Lecture 10

Common Knowledge and Perfect Recall.

Introduction

- Traditional game theory typically assumes certain aspects about the informational environment.
  - Assumes much of the game structure and equilibrium strategy is “common knowledge”
  - Assumes players never forget information they previously possessed, “perfect recall”
- In this lecture we examine, briefly, consequences of relaxing these assumptions.

Common Knowledge

- We say that some information, p, is common knowledge when it is the case that, for all the agents involved,
- each agent knows p,
- each agent knows that each agent knows p
- each agent know that each agent knows that each agent knows p
- ….. ad infinitum

The Email Game

- Suppose that Ariel and Asher would like to meet up in Strasbourg
- Ariel is in Israel, Asher is in Chicago
- Ariel sends Asher an email suggesting they meet in front of the cathedral at 12 noon.
- Is it CK that they will meet at the cathedral at noon?

The Email Game (ctd)

- Asher responds by email to Ariel agreeing.
- Is it CK now?
- Ariel responds, Got your email.
- What about now?
- Can it ever be CK?
- What if Ariel and Asher make their plans in person?

The Intrusive Anthropologist

- There is an isolated island of five people.
- They have a genetic condition that induces a blue dot on their forehead with probability 1%
- Should any agent know she had the blue dot, she would be so shy she would never be seen in public.
- In fact, they all have blue dots on their forehead.
- There are no mirrors. No one would presume to mention the blue dot of their neighbor.
- They regularly meet each day at 4 for tea. Very pleasant….

The Intrusive Anthropologist (ctd)

- One day, Margaret Meddle arrives to the island and learns their customs.
- Before she leaves, she mentions casually to everyone, “At least one of you has a blue dot!”
- Does this have any impact?
- Each agent already knew this so how could it?

The Intrusive Anthropologist (ctd)

- Consider the case of an island with one agent,
  - obviously, the announcement would convey information
- What about two agents?
  - Could be {(NBD, NBD), (BD, NBD), (NBD,BD),(BD,BD)}={a,b,c,d}
  - Agent 1 knows a and b are not possible, and hopes that c is true, not d. Agent 2 knows a and c are not possible and hopes b is true and not d.

The Intrusive Anthropologist (ctd)

- That is Agent 1 knows c or d, Agent 2 knows b or d.
- Before MM, the agents do not know what the other agents know.
- So Agent 1 knows that Agent 2 knows EITHER {a,c} or {b,d}.
- If 2 knows {a,c}, then MM's info tells him what?
- If 2 knows {b,d} then what?
- Then what should 1 do?

The Paradox of the Surprise Quiz

- On Friday, a teacher announces that there will be a surprise quiz the next week.
- Is this possible?

Cheryl's Birthday

- Cheryl, Albert and Bernard are newly made friends.
- Cheryl tells them her birthday is one of 10 dates:

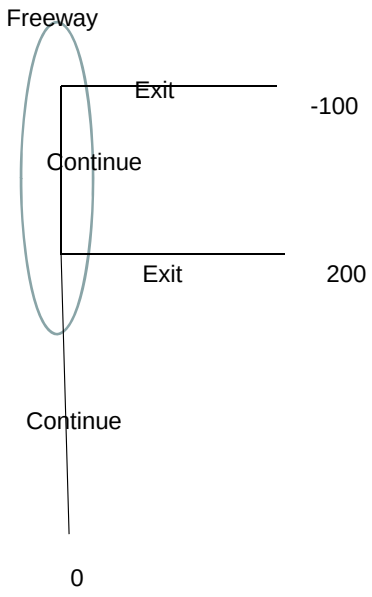
May 15	May 16
May 19	June 17
June 18	July 14
July 16	Aug 14
Aug 15	Aug 17

Cheryl's Birthday

- Cheryl tells Albert the month and Bernard the day her birthday is on.
- Albert says, "I do not know when Cheryl's birthday is but I know Bernard does not know either."
- Bernard says, "Now I know"
- Albert says, "Now I know."
- When is Cheryl's birthday?

The Absentminded Driver

- This is from Piccione and Rubenstein.
- John has a lot on his mind. He is planning on driving home, and knows he must take the second exit and definitely not the first exit.
- He knows, that once he gets on the freeway, he will not remember if he has passed an exit.
- What should he do?



Never Exit?

- John has to make his plan upfront.
- If his plan is “exit” he will go to the badlands.
- Therefore, he should just decide, “Continue”
- But what about when he is on the freeway?
- Will he change his mind?