Lecture 5

Strategies and Games: Dominated and Dominant strategies

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Lecture Outline

- Simultaneous versus Dynamic Games
- Simultaneous? Not necessarily
  - examples
- Strategies
- Dominant and dominated strategies
  - examples
- Eliminating dominated strategies
- Solving or simplifying games.

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Sequential versus Simultaneous Games

- Games may be purely sequential (like the dynamic games we have looked at), purely simultaneous, or any mix of both.
- In sequential move games, one player moves, then another player observes that move and makes a move etc.
- Simultaneous move games are not always literally simultaneous. They are games where when it is the turn of one player to choose an action, that player does not know what the other player has chosen to do.

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Examples

- Sequential move games:
  – Chess and checkers;
  – you cut and I choose;
  – the long jump;
  – HORSE;
  – market leader games.

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Examples

- Simultaneous move games:
  - serve and return in tennis;
  - pricing games;
  - rock, paper, scissors
  - Traveling to the eastern shore on a holiday weekend.

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Strategies

- A player’s strategy set is just the collection of all the choices that player could make when it is her turn to move.
- Examples:
  - {Rock, Paper, Scissors}
  - {Real number line} (bargaining offer, set price}
  - {Accept offer, reject offer}
  - {Serve to forehand, Serve to backhand.}

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Matrix Games

- With two players, simultaneous move games are most easily represented by matrices showing strategies in columns and rows and payoffs in cells.

A Matrix Game

Firm 2 Firm 1	Price High	Price Low
Price High	(500, 500)	(100,700)
Price Low	(700,100)	(200,200)

Many Strategies

- Matrix games can accommodate a large number of strategies and different numbers for different players:
  - – Consider a coordination game. Where to meet in NYC without a plan.

Meeting in NYC

Me Nicholas	Met (1)	Met(2)	Empire State B	Times Square
Met	(1,1)	(0,0)	(0,0)	(0,0)
ESB	(0,0)	(0,0)	(1,1)	(0,0)
TS	(0,0)	(0,0)	(0,0)	(1,1)

Dominated Strategies

- A strategy, A, is said to be “dominated” by another strategy, B, if *for every move by the rival*, B gives a better payoff than A.
- Should A ever be played? NO!
- Then let’s get rid of it.

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A Matrix Game

Firm 2 Firm 1	Price High	Price Low
Price High	(500, 500)	(100,700)
Price Low	(700,100)	(200,200)

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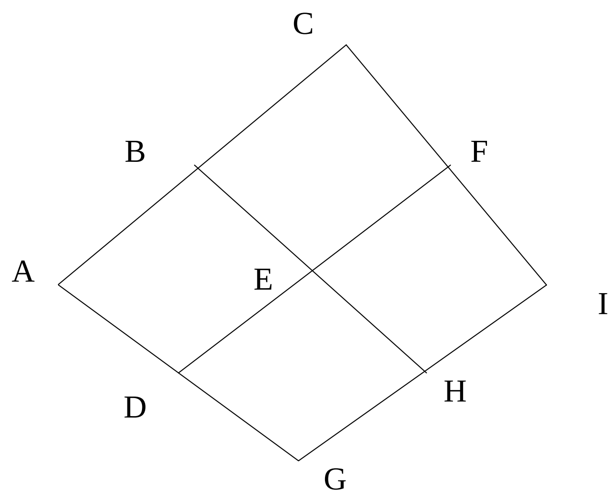
The Missile Game



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The Missile Game

- In this game, the A team must send out an anti-missile device to intercept the I team missile within 3 segments.
- The I team sends out a missile to destroy the A ship.
- Missiles must follow the paths on the grid.

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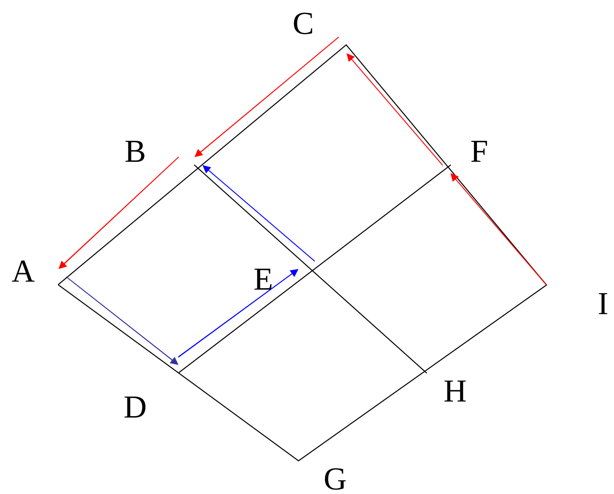
The Missile Game



The Missile Game

- Only the first 3 segments matter.
- Each ship has 8 possible choices:
  - {ABCF,ABEF,ABEH,ABED,ADGH,ADEH,ADEF,ADEB}
  - {IFCB,IFEB,IFED, IFEH, IHGD, IHED, IHEB, IHEF}
- Thus, this is an 8 by 8 “game”.

	B1	B2	B3	B4	B5	B6	B7	B8
A1	H							H
A2		H	H		H	H	H	H
A3		H	H	H		H	H	H
A4		H	H	H	H	H	H	H
A5				H	H			
A6		H	H	H		H	H	H
A7		H	H	H		H	H	H
A8	H	H	H	H		H	H	H

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	B1	B2	B3	B4	B5	B6	B7	B8
A4		H	H	H	H	H	H	H
A8	H	H	H	H		H	H	H

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The Much Simplified Game.

	B1	B5
A4		H
A8	H	

The Missile Game

- At this point, we have gone as far as we can using elimination of dominated strategies.
- But note how much simpler the game has become to play.
- How this game will be played now we will examine in the next lecture.

Dominant Strategies

- A “Dominant Strategy” is a strategy that does better than any other strategy that you may have, *no matter what strategy choice your opponent makes.*
- The important point is you compare YOUR OWN payoff to determine if a strategy is dominant.

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Example 1

- Strategy R2 is dominant (first number is row player’s payoff)

	C1	C2	C3	C4
R1	(1,1)	(5,2)	(0,10)	(3,4)
R2	(2,6)	(5.5,7)	(1,1)	(3.5,4)

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Example 2

- Strategy R2 is NOT dominant (first number is row player's payoff). Does Column player have a DS?

	C1	C2	C3	C4
R1	(1,1)	(5,2)	(0,10)	(3,4)
R2	(2,6)	(5.5,7)	(3.5,4)	(1,1)

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Example 3

- What would have to be the value of x for R2 to become a DS?

	C1	C2	C3	C4
R1	(1,1)	(5,2)	(0,10)	(3,4)
R2	(2,6)	(5.5,7)	(x,4)	(3.5,1)

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The Prisoners' Dilemma

- Bonny and Clyde are captured by the police and placed in separate cells.
- The police have enough evidence to put them both away for one year.
- If they can get one of them to turn state's evidence, they can pin a bank robbery on them as well for which the sentence is 10 years.
- They make each player the following offer: Confess. If we need your evidence to convict your partner, you will get off free. If we do not need it, you will get the minimum sentence of 5 years.
- How does this game look? How is it played?

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The Prisoners' Dilemma

Bonnie Clyde	Stay Silent	Confess
Stay Silent	(-1,-1)	(-10,0)
Confess	(0,-10)	(-5,-5)

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The Prisoners' Dilemma

Bonnie Clyde	Stay Silent	Confess
Stay Silent	(-1,-1)	(-10,0)
Confess	(0,-10)	(-5,-5)

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Running on a Full Count with Two Out

- The situation: A base runner is on first. The batter has three balls and two strikes.
- The Choice: Run on the pitch, wait to see if it is a hit.
- Outcomes: Strikeout, walk, fly ball, single, multi-base hit, HR.

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The Matrix Game Version

	SO	W	FB	S	MB	HR
Wait	Inning ends	Advance to 2	Inning ends	Advance to 2	Advance to 3	Score
Go	Inning ends	Advance to 2	Inning ends	Advance to 3	Score	Score

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Climate Change Dilemma

- The US along with Europe and the rest of the world (ROW) is concerned that climate change, exacerbated by global use of hydrocarbons, will adversely affect its future prosperity.
- If all three groups reduce emissions, the threat vanishes.
- If two of the three, the threat diminishes.
- If only one reduces, the threat remains.

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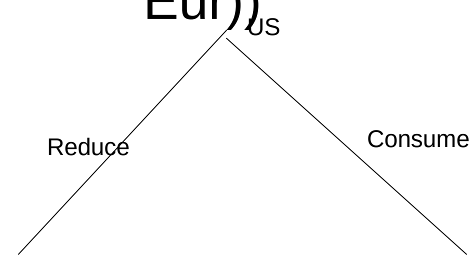
The Game (Payoffs –(US, ROW, Eur))



Eur ROW	R	C
R	(10,10,10)	(4,4,12)
C	(4,12,4)	(0,6,6)

Eur ROW	R	C
R	(12,4,4)	(6,0,6)
C	(6,6,0)	(0,0,0)

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Repeated Prisoners' Dilemma

- PD games are frustrating because they illustrate situations where even though we all know there is a better outcome, strategic incentives prevent us from achieving them.
- What if the game was known to be repeated? Could that get us to cooperate in the first games at least?
- Consider the PD game between BB Lean and Rainbow's End in the book played twice.
- Back to front reasoning suggest that it still is a problem:

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A pricing dilemma: The one stage game.

	80	70
80	$72K, $72K	$24K, $110K
70	$110K, $24K	$70K,$70K

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Analyzing the Final Stage

- Suppose the game is played first once, then a second time.
- However the game is played the first time, going forward, the firms still face a PD situation.
- To see this, let the first stage play give profits $xK and $yK (that is, could be (72,72), (24,110), (110,24) or (70,70)
- Now analyze the final stage.

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A pricing dilemma: The Last period of a two stage game (first game played so that profits were ($xK,$yK).

	80	70
80	$72+xK, $72+yK	$24+xK, $110+yK
70	$110+xK, $24+yK	$70+xK,$70+yK

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Resolving the PD

- In experiments, repeating the PD increases the amount of cooperation.
- Why is this?
- What if there was a possibility that one or both of the players was just a nice generous guy?
- (Recall the Insane pirate from earlier lectures)
- How might even mean people play for a while? Why?

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