_{-1} T + \alpha_{RT} R_{-1} T\\ &\quad + Y^{-1/\rho}\{\tfrac{1}{2}\alpha_{KK} K_{-1}^2 + \alpha_{KR} K_{-1} R_{-1} + \tfrac{1}{2}\alpha_{RR} R_{-1}^2 + \tfrac{1}{2}\alpha_{\dot K\dot K}\Delta K^2 + \tfrac{1}{2}\alpha_{\dot R\dot R}\Delta R^2\}. \end{aligned}$$

For reasons of interpretation of the function $G(.)$ we note that in general (as is not difficult to see) the normalized variable cost function net of the value of the 'old' stocks corresponding to a homothetic production function is of the form

$$G_*\left(p^L, q^K, \frac{K_{-1}}{H(Y)}, \frac{R_{-1}}{H(Y)}, \frac{\Delta K}{H(Y)}, \frac{\Delta R}{H(Y)}, T\right) H(Y), \qquad (4.2)$$

[12] We have estimated the model with alternative discount rates and found the results to be insensitive to this assumption. As discussed in more detail in Appendix B, in taking the discount rate to be constant we are able to reparameterize the model such that we are able to estimate the explicit solution for the quasi-fixed factors rather than just a set of Euler equations; cp. also Prucha and Nadiri (1986) on implied trade-offs.

where $H(Y)$ is a function in Y. The scale elasticity is then given by $H(Y)/[Y(\mathrm{d}H/\mathrm{d}Y)]$. In case $H(Y) = Y^{1/\rho}$, the technology is homogeneous of degree ρ. Consequently the function $G(.)$ defined in (4.1) can be viewed as a second-order approximation of the normalized variable cost function net of the value of the 'old' stocks corresponding to some general homogeneous technology (where parameter restrictions such that the marginal adjustment costs at $\Delta K = \Delta R = 0$ are zero have been imposed).[13] The convexity of $G(.)$ in K, R, ΔK, ΔR and the concavity in p^L and q^K implies that $\alpha_{KK} > 0$, $\alpha_{RR} > 0$, $\alpha_{KK}\alpha_{RR} - \alpha_{KR}^2 > 0$, $\alpha_{\dot{K}\dot{K}} > 0$, $\alpha_{\dot{R}\dot{R}} > 0$, $\alpha_{LL} < 0$, $\alpha_{K^\circ K^\circ} < 0$, $\alpha_{LL}\alpha_{K^\circ K^\circ} - \alpha_{LK^\circ}^2 > 0$.

We assume that the firm determines its inputs according to a certainty equivalence feedback control policy and holds static expectations on relative prices, output, and the technology. Utilizing, e.g., the expressions for the optimal control solution of a linear quadratic optimal control problem given in Madan and Prucha (1989), it then follows that we can describe the firm's optimal quasi-fixed factor inputs in period t corresponding to the technology defined by (4.1) by the following accelerator equations:

$$\Delta K_t = m_{KK}(K_t^* - K_{t-1}) + m_{KR}(R_t^* - R_{t-1}), \tag{4.3a}$$

$$\Delta R_t = m_{RK}(K_t^* - K_{t-1}) + m_{RR}(R_t^* - R_{t-1}), \tag{4.3b}$$

with

$$\begin{bmatrix} K_t^* \\ R_t^* \end{bmatrix} = -\begin{bmatrix} \alpha_{KK} & \alpha_{KR} \\ \alpha_{KR} & \alpha_{RR} \end{bmatrix}^{-1} \times \begin{bmatrix} \alpha_K + \alpha_{KT}T_t + \alpha_{KL}\hat{p}_t^L + \hat{q}_t^K(1 + r + \alpha_{KK^\circ}) \\ \alpha_R + \alpha_{RT}T_t + \alpha_{RL}\hat{p}_t^L + \hat{c}_t^R \end{bmatrix} \hat{Y}_t^{1/\rho}.$$

In the above equations expectations are characterized with a carat ($\hat{}$), and the accelerator coefficients $M = (m_{ij})_{i,j=K,R}$ have to satisfy the following matrix equation: $BM^2 + (A + rB)M - A = 0$ with $A = (\alpha_{ij})_{i,j=K,R}$ and where B is the diagonal matrix with elements $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ in the diagonal. The firm's demand equations for the variable factors and the firm's optimal choice for the 'old' stock (to be left over from the beginning-of-period capital stock) can be derived from

[13] We note that $G(.)$ defined in (4.1) is a generalization of the normalized variable cost function introduced by Denny, Fuss, and Waverman (1981b) and Morrison and Berndt (1981) for constant returns to scale technologies. Nadiri and Prucha (1984, 1990b) generalized the latter function to homothetic technologies. In imposing parameter restrictions such that the marginal adjustment costs are zero for zero net investment we follow that literature.

(4.1) as $M_t = G_t - p_t^L L_t + q_t^K K_t^\circ$, $L_t = \partial G_t/\partial p_t^L$, and $K_t^\circ = -\partial G_t/\partial q_t^K$:

$$\begin{aligned} M_t = {} & \{\alpha_0 - \tfrac{1}{2}\alpha_{K^\circ K^\circ}(\hat{q}_t^K)^2 - \alpha_{LK^\circ}\hat{p}_t^L\hat{q}_t^K - \tfrac{1}{2}\alpha_{LL}(\hat{p}_t^L)^2\}\,\hat{Y}_t^{1/\rho} \\ & + \alpha_K K_{t-1} + \alpha_R R_{t-1} + \alpha_{KT}K_{t-1}T_t + \alpha_{RT}R_{t-1}T_t \\ & + \{\tfrac{1}{2}\alpha_{KK}K_{t-1}^2 + \alpha_{KR}K_{t-1}R_{t-1} + \tfrac{1}{2}\alpha_{RR}R_{t-1}^2 \\ & + \tfrac{1}{2}\alpha_{\dot{K}\dot{K}}\Delta K_t^2 + \tfrac{1}{2}\alpha_{\dot{R}\dot{R}}\Delta R_t^2\}/\hat{Y}_t^{1/\rho}, \end{aligned} \tag{4.4a}$$

$$L_t = \{\alpha_L + \alpha_{LT}T_t + \alpha_{LK^\circ}\hat{q}_t^K + \alpha_{LL}(\hat{p}_t^L)^2\}\,\hat{Y}_t^{1/\rho} + \alpha_{KL}K_{t-1} + \alpha_{RL}R_{t-1}, \tag{4.4b}$$

$$K_t^\circ = -\{\alpha_{LK^\circ}\hat{p}_t^L + \alpha_{K^\circ K^\circ}\hat{q}_t^K\}\,\hat{Y}_t^{1/\rho} - \alpha_{KK^\circ}K_{t-1} - \alpha_{RK^\circ}R_{t-1}. \tag{4.5}$$

Recall also from (2.2) that

$$K_t = I_t^K + K_t^\circ. \tag{4.6}$$

Eq. (4.5) provides an economic model for K_t° and hence for the depreciation rate of capital δ_t^K; recall that the depreciation rate of capital is implicitly defined by $K_t^\circ = (1 - \delta_t^K)K_{t-1}$. Eq. (4.5) explains K_t° as a function of relative prices, output, and lagged stocks. The case of a constant and exogenously given depreciation rate is contained as a special case with $\alpha_{LK^\circ} = \alpha_{K^\circ K^\circ} = \alpha_{RK^\circ} = 0$, and $\alpha_{KK^\circ} = -(1 - \delta^K)$.

For purposes of estimation it proves advantageous to reparameterize the model. More specifically, instead of estimating the parameter matrices A and B, it proves advantageous to estimate the matrices $C = (c_{ij})_{i,j=K,R} = -BM$ and B (and to express the elements of A as functions of the elements of B and C). This approach is explained in more detail in Appendix B. The matrix C is found to be symmetric and negative definite.

For purposes of estimation we also add stochastic disturbance terms to each of the factor demand equations in (4.3) and (4.4). Those disturbances can be viewed as random errors of optimization and errors in the data.[14] Analogously to the approach taken by Epstein and Denny (1980) we assume that Eq. (4.5) for K_t° holds exactly. This assumption is clearly strong. It facilitates that the unobservable stocks K_t and K_t° can, at least in principle, be expressed as functions of observable variables and the unknown model parameters. More specifically, by solving (4.5) together with the identity (4.6) recursively for K_t and K_t° from some given initial capital stock, say K_0, we can express K_t as a function of $I_t^K, I_{t-1}^K, \ldots, K_0, R_{t-1}, R_{t-2}, \ldots,$ the exogenous variables and the model parameters. Consequently, upon replacing K_t and K_{t-1} in (4.3) and (4.4) by the obtained expressions we can, at least in principle, rewrite the system of factor demand equations as a dynamic system of equations that determines I_t^K, R_t, M_t,

[14] The error in the materials equation may also be viewed as a random shock observed by the firm but not by the researcher; cp., e.g., Epstein and Yatchew (1985). In the labor equation we also corrected for first-order autocorrelation of the disturbances.

and L_t, and where in the so obtained system *all variables are observable.* (If the initial stock is unobserved, we may treat it as an additional parameter.)

Of course, for the actual numerical computation of estimators of the model parameters it is generally not necessary to solve (4.5) and (4.6) analytically for K_t (and K_t°). Numerical algorithms for the computation of estimators that are defined as optimizers of some statistical objective function generally require the numerical evaluation of the statistical objective function for different sets of parameter values. For any given set of parameter values we can solve (4.5) and (4.6) numerically for K_t (and K_t°). Hence, rather than to substitute the analytic solution for K_t we can, in evaluating the statistical objective function, first solve (4.5) and (4.6) numerically and then substitute the numerical solution for K_t.

The statistical objective function underlying the parameter estimates reported in the next section is the Gaussian full information maximum likelihood (FIML) function. We used the subroutine VA10AD from the Harwell program library to numerically maximize this function, i.e., to calculate the FIML estimates. We note that the factor demand system (4.3) and (4.4) in conjunction with (4.5) and (4.6) may be viewed as a system of equations with implicitly defined variables.[15]

4.2. Parameter estimates and elasticities

We have estimated two versions of model (4.3)–(4.6) from U.S. electrical machinery industry data. In one version we have imposed the parameter restrictions $\alpha_{LK^\circ} = \alpha_{K^\circ K^\circ} = \alpha_{RK^\circ} = 0$ which implies that $K_t^\circ = -\alpha_{KK^\circ}K_{t-1}$. I.e., in this case the depreciation rate of capital is constant with $\delta^K = 1 - \alpha_{KK^\circ}$. In the other version no parameter restrictions are imposed, and the depreciation rate of capital is permitted to depend on relative prices, output and lagged stocks. We refer to those versions as models 2 and 3, respectively. We note that for both models 2 and 3 the depreciation rate is estimated, and the respective capital stock series are determined consistently with the estimated model parameters from gross investment data during estimation. To contrast these results we also report the parameter estimates presented in Nadiri and Prucha (1990a) for a model with exogenous capital depreciation rate based on the capital stock series provided in the OBA data bank. We refer to this latter model as model 1. It corresponds to (4.3)–(4.6) with $\alpha_{LK^\circ} = \alpha_{K^\circ K^\circ} = \alpha_{RK^\circ} = 0$ and with α_{KK° replaced by $-(1 - \delta_t^K)$ where δ_t^K is defined by the OBA capital stock series.

The underlying data for the U.S. electrical machinery industry are described in Appendix C and are the same as those used in Nadiri and Prucha (1990a). Expectations on (relative) prices were set equal to current (relative) prices.

[15] For an estimation algorithm for general systems of equations with implicitly defined variables that evaluates the gradient of the objective function from analytic expressions see, e.g., Prucha and Nadiri (1988).

Expectations on gross output were calculated as follows. We first estimated a first-order autoregressive model for output, and then used this model to predict Y_t. Time was used for the technology index T. In estimating models 2 and 3 we used as the initial capital stock the corresponding value of the OBA capital stock.

FIML estimates of the structural parameter estimates are given in Table 1. Asymptotic 't'-ratios are given in parentheses. The underlying estimator for the variance–covariance matrix for the FIML estimator takes into account that expectations on gross output depend on pre-estimated parameters; for more

Table 1
Full information maximum likelihood estimates of the parameters for the U.S. electrical machinery industry: 1960–1980

Parameters	*Model 1:* OBA capital stock		*Model 2:* Estimated capital stock and constant depreciation rate		*Model 3:* Estimated capital stock and variable depreciation rate	
α_0	1.83	(6.61)	1.87	(6.40)	1.86	(5.07)
$1/\rho$	0.82	(11.32)	0.86	(11.28)	0.84	(10.39)
α_K	− 0.95	(2.76)	− 0.82	(2.09)	− 0.73	(1.65)
α_R	− 0.65	(2.01)	− 0.77	(1.76)	− 0.81	(1.84)
α_{KT}	− 0.19	(3.58)	− 0.20	(4.34)	− 0.17	(3.38)
α_{RT}	0.22	(2.83)	0.27	(2.12)	0.23	(2.51)
c_{KK}	− 2.05	(2.66)	− 1.71	(2.58)	− 1.41	(1.93)
c_{RR}	− 2.10	(1.03)	− 2.45	(1.75)	− 2.27	(1.96)
c_{RK}	0.15	(0.71)	0.15	(0.59)	0.01	(0.07)
$\alpha_{\dot{K}\dot{K}}$	8.71	(2.61)	8.01	(3.13)	7.30	(2.79)
$\alpha_{\dot{R}\dot{R}}$	13.83	(1.75)	16.20	(1.56)	15.27	(1.82)
α_L	1.91	(22.23)	1.94	(19.07)	1.88	(12.91)
α_{LL}	− 0.48	(3.56)	− 0.44	(3.29)	− 0.52	(2.67)
α_{KL}	0.29	(2.25)	0.32	(2.78)	0.35	(2.57)
α_{RL}	− 0.52	(3.97)	− 0.57	(3.73)	− 0.56	(3.40)
α_{LT}	− 0.28	(6.99)	− 0.34	(4.36)	− 0.32	(3.72)
α_{KK°			− 0.96	(42.14)	− 0.998	(24.66)
$\alpha_{K^\circ K^\circ}$					− 0.003	(0.09)
α_{LK°					0.028	(0.52)
α_{RK°					0.023	(0.40)
Log of likelihood		222.10		223.62		224.16
M equation: R^2		0.85		0.84		0.84
L equation: R^2		0.65		0.65		0.65
I^K equation: R^2		0.91		0.89		0.89
I^R equation: R^2		0.86		0.86		0.86

Absolute values of the asymptotic 't'-ratios are given in parentheses. The R^2 values correspond to the squared correlation coefficients between the actual M, L, I^K, I^R variables and their fitted values calculated from the reduced form.

details see Appendix D. The parameter estimates satisfy the theoretical restrictions for all models. In particular, the estimates for c_{KK}, c_{RR}, α_{LL}, and $\alpha_{K^\circ K^\circ}$ are negative and those for $\alpha_{\dot{K}\dot{K}}$, $\alpha_{\dot{R}\dot{R}}$, $c_{KK}c_{RR} - c_{KR}^2$, and $\alpha_{LL}\alpha_{K^\circ K^\circ} - \alpha_{LK^\circ}^2$ are positive. The squared correlation coefficients between actual and fitted data are quite high and very similar across models. (Fitted values are calculated from the reduced form.)

To discriminate between the three models observe that, as pointed out above, model 2 is a special case of model 3 with parameter restrictions $\alpha_{LK^\circ} = \alpha_{K^\circ K^\circ} = \alpha_{RK^\circ} = 0$. The corresponding likelihood ratio test statistic is distributed chi-square with three degrees of freedom. Given the observed value of 1.08 for the likelihood ratio test statistic and a critical value of 7.81 we accept model 2 over model 3 at the 5 percent significance level. Next observe that if we replace in (4.3)–(4.6) α_{KK° by $\vartheta\alpha_{KK^\circ} - (1 - \vartheta)(1 - \delta_t^K)$, where δ_t^K is defined by the OBA capital stock series, we obtain a 'combined' model that contains models 1 and 2 as special cases corresponding to $\vartheta = 0$ and $\vartheta = 1$, respectively. (Note, since δ_t^K is observed ϑ is identified.) The FIML estimate for ϑ from the combined model is 0.85 with an estimated standard error of 0.08, leading us to accept model 2 over model 1.

It seems of interest to also test the specification of model 2 against some general alternative. In particular, we may test if the stochastic disturbances in the respective demand equations are orthogonal to elements of the information set. More specifically, we use 16 instruments (consisting of lagged endogenous variables, prices, lagged output, time, as well as quadratic terms of the exogenous variables); this results, given we have four stochastic equations, in a total of 64 orthogonality conditions. Following, e.g., White (1987), we define our test statistic as a quadratic form of corresponding sample moments between the estimated disturbances, based on our FIML estimates, and the respective instruments. A more detailed discussion of the test statistic is given in Appendix D; again the statistic has to take into account that expectations on gross output depend on pre-estimated parameters. The statistic is asymptotically distributed chi-square with 64 degrees of freedom. The observed value for our test statistic is 58.16, compared to a critical value of approximately 84. Thus, also this test leads us to accept model 2, i.e., to accept the hypothesis of a constant depreciation rate for capital for the U.S. electrical machinery industry.

The parameter estimates per se are difficult to interpret. Consequently we present in the following estimates for various implied characteristics for the estimated factor demand systems. For purposes of comparison we not only report estimates for model 2 but also for models 1 and 3.

The adjustment cost coefficients $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$ are in magnitude similar across models. For models 1, 2, and 3 the implied accelerator coefficients m_{KK} and m_{RR} are, respectively, 0.24 and 0.15, 0.21 and 0.15, and 0.19 and 0.15. The cross-adjustment coefficients m_{KR} and m_{RK} are small and (in absolute value) less than 0.02 for all models. For a further interpretation of the adjustment cost coefficients observe that the normalized adjustment cost for capital and R&D in any period is given by $0.5\alpha_{\dot{K}\dot{K}}\Delta K^2/Y^{1/\rho}$ and $0.5\alpha_{\dot{R}\dot{R}}\Delta R^2/Y^{1/\rho}$, respectively. For model 2 the sample average of the ratio of adjustment costs to gross investment is 0.14 and 0.15 for capital and R&D, respectively. The estimates for models 1 and 3 are similar.

Our specification does not impose a priori constant returns to scale. Rather, we estimate the scale elasticity ρ from the data. The implied scale estimates are similar, i.e., 1.22, 1.16, and 1.19 for models 1, 2, and 3.

The own- and cross-price elasticities and output elasticities of labor, materials, capital, R&D, the capital left over at the end of the period, and gross capital investment for 1976 are reported in Table 2. They are calculated for the short run (SR), intermediate run (IR), and long run (LR).[16] We note that the elasticities are stable over time. Estimated standard errors for respective long-run elasticities are given in parentheses.

All of the own-price elasticities are negative. The magnitudes of the own- and cross-price elasticities are generally similar across models. However, the results also point to some important differences that may arise in case the capital depreciation rate is endogenously determined: One interesting difference can be observed in comparing the long-run elasticities of labor with respect to the price of capital, ε_{Lq^K}, and the long-run elasticity of capital with respect to the price of labor, ε_{Kp^L}. For models 1 and 2 both ε_{Lq^K} and ε_{Kp^L} are negative, which reflects the fact that in the long run $\partial L/\partial q^K = (r + \delta^K)\, \partial K/\partial p^L$. (Recall that q^K denotes the after-tax acquisition price and not the rental price.) However, for model 3 the elasticity ε_{Lq^K} is positive while ε_{Kp^L} is negative. At first glance this may seem inadmissible. However, as is demonstrated in Appendix D of the original version of this paper (Prucha and Nadiri, 1990), if we allow for the depreciation rate of capital to be endogenously determined as in model 3 we have the following long-run relationship: $\partial L/\partial q^K = (1 + r)\, \partial K/\partial p^L - \partial K^{\circ}/\partial p^L$. Therefore, in the case of an endogenous depreciation rate the sign of $\partial L/\partial q^K$ may differ from that of $\partial K/\partial p^L$. The sign of $\partial L/\partial q^K$ depends on the relative magnitudes of $\partial K/\partial p^L$ and $\partial K^{\circ}/\partial p^L$, as opposed to the case of an exogenous depreciation rate where $\partial K^{\circ}/\partial p^L = (1 - \delta^K)\, \partial K/\partial p^L$ and hence $\partial L/\partial q^K = (r + \delta^K)\, \partial K/\partial p^L$. Another interesting difference can be observed in comparing the long-run elasticities of gross capital investment with respect to the price of labor. For model 2 this elasticity is negative, while for model 3 it is positive. Again this can be explained from the fact that in the case of an exogenous depreciation rate we have $\partial I^K/\partial p^L = \delta^K \partial K/\partial p^L$, while in case of an endogenous depreciation rate we have more generally $\partial I^K/\partial p^L = \partial K/\partial p^L - \partial K^{\circ}/\partial p^L$. As a consequence $\partial I^K/\partial p^L$ can be positive while both $\partial K/\partial p^L$ and $\partial K^{\circ}/\partial p^L$ are negative. The switch in the long-run

[16] For $Z = M, L, K, R, K^{\circ}, I^K, I^R$ let $Z_{t,\tau}$ denote the optimal plan value for Z in period $t + \tau$ corresponding to the firm's optimization problem in period t. Short-run, intermediate-run, and long-run elasticities then refer to the elasticities of $Z_{t,\tau}$ for $\tau = 0, 1,$ and ∞, respectively.

elasticity of gross capital investment with respect to the price of materials from positive for model 2 to negative for model 3 can be explained analogously.

The pattern of the output elasticities reveals that the variable factors of production, labor, and materials respond strongly in the short run to changes in output; in the short run they overshoot their long run equilibrium values. The output elasticities of the quasi-fixed factors, capital and R&D, are small in the short run but increase over time. In general the respective output elasticities are similar across models.

4.3. Capital stock and capital depreciation rate

The modeling approach discussed in this paper generates estimates for the capital stock and depreciation rate as a by-product of the estimation process. In Table 3 we report those estimates for model 2. As discussed, the test results reported above imply that for the U.S. machinery industry we accept model 2, i.e., we accept the hypothesis of a constant depreciation rate. Still, as an illustration we also report estimates corresponding to model 3, where the depreciation rate is allowed to vary over time, and for reasons of comparison we also report the OBA capital stock and depreciation rate series that underlie the estimates of model 1. For model 2 the estimated depreciation rate is 0.038 with an estimated standard error of 0.023. For model 3 the estimated depreciation rate is on average again 0.038 as compared to 0.055 for the OBA capital stock series. This translates into a difference of 16 percent in magnitude between the former and latter capital stock series at the end of the sample period. Still, all of the respective correlation coefficients between the respective capital stock series exceed 0.99. We note that the pattern of depreciation rates calculated from model 3 shows declines in 1974 and 1975 as well as in 1980, reflecting periods of slow growth and recession in the U.S. electrical machinery industry.

It seems of interest to discuss the magnitude of the estimated depreciation rate as it relates to the shape of the efficiency function and to the average survival time of capital in more detail. Let $K_t = \sum_{i=0}^{\infty} \phi_i I_{t-i}^K$ where $\phi_i \geqslant 0$ denotes the efficiency function. Assume that the ϕ_i are nonincreasing, $\phi_0 = 1$, $\phi_i > 0$ for $i = 0, \ldots, m$ and $\phi_i = 0$ for $i > m$, where m is the maximal survival time (which may possibly be infinite). Given $K_t = I_t^K + (1 - \delta_t^K)K_{t-1}$ it follows that $\delta_t^K = [\sum_{i=0}^{m}(\phi_i - \phi_{i+1})I_{t-i-1}^K]/[\sum_{i=0}^{m} \phi_i I_{t-i-1}^K]$. The average survival time is given by $\sum_{i=0}^{m}(\phi_i - \phi_{i+1})i$. Clearly if gross investment grows at a constant rate, i.e., $I_t^K = (1 + \rho_I)^t I_0^K$, and the efficiency function does not depend on t, the depreciation rate is constant over time and given by $\delta_t^K = [\sum_{i=0}^{m}(\phi_i - \phi_{i+1}) \times (1 + \rho_I)^{-i}]/[\sum_{i=0}^{m} \phi_i(1 + \rho_I)^{-i}]$. That is, the depreciation rate is only a function of $\phi_0, \ldots, \phi_m$ and the growth rate of gross investment (and hence constant) regardless of the shape of the efficiency function. We consider two

Table 2
Short-run, intermediate-run, and long-run elasticities in the U.S. electrical machinery industry: 1976

Elasticity	*Model 1:* OBA capital stock			*Model 2:* Estimated capital stock and constant depreciation rate			*Model 3:* Estimated capital stock and variable depreciation rate		
	SR	IR	LR	SR	IR	LR	SR	IR	LR
Price elasticities of materials									
ε_{Mp^M}	−0.32	−0.40	−0.64 (0.40)	−0.29	−0.40	−0.75 (0.64)	−0.26	−0.37	−0.80 (1.00)
ε_{Mp^L}	0.36	0.42	0.65 (0.43)	0.32	0.41	0.75 (0.66)	0.33	0.43	0.88 (1.12)
ε_{Mq^K}	−0.01	0.02	0.09 (0.04)	−0.02	0.02	0.10 (0.04)	−0.04	−0.02	0.03 (0.10)
ε_{Mq^R}	−0.01	−0.02	−0.08 (0.07)	−0.01	−0.02	−0.08 (0.09)	−0.01	−0.02	−0.10 (0.14)
Price elasticities of labor									
ε_{Lp^M}	0.47	0.55	0.90 (0.58)	0.44	0.54	0.97 (0.65)	0.46	0.57	1.10 (0.87)
ε_{Lp^L}	−0.48	−0.58	−1.12 (0.65)	−0.45	−0.56	−1.14 (0.70)	−0.52	−0.66	−1.40 (1.14)
ε_{Lq^K}	0.00	−0.02	−0.05 (0.05)	0.00	−0.02	−0.07 (0.05)	0.05	0.04	0.02 (0.20)
ε_{Lq^R}	0.00	0.04	0.27 (0.11)	0.00	0.04	0.24 (0.09)	0.00	0.04	0.27 (0.18)
Price elasticities of capital									
ε_{Kp^M}	0.10	0.18	0.38 (0.14)	0.11	0.19	0.44 (0.12)	0.13	0.23	0.59 (0.25)
ε_{Kp^L}	−0.05	−0.09	−0.17 (0.12)	−0.07	−0.12	−0.24 (0.14)	−0.10	−0.18	−0.47 (0.33)
ε_{Kq^K}	−0.04	−0.08	−0.18 (0.06)	−0.04	−0.07	−0.17 (0.04)	−0.03	−0.05	−0.13 (0.09)
ε_{Kq^R}	0.00	−0.01	−0.04 (0.07)	0.00	−0.01	−0.04 (0.07)	0.00	0.00	0.00 (0.10)
Price elasticities of R&D									
ε_{Rp^M}	−0.05	−0.09	−0.27 (0.14)	−0.05	−0.09	−0.28 (0.16)	−0.05	−0.09	−0.32 (0.20)
ε_{Rp^L}	0.11	0.20	0.65 (0.14)	0.10	0.18	0.61 (0.17)	0.11	0.20	0.71 (0.26)
ε_{Rq^K}	0.00	−0.01	−0.03 (0.05)	0.00	−0.01	−0.03 (0.06)	−0.01	−0.02	−0.05 (0.13)
ε_{Rq^R}	−0.05	−0.10	−0.34 (0.10)	−0.05	−0.09	−0.30 (0.11)	−0.05	−0.09	−0.33 (0.11)

Price elasticities of capital left over at end of period									
$\varepsilon_{K^o p^M}$	0.00	0.10	0.38 (0.14)	0.00	0.11	0.44 (0.12)	0.02	0.15	0.63 (0.29)
$\varepsilon_{K^o p^L}$	0.00	− 0.05	− 0.17 (0.12)	0.00	− 0.07	− 0.24 (0.14)	− 0.02	− 0.13	− 0.52 (0.39)
$\varepsilon_{K^o q^K}$	0.00	− 0.04	− 0.18 (0.06)	0.00	− 0.04	− 0.17 (0.04)	0.00	− 0.03	− 0.13 (0.12)
$\varepsilon_{K^o q^R}$	0.00	0.00	− 0.04 (0.06)	0.00	0.00	− 0.04 (0.07)	0.00	0.00	0.01 (0.01)
Price elasticities of gross capital investment									
$\varepsilon_{I^K p^M}$	1.33	1.13	0.38 (0.13)	1.70	1.49	0.44 (0.14)	1.76	1.49	− 0.52 (1.75)
$\varepsilon_{I^K p^L}$	− 0.71	− 0.59	− 0.17 (0.14)	− 1.05	− 0.91	− 0.24 (0.13)	− 1.30	− 1.05	0.83 (2.92)
$\varepsilon_{I^K q^K}$	− 0.57	− 0.49	− 0.18 (0.12)	− 0.46	− 0.54	− 0.17 (0.03)	− 0.48	− 0.44	− 0.12 (1.51)
$\varepsilon_{I^K q^R}$	− 0.05	− 0.06	− 0.04 (0.14)	− 0.06	− 0.06	− 0.04 (0.06)	0.00	− 0.02	− 0.19 (0.56)
Technical change									
ε_{MT}	0.07	0.01	− 0.22 (0.37)	0.12	0.02	− 0.33 (0.55)	0.11	0.02	− 0.37 (0.94)
ε_{LT}	− 0.54	− 0.44	− 0.02 (0.16)	− 0.67	− 0.55	− 0.02 (0.19)	− 0.62	− 0.50	0.12 (0.35)
ε_{KT}	0.07	0.13	0.25 (0.10)	0.08	0.14	0.31 (0.10)	0.09	0.16	0.43 0.20)
ε_{RT}	− 0.09	− 0.16	− 0.52 (0.14)	− 0.09	− 0.16	− 0.54 (0.14)	− 0.09	− 0.16	− 0.57 (0.19)
$\varepsilon_{K^o T}$	0.00	0.07	0.25 (0.10)	0.00	0.08	0.31 (0.10)	0.00	0.10	0.45 (0.23)
$\varepsilon_{I^K T}$	0.95	0.80	0.25 (0.10)	1.27	1.11	0.31 (0.13)	1.43	1.23	− 0.30 (1.68)
Output elasticities									
ε_{MY}	1.19	1.07	0.82 (0.05)	1.31	1.18	0.86 (0.06)	1.28	1.16	0.84 (0.06)
ε_{LY}	1.07	1.06	0.82 (0.05)	1.08	1.07	0.86 (0.06)	1.01	1.01	0.84 (0.06)
ε_{KY}	0.19	0.34	0.82 (0.05)	0.19	0.33	0.86 (0.06)	0.18	0.32	0.84 (0.06)
ε_{RY}	0.12	0.23	0.82 (0.05)	0.13	0.24	0.86 (0.06)	0.13	0.24	0.84 (0.06)
$\varepsilon_{K^o Y}$	0.00	0.19	0.82 (0.05)	0.00	0.19	0.86 (0.06)	− 0.01	0.17	0.84 (0.06)
$\varepsilon_{I^K Y}$	2.52	2.20	0.82 (0.04)	2.90	2.61	0.86 (0.06)	3.00	2.72	0.84 (0.10)

ε_{Zs} is the elasticity of Z = materials (M), labor (L), capital (K), R&D (R), capital left over a the end of the period (K^o), and gross capital investment (I^K) with respect to s = price of materials (p^M), price of labor (p^L), price of capital (q^K), price of R&D (q^R), and output (Y). The symbols SR, IR, and LR refer to the short, intermediate, and the long run. The numbers in parentheses next to the respective long-run elasticities represent estimated standard errors.

Table 3
Comparison of OBA and estimated capital stock data in the U.S. electrical machinery industry: 1960–1980

	Model 1: OBA capital stock			*Model 2:* Estimated capital stock and constant depreciation rate			*Model 3:* Estimated capital stock and variable depreciation rate		
Year	K	K^o	δ^K	K	K^o	δ^K	K	K^o	δ^K
1959	0.536	0.489	0.055	0.544	0.497	0.038	0.550	0.502	0.028
1960	0.561	0.506	0.055	0.576	0.523	0.038	0.588	0.533	0.030
1961	0.587	0.530	0.055	0.613	0.556	0.038	0.626	0.569	0.033
1962	0.612	0.555	0.055	0.647	0.590	0.038	0.661	0.604	0.035
1963	0.639	0.578	0.055	0.683	0.622	0.038	0.698	0.637	0.036
1964	0.669	0.604	0.055	0.722	0.657	0.038	0.737	0.672	0.038
1965	0.720	0.632	0.055	0.782	0.694	0.038	0.796	0.708	0.040
1966	0.796	0.681	0.054	0.867	0.752	0.038	0.877	0.763	0.039
1967	0.876	0.754	0.052	0.955	0.833	0.038	0.963	0.841	0.040
1968	0.943	0.831	0.051	1.030	0.918	0.038	1.036	0.924	0.041
1969	1.016	0.896	0.050	1.110	0.991	0.038	1.114	0.994	0.041
1970	1.069	0.965	0.050	1.172	1.068	0.038	1.174	1.070	0.039
1971	1.108	1.015	0.051	1.220	1.127	0.038	1.221	1.128	0.040
1972	1.143	1.050	0.052	1.266	1.173	0.038	1.263	1.170	0.041
1973	1.208	1.081	0.054	1.344	1.217	0.038	1.335	1.209	0.043
1974	1.281	1.142	0.055	1.432	1.292	0.038	1.422	1.283	0.040
1975	1.305	1.211	0.055	1.471	1.377	0.038	1.465	1.371	0.036
1976	1.337	1.230	0.057	1.521	1.415	0.038	1.516	1.409	0.038
1977	1.389	1.258	0.059	1.593	1.463	0.038	1.587	1.457	0.039
1978	1.460	1.306	0.060	1.687	1.532	0.038	1.681	1.526	0.039
1979	1.549	1.372	0.060	1.799	1.622	0.038	1.794	1.617	0.038
1980	1.678	1.456	0.061	1.952	1.730	0.038	1.952	1.729	0.036

K, K^o, and δ^K denote, respectively, the end-of-period capital stock, the capital stock left over at the end of the period from the beginning-of-period capital stock, and the depreciation rate of capital.

'limiting' cases. In case of a one-hoss shay efficiency function, i.e., $\phi_i = 1$ for $i = 1, \ldots, m$, the depreciation rate equals $\delta_t^K = 1/[\sum_{i=0}^{m}(1 + \rho_I)^i]$, and the average survival time equals the maximal survival time m. In case of a geometrically declining efficiency function, i.e., $\phi_i = (1 - \delta)^i$, the depreciation rate is constant regardless of the pattern of investment and given by $\delta_t^K = \delta$, and the average survival time equals $(1 - \delta)/\delta$. The average growth rate of gross investment in our sample is 9 percent. Corresponding to this growth rate and our estimate of a depreciation rate of 0.038, the implied average survival times for the two 'limiting' cases are approximately 13 and 25 years, respectively.

The assumption of a constant depreciation rate has a long history and has been the subject of considerable debate.[17] (As remarked above, both a geometrically declining efficiency function and a constant growth rate of gross investment imply a constant depreciation rate.) While for the U.S. electrical machinery industry we accept the hypothesis of a constant depreciation rate, the model considered in this paper allows in general for quasi-fixed factors with a variable depreciation rate. Replacement investment is defined as the difference between the initial stocks and what is left over from those stocks at the end of the period, i.e., $I_t^{KR} = K_{t-1} - K_t^0$. Net investment is defined as the difference between gross investment and replacement investment, i.e., $I_t^{KE} = I_t^K - I_t^{KR}$ or $I_t^{KE} = K_t - K_{t-1}$. In the case of a variable depreciation rate both K_t and K_t^0 are endogenously determined by the firm; hence also I_t^{KR} and I_t^{KE} are endogenously determined. That is, as a by-product, our specification then also yields a structural model for the endogenous determination of replacement investment versus expansion investment. We repeat that at the estimation stage only gross investment enters as an observed variable. Stocks are generated internally and hence are generated consistently with replacement investment. As pointed out by Jorgenson (1974) some of the previous studies on replacement investment were not fully consistent in that they employed capital stock data that have been generated under a different set of assumptions than those maintained in those studies. Our approach is not subject to the same criticism and hence allows for a proper test of the constancy of depreciation rates.

In Table 4 we present the ratio of net investment to gross investment for the period 1960 to 1980. The ratios implied by model 2 and 3 are much higher than the ratio implied by the OBA capital stock series. This implies (consistent with our previous remarks) a much higher rate of capital accumulation as compared to the OBA capital stock series. We note that the patterns of the net to gross investment ratio over time seem quite similar across the models and the ratio generally drops in years of slow output growth. The correlation coefficients between the ratios corresponding to model 1 and models 2 and 3 are, respectively, 0.97 and 0.84. The correlation coefficient between the ratios corresponding to model 2 and model 3 is 0.94.

4.4. Technical change and capacity utilization

Given our estimate of the normalized variable cost function net of the 'old stocks', G, defined in (4.1) we can use the expressions in (3.3) to compute

[17] The assumption of a constant depreciation rate has been challenged, among others, by Feldstein and Foot (1971), Eisner (1972), Eisner and Nadiri (1968, 1970), Feldstein (1974), Feldstein and Rothschild (1974), and Bitros and Kelejian (1974); and was forcefully defended by Jorgenson (1974). Recently the validity of the geometric depreciation assumption has been tested in several papers by Hulten and Wykoff (1980, 1981a, b, c) based on a sample of used asset transaction prices.

Table 4
Ratio of net investment to gross investment in the U.S. electrical machinery industry: 1960–1980

Year	*Model 1:* OBA capital stock	*Model 2:* Estimated capital stock and constant depreciation rate	*Model 3:* Estimated capital stock and variable depreciation rate
1960	0.46 (0.00)	0.62 (0.22)	0.69 (0.24)
1961	0.45 (0.00)	0.61 (0.23)	0.66 (0.22)
1962	0.43 (0.00)	0.59 (0.24)	0.62 (0.22)
1963	0.45 (0.00)	0.59 (0.24)	0.60 (0.22)
1964	0.46 (0.00)	0.60 (0.24)	0.59 (0.19)
1965	0.58 (0.00)	0.68 (0.19)	0.67 (0.14)
1966	0.66 (0.00)	0.74 (0.16)	0.72 (0.12)
1967	0.66 (0.00)	0.73 (0.16)	0.70 (0.13)
1968	0.60 (0.00)	0.67 (0.19)	0.65 (0.17)
1969	0.60 (0.00)	0.67 (0.20)	0.65 (0.17)
1970	0.51 (0.00)	0.59 (0.24)	0.57 (0.19)
1971	0.41 (0.00)	0.51 (0.29)	0.48 (0.22)
1972	0.38 (0.00)	0.50 (0.30)	0.47 (0.22)
1973	0.51 (0.00)	0.62 (0.23)	0.58 (0.17)
1974	0.53 (0.00)	0.63 (0.22)	0.61 (0.16)
1975	0.25 (0.00)	0.41 (0.35)	0.41 (0.25)
1976	0.30 (0.00)	0.47 (0.31)	0.49 (0.23)
1977	0.40 (0.00)	0.55 (0.26)	0.56 (0.20)
1978	0.46 (0.00)	0.60 (0.23)	0.61 (0.18)
1979	0.50 (0.00)	0.63 (0.22)	0.65 (0.17)
1980	0.58 (0.00)	0.69 (0.18)	0.70 (0.15)

The numbers in parentheses represent estimated standard errors.

estimates for technical change. As reported in Table 5 our estimates of pure (input based) technical change from model 2 is 0.69. For reasons of comparison and illustration we also report estimates for technical change based on models 1 and 3. Those estimates are, respectively, 0.60 and 0.66. As discussed in Section 3.3, the traditional measure of total factor productivity only equals technical change if, in particular, producers are in long-run equilibrium, the technology exhibits constant returns to scale, input and output markets are perfectly competitive, and factors are utilized at a constant rate. In Table 5 we also report estimates of the traditional measure of total factor productivity. Those estimates are approximately three times larger than our estimates of pure technical change. Based on the decomposition formula (3.9) given in Section 3.3 and based on the estimates of the respective models we also provide a decomposition of the sources for this difference. The main source of the difference is the scale effect which represents about 40 percent of the growth in the traditional total factor productivity measure. The remainder of the difference is mainly due to the presence of adjustment costs. The measures of total factor productivity differ

Table 5
Decomposition of total factor productivity growth in the U.S. electrical machinery industry in percentages: 1960–1980

	Model 1: OBA capital stock	*Model 2:* Estimated capital stock and constant depreciation rate	*Model 3:* Estimated capital stock and variable depreciation rate
Technical change	0.60	0.69	0.66
Scale effect	1.04	0.83	0.93
Adjustment cost effects			
Temporary equilibrium effect	0.33	0.42	0.39
Direct adjustment cost effect	0.03	0.02	0.02
Variable depreciation effect	0.00	0.00	0.02
Unexplained residual	0.04	0.03	− 0.03
Total factor productivity	2.04	1.99	1.99

across the models since they are based on different capital stock series. Comparing the decomposition between models 2 and 3 shows that allowing for the depreciation rate to vary increases the scale effect, lowers the adjustment cost effect and decreases the estimate for pure technical change. For the data set under consideration the variable depreciation rate effect is small.

We note that for all models estimated pure technical change exhibits a very smooth pattern and increases over time. In particular, for model 2 the estimate of pure technical change is 0.60 in 1960 and 0.94 in 1980 with a low of 0.51 in 1964.

In Table 6 we report estimates of capacity utilization based on the cost ratio C/C^{+} defined in Eq. (3.7) given in Section 3.2. Those estimates are similar across models. For all models capacity utilization drops approximately 10 percent in 1975, reflecting a 14 percent decline in gross output in the U.S. electrical machinery industry in that year. Comparing the capacity utilization estimates corresponding to models 2 and 3 we see that the estimates corresponding to the latter are generally somewhat smaller. The correlation coefficients between the respective capacity utilization series all exceed 0.99.

5. Conclusion

In this paper we have specified a general dynamic factor demand model where the firm can choose the depreciation rate of some (or all) of the quasi-fixed

Table 6
Capacity utilization in the U.S. electrical machinery industry: 1960–1980

Year	*Model 1:* OBA capital stock	*Model 2:* Estimated capital stock and constant depreciation rate	*Model 3:* Estimated capital stock and variable depreciation rate
1960	1.072 (0.041)	1.089 (0.047)	1.093 (0.057)
1961	1.091 (0.035)	1.107 (0.045)	1.107 (0.056)
1962	1.099 (0.032)	1.109 (0.038)	1.107 (0.052)
1963	1.111 (0.029)	1.122 (0.038)	1.118 (0.053)
1964	1.119 (0.028)	1.130 (0.039)	1.125 (0.055)
1965	1.118 (0.028)	1.125 (0.031)	1.121 (0.050)
1966	1.115 (0.030)	1.119 (0.030)	1.116 (0.048)
1967	1.119 (0.029)	1.126 (0.032)	1.121 (0.052)
1968	1.115 (0.029)	1.124 (0.039)	1.118 (0.057)
1969	1.113 (0.030)	1.123 (0.044)	1.117 (0.062)
1970	1.098 (0.037)	1.112 (0.068)	1.104 (0.084)
1971	1.071 (0.047)	1.086 (0.091)	1.078 (0.105)
1972	1.093 (0.041)	1.107 (0.072)	1.099 (0.089)
1973	1.111 (0.038)	1.122 (0.053)	1.114 (0.075)
1974	1.099 (0.046)	1.112 (0.071)	1.104 (0.093)
1975	0.995 (0.076)	1.010 (0.148)	1.009 (0.160)
1976	1.059 (0.056)	1.074 (0.107)	1.068 (0.125)
1977	1.102 (0.047)	1.116 (0.070)	1.107 (0.094)
1978	1.113 (0.047)	1.126 (0.061)	1.117 (0.087)
1979	1.125 (0.050)	1.137 (0.057)	1.127 (0.083)
1980	1.128 (0.054)	1.142 (0.063)	1.132 (0.091)

The numbers in parentheses represent estimated standard errors.

factors optimally. The case of an exogenously given constant depreciation rate is contained as a special case, thus permitting a formal test of the hypothesis of a constant depreciation rate. The model allows for multiple outputs, variable inputs, and for the quasi-fixed factors to become productive immediately or with a lag. Based on the model we discuss primal and dual measures of technical change. Those measures extend various measures previously considered in the literature. We also deduce a measure of capacity utilization and explore the sources of bias for the traditional measure of total factor productivity growth.

As an illustration we apply the model to data from the U.S. electrical machinery industry. We have estimated two versions of the model. The more general version of the model permits the depreciation rate of capital to be determined as a function of output and relative prices. For the other versions of the model we have imposed parameter restrictions such that the depreciation rate of capital is constant. We note that for both versions of the model the depreciation rate is estimated and the respective capital stocks are generated internally during estimation in a theoretically consistent fashion from the gross investment series. For purposes of comparison we also report the estimates obtained in Nadiri and Prucha (1990a) from a model with exogenous depreciation rate that utilizes the capital stock series published by OBA. We refer to those models as, respectively, model 3, 2, and 1.

Based on our tests we accept model 2 corresponding to a constant depreciation rate of capital for the U.S. electrical machinery industry. However, for purposes of illustration and comparison we not only report price and output elasticities, estimates of technical change, etc., for model 2 but also for models 1 and 3. On the whole the price and output elasticities are similar across models. However, some interesting differences can be observed. In particular, as explained in more detail in the text, when the depreciation rate is permitted to be endogenously determined the sign of the long-run cross-price elasticities of the variable factors and capital need not be the same. In fact, we find the estimated long-run cross-price elasticities of capital and labor to be of opposite sign. Related to this phenomenon we also observe that the long-run elasticity of gross investment with respect to the prices of labor and materials changes between models 2 and 3 from -0.24 to 0.83 and 0.44 to -0.52, respectively.

For both models 2 and 3 the depreciation rate is estimated on average to be 0.038 as compared to 0.055 for the OBA capital stock series. This translates into a sizable difference of 16 percent in the level of the capital stock at the end of the sample period. Also the ratio of net to gross investment implied by models 2 and 3 is much higher than the ratio implied by the OBA data. All these ratios show sensitivity to the growth in output.

Our estimate of pure technical change is approximately 0.6, which is approximately one third of the estimate implied by the traditional measure of total factor productivity growth. I.e., the traditional measure of total factor productivity growth significantly overestimates the rate of technical change.

Although the model considered here is quite general, several extensions of the theoretical model seem of interest. In particular, variations in the rate of utilization of an input can be achieved by varying the numbers of hours the input is employed and/or by changing the intensity or speed with which the input is used in the production process. An increase in the intensity or speed with which capital is operated will typically result in an increase in the rate of depreciation of capital. An increase in the length of time capital is employed will typically result in increased costs due to shift and overtime premiums and an increase in the rate of depreciation of capital. It seems of interest to incorporate both cost aspects into the model.

At the empirical stage it may be interesting to include more quasi-fixed factors by distinguishing between production and nonproduction workers and by differentiating between different types of capital. Also it seems desirable to endogenize the depreciation rate of R&D. Another extension would be to allow for more general patterns of expectations. Furthermore, the model can be reformulated in a profit-maximizing setting to explore the existence of markups in different industries.

Appendix A: Decomposition of the traditional measure of TFP growth

In the following we give a proof for the decomposition of the Törnquist approximation of the traditional measure of total factor productivity growth, ΔTFP, presented in Section 3.3. We consider the case of a single-output good and maintain that all assumptions stated in the text hold. We shall utilize the following lemma; for a proof see Prucha and Nadiri (1990).

Lemma A.1. *Let* $\dot{u} = \dot{w} - \dot{v}$ *where* $\dot{w} = \varepsilon \sum_{i=1}^{m+k} (\alpha_i/\alpha)\dot{\eta}_i + \lambda$ *and* $\dot{v} = \sum_{i=1}^{m} (\beta_i/\beta)\dot{\eta}_i$ *with* $\alpha = \sum_{i=1}^{m+k} \alpha_i$ *and* $\beta = \sum_{i=1}^{m} \beta_i$. *Then*

$$\dot{u} = (1 - 1/\varepsilon)\dot{w} + \sum_{i=1}^{m} [(\alpha_i - \beta_i)/\alpha](\dot{\eta}_i - \dot{v}) + \sum_{i=m+1}^{m+k} (\alpha_i/\alpha)(\dot{\eta}_i - \dot{v}) + (1/\varepsilon)\lambda.$$

We shall utilize furthermore the following relationships $\partial Y/\partial M = p^M/[\partial G/\partial Y]$, $\partial Y/\partial L = p^L/[\partial G/\partial Y]$, $\partial Y/\partial K^o = -q^K/[\partial G/\partial Y]$, $\partial Y/\partial Z = -[\partial G/\partial Z]/[\partial G/\partial Y]$ for $Z = \underline{K}, \underline{R}, \Delta K, \Delta R$. These relationships are obtained by respective differentiation of (3.1), which yields $\partial Y/\partial M = 1/[\partial M/\partial Y]$ and $\partial Y/\partial Z = -[\partial M/\partial Z]/[\partial M/\partial Y]$ for $Z = L, K^0, \underline{K}, \underline{R}, \Delta K, \Delta R$, and by utilizing the results concerning the derivatives of M given before (3.3). (Recall that $p^M = 1$.)

Now consider the following decomposition of output growth based on a translog expansion of the production function:

$$\Delta \ln Y_t = \tfrac{1}{2}[\Delta \ln Y_t^t + \Delta \ln Y_t^{t-1}], \tag{A.1}$$

$$\begin{aligned}\Delta \ln Y_t^\tau = {} & \varepsilon_{YM}(\tau)\Delta \ln M_t + \varepsilon_{YL}(\tau)\Delta \ln L_t + \varepsilon_{YK^o}(\tau)\Delta \ln K_t^o + \varepsilon_{YK}(\tau)\Delta \ln \underline{K}_t \\ & + \varepsilon_{YR}(\tau)\Delta \ln \underline{R}_t + \varepsilon_{Y\Delta K}(\tau)\Delta \ln \Delta K_t + \varepsilon_{Y\Delta R}(\tau)\Delta \ln \Delta R_t + \lambda_Y(\tau),\end{aligned}$$

with $\tau = t,\ t-1$, and where the $\varepsilon_{YZ}(\tau) = [\partial Y_\tau/\partial Z_\tau]\ [Z_\tau/Y_\tau]$ with $Z = M, L, K^o, \underline{K}, \underline{R}, \Delta K, \Delta R$ denote output elasticities, and output-based technical change λ_Y is defined by (3.2). (For notational convenience we do not underline the subscripts K and R in denoting the output elasticity with respect to $\underline{K}$ and $\underline{R}$.) In light of (3.3) and (3.4) the scale elasticity is given by $\rho(\tau) = C_\tau/[(\partial G_\tau/\partial Y_\tau)Y_\tau]$, where C denotes total shadow cost. Using the derivative relationships developed after Lemma A.1 it follows that

$$\varepsilon_{YM}(\tau) = \rho(\tau)[p_\tau^M M_\tau/C_\tau], \qquad \varepsilon_{YL}(\tau) = \rho(\tau)[p_\tau^L L_\tau/C_\tau], \tag{A.2}$$

$$\varepsilon_{YK^o}(\tau) = \rho(\tau)[-q_\tau^K K_\tau^o/C_\tau], \quad \varepsilon_{YZ}(\tau) = \rho(\tau)[(-\partial G_\tau/\partial Z_\tau)Z_\tau]/C_\tau,$$

with $Z = \underline{K}, \underline{R}, \Delta K, \Delta R$. Observe that $\Delta TFP_t = \Delta \ln Y_t - \Delta \ln N_t = \frac{1}{2}\sum_{\tau=t,\,t-1} [\Delta \ln Y_t^\tau - \Delta \ln N_t^\tau]$. A general decomposition of the Törnquist approximation of the traditional measure of total factor productivity growth is now readily obtained by applying Lemma A.1 to $\Delta \ln Y_t^\tau - \Delta \ln N_t^\tau$ for $\tau = t, t-1$, utilizing (A.1) and (A.2) to express $\Delta \ln Y_t^\tau$, using the definition of $\Delta \ln N_t^\tau$ in (3.8), and observing that $c^K \underline{K} = (1+r)q^K \underline{K} - q^K K^o$. The decomposition given in (3.9) follows under the additional assumption that $\rho(t) = \rho(t-1)$.

Appendix B: Estimated system of factor demand equations

As noted in the text, the accelerator coefficients have to satisfy the following matrix equation: $BM^2 + (A + rB)M - A = 0$. In general this equation cannot be solved for M in terms of A and B. The equation can, however, be solved for A in terms of M and B: $A = BM(M + rI)(I - M)^{-1}$. Since the real discount rate r was assumed to be constant, the matrix M is constant over the sample. Hence, instead of estimating the elements of A and B, we may estimate those of M and B. To impose the symmetry of C we can also estimate B and C instead of B and M. Let $D = (d_{ij})_{i,\,j=K,\,R} = -MA^{-1}$ and observe that $A = C - (1+r)\times [B - B(C+B)^{-1}B]$ and that $D = B^{-1} + (1+r)\,(C - rB)^{-1}$ is symmetric.[18] Analogously as in Nadiri and Prucha (1990b) it is then readily seen that we can write (4.3) equivalently as

$$\begin{aligned}
\Delta K_t = {} & d_{KK}[\alpha_K + \alpha_{KT} T_t + \alpha_{KL}\hat{p}_t^L + \hat{q}_t^K(1 + r + \alpha_{K^oK^o})]\,\hat{Y}_t^{1/\rho} \\
& + d_{KR}[\alpha_R + \alpha_{RT} T_t + \alpha_{RL}\hat{p}_t^L + \hat{c}_t^R]\,\hat{Y}_t^{1/\rho} \\
& + [c_{KK}/\alpha_{\dot{K}\dot{K}}]K_{t-1} + [c_{KR}/\alpha_{\dot{K}\dot{K}}]R_{t-1}, \\
\Delta R_t = {} & d_{KR}[\alpha_K + \alpha_{KT} T_t + \alpha_{KL}\hat{p}_t^L + \hat{q}_t^K(1 + r + \alpha_{K^oK^o})]\,\hat{Y}_t^{1/\rho} \\
& + d_{RR}[\alpha_R + \alpha_{RT} T_t + \alpha_{RL}\hat{p}_t^L + \hat{c}_t^R]\,\hat{Y}_t^{1/\rho} \\
& + [c_{KR}/\alpha_{\dot{R}\dot{R}}]K_{t-1} + [c_{RR}/\alpha_{\dot{R}\dot{R}}]R_{t-1},
\end{aligned}$$

where

$$\begin{aligned}
d_{KK} &= 1/\alpha_{\dot{K}\dot{K}} + (1+r)[c_{RR} - r\alpha_{\dot{R}\dot{R}}]/e, \\
d_{RR} &= 1/\alpha_{\dot{R}\dot{R}} + (1+r)[c_{KK} - r\alpha_{\dot{K}\dot{K}}]/e, \\
d_{KR} &= -(1+r)c_{KR}/e,
\end{aligned}$$

and

$$e = (c_{KK} - r\alpha_{\dot{K}\dot{K}})(c_{RR} - r\alpha_{\dot{R}\dot{R}}) - c_{KR}^2.$$

[18] The reparametrization approach was first suggested by Epstein and Yatchew (1985) for a somewhat different model with a similar algebra. It was further generalized by Madan and Prucha (1989); for further application see, e.g., Mohnen, Nadiri and Prucha (1986) and Nadiri and Prucha (1990b).

Furthermore, we can express α_{KK}, α_{RR}, α_{KR} in (4.4) as

$$\alpha_{KK} = c_{KK} - (1 + r)[\alpha_{\dot{K}\dot{K}} - (\alpha_{\dot{K}\dot{K}})^2 (\alpha_{\dot{R}\dot{R}} + c_{RR})/f],$$

$$\alpha_{RR} = c_{RR} - (1 + r)[\alpha_{\dot{R}\dot{R}} - (\alpha_{\dot{R}\dot{R}})^2 (\alpha_{\dot{K}\dot{K}} + c_{KK})/f],$$

$$\alpha_{KR} = c_{KR} - (1 + r)(\alpha_{\dot{K}\dot{K}}\alpha_{\dot{R}\dot{R}}c_{KR})/f,$$

and

$$f = (\alpha_{\dot{K}\dot{K}} + c_{KK})(\alpha_{\dot{R}\dot{R}} + c_{RR}) - c_{KR}^2.$$

Once the model has been estimated in the reparameterized form we can obtain estimates for the original model parameters via $A = C - (1 + r) \times [B - B(C + B)^{-1}B]$.

Appendix C: Data sources and construction of variables

Gross Output: Data on gross output in current and constant 1972 dollars were obtained from the U.S. Department of Commerce, Office of Business Analysis (OBA) database and correspond to the gross output series of the U.S. Department of Commerce, Bureau of Industrial Economics.

Labor: Total hours worked were derived as the sum of hours worked by production workers and nonproduction workers. Hours worked by production workers were obtained directly from the OBA database. Hours worked by nonproduction workers were calculated as the number of nonproduction workers * hours worked per week * 52. The number of nonproduction workers was obtained from the OBA database. Weekly hours worked of nonproduction workers were taken to be 39.7. A series of total compensation in current dollars was calculated by multiplying the total payroll series from the OBA database with the ratio of compensation of employees to wages and salaries from U.S. Department of Commerce, Bureau of Economic Analysis (1981, 1984).

Materials: Materials in current dollars were obtained from the OBA database. Materials in constant 1972 dollars were calculated using deflators provided by the U.S. Department of Commerce, Bureau of Economic Analysis.

Value Added: Value added in current and constant 1972 dollars was calculated by subtracting materials from gross output.

Capital: The net capital stock series in 1972 dollars (utilized only in the estimation of model 1) and the current and constant 1972 dollar gross investment series were taken from the OBA database. The method by which the OBA capital stock series is constructed is described in the U.S. Department of Labor, Bureau of Labor Statistics (1979). The user cost of capital was constructed as $c^K = q^K(r + \delta^K)$ with $q^K = p^{IK}/(1 - u)$, where p^{IK} is the investment deflator, u is the corporate tax rate taken from Pechman (1983), and $r = 0.05$.

R&D: The stock of total R&D is constructed by the perpetual inventory method with a depreciation rate $\delta^R = 0.1$. The 1958 benchmark was obtained by dividing total R&D expenditures by δ^R plus the growth rate of real value added. The nominal R&D expenditures are taken from National Science Foundation (1984) and earlier issues. To avoid double counting we subtracted the labor and material components of R&D from the labor and material inputs. The GDP deflator for total manufacturing is used as the deflator for R&D expenditures, p^{IR}. All R&D expenditures were taken to be immediately expensible. The user cost for R&D was hence constructed as $c^R = p^{IR}(r + \delta^R)$.

All constant dollar variables were normalized by respective sample means. Prices were constructed conformably from current and constant dollar variables. Furthermore all prices were normalized by the price of materials.

Appendix D: Variance-covariance matrix estimator and specification test statistic

Suppose the probability law of some random vector depends on the parameter vectors θ and γ, and let $\underline{l}_t(\theta, \gamma)$ denote the corresponding (conditional) log-likelihood function in period t and let $s_t(\theta, \gamma) = -\partial \underline{l}_t(\theta, \gamma)/\partial\theta$. Suppose further that we observe another random vector whose probability law only depends on γ, and let $\bar{l}_t(\gamma)$ denote the corresponding log-likelihood function in period t and let $h_t(\gamma) = -\partial\bar{l}_t(\gamma)/\partial\gamma$. Next suppose that θ and γ are estimated in two stages from samples of size n. In the first stage γ is estimated by the maximum likelihood estimator, say $\hat{\gamma}_n$, corresponding to the latter log-likelihood function, $\bar{l}_t$. In the second stage θ is estimated by the maximum likelihood estimator, say $\hat{\theta}_n$, corresponding to the former log-likelihood function, $\underline{l}_t$, where γ is replaced by the first-stage estimator $\hat{\gamma}_n$. More specifically, $\hat{\gamma}_n$ and $\hat{\theta}_n$ satisfy the following first-order conditions: $\sum_{t=1}^n h_t(\hat{\gamma}_n) = 0$ and $\sum_{t=1}^n s_t(\hat{\theta}_n, \hat{\gamma}_n) = 0$. Under assumptions as, e.g., in White (1987) it is then not difficult to show that $n^{1/2}(\hat{\theta}_n - \theta) \to \mathrm{N}(0, \Phi_\theta)$, where Φ_θ can be estimated consistently by

$$\hat{\Phi}_\theta = \hat{S}_{1n}^{-1}[\hat{S}_{1n} - \hat{S}_{2n}\hat{H}_n^{-1}\hat{V}_{hs} - \hat{V}_{sh}\hat{H}_n^{-1}\hat{S}'_{2n} + \hat{S}_{2n}\hat{H}_n^{-1}\hat{S}'_{2n}]\hat{S}_{1n}^{-1},$$

with

$$\hat{S}_{1n} = n^{-1}\sum_{t=1}^{n} \partial s_t(\hat{\theta}_n, \hat{\gamma}_n)/\partial\theta, \qquad \hat{S}_{2n} = n^{-1}\sum_{t=1}^{n} \partial s_t(\hat{\theta}_n, \hat{\gamma}_n)/\partial\gamma,$$

$$\hat{H}_n = n^{-1}\sum_{t=1}^{n} \partial h_t(\hat{\gamma}_n)/\partial\gamma, \qquad \hat{V}_{sh} = \hat{V}'_{hs} = n^{-1}\sum_{t=1}^{n} s_t(\hat{\theta}_n, \hat{\gamma}_n)h'_t(\hat{\gamma}_n).$$

In the context of Section 4 the estimator $\hat{\theta}_n$ denotes the FIML estimator for the parameters of the factor demand equations (4.3)–(4.6), and $\hat{\gamma}_n$ denotes the vector of FIML (OLS) estimators for the vector of parameters of the autoregressive process from which the expectations on output are calculated.

Next consider moment conditions of the form $\mathrm{E}m_t(\theta, \gamma) = 0$ where m_t is of dimension $p \times 1$. Given those moment conditions hold we expect the sample averages $\hat{m}_n = n^{-1}\sum_{t=1}^{n} m_t(\hat{\theta}_n, \hat{\gamma}_n)$ to be close to zero. Following, e.g., White (1987) it is not difficult to show that under a set of assumptions as, e.g., maintained in that paper the test statistic $M_m = n\hat{m}_n'\hat{\Psi}_n^{-1}\hat{m}_n$ is distributed asymptotically chi-square with p degrees of freedom, with $\hat{\Psi}_n$ given by

$$\begin{aligned}\hat{\Psi}_n = \hat{V}_{mm} &- \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{M}'_{1n} + \hat{V}_{mh}\hat{H}_n^{-1}\hat{S}'_{2n}\hat{S}_{1n}^{-1}\hat{M}'_{1n} \\ &- \hat{V}_{mh}\hat{H}_n^{-1}\hat{M}'_{2n} - \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{V}_{sh}\hat{H}_n^{-1}\hat{S}'_{2n}\hat{S}_{1n}^{-1}\hat{M}'_{1n} \\ &+ \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{V}_{sh}\hat{H}_n^{-1}\hat{M}'_{2n} + \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{S}_{2n}\hat{H}_n^{-1}\hat{V}_{hm} \\ &- \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{S}_{2n}\hat{H}_n^{-1}\hat{V}_{hs}\hat{S}_{1n}^{-1}\hat{M}'_{1n} \\ &+ \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{S}_{2n}\hat{H}_n^{-1}\hat{S}'_{2n}\hat{S}_{1n}^{-1}\hat{M}'_{1n} - \hat{M}_{1n}\hat{S}_{1n}^{-1}\hat{S}_{2n}\hat{H}_n^{-1}\hat{M}'_{2n} \\ &- \hat{M}_{2n}\hat{H}_n^{-1}\hat{V}_{hm} + \hat{M}_{2n}\hat{H}_n^{-1}\hat{V}_{hs}\hat{S}_{1n}^{-1}\hat{M}'_{1n} \\ &- \hat{M}_{2n}\hat{H}_n^{-1}\hat{S}'_{2n}\hat{S}_{1n}^{-1}\hat{M}'_{1n} + \hat{M}_{2n}\hat{H}_n^{-1}\hat{M}'_{2n},\end{aligned}$$

where

$$\hat{M}_{1n} = n^{-1}\sum_{t=1}^{n} \partial m_t(\hat{\theta}_n, \hat{\gamma}_n)/\partial\theta, \qquad \hat{M}_{2n} = n^{-1}\sum_{t=1}^{n} \partial m_t(\hat{\theta}_n, \hat{\gamma}_n)/\partial\gamma,$$

$$\hat{V}_{mm} = n^{-1}\sum_{t=1}^{n} m_t(\hat{\theta}_n, \hat{\gamma}_n)m_t'(\hat{\theta}_n, \hat{\gamma}_n), \quad \hat{V}_{mh} = \hat{V}'_{hm} = n^{-1}\sum_{t=1}^{n} m_t(\hat{\theta}_n, \hat{\gamma}_n)h_t'(\hat{\gamma}_n),$$

and the other terms are defined above.

In the context of Section 4 the moment conditions $\mathrm{E}m_t(\theta, \gamma) = 0$ represent orthogonality conditions between the stochastic disturbances and respective instruments. In evaluating the above expressions we computed all first-order derivatives (from the log-likelihood functions) analytically. All second-order derivatives were calculated by differentiating the analytic first-order derivatives numerically.

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