Lecture 6

Equilibrium and Chance

1

Lecture Outline

- http://www.youtube.com/watch?v=3EkBuKQEkio
- Best Responses.
- The “Beautiful Equilibrium”, Nash Equilibrium
  - defining the NE
  - finding NE -examples
  - How many?
  - Are there any?
- Randomizing behavior
  - Motivation
  - explanation
  - computation.
  - examples.

2

Best Responses

- Even in simultaneous move games, it is always worth asking yourself, “For any particular strategy that my rival MIGHT choose, what would I choose to do best against it?”
- For example, even though you do not KNOW your opponent is running the ball, you do want to know that a safety or linebacker blitz is the best defense against it.
- Even though you do not know your opponent is choosing Rock, you want to recognize that Paper is the best strategy against that.

3

Best Responses

- For any particular strategy your opponent has, a best response to that strategy is the strategy that YOU have that yields you the highest payoff when the opponent chooses to play that strategy.
- Best responses are important building blocks in developing an idea of how to play a game.
- They are like a collection of “What if?” statements.
  - What if my rival throws a screen?
  - What if my rival runs a draw?
  - What if my rival throws it deep?
  - etc.

4

Best Responses and Dominant Strategies

- If you have a Dominant strategy then that strategy is a Best Response to ALL of your rival's strategies.
- However, many games do not have Dominant strategies.
- In that case, when you have your collection of best responses, you can start assessing which strategy your opponent is most likely to play, and choose your own strategy accordingly.

5

Computing BRs

	C1	C2	C3	C4
R1	(1,2)	(-5,2)	(4,3)	(3,0)
R2	(-10,4)	(30,2)	(40,3)	(12,9)
R3	(6,5)	(9,30)	(3,6)	(3,7)

6

A Pricing Game.

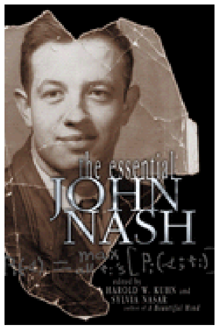
RE \ BBLean	38	39	40	41	42
38	36720 36720	**36860** 38160	36800 39600	36540 41040	36080 42480
39	38160 **36860**	38380 38380	**38400** 39900	38220 41420	37840 42940
40	39600 36800	39900 **38400**	**40000** **40000**	39900 **41600**	39600 43200
41	41040 36540	41420 38220	**41600** 39900	41580 41580	41360 **43260**
42	42480 36080	42940 37840	43200 39600	**43260** 41360	43120 43120

7

Using Best Responses

- Suppose your opponent has two strategies (eg. Run or Pass)
- Blitz is a BR to a Run and Cover 2 is a BR to the Pass.
- What should you do?
  - Answer 1: You might just guess that since he ran (say) 30% of the time and passed 70%, then Cover 2.
  - Answer 2: Or you might try to figure out HIS Best Response.
- But how is he going to guess what YOU are going to do?

8


the essential
JOHN
NASH
edited by
HAROLD W. KUHN and
SYLVIA NASAR


JOHN NASH

9

10

- www.gametheory.net
- http://mitworld.mit.edu/video/39
- http://www.veoh.com/collection/s274425/watch/e107370y6p2Dpwx
- http://www.gametheory.net/media/Beautiful.mov

11

A Nash Equilibrium

- In games that cannot be solved by using dominant strategies or by eliminating dominated strategies, John Nash proposed the following idea
- Look for a profile of strategies of all players all of which are best responses to each other.
- This is known as a Nash Equilibrium:

12

Nash Equilibrium Example

- There are two pure strategy NE. in this game:

	C1	C2
R1	(2,3)	(1,2)
R2	(1,0)	(2,5)

13

Example

- Consider the pricing game example from the book
- B.B. Lean and Rainbow's End are pricing clothing against each other
- as one company lowers its price, it gains more sales both from non buyers and from its rival.
- The next chart shows best responses and how to use them to find the NE.

14

A Pricing Game.

RE \ BBLean	38	39	40	41	42
38	36720 / 36720	**36860** / 38160	36800 / 39600	36540 / 41040	36080 / 42480
39	38160 / **36860**	38380 / 38380	**38400** / 39900	38220 / 41420	37840 / 42940
40	39600 / 36800	39900 / **38400**	**40000** / **40000**	39900 / **41600**	39600 / 43200
41	41040 / 36540	41420 / 38220	**41600** / 39900	41580 / 41580	41360 / **43260**
42	42480 / 36080	42940 / 37840	43200 / 39600	**43260** / 41360	43120 / 43120

15

The Hunting Game

- Another game from the book is the “coordination” game between Fred and Barney.
- They have to decide on their own what they are going to hunt.
- If their decisions agree, then they will bag a big game, else a rabbit or go hungry.

16

The Hunting Game

	Stag	Bison	Rabbit
Stag	**(3,3)**	(0,0)	(0,1)
Bison	(0,0)	**(3,3)**	(0,1)
Rabbit	(1,0)	(1,0)	**(1,1)**

17

Multiple NE

- There are many NE of this game. Which will they choose?
- Convention? The "Best"
- History
- The role of society and culture.
- Focal Points?
- Location games.

18

The Game of Chicken

- This game has come up in many guises, the most famous is The Rebel but here is a funny one from Footloose.

- http://www.youtube.com/watch?v=JA1wrvqDRNw

19

CHICKEN!

	Swerve	Straight
Swerve	(0,0)	(-5,10)
Straight	(10,-5)	(-10,-10)

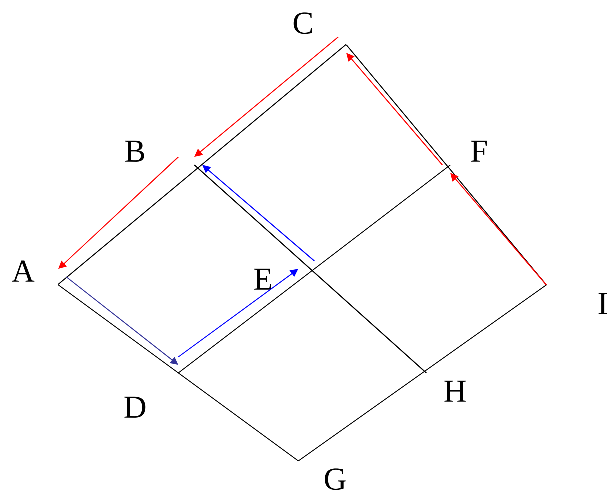
20

Are there always NE?

- Recall the missile game:
- By eliminating dominated strategies we were able to simplify this game significantly
- But we still did not have it completely figured out.

21

The Missile Game



22

	B1	B2	B3	B4	B5	B6	B7	B8
A1	H							H
A2		H	H		H	H	H	H
A3		H	H	H		H	H	H
A4		H	H	H	H	H	H	H
A5				H	H			
A6		H	H	H		H	H	H
A7		H	H	H		H	H	H
A8	H	H	H	H		H	H	H

23

The Much Simplified Game.

	B1	B5
A4		H
A8	H	

24

Examples

- There are many strategic situations of this type:
- the serve and return part of tennis
- the penalty kick
- http://www.youtube.com/watch?v=S2iNGFRtLkI
- most defense and offense in football
- pricing discount games
- Vizzini and the dread pirate Roberts.

25

The Penalty Kick Game

Goalie Kicker	Left	Right
Left	(58%,42%)	(95%,5%)
Right	(93%,7%)	(70%,30%)

26

The Penalty Kick Game

- There is no (pure strategy) NE of this game.
- Whenever Kicker chooses left, Goalie wants to choose left, but when Goalie chooses left kicker wants to choose R.
- How might this game be played?
- Note that one player wants to mimic, the other wants to avoid being mimicked.
- Perhaps this can be achieved by “mixing” it up.

27

The Kicker’s Viewpoint

- If Kicker always chose L, the goalie would figure this out and drive the kicker to 58% by selecting L.
- If Kicker always chose R, the goalie would figure this out and drive the kicker to 70% by selecting R.
- What if Kicker mixed for starters, say 50-50?

28

The Kicker's Viewpoint

- A 50-50 mix on left or right would give the kicker:
  - .5*58+.5*93=75.5% if goalie chooses L
  - .5*95+.5*70=82.5 if goalie chooses R.
- Both are better than 70 or 58 and it might be reasonable to expect the goalie would choose Left (why?)

29

The Kicker's Viewpoint

- Both are better than 70 or 58 and it might be reasonable to expect the goalie would choose Left (why?)
- Can the Kicker do better than mixing 50-50? What if he chose L 38.3% of the time and R 61.7%?
  - when G goes L, K gets .38*58+.62*93=79.6%
  - when G goes R, K gets .38*95+.62*70=79.6%

30

Why Be Unpredictable?

- Notice that Kicker gets the same probability of a goal with L as with R.
- Why bother mixing then? Isn't it too much trouble?
- If Kicker did NOT mix, say chose R with probability 1 (ie for certain), then we know what G would end up doing, (ie R)
- Mixing is the only way Kicker can keep Goalie from copying him and winning.

The Mixing Equilibrium

- What about the Goalie?
- Same argument, if G chose one side for sure, the Kicker would choose the opposite.

Maximin--Minimax

- Let's look at the Goalie more carefully.
- If Goalie chooses L, the worst case scenario is if K chooses R, and the Goalie loses 93% of the time.
- If Goalie chooses R, the worst case scenario is if K chooses L, and the Goalie loses 95% of the time.
- If Goalie chooses 50-50, the worst case scenario is if K chooses R, and the Goalie loses 81.5% of the time which is better.
- The mix the MINimizes the MAXimum loss for the goalie is Left 41.7% and Right 58.3% yielding a Minimax of 79.6%.

MAXimin

- We already computed the mix that MAXimized the MINimum win percent for the goalie as Left (38.3%) and Right (61.7%)=79.6%
- Notice that the Minimax for the goalie is the same as the Maximin for the Kicker
  - This is not an accident
  - This game has the property that every time one player does better, the other does worse.
  - It is a "zero-sum" game.
- In Zero-sum games, the best worse case scenario for one player is the best worse case scenario for the other.
- Other zero sum games: the missile game, Princess Bride, Rock Paper Scissors.

MAXimin

- Notice also, that at the Minimax, the goalie has the same probability of success when the Kicker goes Left as when the Kicker goes Right.
- If this were not true, then the goalie would always do better by changing the probabilities of one of her strategies.

35

The Problem with 50-50

- Why not just flip a fair coin?
- The reason 50-50 is not the minimax/maximin solution is because the game is not symmetric.
- The kicker has generally a higher probability of success on the right than on the left.

36

Computing Minimax/Maximin

- There are two ways that the minimax equilibrium can be computed.
- The first is to find the probability mixing for player 1(say) so that the probability of winning is the same no matter which strategy player 2 uses.
- Then do the same thing for player 2.

37

Computing Minimax/Maximin

- A more useful way is to use a graph. Consider the general type of mixing game

	C1	C2
R1	(A,100-A)	(B,100-B)
R2	(C,100-C)	(D,100-D)

38

Computing

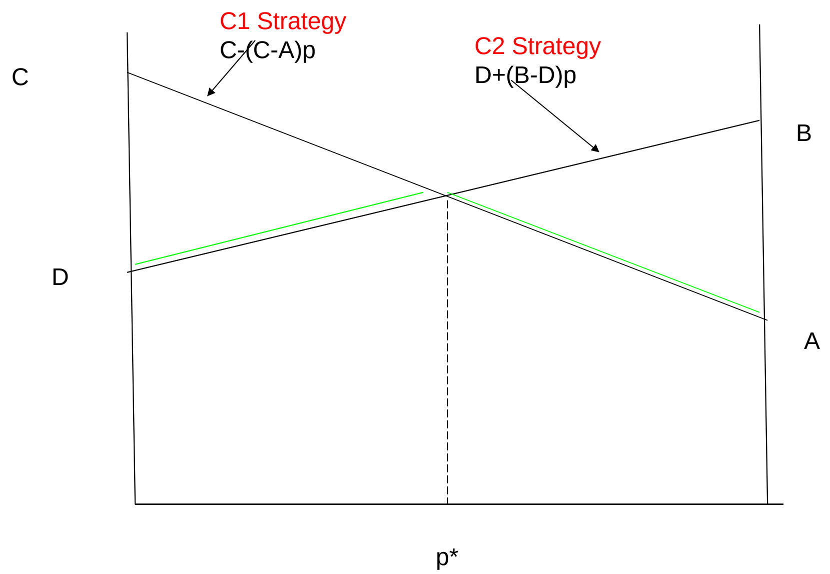
- For this game not to have a pure strategy equilibrium, we need C>A, B>D, 100-A>100-B (or B>A), and 100-D>100-C (or C>D).
- Collecting these we get
- C>A, B>A
- C>D, B>D

39

Computing

- Suppose row player choose R1 with probability *p.*
- Probability of success when Column player chooses C1 is
  - p*A+(1-p)C or C-(C-A)p
- Probability of success when Column player chooses C2 is
  - p*B+(1-p)D or D+(B-D)p
- Graphing these lines we get

40

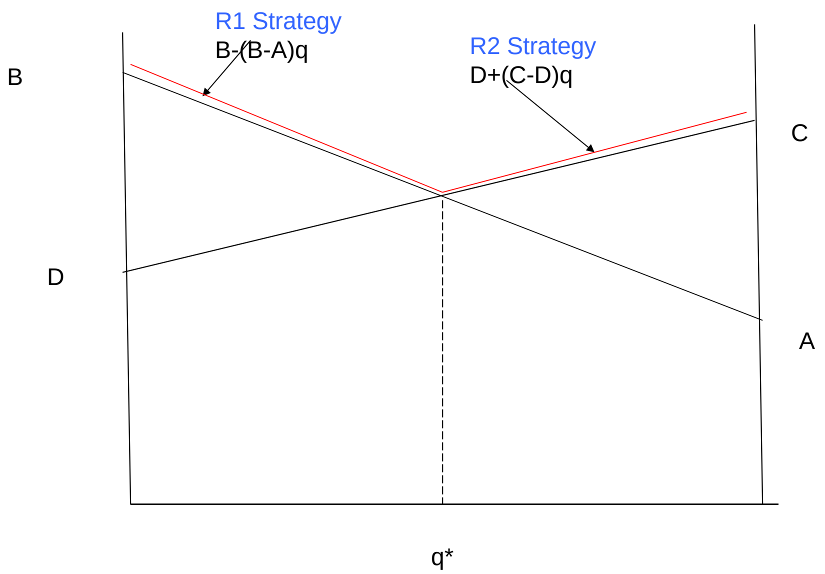


41

Computing

- Similarly, suppose column player choose C1 with probability *q.*
- Probability of success (of ROW) when Row player chooses R1 is
  - q*A+(1-q) B or B-(B-A)q
- Probability of success (of ROW) when Row player chooses R2 is
  - q*C+(1-q) D or D+(C-D)q
- The red line shows the maximum probability (which is the worst case for column).
- The kink is at the lowest worst case, with probability mix q*

42



Computing Minimax/Maximin

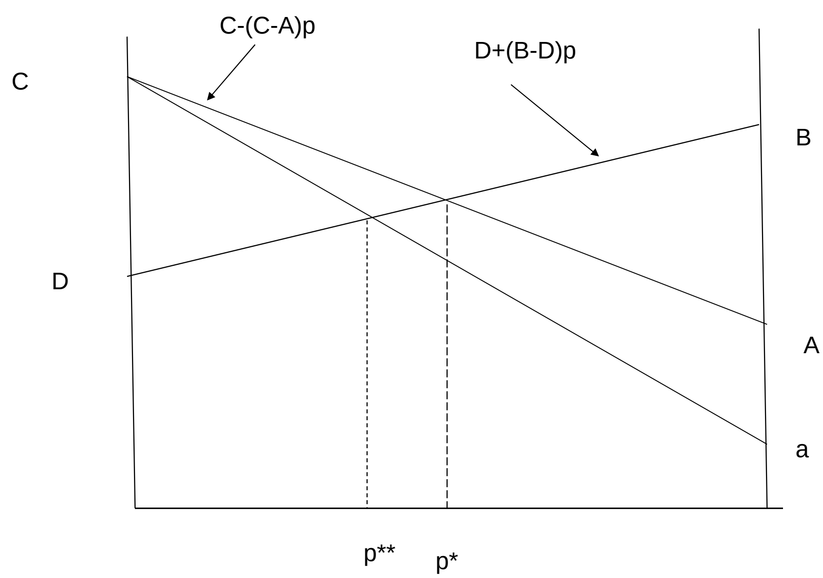
- Recall the PK game:

	C1	C2
R1	(A=58%,42%)	(B=95%,5%)
R2	(C=93%,7%)	(D=70%,30%)

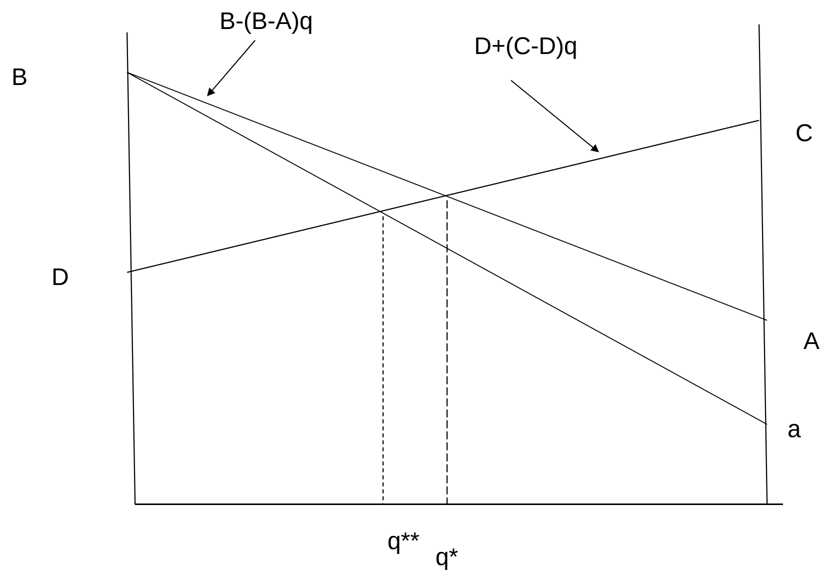
Looking at effects in Minimax games

- We could use the graph to compute *p*=38.3%* and *q*=41.7%* by finding the intersection points of the lines.
- But we can use the graph to see some more interesting effects.
- Suppose the goalie becomes better at saving balls kicked to his left when he guesses correctly. (so *A* falls to *a<A)*
- What happens to the strategies?

45



46



47

Effects of Improving Skill

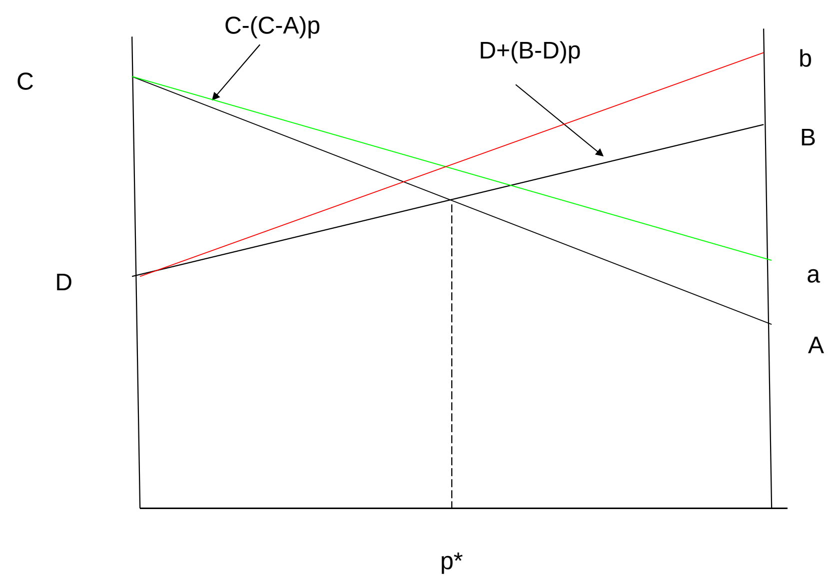
- Notice that if goalie becomes better at saving balls kicked to the left, then, as we might expect, kicker kicks less frequently to the left (*p*** < *p**)
- More intriguing is the prediction that the goalie also guesses less frequently to the left (*q***<*q**).
- This is precisely because the kicker does not go left as much.

48

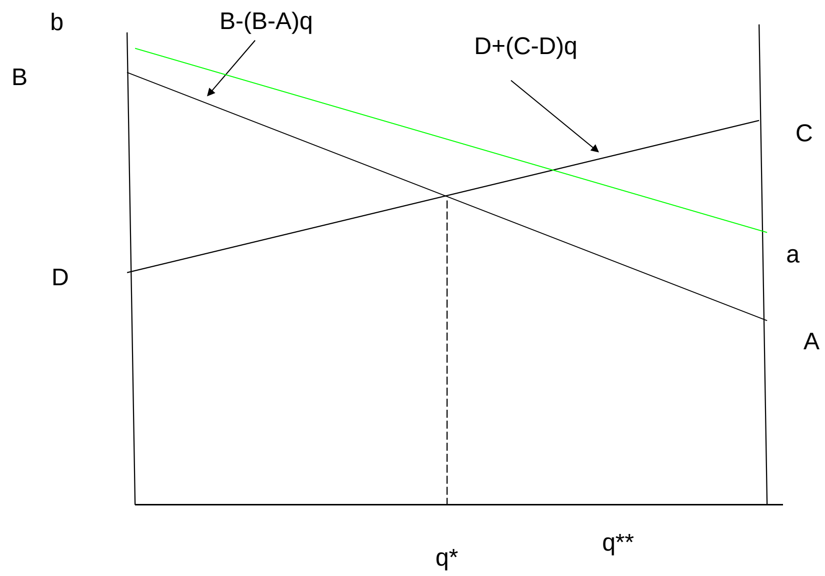
Effects of Improving Skills

- What if the kicker becomes more accurate when kicking to the left so he misses the goal less often?
- In this case, both A and B may go up.

49



50



51

Improving Skills

- In this case, the keeper clearly goes to the left more often.
- But it is not clear what the kicker does. He may go left more often or he may go right.
- As drawn, the kicker chooses exactly the same as before but in general, it will depend on how much A and B change.
- If A goes up a lot relative to B, then the kicker increases the shots to the left.
- However, if instead B goes up more then the goalie increases the guesses to the right.
- Suppose only B goes up. Then for sure p* goes down. Why? The rise in B makes Right less attractive for the Keeper so q* falls. Therefore, the kicker wants to increase the chances of avoiding him.
- http://www.nytimes.com/2006/06/18/sports/soccer/18score.html?ex=1308283200&en=67391adea0395a75&ei=5088&partner=rssnyt&emc=rss

52

Do Soccer Players Really Mix?

Proportion of Left		
Kicker	Best	38.3%
	Actual	40%
Keeper	Best	41.7%
	Actual	42.3%

53

What Game is This?

	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

54

Questions

- Is there a deterministic equilibrium?
- Are there mixed strategy equilibria?
- Can you guess?

- http://video.yahoo.com/watch/1185303/4223046

Do Professional Athletes Actually Play Minimax?

- In a new paper, Kovash and Levitt examine behavior of baseball pitchers and football offenses to determine if they use minimax strategies.
  - PROFESSIONALS DO NOT PLAY MINIMAX: EVIDENCE FROM MAJOR LEAGUE BASEBALL AND THE NATIONAL FOOTBALL LEAGUE: NBER WP 15347. Sept. 2009
- http://www.nber.org/papers/w15347
- Notice both situations, a pitcher facing a batter and an offense facing a defense, are “zero-sum” situations
  - any gain made by one side is a direct one for one loss imposed on the other

How do we tell?

- We cannot literally read the minds of the decision-makers so there is no direct way of confirming if the players randomize in the way that theory predicts.
- However, Minimax theory does predict that we should observe some related results. If those results are not observed, we should be able to conclude the theory is not predicting behavior.

57

Predictions of Game Theory

- Our analysis from before predicts at least three features of equilibrium behavior:
  - 1) all strategies that are selected give the same probability of success.
  - 2) Every strategy that is selected must do no worse than any strategy that is not used
  - 3) Over the repetition of play, strategies should be serially independent (they should not exhibit negative or positive serial correlation).
- Observations 1) and 3) can be tested with data.

58

Baseball: The pitcher-batter duel

- Authors collected data on the types of pitches thrown to batters in similar situations.
- They have over 3M observations.
- Use On-base Percentage and Slugging (OPS) as a measure of success.
  - When a pitch type increases OPS the batter increases the chance of winning and pitcher decreases chance of winning.
- on 0-2 counts, non-fastballs lead to an OPS that is 100 points *lower* than do fastballs.

59

Baseball

- this bias towards fastballs persists even when factors such as who the P and B are, innings pitched etc are taken into account.
- Paper also finds *negative serial correlation.*
- Chances a fastball is selected given a fastball was selected in a similar situation in the past is 4.1% lower.
- If the batter is aware of this, he could conceivably raise his OPS by approx. .006 or about 10-15 runs per season!

60

Football.

- Similar test was applied to frequency of selection by offense of run versus pass in similar situations.
- On average, pass plays generate .066 more points than *run* plays!
- There is strong negative correlation. Teams that passed on the previous situation are 10% less likely to pass in the current situation!
- Defenses that adjusted for this could increase their final outcome by approx 1 point per game leading to approx ½ more wins per season!

61