Lecture 11

Voting

Outline

Hanging Chads Again

- Did Ralph Nader cause the Bush presidency?

A Paradox

Left	Middle	Right
40	25	35
Robespierre	Danton	Lafarge
D	L	R
L	R	D

A Paradox

- Consider Robespierre versus Danton
  - R wins 75:25
- R versus L
  - L wins 60:40
- L versus D
  - D wins 65:35
- Depending on who runs against whom, the outcome can be very different.

A Paradox

- Does the fact that R wins by the most make that more acceptable?
- In the current French system, there is first a multi-person election and, if no majority, then a “runoff
- who would win in this case?
  - Chirac vs. Jospin vs. LePen

Condorcet Rules

- Condorcet suggested the following rule.
- Have all voters list their entire ranking.
- Use the ranking to determine who beats whom on pair-wise comparisons.
- The winner is the one with the smallest maximum votes against.

Approval Voting

- Suppose there is a list of (say) 6 candidates
- Voters are asked to vote for any of the candidates they find acceptable.
- Which candidates win depends on the rule.
  - if only a fixed number of slots, say, 3 then the top 3 vote getters win
  - if a minimum number of acceptable votes must win to get elected
  - eg. proposed voting for HOF

Borda Count

- All candidates are ranked by voters
- Each ranking gets a certain number of points.
  - eg. 1st gets 10 points, 2nd gets 8 etc.
- winner(s) are those with the most points.
- Example: Suppose we do a Borda Count with points, 3,2,1 on the R,D L example.

A Paradox

Robespierre	Danton	Lafarge
40*3+35*2+25=215	25*3+40*2+35=190	35*3+25*2+40=195
40	25	35
Robespierre	Danton	Lafarge
D	L	R
L	R	D

Borda Count

- Sincere voting would lead to a victory for Robespierre with 215
- Again, this is the worst outcome for the Center party.
- If instead of voting sincerely, they lied and ranked Lafarge first, they could ensure that their least favorite candidate loses. (As Lafarge would then get 60*3+40=220)

Arrow's Impossibility Theorem

- Observe that all of these rules create incentives for voters not to vote sincerely
  - that is, they may strategically misrepresent their true desires.
- Kenneth Arrow (Nobel Prize, 1972) demonstrated that this was a general truth
  - whenever there are three or more alternatives, (and agents' true preferences could take any ranking) there will be an incentive to lie in any voting scheme.

Why Vote?

- Note that Arrow's Theorem does NOT apply if there are only two choices.
- In that case, majority voting induces sincere behavior: it is always a best response to vote truthfully.
- (You can probably prove this for yourself)

Why Vote?: Are you 'pivotal?'

- However, when does your vote matter?
- Suppose there are exactly 2n+1 people including you.
- Your vote only has an impact if the other 2n people split exactly evenly.
- In this case, we say you are 'pivotal'
- (What about the other voters?)

When Are you ‘pivotal?’

- Suppose everyone has a 50% chance of preferring one candidate over the other.
- What are the chances that you are pivotal?
- That is, what are the chances the other 2n voters split exactly evenly?
- How many ways can you divide 2n into two equal groups? (2n!)/(n!n!)
- Each grouping occurs with probability $(1/2)^n(1/2)^n = (1/4)^n$
- Now suppose it costs you $5 to vote. In a group of 100,000 other people what would it have to be worth to you to have your side win?

Probability of Pivotal.

(n,2n)	Probability
(2,4)	.375
(4,8)	.27
(8,16)	.20
(50,100)	.08
(500,1000)	.025
(50,000,100,000)	.0025

Order matters

- Current system:
  - determine guilt or innocence, then fix sentence
- Roman Tradition
  - Go through sentences from most severe to least and decide on whether appropriate
- Mandatory Sentencing
  - Specify mandatory sentence then decide guilty or innocent

Order matters

	Judge A	Judge B	Judge C
Best	Death Sentence	Life in Prison	Acquittal
Middle	Life in Prison	Acquittal	Death Sentence
Worst	Acquittal	Death Sentence	Life in Prison

Current System

- Judge B recognizes that if the defendant is found guilty, then the two of the three judges will pronounce the death penalty.
- Since the Death Penalty is the worst outcome for him, he can force an acquittal
- Thus, this system generates an Acquittal

Roman Tradition

- Since more judges prefer the life sentence to Acquittal, if the death sentence is not pronounced in the first stage, that will occur
- Two out of three judges prefer the death penalty to a life sentence, so under this system, the Death penalty is imposed

Mandatory Sentencing

- If the judges decide that a life sentence should be mandatory if the defendant is found guilty, then since life outvotes acquittal, the defendant would get life
- If the judges decide that the death penalty should be mandatory if the defendant is found guilty, then since acquittal outvotes death, the defendant would be acquitted.
- Acquitted is outvoted by life, so the judges will select a mandatory life sentence.

Summary

- This is another example of strategic voting.
- It illustrates that the procedure adopted can have an effect on the outcome when we consider the incentives it creates for vote manipulation.