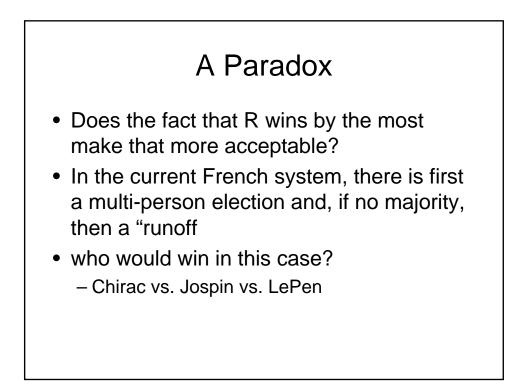


A Paradox				
Left	Middle	Right		
40	25	35		
Robespierre	Danton	Lafarge		
D	L	R		
L	R	D		

A Paradox

- Consider Robespierre versus Danton – R wins 75:25
- R versus L
 - L wins 60:40
- L versus D
 - D wins 65:35
- Depending on who runs against whom, the outcome can be very different.



Condorcet Rules

- Condorcet suggested the following rule.
- Have all voters list their entire ranking.
- Use the ranking to determine who beats whom on pair-wise comparisons.
- The winner is the one with the smallest maximum votes against.

Approval Voting

- Suppose there is a list of (say) 6 candidates
- Voters are asked to vote for any of the candidates they find acceptable.
- Which candidates win depends on the rule.
 - if only a fixed number of slots, say, 3 then the top 3 vote getters win
 - if a minimum number of acceptable votes must win to get elected
 - eg. proposed voting for HOF

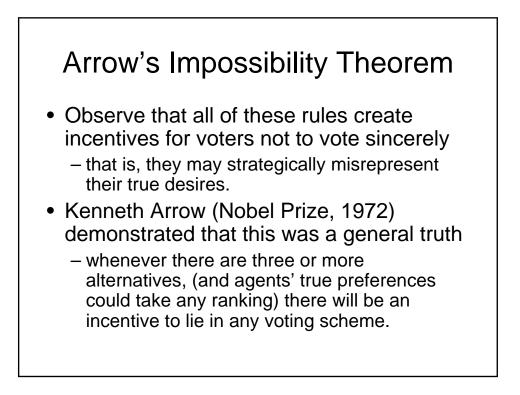
Borda Count

- All candidates are ranked by voters
- Each ranking gets a certain number of points.
 - eg. 1st gets 10 points, 2nd gets 8 etc.
- winner(s) are those with the most points.
- Example: Suppose we do a Borda Count with points, 3,2,1 on the R,D L example.

A Paradox				
Robespierre	Danton	Lafarge		
40*3+35*2+25= 215	25*3+40*2+35= 190	35*3+25*2+40= 195		
40	25	35		
Robespierre	Danton	Lafarge		
D	L	R		
L	R	D		

Borda Count

- Sincere voting would lead to a victory for Robespierre with 215
- Again, this is the worst outcome for the Center party.
- If instead of voting sincerely, they lied and ranked Lafarge first, they could ensure that their least favorite candidate loses. (As Lafarge would then get 60*3+40=220)

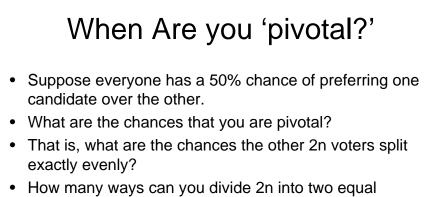


Why Vote?

- Note that Arrow's Theorem does NOT apply if there are only two choices.
- In that case, majority voting induces sincere behavior: it is always a best response to vote truthfully.
- (You can probably prove this for yourself)

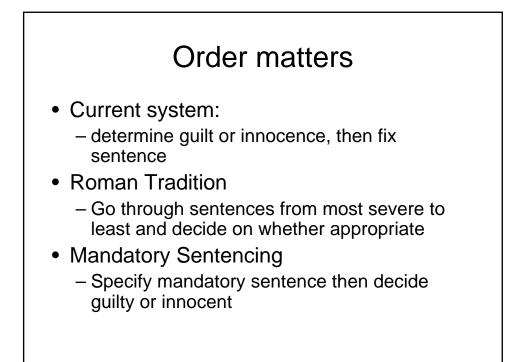
Why Vote?: Are you 'pivotal?'

- However, when does your vote matter?
- Suppose there are exactly 2n+1 people including you.
- Your vote only has an impact if the other 2n people split exactly evenly.
- In this case, we say you are 'pivotal'
- (What about the other voters?)



- groups? (2n!)/(n!n!)
- Each grouping occurs with probability $(1/2)^n(1/2)^n = (1/4)^n$
- Now suppose it costs you \$5 to vote. In a group of 100,000 other people what would it have to be worth to you to have your side win?

Probability of Pivotal.				
(n,2n)	Probability			
(2,4)	.375			
(4,8)	.27			
(8,16)	.20			
(50,100)	.08			
(500,1000)	.025			
(50,000,100,000)	.0025			



Order matters					
	Judge A	Judge B	Judge C		
Best	Death Sentence	Life in Prison	Acquittal		
Middle	Life in Prison	Acquittal	Death Sentence		
Worst	Acquittal	Death Sentence	Life in Prison		

Current System

- Judge B recognizes that if the defendant is found guilty, then the two of the three judges will pronounce the death penalty.
- Since the Death Penalty is the worst outcome for him, he can force an acquittal
- Thus, this system generates an Acquittal

Roman Tradition

- Since more judges prefer the life sentence to Acquittal, if the death sentence is not pronounced in the first stage, that will occur
- Two out of three judges prefer the death penalty to a life sentence, so under this system, the Death penalty is imposed

Mandatory Sentencing

- If the judges decide that a life sentence should be mandatory if the defendant is found guilty, then since life outvotes acquittal, the defendant would get life
- If the judges decide that the death penalty should be mandatory if the defendant is found guilty, then since acquittal outvotes death, the defendant would be acquitted.
- Acquitted is outvoted by life, so the judges will select a mandatory life sentence.

