Platform Competition With Cash-back Rebates Under No Surcharge Rules

Marius Schwartz *

and

Daniel R. Vincent **

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Abstract

We analyze competing strategic platforms setting fees to a local monopolist merchant and cash-back rebates to end users, when the merchant may not surcharge platforms' customers, a rule imposed by some credit card networks. Each platform has an incentive to gain transactions by increasing the spread between its merchant fee and user rebate above its rival's spread. This incentive yields non-existence of pure strategy equilibrium in many natural environments. In some circumstances, there is a mixed strategy equilibrium where platforms choose fee structures that induce the merchant to accept only one platform with equal probability, a form of monopolistic market allocation.

Keywords: Platform price competition; rebates; no surcharge; payment networks; credit cards.

JEL Codes: L13, L41, L42, D43.

*Department of Economics, Georgetown University, Washington, DC 20057, mariusschwartz@mac.com. ** Department of Economics, University of Maryland, College Park, MD 20742, dvincent@umd.edu. Both authors have consulted in regulatory and litigation matters involving credit cards. For helpful comments we thank Andre Boik and Julian Wright.

1 Introduction

In prominent sectors of the economy, platforms intermediate transactions between end users and merchants. Sometimes, as with search engines, the platform charges only one side (advertisers) and has no financial interactions with users. In other cases, notably payment cards, the platform sets fees also to users, often negative in the form of cash-back rebates or other rewards. Under certain conditions only the sum of platform fees matters, not its division between the two sides, a property known as neutrality of the pricing structure (Carlton and Frankel, 1995; Rochet and Tirole, 2002; Gans and King, 2003)[1]. Neutrality is absent, however, when a merchant cannot freely adjust its price(s) in a response to platform fees. This rigidity can be due to exogenous factors such as transaction costs of setting different prices to platform users versus other customers, or a platform's *no-surcharge rule* (NSR) that bars a merchant from charging a higher price to platform users. Such restrictions have long been used in credit card networks, and more recently in 'new economy' sectors, for example, online travel booking sites, triggering extensive regulatory and antitrust scrutiny (Bender and Fairless, 2014; Assaf and Moskowitz, 2015; Gonzales-Diaz and Bennett, 2015; Montovani, et. al., 2018).

A merchant's inability to surcharge reduces demand elasticity for platform transactions with respect to the platform fee. First, the pass-through rate from a platform's fee to the merchants price is dampened because any price increase must extend to transactions for which the merchant's cost is unchanged. Second, if those transactions are substitutes for the platform's transactions, a given uniform price increase by the merchant will reduce platform sales by less than a selective price increase for that platform alone. Reduced demand elasticity leads the platform to raise its merchant fee. This logic underlies regulatory concerns that fees charged to merchants for certain payment cards under a NSR are excessive and harm non-card customers such as cash users (Katz, 2001; Farrell, 2006; Schwartz and Vincent, 2006). It also permeates antitrust concerns that NSRs adopted by rival platforms will induce anti-competitively high fees to merchants (Boik and Corts, 2016; Carlton and Winter, 2017).

This one-sided reasoning is correct as far as it goes, but yields an incomplete understanding of equilibrium pricing under a (merchant-side) NSR when competing platforms set fees also to end users. Yet the issue is important for both economic theory and public policy. For example, the one-sided logic and concern underlay the U.S. district court's important decision in *United States v. American Express* (2015).

[1] We describe the conditions required for neutrality in Section 2.

The decision prohibited various nondiscriminatory provisions imposed by Amex, including a NSR, that prevent merchants from steering customers to use competing cards (with potentially lower merchant fees), arguing that such 'no-steering' provisions anti-competitively induce higher fees to merchants and ultimately harm card users as well.[2] In overturning this decision, the appellate court wrote: "The District Court erred in concluding that 'increases in *merchant* pricing are properly viewed as changes to the net price charged across Amex's integrated platform,' [...] because merchant pricing is only one half of the pertinent equation." The appellate court added: "Because the two sides of the platform cannot be considered in isolation, it was error for the District Court to discard evidence [of 'two-sided price' calculations]" (*United States v. Amex*, 2016, p. 49).[3]

To assess the welfare properties of such restraints requires an understanding of how they affect behavior. This paper analyzes the two-sided pricing incentives created by constraints on a merchant's ability to charge differential prices for purchases via competing platforms when platforms can offer per-unit rebates to end users. The constraints may arise from contractual restrictions (as in *Amex*) or from a merchant's intrinsic reluctance to charge different prices for a given good. For brevity, we refer to all such constraints as a no-surcharge rule. Our analysis is motivated most closely by the credit card sector, which in 2015 accounted for almost eleven trillion dollars of sales globally (Carlton and Winter, 2017). However, its relevance may extend to other multi-sided platforms.

A useful starting point for grasping the pricing incentives is Schwartz and Vincent (2006), who consider a monopoly card platform facing a merchant that serves card

[2] "By suppressing the incentives of its network rivals to offer merchants, and by extension their customers, lower priced payment options at the point of sale [...] American Express's merchant restraints harm interbrand competition." (pp. 100-101.) "American Express's merchant restraints have allowed all four networks to raise their swipe fees more easily and more profitably than would have been possible were merchants permitted to influence their customers' payment decisions." (p. 111.) The nondiscriminatory provisions are framed broadly to include various merchant conduct that would discourage payments via Amex credit or charge cards. Most relevant for our purposes, the merchant may not "impose any restrictions, conditions, disadvantages *or fees* [...] that are not imposed equally on all Other Payment Products, except for electronic funds transfer, or cash and check" (*Id.*, pp. 25-26, emphasis added).

[3] Economically, the impact of such provisions on the fee to card users is clearly relevant for a full welfare analysis. We take no position on the overall merits of the appellate court's decision, which raises also legal issues such as evidential considerations and the appropriate burden of proof. For a critique of the appellate court's position see Carlton and Winter (2017). In October 2017, the U.S. Supreme Court agreed to hear an appeal of the Second Circuit's decision.

and cash customers, with both groups exhibiting elastic demand for transactions. By raising its merchant fee and cutting the user fee – or increasing the rebate – equally, the platform can maintain its margin and profitably boost transactions volume since the NSR leads the merchant to raise price by less than the upward shift in card users' demand. With multiple strategic platforms (instead of a 'passive' payment mode, cash), a similar logic implies that each platform wishes to increase the spread between its merchant fee and its end-user rebate. However, since each platform vies to have a *greater spread* between its fees than the other platform, the implications for equilibrium are subtle.

Our model has two symmetric platforms offering intermediary services viewed by end users as differentiated substitutes. We set aside potential efficiency roles of a NSR, such as preventing free riding or hold-up problems (e.g., Wright, 2003; Bourguignon, Gomes and Tirole, 2014), and abstract from downstream competition by assuming a local monopolist merchant. Each platform (card network for concreteness) sets per-transaction fees to the merchant and to its end users (cardholders), potentially with a NSR, and the merchant decides which platform(s) to accept. We examine two timing structures. In the first, both platforms simultaneously set merchant and cardholder fees; the merchant accepts both platforms, one, or none; and lastly, the merchant sets its price(s). Partly as a robustness check, we consider an alternative timing: cardholder terms are set after the merchant decides whether to accept platforms. Under both scenarios, the conclusions are similar: NSR pricing restraints – whether present for both platforms or only one – create such strong incentives for each competing platform to persistently outdo its rival in offering rebates to cardholders and funding them with fees to merchants that stable (equilibrium) outcomes in the sense of deterministic prices are not achievable.

This result holds under both of our timing structures. However, in the case where platforms set cardholder terms after merchant acceptance decisions, and if the platforms are sufficiently close substitutes, we show there exists an interesting *mixed strategy* equilibrium. Platforms offer a NSR and sufficiently different merchant fees and the merchant accepts only one platform with equal probabilities. Viewed in this light, setting sufficiently disparate merchant fees along with a NSR can serve as a profitable mechanism to achieve probabilistic market allocation among platforms.

Our paper is related to two broad literatures: Two-Sided Markets, and Most Favored Nation (MFN) clauses. Our analysis is two-sided in the key sense that a platform's pricing structure matters, but we ignore other central issues in that literature, such as the role of platform fees in attracting participation on both sides and the use of fixed fees as well as usage fees (e.g., Armstrong, 2006; Rochet and Tirole, 2006; Rysman, 2009). That literature has not encountered our non-existence of equilibrium under a NSR and competing platforms. Rochet and Tirole (2002) analyze a NSR with a monopoly card platform. Competing platforms are analyzed by Rochet and Tirole (2003), Guthrie and Wright (2007), and Edelman and Wright (2015). In those models, however, there is an upper limit on the spread between a platform's merchant fee and user fee for various reasons (explained in Section 4) – a property that seems implausible with cash-back rebates.[4]

MFN clauses, such as the NSR, are contractual provisions that specify 'non-discriminatory' terms between various agents, often uniform pricing. They can arise in diverse settings and perform efficient roles (e.g., reduce transaction costs or delays in purchasing) or anti-competitive roles (e.g., prevent selective discounts that undermine collusion). For a comprehensive survey see LEAR (2012). This literature mostly considers retail MFNs, involving a firm and its direct customers. There is less formal work on MFNs imposed by firms at different vertical stages. Carlton and Winter (2017) show how vertical MFNs can induce higher prices to downstream merchants. However, their model of competing platforms does not consider user rebates, leaving the existence of equilibrium when this option is available as an open question.

The closest analysis to ours is a brief treatment by Boik and Corts (2016). Like us, they consider differentiated platforms facing a monopolist merchant. In the bulk of their paper, platforms set fees only to the merchant, a simplification that lets them tackle a rich set of issues we ignore, including a NSR's effect on entry by a lower-quality, lower-priced platform. In an appendix, however, they provide a linear demand example where platforms offer rebates to their end users, and posit that a pure-strategy equilibrium does not exist, for the same basic force we identify: each platform vies to increase the spread between its merchant fee and user rebate above the other platform's spread. We prove the non-existence result in a more general environment, show it is robust to alternative timing, and explicitly incorporate the merchant's acceptance behavior. The latter is important; for example, one might conjecture that an equilibrium is pinned down by the merchant's threat to reject a platform whose fee-spread is excessive, but we show that this intuition is incorrect. The merchant's acceptance behavior also is central for our second main result, on the mixed-strategy equilibrium under sequential timing.

[4]Caillaud and Julien (2003), another pioneering article on platform competition, assumes that a platform can only charge a total fee, not separate fees to each side.

Section 2 describes the setting. Section 3 analyzes the effect of a NSR when each network sets its fees to the merchant and card users simultaneously and proves that pure strategy equilibria cannot exist. Section 4 shows the same result in the alternative case where merchant fees are set first and user fees are set after the merchant sets its price(s). It also characterizes a mixed-strategy equilibrium with probabilistic market allocation. All proofs are in the Appendix.

2 The Model With Simultaneous Platform Fees

2.1 Agents, Prices and Payoffs

We examine an economic environment where two classes of complementary products are required in fixed proportions to generate a transaction for final consumption; the providers interact with each other through pricing; and each provider also interacts with the end-users through pricing. The motivating context is payment systems. In order for a merchant to complete a transaction, it often must combine its services with a payment instrument such as a credit card offered by a platform/card network. We consider two differentiated competing platforms and a local monopolist merchant. Other papers, such as Rochet and Tirole (2003), assume a continuum of merchants (and uniform pricing by platforms to merchants) while Guthrie and Wright (2007) assume duopoly merchants. Our market structure lets us focus on strategic interaction between platforms while abstracting from strategic interaction among downstream merchants who, nevertheless, possess some power over price.

Platforms 1 and 2 offer differentiated payment services to a single downstream merchant. Let P_i be the total per unit price paid by a consumer for a purchase via platform i. Demand for sales made on platform i, $D^i(P_1, P_2)$, is twice continuously differentiable and satisfies the properties:

$$-\frac{\partial D^i(P_1, P_2)}{\partial P_i} > \frac{\partial D^i(P_1, P_2)}{\partial P_j} \geq 0, \quad D^1(x, y) = D^2(y, x).$$

The first condition implies that the two platforms are (imperfect) gross substitutes with demand varying strictly with price. The second condition indicates that we restrict attention to symmetric platforms. This representation of demand is a 'reduced form' characterization that abstracts from the micro-structure underlying consumers' choice of payment mode. The assumption $\partial D^i/\partial P_i < 0$ can reflect substitution between platforms in response to i's price increase or, as in Schwartz and Vincent (2006), elastic demand for the merchant's good.

We assume for every price, P_1, there is a choke price, $P_2(P_1)$ such that

$$D^2(P_1, P_2(P_1)) = 0.$$

Symmetry implies a similar property for platform 2. The assumption of gross substitutes implies that $P_2(\cdot)$ is weakly increasing. Define

$$\bar{D}^1(P_1) \equiv D^1(P_1, P_2(P_1))$$

to be the demand for platform 1 sales when platform 2 is not available. Gross substitutes implies that for all (P_1, P_2) such that $D^2(P_1, P_2) > 0$,

$$\bar{D}^1(P_1) > D^1(P_1, P_2)$$

and similarly for platform 2.

Each platform i sets a per transaction fee f_i to cardholders (typically negative, i.e., rebates) and m_i to the merchant.[5] If the merchant accepts a platform, the merchant sets a price p_i for a purchase through that platform and the cardholder total price is

$$P_i \equiv p_i + f_i$$

and define platform i's total fee as

$$t_i \equiv f_i + m_i.$$

Marginal costs for both platforms and merchant are assumed constant and normalized to zero. Platform profits are

$$(f_i + m_i)q_i = t_i q_i.$$

The merchant's outside option – its profit from carrying no platform – is also normalized to zero. If the merchant adopts both platforms, using $m_i = t_i - f_i$, merchant profits can be expressed in terms of (t_1, t_2) and total cardholder prices, (P_1, P_2):

$$\begin{aligned} \Pi(P_1, P_2; t_1, t_2) &= (p_1 - m_1)D^1(p_1 + f_1, p_2 + f_2) + (p_2 - m_2)D^2(p_1 + f_1, p_2 + f_2) \\ &= (P_1 - t_1)D^1(P_1, P_2) + (P_2 - t_2)D^2(P_1, P_2). \end{aligned} \tag{1}$$

Here and in Section 3, we consider the following price-setting game:

[5] In the literature on payment systems, the merchant fee is often termed 'the merchant discount'.

Simultaneous Fees Game

1) Both platforms simultaneously select cardholder and merchant fees, (f_i, m_i).

2) The merchant observes all fees and accepts both platforms, one or neither.

3) For each accepted platform, i, the merchant sets a price, p_i, potentially subject to restrictions, for its product and the cardholder's price is $P_i = p_i + f_i$.

4) For each accepted platform, i, consumers observe (f_i, p_i): If both platforms are accepted, transactions via platform i are given by $D^i(p_1 + f_1, p_2 + f_2)$; If only platform i is accepted, its transactions are given by $\bar{D}^i(p_i + f_i)$ and none occur via platform j.

This timing captures a sense in which a merchant is able to change its consumer prices more rapidly than platforms can alter their fees either to merchants or consumers. In Section 4 we examine an alternative timing where platforms set cardholder fees *after* merchants set consumer prices.

2.2 Unrestricted Merchant Pricing

Start with the benchmark case where the merchant is free to set differential prices, (p_1, p_2). Using (1), the merchant's profit maximization problem can be equivalently expressed as, given platform fees, selecting total cardholder prices, (P_1, P_2), rather than merchant prices, (p_1, p_2). This representation illustrates the well-known 'neutrality' property that, with no restriction on merchant pricing (including no NSR), equilibrium cardholder prices, P_i, depend solely on total fees, (t_1, t_2), and are independent of the platform's fee *structure* – the split between cardholder fees and merchant fees. The merchant's optimal quantities, therefore, depend solely on t_i:

$$q_i(t_1, t_2) \equiv D^i(P_1(t_1, t_2), P_2(t_1, t_2)).$$

Since platform i's profit margin also depends only on t_i and not on f_i, m_i separately, the neutrality property follows.[6]

[6]In our setting, where platforms charge only usage (i.e., per-transaction) fees, neutrality requires that a shift in a platform's fee structure – say, an increase of Δ to the merchant and decrease of Δ to the cardholder – will yield an equal and offsetting change in the merchant's price to the cardholder. Neutrality thus implies that the merchant price can adjust freely, unimpeded by contractual restrictions or other factors. In environments where platforms charge fixed fees, neutrality can break down even if the merchant's price can adjust freely (Rochet and Tirole, 2006).

Neutrality thus implies that, with no pricing constraints on the merchant, rival platforms can be thought of as playing a strategic game solely in total fees, t_1, t_2. For a platform i, define an induced best response function from the neutrality environment as

$$r^i(t_j) \equiv argmax_{t_i} t_i q_i(t_1, t_2),$$

and denote the partial derivatives of the merchant's profit function with respect to price as

$$\Pi_i \equiv \frac{\partial \Pi}{\partial P_i}, \quad \Pi_{ij} \equiv \frac{\partial \Pi_i}{\partial P_j}.$$

For the remainder of this Section and Section 3, we assume:

A1) Platform profits are strictly quasi-concave in t_i for all t_j and smoothly supermodular in (t_1, t_2), and $r^i(t_j)$ is continuously differentiable with $r^{i\prime}(t_j) \in [0, 1)$.

A2) For all t_1, t_2, Π is strictly quasi-concave in (P_1, P_2) and there is a unique $(\hat{P}_1(t_1, t_2), \hat{P}_2(t_1, t_2))$ such that $\Pi_i(\hat{P}_1(t_1, t_2), \hat{P}_2(t_1, t_2)) = 0, i = 1, 2$.

A3) For all t_1, t_2, P_1, P_2, $\Pi_{ii} + \Pi_{ij} < 0$.

A4) For any fixed (t_1, t_2), if a merchant accepts only one platform, that platform's sales are (weakly) higher than its sales when the merchant accepts both platforms.[7]

Assumption A1) implies that this is a game in strategic complements and there is a unique equilibrium in t_1, t_2 (see Vives, 2001, p. 47). The remaining assumptions allow us to focus primarily on first order conditions to conduct the proofs.

Under mild conditions, assumptions A1)-A4) are satisfied for two commonly used demand systems:

Linear Demand System (LDS): Demand for platform 1 given by

$$D^1(P_1, P_2) = \frac{1 - \gamma - P_1 + \gamma P_2}{1 - \gamma^2}, \tag{2}$$

and symmetrically for platform 2. This differentiated products system can be generated by a representative consumer with quadratic/quasi-linear consumer preferences where $\gamma = 0$ implies that demands are unrelated and $1 > \gamma > 0$ corresponds to

[7]Chen and Riordan (2015) prove that a similar property holds quite generally when demand is generated by a discrete choice model.

platforms as imperfect substitutes (see Vives, 2001). Direct calculations show that assumptions A1)-A4) hold (see Schwartz and Vincent, 2017).

Independent Demands (ID): Consumer preferences are quasi-linear

$$u(q_1, q_2, y) = V(q_1) + V(q_2) + y, \quad V' > 0, V'' < 0,$$

and demand for transactions on platform i is given by

$$D^i(P_1, P_2) = V'^{-1}(P_1).$$

In the **ID** case, if merchant profits are concave in prices for each platform use, then A2) is satisfied and, since $\Pi_{ij} = 0$, A3) is also satisfied. Since $r^{i\prime}(t_j) = 0$, A1) holds trivially. A4) also holds trivially since demand for good i is unaffected by good j.

A variant of the **ID** case corresponds to a model examined in Schwartz and Vincent (2006) where one platform is interpreted as cash and its corresponding fees are fixed at 0 (so one platform is non-strategic).[8] They examine the fee-setting behavior of the other (card) platform when the merchant must charge equal prices for both means of payments. In the next section, we examine the effects of a similar constraint where *both* platforms can set fees, to the merchant and to consumers.

3 No Equilibrium Under No Surcharge Rule(s)

If the merchant accepts both platforms and the NSR applies to both platforms, the merchant's prices for purchases on either platform must be equal, $p_1 = p_2$. We take this restriction as exogenous to the environment.[9] It only has force if a merchant accepts both platforms. Later in this section we consider a NSR effective only for a single platform, i, so the restriction is tantamount to the inequality, $p_i \leq p_j$.

Fix any f_1, f_2. Given $p_1 = p_2$, total consumer prices can differ solely because of different platform fees to consumers:

$$P_2 = P_1 + f_2 - f_1. \tag{3}$$

[8]That model is not exactly nested in this one, though, since the quantity of cash sales at equal prices is not required to equal those of the other platform, that is, demands can be asymmetric.

[9]Our analysis therefore considers sub-games conditional on a NSR being in place (for one platform or both), either contractually or due to transaction costs. As we show, a NSR yields to non-existence of pure-strategy equilibrium, making it difficult to evaluate platforms' incentives to impose a contractual NSR.

Since $m_i = t_i - f_i$, we can represent the strategic choice of a platform equivalently as setting (f_i, m_i) or setting (f_i, t_i). In what follows, we use the latter representation.

Assumptions A2) and A3) on the merchant's profit function can now be used to derive the impact of a platform's fee structure on sales given a NSR. As long as the merchant continues to accept both platforms, an increase in spread between a platform's cardholder and merchant fees, holding the total fee constant, will increase that platform's sales and benefit that platform.

Lemma 1. *If Platform 1 lowers f_1 by (small) $\delta > 0$ holding t_1 fixed and the merchant accepts the NSR, then sales on platform 1 rise and sales on platform 2 fall.*

Will a merchant accept an increase in a platform fee spread? Under a NSR, the merchant's profit maximization problem can be expressed as the constrained problem

$$max_{P_1,P_2} \;\; \Pi(P_1, P_2; t_1, t_2) \qquad (CP)$$
$$s.t. \;\; P_1 + f_2 - f_1 - P_2 = 0,$$

where $\Pi(P_1, P_2; t_1, t_2)$ is defined in (1). Let the lagrangian associated with (CP) be

$$L(P_1, P_2, \lambda; t_1, t_2, f_1, f_2) = \Pi(P_1, P_2; t_1, t_2) + \lambda(P_1 + f_2 - f_1 - P_2) \qquad (4)$$

and denote the solution to (CP) (including the associated lagrange multiplier on the constraint) by $(\hat{P}_1, \hat{P}_2, \hat{\lambda})$. Since the NSR here is an equality constraint, the lagrange multiplier can take either sign.[10] In the formulation above, $\hat{\lambda} > 0$ implies that, (locally) given (t_1, t_2, f_1, f_2), the merchant would prefer to charge a higher total price for platform 2 but is prevented from doing so by the NSR.[11] When fees are such that $\hat{\lambda} > 0$, Lemma 2 exploits the envelope theorem to show that, holding t_1, t_2, f_2 fixed, merchant profits increase as f_1 falls. Intuitively, with t_1 fixed, a fall in cardholder fee f_1 requires an equal rise in merchant fee m_1. These changes raise demand for transactions on platform 1 and also raise the merchant's marginal cost of these transactions. Both effects increase the merchant's unconstrained optimal price for platform 1, relaxing the effect of the constraint and thus benefitting the merchant. The same logic implies that a fall in cardholder fee and rise in merchant fee of platform 2 exacerbate the constraint on the merchant.

[10] Since the constraint is linear, the usual constraint qualification in the Lagrange Theorem is satisfied.

[11] Strict quasi-concavity of Π, from assumption A2) also implies the merchant's optimal p_2 without the NSR is strictly higher than the constrained merchant price, $\hat{P}_2 - f_2$.

Lemma 2. *Suppose $\hat{\lambda}$ is strictly positive (at current fees, the merchant prefers to charge a higher price to platform 2). Holding (t_1, t_2, f_2) fixed, merchant profits increase as f_1 falls. Holding (t_1, f_1, t_2) fixed, merchant profits decrease as f_2 falls.*

For a given profile of fees, (t_1, t_2, f_1, f_2), if a merchant accepts only a single platform i, a NSR has no force and neutrality implies that the merchant's maximal profit is given by

$$\Pi^S(t_i) \equiv max_P \;\; (P - t_i)\bar{D}^i(P). \tag{5}$$

The envelope theorem implies that stand-alone profits are decreasing in t_i, therefore, if a merchant avoids a NSR by accepting only one platform, it will accept the platform with the lower total fee. Thus, when offered fees (t_1, t_2, f_1, f_2) and a NSR, the merchant's best alternative to accepting both platforms is either its outside option, 0, or $\Pi^S(min\{t_1, t_2\})$. This feature along with Lemmas 1 and 2 imply that at any pure strategy equilibrium where the NSR is accepted, the NSR cannot bind on the merchant:

Lemma 3. *In a pure strategy equilibrium in platform fees when the merchant accepts a NSR from both platforms, the lagrange multiplier in the solution to (CP) satisfies $\hat{\lambda} = 0$.*

The forces at work in Lemmas 1 through 3 are as follows. Whenever a NSR binds on the merchant, two properties will hold. First, conditional on merchant acceptance, either platform gains by raising its merchant fee and cutting its user fee equally. With this change, platform i's total fee remains constant, but expanding the spread $(m_i - f_i)$ increases transactions on platform i: its users' willingness to pay increases by the cut in f_i whereas the merchant raises price by less since the price rise must apply also to the other platform's users. Second, expanding the fee spread will raise the merchant's preferred price for i's transactions and, hence, *mitigate* the NSR constraint if and only if the merchant initially preferred to charge a lower price for i than for the other platform because the merchant's preferred prices would move closer. An increase in the fee spread of platform i therefore relaxes the NSR constraint, benefitting the merchant. It follows that in any pure-strategy equilibrium a NSR cannot bind; if it did, the platform with the lower fee spread could profitably deviate by increasing its fee spread while retaining merchant acceptance.

Lemmas 1 through 3 can now be combined to obtain our main result.

Proposition 1. *In the Simultaneous Fees Game under Assumptions A1) through A4), if an NSR is present for both platforms, there is no equilibrium in pure strategies.*

The logic is the following. First, observe that in a candidate pure-strategy equilibrium both platforms must be accepted. If only platform j were accepted, platform i could deviate by mimicking j's fees, hence the NSR would not bind. The merchant then would strictly prefer to accept both platforms since they are differentiated, and this differentiation implies positive profit for the newly accepted platform.

Next, consider a candidate equilibrium with both platforms accepted. Given the platform fees, the NSR again cannot bind on the merchant (Lemma 3). Therefore, the fee structure is neutral and only the total fees (t_1, t_2) matter. Suppose total fees are equal. Since the platforms are symmetric and differentiated, the merchant strictly prefers to accept both instead of just one. Thus, either platform could profitably deviate by raising its fee spread slightly until the NSR starts to bind (Lemma 1) without being dropped by the merchant, breaking the candidate equilibrium. Now consider unequal total fees, say $t_2 > t_1$. In a candidate equilibrium, platform 1's fee must weakly exceed its best-response to 2's fee, $t_1 \geq r_1(t_2)$: if $t_1 < r_1(t_2)$, then 1 could profitably raise t_1 and the merchant would continue accepting it given $t_2 > t_1$. In turn, $t_1 \geq r_1(t_2)$ implies $t_2 > r_2(t_1)$, so that platform 2 would strictly prefer to cut its fee:

$$t_2 - r_2(t_1) > t_1 - r_2(t_1) \geq t_1 - r_1(t_2) \geq 0,$$

where the first inequality follows given $t_2 > t_1$, the second follows since $r_2(t_1) \leq r_1(t_2)$ given $t_2 > t_1$, platform symmetry and the assumption ${r^j}'(\cdot) \geq 0$, and the final inequality was explained earlier.

To this point, we assumed a NSR for both platforms. Suppose instead only one platform operates under a NSR. For example, Visa and MasterCard dropped their no-steering rules in a 2010 settlement with the Department of Justice, while Amex litigated and retained its rules. The merchant's profit maximization problem is an obvious modification of (CP) where the constraint becomes the inequality constraint (assuming the NSR is present only for platform 2):

$$P_2 \leq P_1 + f_2 - f_1$$

and the lagrange multiplier in (4) must be non-negative. Proposition 2 demonstrates that with a single NSR, equilibrium existence continues to fail.

Proposition 2. *In the Simultaneous Fees Game under Assumptions A1) through A4), if the NSR is present for only one platform, there is no equilibrium in pure strategies.*

The argument is similar to the logic underlying Proposition 1. If a NSR binds in equilibrium, it must clearly bind on the price of the platform with the NSR, say, platform 2. Platform 1 then has the incentive described by Lemma 1 to increase its fee spread and thereby increase transactions, and the merchant would accept such a change (Lemma 2). This implies that the NSR cannot bind in equilibrium (Lemma 3) and the proof from Proposition 1 now proceeds in the same fashion.

Propositions 1 and 2 show that pricing restraints such as no surcharge rules raise important questions about the stability of a pricing game with competing platforms. The driving force is that with the NSR, no matter the size of the gap between merchant and cardholder fees, if competing platforms have equal gaps (so the NSR is not locally binding) each platform wants to increase its gap and divert sales from the rival.

It is worth examining why some other analyses of platform competition under a NSR have not encountered our non-existence of equilibrium. In Rochet and Tirole (2003), merchants' decisions to accept a payment card depend solely on the card network's merchant fee and are unaffected by the fee to cardholders, so raising the former while lowering the latter equally will cause some merchants to refuse the card and platform transactions will not necessarily rise. Networks therefore lack the persistent incentive to increase their fee spread that arises in our model.

In Guthrie and Wright (2007), merchants internalize buyers' benefits from card use, which biases platforms to tilt the fee structure against merchants (as is the case here). However, there is an endogenous limit on the maximal rebate to card users because the merchant must charge the same price to cash users and card users. In their model, consumers have unit demands and to increase platform sales requires convincing more customers to convert from using cash to cards. Since consumers are heterogeneous in their value for card use, if the merchant must charge the same price not only to all card users but to cash users as well, it becomes more costly to draw in cash users, and simultaneously increasingly costly for platforms to induce merchants to raise the common price to remaining cash customers. When, instead, the NSR constraint governs only the merchant's prices to users of the same payment mode (cards), leaving the merchant free to charge separate prices to users of other means of payment, this limiting effect is absent and the source of our equilibrium non-existence emerges. An equivalent result would arise if there were only card users in the market. Finally, Edelman and Wright (2015) assume that increased expenditure on card holder 'rewards' by a platform delivers benefits to its users at a diminishing rate, which caps the benefit offered to users (and the fee to merchants). As they point out, however, this assumption rules out cash-back rebates.

4 Carduser Fees Set After Merchant Prices

One candidate explanation for non-existence of pure strategy equilibrium is that the strategic structure has been misspecified. We explore this possibility by considering a related game where the timing is modified so that platforms set fees to card users after setting fees to the merchant. The underlying logic is robust to this new specification: the incentive remains for each platform to exploit a NSR by increasing its fee spread so as to shift transactions to itself from the rival and non-existence of pure-strategy equilibrium persists. However, we also demonstrate that a mixed-strategy can exist where the merchant single-homes but randomizes over which platform it accepts.

Beyond providing a robustness check on the results of Section 3, the alternative timing structure where platforms choose cardholder fees *after* the merchant sets its price may better reflect certain economic situations. If contracts between platforms and merchants and platforms and consumers are relatively long-term, de facto or de jure, while merchant prices can be altered more quickly and easily, the simultaneous pricing game may be a close representation of the interactions among agents. In other circumstances, however, contracts between merchants and platforms may extend over a longer period than contracts between platforms and cardholders and, furthermore, the true fees between cardholders and platforms are not likely to be known (and credibly committed) to merchants before they set their prices. If so, then any initial cardholder fees are not generally sequentially rational.

The timing structure in the following game more accurately captures these features:

Sequential Fees Game

1) Both platforms simultaneously select merchant fees, m_i.

2) The merchant observes both fees and accepts both, one or neither platform.

3) For each accepted platform, i, the merchant sets a price, p_i, potentially subject to restrictions.

4) Merchant price(s) are observed and each accepted platform, i, sets cardholder fee, f_i.

5) For each accepted platform, i, consumers observe (f_i, p_i): If both platforms are accepted, transactions via platform i are given by $D^i(p_1 + f_1, p_2 + f_2)$; If only

platform i is accepted, its transactions are given by $\bar{D}^i(p_i + f_i)$ and none occur via platform j.

In this alternative pricing game, given the merchant fees set by the platform and the subsequent merchant prices, the two platforms play a subgame in cardholder fees, f_1, f_2. In order to apply the natural solution concept, subgame perfection, we need to determine how the equilibria of these subgames vary with the the prior selected fees and prices, (m_1, m_2) and (p_1, p_2). This requirement restricts our ability to provide general results; however, the linear demands case (**LDS**) is tractable and offers useful insights. In particular, merchant and platform best responses with a NSR mirror in many ways those of the simultaneous structure and indicate that the leapfrogging incentive for increasing the gap between merchant fees and cardholder fees that leads to non-existence is again present.

4.1 Equilibrium Behavior in the Continuation Game

With or without the NSR, the subgame perfect equilibrium is obtained by first finding the equilibrium of the subgame where platforms select cardholder fees given merchant prices and merchant fees, (m_1, m_2). Anticipating these equilibrium fees and given any pair of merchant fees: if there is no NSR and both platforms are accepted, the merchant then selects (p_1, p_2); if there is an NSR and both platforms are accepted, the merchant selects a single price, p; if the merchant accepts only a single platform, i, the merchant selects p_i. Anticipating this behavior, the platforms select (m_1, m_2).

In the **LDS** model, platform demand is given by (2). Using $P_i = p_i + f_i$, we can generate platform best responses in f_i as

$$f_i(f_j) = (1 - \gamma + \gamma p_j - p_i - m_i + \gamma f_j)/2$$

This is a familiar linear game in strategic complements (see for example Vives, 2001 pp. 159-160) and the equilibrium in cardholder fees satisfies

$$f_i(p_1, p_2, m_1, m_2) = \frac{2A_i + \gamma A_j}{4 - \gamma^2}, \tag{6}$$

where

$$A_i \equiv 1 - \gamma + \gamma p_j - p_i - m_i.$$

Setting $p_1 = p_2$ yields the equilibrium cardholder fees with a NSR in place and setting $\gamma = 0$ yields the fees with a single platform. With this equilibrium behavior, a merchant then selects prices to maximize profits. This analysis enables us to characterize the continuation equilibrium in any subgame given merchant fees, (m_1, m_2) and a merchant's acceptance decision over platforms.

No NSR. In a market with no NSR, neutrality implies that quantities and profits depend only on the total fees, (t_1, t_2). To see this, fix any m_1, m_2. Suppose p_1, p_2, f_1, f_2 are the equilibrium prices that maximize merchant profits given that the subsequent f_i are determined by (6). The corresponding quantities are $q_i = D^i(p_1 + f_1, p_2 + f_2)$. Now suppose a different pair of merchant fees, $\tilde{m}_1, \tilde{m}_2$, are offered. If the merchant sets prices $\tilde{p}_i = p_i + (\tilde{m}_i - m_i)$ and the platforms each set cardholder fees $\tilde{f}_i = f_i - (\tilde{m}_i - m_i)$, then quantities and platform and merchant profits are the same as in the original equilibrium and therefore, this new profile of price and fees forms an equilibrium with the same outcome. This implies that even though platforms set merchant fees first, the outcome ultimately mirrors a vertical chain where the merchant acts as an upstream price-setter, setting p_1, p_2 and platforms then react in an imperfectly competitive way. The equilibrium margins and quantities are then[12]

$$p_i - m_i = \frac{1}{2}, \quad q_i = \frac{1}{2(1+\gamma)(2-\gamma)}.$$

The optimal merchant pricing then yields platform margins as

$$f_i + m_i = \frac{1-\gamma}{2(2-\gamma)}.$$

Observe that, consistent with neutrality, platform margins are independent of the initial stage offer of merchant fees, (m_1, m_2), and platform margins vanish as the degree of differentiation between platforms vanishes (γ approaches one).

Single Platform. If the merchant accepts only a single platform i, the NSR is irrelevant and once again neutrality implies that quantities and profits depend only on the total fee, t_i. The resulting game is one with an upstream supplier, in this case, the merchant, and a downstream firm, platform i. This is a familiar vertical chain with linear demand and double marginalization because of linear pricing. The merchant's subsequent profits are independent of the merchant fee, m_i, agreed to in the first stage. For the **LDS** model, merchant profits are $\frac{1}{8}$ while the profits of the accepted platform i are $\frac{1}{16}$.

NSR in Force, Two Platforms. The equilibrium of the merchant pricing subgame is computed by setting $p_1 = p_2 = p$, determining equilibrium f_i and solving

[12] All calculations for this section are available from the authors.

the merchant's maximization problem in p. This yields an equilibrium common price

$$p(m_1, m_2) = \frac{1 + m_1 + m_2}{2},$$

and quantity:

$$q_i = max\{0, \frac{1}{2(2-\gamma)(1+\gamma)} + \frac{m_i - m_j}{2(2+\gamma)(1-\gamma)}\}. \tag{7}$$

As in the initial game with alternative timing, when a NSR is in place, sales on platform i increase in the difference between platform i's merchant fee and that of its rival (Lemma 1).[13]

The equilibrium prices imply that, under a NSR, merchant profits are *decreasing* in the difference in merchant fees (by the same logic as in Lemma 2 of Section 3):

$$\frac{1}{2(2-\gamma)(1+\gamma)} - \frac{(m_1 - m_2)^2}{2(2+\gamma)(1-\gamma)}. \tag{8}$$

Using the equilibrium price and *cardholder* fees, the profit margin of each platform for a given profile of merchant fees is

$$f_i + m_i = \frac{1-\gamma}{2-\gamma} + (m_i - m_j)\frac{1+\gamma}{2+\gamma}. \tag{9}$$

Platform i's margin and sales increase in $m_i - m_j$, therefore, its profits increase in this difference as well.

4.2 Behavior in the Full Game

These properties of the continuation game immediately imply a result mirroring Propositions 1 and 2 for the *Simultaneous Fees Game.*

Proposition 3. *In the Sequential Fees Game with linear demands, if an NSR is present, there is no pure strategy equilibrium.*

We can, however, construct a mixed strategy equilibrium in this game. With a single accepted platform, the Sequential Pricing Game collapses to effectively a two-stage pricing game where the merchant sets price and the platform then sets an optimal cardholder fee (and therefore optimal total fee, t_i). Define Δ^* to be the

[13] Intuitively, if $m_i > m_j$, then platform i will have a greater incentive to increase sales, leading it to subsequently choose $f_i < f_j$. With this profile of fees, the NSR constraint binds on the merchant for platform i's transactions.

maximum difference in platform fees to the merchant such that the merchant is just willing to accept both platforms with a NSR compared to accepting a single platform (yielding profit $\frac{1}{8}$). Using (8), this implies

$$\Delta^* = \left(\frac{(1-\gamma)(2+\gamma)}{(2-\gamma)(1+\gamma)} - \frac{(1-\gamma)(2+\gamma)}{4} \right)^{1/2}$$

Note that as γ approaches 1 (perfect substitutes), Δ^* approaches zero. Economically, as the platforms become closer substitutes, the merchant's incremental profit from accepting a second platform decreases. Therefore, to maintain the merchant's willingness to accept a second platform under a binding NSR, the burden of the NSR must be eased, requiring a smaller gap between the platforms' merchant fees.

As platforms become close enough substitutes, there is a mixed strategy equilibrium with a NSR such that the merchant accepts only a single platform, each with equal probability. In such an equilibrium, the platforms exploit the NSR to weaken the strong competition between them by inducing the merchant to single-home:

Proposition 4. *Suppose $\hat{m}_i > \hat{m}_j + \Delta^*$. In the Sequential Fees Game with linear demands and a NSR present for both platforms, as γ approaches 1, it is an equilibrium for platforms to offer $(\hat{m}_1, \hat{m}_2)$ along with a NSR. The merchant adopts a single platform, rejecting each platform with equal probability.*

The logic for Proposition 4 is a follows. By accepting a single platform, the merchant renders a NSR irrelevant. Moreover, if a single platform is accepted, neutrality implies that only that platform's total fee matters, and platform symmetry implies the total fee would be the same whichever platform is accepted. (Regardless of a platform's *merchant fee*, the platform's unique equilibrium *total fee* is determined by its subsequent choice of cardholder fee, set after the merchant's price to consumers.) Thus, the merchant is indifferent between the platforms, justifying randomization over acceptance.[14] The merchant prefers this to accepting both platforms with a NSR when their merchant fees differ enough, because the merchant's preferred prices to consumers will then differ sufficiently that satisfying the NSR becomes too onerous. By adopting a NSR with sufficiently disparate merchant fees, the platforms therefore can ensure that only one of them will be accepted. Finally, when platforms are sufficiently close substitutes, their margins will be arbitrarily small if both are accepted

[14] By contrast, in the Simultaneous Fees Game, each platform commits initially to both fees, hence the mixed-strategy equilibrium is ruled out: either platform would undercut the other's total fee slightly and be accepted with probability one.

(which could be achieved by offering similar merchant fees), whereas if only one is accepted, its profit is positive and independent of the degree of substitutability.

5 Discussion

In markets such as credit cards, where a platform sets fees both to merchants and its end users who transact with the merchants, restrictions on merchant ability to surcharge platform customers create clear incentives for a platform to raise its fee to merchants and lower its fee to users or grant rebates. However, the equilibrium implications of this incentive have gone virtually unexplored in the case of *competing platforms*.[15] We analyze this case for platforms supplying differentiated substitute services under the traditional assumption that rational consumers care about the total cost of purchases – merchant's price plus platform's fee. No-surcharge restrictions lead to non-existence of pure strategy equilibria under two alternative timing of moves. The basic force is that strategic platforms will try to outdo each other's spread between the merchant fee and user rebate. In some circumstances, there is a mixed strategy equilibrium in which platforms offer very disparate merchant fees along with no-surcharge rules and the merchant randomizes over which platform to accept, an outcome with the flavor of probabilistic market allocation by platforms.

What should one make of these findings for our motivating case of credit card platforms? Some anecdotal evidence is consistent with the mixed-strategy outcome, insofar as several large merchants accept only a single card. For example, the major retailing club Costco historically accepted only a single card, originally Discover, then American Express prior to 2014, and more recently Visa.[16] However, many merchants still accept multiple cards, and their acceptance decisions as well as platforms' fees appear more stable than might be expected under non-existence of a pure strategy equilibrium.

What, then, serves as the brake on each platform's incentive under a NSR to

[15] Except for the short treatment in Boik and Corts (2016), as noted in the Introduction.

[16] Prior to 2011, Neiman Marcus accepted only American Express of the four major card platforms.<http://www.wsj.com/articles/SB10001424052970204505304577000103355671444>. The Second Circuit Court opinion noted that almost one-third of all merchants that accept cards do not accept American Express. Details on these single-homing examples can be found at <https://consumerist.com/2014/11/06/costco-may-finally-start-accepting-something-other-than-american-express/> and <http://www.usatoday.com/story/money/2016/06/13/walmart-canada-will-stop-accepting-visa-cards/85826704/>.

continually leapfrog its rival's fee spread, by increasing its own cardholder rebates funded by higher merchant fees? The literature offers potential explanations, but none is entirely convincing. Certain cardholder rewards, such as airline miles, may exhibit diminishing returns, which precludes a platform from increasing its transaction volume while maintaining its total margin by raising its merchant fee and rewards equally. However, this diminishing returns feature seems implausible for cash-back rebates. Alternatively, a platform's incentive to increase its fee spread – leading the merchant to raise its price to all card users – might be limited if the merchant is unable to charge a lower price to its non-card customers. However, non-card payment modes have been exempt from card networks' no-surcharge rules (for example, Amex exempted "electronic funds transfer, or cash and check". See Footnote 1). Potentially, exogenous factors might limit merchants' ability to set differential prices for non-card customers. But such exogenous price coherence is at odds with card networks' significant efforts to maintain their contractual no-surcharge rules, and with the fact that surcharging of card transactions has often occurred once permitted (Bourguignon, Gomes and Tirole, 2014).

Stepping beyond credit cards, our analysis may have relevance to other platforms that set fees to both sides and there are direct payment flows between those sides. In the presence of frictions that limit merchants' pricing flexibility, the main message of our paper is that platforms will have strong incentives to outdo each other's fee spread, posing a serious challenge to reaching deterministic equilibria in fees.

6 Appendix

6.1 Proof of Lemma 1

Proof. Fix t_1, t_2, f_1, f_2. Set $d = f_1 - f_2$ and note from (3) that $(\hat{P}_1, \hat{P}_1 - d)$ are thus the merchant's optimal price(s) under the NSR at these prices. This pair is unique by A2). By definition,

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d) + \Pi_2(\hat{P}_1, \hat{P}_1 - d) = 0. \tag{10}$$

Now suppose platform 1 changes its cardholder fee to $f_1 - \delta$ and suppose the merchant chose to raise the common price by δ so that the new cardholder prices become $(\hat{P}_1, \hat{P}_1 - d + \delta)$. That is, the total cardholder price for platform 1 stays constant, while the total price for platform 2 goes up by δ because the common merchant price is raised by δ. Observe that by the fundamental theorem of calculus,

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d + \delta) = \Pi_1(\hat{P}_1, \hat{P}_1 - d) + \int_0^\delta \Pi_{12}(\hat{P}_1, \hat{P}_1 - d + \tau)d\tau$$

and

$$\Pi_2(\hat{P}_1, \hat{P}_1 - d + \delta) = \Pi_2(\hat{P}_1, \hat{P}_1 - d) + \int_0^\delta \Pi_{22}(\hat{P}_1, \hat{P}_1 - d + \tau)d\tau$$

Summing the two expressions and using the first order conditions from (10) to eliminate the first term on the right side of each equation, gives

$$\Pi_1(\hat{P}_1, \hat{P}_1 - d + \delta) + \Pi_2(\hat{P}_1, \hat{P}_1 - d + \delta) = \int_0^\delta \Pi_{12}(\hat{P}_1, \hat{P}_1 - d + \tau) + \Pi_{22}(\hat{P}_1, \hat{P}_1 - d + \tau)d\tau.$$

Assumption A3) implies this is negative. Since the left side is the derivative of merchant profits with respect to price, this means the merchant prefers to lower prices from this point.

Consider the price profile, $(\hat{P}_1 - \delta, \hat{P}_1 - d)$ so that the cardholder price for platform 2 remains the same but for platform 1 falls by δ. A similar argument to above yields

$$\begin{aligned}
\Pi_1(\hat{P}_1 - \delta, \hat{P}_1 - d) + \Pi_2(\hat{P}_1 - \delta, \hat{P}_1 - d) &= \int_0^{-\delta} \Pi_{11}(\hat{P}_1 + \tau, \hat{P}_1 - d) + \Pi_{21}(\hat{P}_1 + \tau, \hat{P}_1 - d)d\tau \\
&= -\int_{-\delta}^{0} \Pi_{11}(\hat{P}_1 + \tau, \hat{P}_1 - d) + \Pi_{21}(\hat{P}_1 + \tau, \hat{P}_1 - d)d\tau.
\end{aligned}$$

The second line just changes the direction of integration and therefore is multiplied by -1. Assumption A3) implies this is positive. Since under the NSR, the derivative of merchant profits with respect to P_1 is positive at the lower end of the interval, $[\hat{P}_1 - \delta, \hat{P}_1]$ and negative at the upper end and since strict quasi-concavity implies that the merchant profits are single-peaked along any ray in (P_1, P_2), this implies that the new optimal cardholder fees involve a lower total price for platform 1 and a higher total price for platform 2, so the gross substitutes property implies that platform 1 use rises and platform 2 use falls.

□

6.2 Proof of Lemma 2

Proof. Partially differentiate the expression in (4) with respect to f_1 and apply the envelope theorem. □

6.3 Proof of Lemma 3

Proof. Suppose, for example, $\hat{\lambda} > 0$. Platform 1 can lower its cardholder fee f_1 while keeping t_1 fixed (that is, raise m_1 by the same amount). This has no effect on the merchant's outside option, $\Pi^S(t_i)$, and by Lemma 2, this raises merchant profits, so the merchant would continue to accept both platforms. By Lemma 1, platform 1 profits rise. A parallel argument holds for platform 2 if $\hat{\lambda} < 0$. □

6.4 Proof of Proposition 1

Proof. Observe that it cannot be an equilibrium for the merchant to accept a single platform. Suppose it was – the merchant accepted Platform 1 at fees (m, f) and total fee $t = m + f$. Let the resulting price be p with consumer price $P = p + f$ generating merchant profits

$$(P - t)\bar{D}(P) = (P - t)D^1(P, P_2(P))).$$

Suppose Platform 2 also offered fees (m, f). If the merchant accepted and responded with consumer prices (P, P), then its margins would remain the same but total demand would rise. To see this, note that

$$D^1(P, P_2(P))) = D^1(P, P_2(P)) + D^2(P, P_2(P))$$

by definition of $P_2(P)$. At consumer prices (P, P), total demand is $D^1(P, P) + D^2(P, P)$. The fundamental theorem of calculus implies that

$$D^1(P,P)+D^2(P,P) = D^1(P,P_2(P))+D^2(P,P_2(P))+\int_{P_2(P)}^{P}\left(\frac{\partial D^1(P,x)}{\partial x}+\frac{\partial D^2(P,x)}{\partial x}\right)dx.$$

Since $P < P_2(P)$, and imperfect substitutes implies $\frac{\partial D^1(P,x)}{\partial x} + \frac{\partial D^2(P,x)}{\partial x} < 0$, this yields

$$D^1(P,P)+D^2(P,P) = D^1(P,P_2(P))+D^2(P,P_2(P))-\int_{P}^{P_2(P)}\left(\frac{\partial D^1(P,x)}{\partial x}+\frac{\partial D^2(P,x)}{\partial x}\right)dx.$$

The second term is strictly positive, so demand strictly increases. Thus, if $t > 0$, Platform 2 could mimic Platform 1, the merchant would strictly prefer to accept both, and Platform 2's profits would be strictly positive. Continuity implies that merchant profits are also strictly higher for t_2 slightly higher than t, so even if $t = 0$, Platform 2 could offer a slightly higher total fee and the merchant would also accept.

We next establish an additional lemma showing that if a platform charges strictly lower total fees under a NSR than its rival, the merchant would continue to accept if the platform raised its total fees slightly:

Lemma 4. *Suppose the profile of fees* (t_1, t_2, f_1, f_2) *are such that* $\hat{\lambda} = 0$ *and* $t_1 < t_2$*. If the merchant accepts the NSR and there are positive sales through both platforms, merchant profits decline in* t_i*. For a small increase in* t_1*, the merchant prefers the NSR to rejecting platform 2.*

Proof. Clearly if the merchant were to reject a platform, it would reject the high total fee platform 2. Recall that $(\hat{P}_1, \hat{P}_2, \hat{\lambda})$ is the solution to (CP) using the lagrangian in (4). The assumption that $\hat{\lambda} = 0$, implies the NSR does not bind and that the derivative of the lagrangian with respect to P_i equals Π_i at $(\hat{P}_1, \hat{P}_2)$ and must equal zero. Assumption A2) then implies that $(\hat{P}_1, \hat{P}_2)$ also represents the optimal prices given (t_1, t_2, f_1, f_2) under no NSR. The envelope theorem implies that the change in merchant profits with respect to t_i is (by partially differentiating (4) with respect to t_i)

$$-D^i(\hat{P}_1, \hat{P}_2) < 0.$$

Let $\bar{P}_1$ be the optimal price offered by the merchant if it rejected platform 2 and sold only through platform 1. Again, the envelope theorem implies that the change in merchant profits with respect to a small increase in t_1 is

$$-\bar{D}^1(\bar{P}_1).$$

Assumption A4) implies that this decline in profits is more than the decline in profits under the NSR, therefore if the low total fee platform 1 raised t_1 slightly (which in this case implies raising m_1 as we are *partially* differentiating) the merchant would continue to accept both platforms and the NSR instead of accepting only platform 1. □

Lemma 3 implies that in any equilibrium, $\hat{\lambda} = 0$, so the NSR does not bind.

First, suppose $t_1 = t_2$ in an equilibrium and let p be the uniform merchant price. Since the NSR does not bind, this must imply $f_1 = f_2$. Suppose not. Consider the equilibrium prices under no NSR. A2) implies this is unique in (t_1, t_2) and neutrality implies that equilibrium quantities and, therefore, consumer prices depend only (t_1, t_2). Symmetry and the hypothesis that $t_1 = t_2$ imply that $p + f_1 = P_1 = P_2 = p + f_2$. This implies $f_1 = f_2$. Since the platforms are not perfect substitutes, then the merchant does strictly better accepting the NSR and both platforms than rejecting one platform. But, then Lemma 1 implies each platform would increase profits by lowering f_i holding t_i fixed so this cannot be an equilibrium.

Therefore, suppose $t_1 < t_2$ and $\hat{\lambda} = 0$. Lemma 1 implies that the merchant's participation constraint must bind, otherwise, each platform has an incentive to raise

m_i and lower f_i. We show that at least one platform will wish to change its t_i thus contradicting the assumption of an optimum.

Lemma 4 implies that $t_1 \geq r^1(t_2)$. To see this, note that if $t_1 < r^1(t_2)$, Lemma 4 shows that platform 1 can raise t_1 and the merchant will still accept the NSR. Since platform profits are strictly quasi-concave in t_1, platform 1 will prefer the higher t_1 so we cannot be at an equilibrium. Therefore, $t_1 \geq r^1(t_2)$.

Symmetry and A1) imply that the equilibrium of the platform game with no NSR is symmetric, say $(\hat{t}, \hat{t})$. Assumption A1) implies $t_2 > t_1$ and $t_1 \geq r^1(t_2)$ if and only if $t_1 > \hat{t}$. This is because the inverse of platform 1's best response function, $r^1(\cdot)$ has slope greater than one in (t_1, t_2) space and, so, crosses the line $t_1 = t_2$ from below at $\hat{t}$.

Assumption A1) also implies that $t_1 \geq \hat{t}$ if and only if $t_1 \geq r^2(t_1)$ ($r^2(\cdot)$ crosses the line $t_1 = t_2$ exactly once and does so at $\hat{t}$ from above since $r^{2\prime}(\cdot) < 1$. Thus, $t_1 > \hat{t}$ if and only if $r^2(t_1) < t_1$). But this then yields, $t_2 > t_1 \geq r^2(t_1)$.[17] Since platform 2 profits are strictly quasi-concave, this implies platform 2 would like to lower t_2 and Lemma 4 implies the merchant would continue to accept the NSR with the lower t_2 since the merchant's single-homing option (accepting only platform 1) is unchanged and merchant profits under an NSR increase with a decline in t_2. □

6.5 Proof of Proposition 2

Proof. Note that the proof of Proposition 1 applies in this case as well if it can be shown that with a single NSR, $\hat{\lambda} = 0$ is necessary for a pure strategy equilibrium. Therefore, suppose $\hat{\lambda} > 0$ at an equilibrium profile of fees such that the NSR from platform 2 is accepted by the merchant. Lemma 2 implies that merchant profits under an NSR rise as f_1 falls holding all other fees constant, thus the merchant continues to accept both platforms (since t_i is held fixed, its outside option has not changed). Lemma 1 implies that since the NSR strictly binds, platform 1 sales rise and platform 2 sales fall as f_1 falls. Thus, it is feasible for platform 1 to lower f_1 and raise m_1 keeping t_1 fixed and its profits would rise. □

[17] In Section 3 we obtained the same conclusion exploiting platform symmetry rather than the condition $r^{2\prime}(\cdot) < 1$.

6.6 Proof of Proposition 3

Proof. Equation (7) implies that, assuming the merchant accepts a NSR, profits of platform i increase in $m_i - m_j$. Suppose a NSR equilibrium exists with, say, $m_1 < m_2$. Merchant profits must be weakly greater under the NSR than what could be earned by rejecting one platform, $(\frac{1}{8})$. Equation (8) illustrates that merchant profits with a NSR strictly increase if platform 1 raises m_1 slightly (and therefore, the merchant would continue to accept the NSR) and equation (9) implies that platform 1 profits rise as well. So $m_1 < m_2$ cannot be a best response. Similarly for the case $m_2 < m_1$. If $m_1 = m_2$, equations (8) and (**??**) imply that the merchant gets strictly higher profits with both platforms than with one (assuming that the platforms are not perfect substitutes, $\gamma < 1$) so each platform could raise its m_i slightly, increase its profits and still have the merchant accept. □

6.7 Proof of Proposition 4

Proof. If the merchant single-homes, neutrality implies that the continuation game is independent of m_i and merchant profits are the same $(\frac{1}{8})$ no matter which platform it selects to single-home with. Therefore, (conditional on single-homing) randomizing over platforms is a best response for the merchant. By definition of Δ^* and equation (8), a merchant will never accept both platforms with this profile of merchant discounts and a NSR. As γ approaches 1, (9) shows that platform profits approach zero when a NSR with both platforms is accepted for any $|m_i - m_j| \leq \Delta^*$, while platform profits under single-homing, $\frac{1}{16}$, are bounded above zero. The proposed equilibrium, then, offers platforms higher equilibrium profits than either a market with no NSR, or one in which merchant discounts are such that the merchant would accept both platforms and a NSR. □

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