## Lecture 3

## Dynamic Games: Introduction

## A Thought Experiment

\&Try to imagine you have no moral principles (hard I know...)
$\mathscr{H}$ Your equally amoral friend find $\$ 100$ left behind in a classroom. Your friend has the money in her hand. You see her pick it up.
$\mathscr{H}$ You both know if you mention it to the prof, he will keep the cash.
$\mathscr{H}$ She offers you $\$ 10$ to keep quiet. What do you do?
What if instead she offered $\$ 25$ ? $\$ 5$ ?

## Lecture Outline

\& Decision Trees again
$\mathscr{H}$ Games against Nature?
$\mathscr{H}$ Solving games back to front
\&Fredo and Charlie Brown
\&Examples:
$\triangle$ Line Item Vetoes
The game of NIM.
Ultimatum games and "Bargaining"
Dictator games
$\triangle$ Cut and choose

## One Person or Many Persons

${ }^{2}$ Consider the decision tree example.
\&What about Sad Sam who never brings his umbrella because he knows it only rains when he forgets it?

## Decision Tree Example



## Dynamic Games.

$\mathscr{H}^{\text {This }}$ is a silly example but it illustrates how one person decision trees become multi-person games.
Sad Sam is now playing a game against nature

## Decision (now Game) Tree Example



## Decision (now Game) Tree Example



Sun

## Back to Front Reasoning Again

\&As before, back to front reasoning helps make this game simpler.
$\mathscr{H}$ In fact, it makes the game very simple.

## Decision (now Game) Tree Example

Rain(0\%)
Bring


80

## Fredo and Charlie Brown

\&Dixit and Nalebuff describe an investment problem.
\&Fredo offers Charlie B an opportunity of a lifetime.
©Invest $\$ 100 \mathrm{~K}$ now, and he will get half of the $\$ 500 \mathrm{~K}$ profits of a project.
$\triangle$ But who is to say F will pay him back?

## Game Tree Analysis



## Solving The Investment Game

\&Using Back to Front Reasoning, how will this game be played?
\&Notice, that there is an outcome both players would prefer.
\&But they cannot achieve it in this world.
\&is there anything that they can do?
©... For later ...

## Game Tree Analysis



## Game Tree Analysis



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## Back to Front Reasoning

$\mathscr{H}$ As with one-person decision trees, the easiest way to make sense of dynamic games is to work "Back to Front"
It Examine all the final decision points (nodes). Determine the decision made there. (This should be "easy")
$\&$ Replace the decision node with the payoffs that result from the projected decsion.
Now move one step back and repeat....
$\mathscr{H}$ Continue until you reach the beginning of the game.

## Back to Front Reasoning.



J

## Back to Front Reasoning.



## Back to Front Reasoning.



## Back to Front Reasoning.



## Back to Front Reasoning.

M


J

## Back To Front Reasoning

\&What remains is a prediction or recommendation of how the game will be (should be?) played.

## Line Item Veto

We usually think that having more to choose from is good.
Here is an example where more choices are bad.
http://news.yahoo.com/s/politico/20090304/pl_ politico/19589
It is because of the dynamic game structure.
Dixit and Nalebuff p. 40. Analysis on board.

## Nim Version 1

\&Game trees can be helpful but may be too complex.
HBack to front reasoning, though, is still important. Consider the game of NIM.

Start with a pile of matches
$\triangle$ Take turns. When it is your turn, you can take either 1 or 2 matches from the pile.
©If it is your turn and there is just 1 match left in the pile, you lose.





## Nim Version 2

HStart with two piles of matches
HTake turns. When it is your turn, you can take 1 or more matches from either pile.
\&You are not allowed to take matches from both piles.
\&If it is your turn and there is just 1 match left in one pile and no matches left in the other pile, you lose.

## NIM Version 3

\&Pearls Before Swine
\&http://www.transience.com.au/pearl.html

## A Simple Bargaining Game

SYou and your business partner have 3 $\$ 100$ bills to divide between you. They cannot be broken.
İYou flip a coin to decide who goes first.
$\mathscr{H}^{2}$ The winner chooses a split.
$\mathscr{H}$ The loser gets to agree or disagree.
\&If disagree, the money goes to the stockholders (not you)

## A Simple Bargaining Game



## Analysis

\&Suppose you lose the coin flip.
$\mathscr{H}$ Which offers do you accept?
$\mathscr{H}($ What if they were $\$ 1000$ bills?)

## Analysis

HSuppose now you have $10 \$ 100$ bills to divide. How does the game proceed?
\&What about $1000 \$ 100$ bills?

## Dictator Game

\&Now suppose you agree on a different way to divide the profits.
$\mathscr{H}$ The winner of the coin flip decides on a split and the loser does not get a say.
\&How is the game played now?

## The Dictator Game



## Discussion

\&How does back to front reasoning predict play will occur in each game?
\&How would you play?
$\mathscr{H}$ and $N$ pp. 51 and 52 describe some laboratory experiments of these games.
\&Very often, the first mover "leaves money on the table? Why?

## First Mover?

\&Is it always an advantage to be the first mover?
$\mathscr{H}$ Consider again the problem of dividing $3 \$ 100$ bills.
$\mathscr{H}$ Now the game is, the winner of the coin toss chooses a split (eg. $(0,3),(1,2)$ etc.)
$\mathscr{H}$ The loser gets to choose which of the two portions she prefers, first or second.
$\mathscr{H}$ See also Dixit and Nalebuff, Red vs. Black.

