

Midterm Exam October, 2009  
HONORS 259L

Each Question is worth 10 points. Explain your reasoning clearly.  
In all matrix games, Row player payoff is shown first.

1. Apply elimination of dominated strategies to solve the following game. Be sure to indicate which rows or columns are dominated and by what other rows or columns in each round of elimination.

	C1	C2	C3	C4	C5
R1	(15,4)	(2,7)	(3,10)	(4,1)	(5,5)
R2	(10,0)	(4,0)	(3,2)	(4,5)	(1,10)
R3	(.5,2)	(3,20)	(3,1)	(0,20)	(1,2)
R4	(1,5)	(2,2)	(2,10)	(4,4)	(3,1)

*R4 is weakly dominated by R1.*

	C1	C2	C3	C4	C5
R1	(15,4)	(2,7)	(3,10)	(4,1)	(5,5)
R2	(10,0)	(4,0)	(3,2)	(4,5)	(1,10)
R3	(.5,2)	(3,20)	(3,1)	(0,20)	(1,2)

*C1 is weakly dominated by C2*

	C2	C3	C4	C5
R1	(2,7)	(3,10)	(4,1)	(5,5)
R2	(4,0)	(3,2)	(4,5)	(1,10)
R3	(3,20)	(3,1)	(0,20)	(1,2)

*R3 is weakly dominated by R2*

	C2	C3	C4	C5
R1	(2,7)	(3,10)	(4,1)	(5,5)
R2	(4,0)	(3,2)	(4,5)	(1,10)

*C2 is weakly dominated by C3 (also C4 is weakly dominated by C5)*

	C3	C5
R1	(3,10)	(5,5)
R2	(3,2)	(1,10)

*R2 is weakly dominated by R1.*

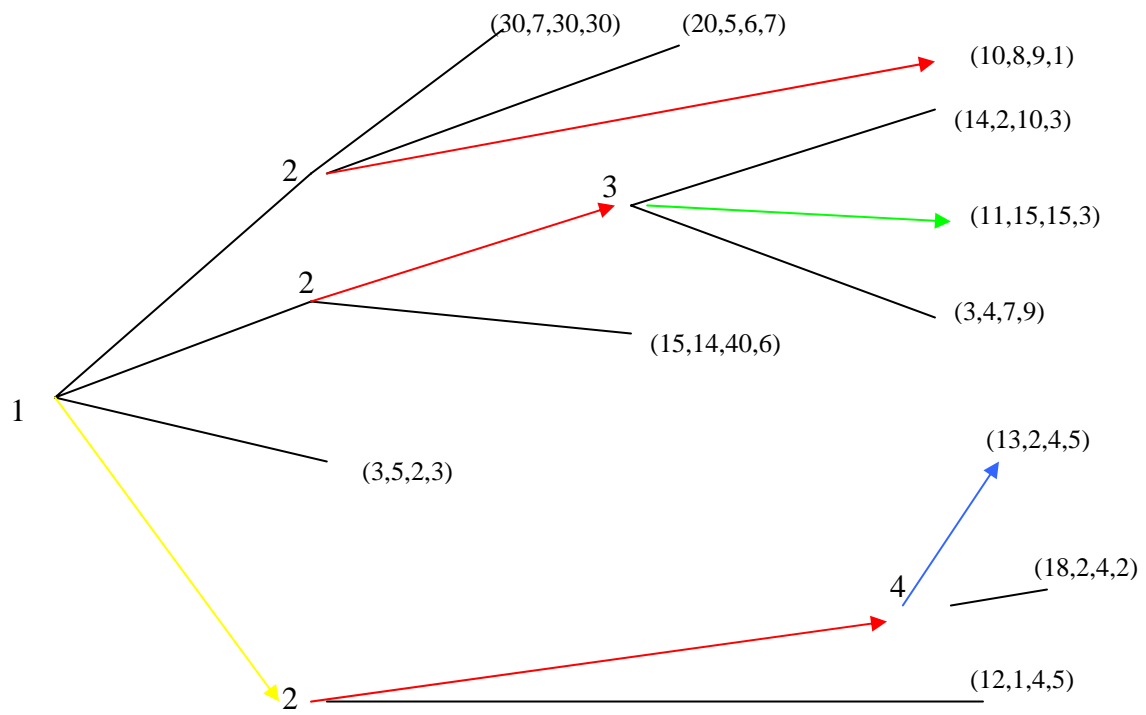
	C3	C5
R1	(3,10)	(5,5)

*C5 is dominated by C3.*

	C3
R1	(3,10)

2. Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1, Player 2, Player 3, Player 4) :

*The arrows show the choice of play at each stage: Final outcome is payoff (13,2,4,5)*



3. After graduating from UMD, you work for a firm for two years. You and the firm recognize that you would be much more valuable to the firm if you studied at night for an MBA. The MBA takes four years of night school and costs \$80,000. The firm is afraid that if it pays for your tuition, you will leave and go to another firm. Recalling that slavery (and therefore contracts that force you to work for any given firm) is illegal. Suggest two ways you might use to persuade your firm to pay your way and discuss some problems with each.
4. Find all the (pure strategy) Nash Equilibria of the following game: *Colored and in Bold*. Note that although R3,C1 involves a choice of a weakly dominated strategy, it is still a NE. Failing to notice that was a loss of 2 points only.

	C1	C2	C3
R1	<b>(2,6)</b>	(1,5)	(1,0)
R2	(0,9)	<b>(2,12)</b>	(0,12)
R3	<b>(2,60)</b>	(-5,6)	(-20,-40)

5. In the game below, Row wants to maximize probabilities and Column player wants to minimize probabilities/ Argue that there is no pure strategy Nash Equilibrium of the

	C1	C2
R1	(50%,50%)	(40%,60%)
R2	(40%,60%)	(60%,40%)

game and find the minimax equilibrium in mixed strategies:

At each cell, for example R1,C1, one of the two players wants to change strategy (for example, column player prefers (40,60) to (50,50)). This is true at all cells so there is no PSNE.

Now, let  $p$  be the probability Row player chooses R1. Then,

When C1 is selected:  $\text{Prob}[\text{Row wins:C1}] = p*50 + (1-p)*40 = 40 + 10*p$

When C2 is selected:  $\text{Prob}[\text{Row wins:C2}] = p*40 + (1-p)*60 = 60 - 20*p$

Equating the two gives  $30*p = 20$ , or  $p = 2/3$

Similarly, let  $q$  be the probability Column player chooses C1. Then,

When R1 is selected:  $\text{Prob}[\text{Column wins:R1}] = q*50 + (1-q)*60 = 60 - 10*q$

When R2 is selected:  $\text{Prob}[\text{Column wins:R2}] = q*60 + (1-q)*40 = 40 + 20*q$

Equating the two gives  $30*q = 20$ , or  $q = 2/3$

Therefore the minimax solution is  $p = 2/3, q = 2/3$  yielding Row player a probability of winning of 46.7%

6. A certain student is attempting to decide how much to study for an upcoming midterm. The student has a part-time job which promises to pay him an extra \$50 if he passes, a further \$20 if he gets a C, a further 15\$ if he gets a B and a further \$10 if he gets an A. The student also has another job tutoring for which he is paid \$12 an hour. He has already studied 2 hours. To pass, he needs to have studied a total of 4 hours. A C requires 5 hours in total, a B requires 6 hours in total, an A requires 7 hours in total.
- a. From this information, characterize an example of each of the following:

i) Opportunity cost *The lost \$12 an hour that the student could get tutoring is an opportunity cost of studying.*

ii) Sunk cost *The two hours already spent is a sunk cost.*

iii) Variable cost *Each hour spent studying is a variable cost.*

- b. Assuming the student only wants to maximize the dollar benefit from his activities, how much should he study? *Recall the principle, if he chooses to study, he should study up to the point where Marginal Benefit = Marginal Cost. Since the marginal benefit of an hour of studying is \$15 if he chooses 6 hours instead of 5 hours while the MB of an hour is \$10 if he chooses 7 hours instead of 6 hours (to get an A) and the marginal cost is always \$12, if he chooses to study, he should choose, 6 hours. This gives a total benefit of  $85 - 48 = 37$  (excluding the sunk cost of two hours.) Note that going for an A would yield  $95 - 60 = 30$  which is less.*
- c. Assuming the student only wants to maximize the dollar benefit from his activities, what would his tutoring wage have to be to lead the student to be satisfied with merely passing the exam? (a range would be fine as an answer or a specific number is also OK.) *The marginal benefit of a C versus just passing is \$20. Thus, since a C costs one extra hour versus passing, if the tutoring wage was more than \$20 he would just try to pass. If it was too high (say more than \$25) then he would not even try to pass.*