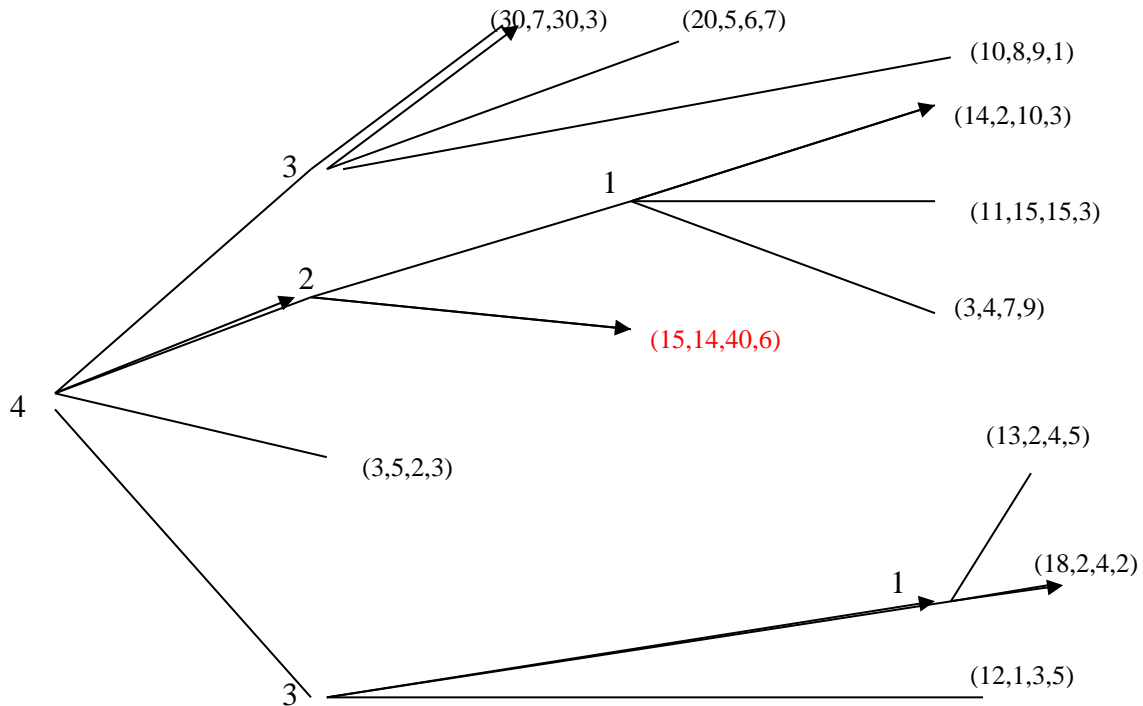


Midterm Exam October, 2010
HONORS 259L

Each Question is worth 10 points. Explain your reasoning clearly.

1. Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1, Player 2, Player 3, Player 4). Pay careful attention to who moves when:



2. Find all the (pure strategy) Nash Equilibria of the following game: Rows best response is in red and column's br is in blue. The jointly colored cells are Nash Equilibria.

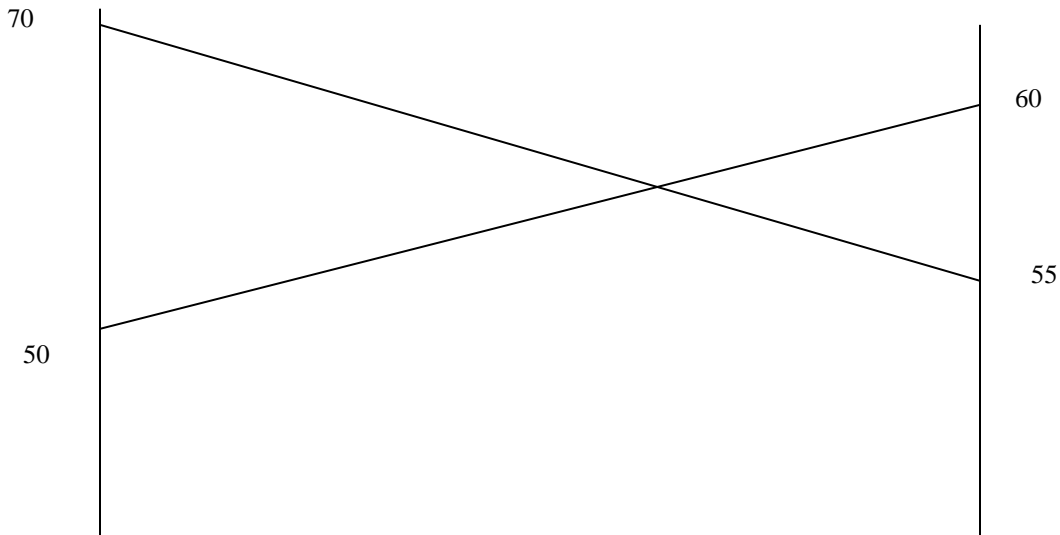
	C1	C2	C3
R1	(2,3)	(-11,5)	(1,10)
R2	(10,1)	(-7,0)	(0,-1)
R3	(10,6)	(-5,7)	(-2,-4)

3. Construct games with the following characteristics.
- A simultaneous move game with three strategies for one player and two strategies for the other player which has no equilibrium in pure strategies (no deterministic equilibrium). Provide support for the claim that the game has no pure strategy equilibrium.
 - A sequential move game with three players where player one has three strategies and players 2 and 3 always have two strategies when they are asked to play. Determine the back to front equilibrium of the game.
4. In the game below, Row wants to maximize probabilities and Column player wants to minimize probabilities. Argue that there is no pure strategy Nash Equilibrium of the game and find the minimax/maximin equilibrium in mixed strategies:

	C1	C2
R1	60%	55%,
R2	50%	70%

At (R1,C1), column wants to choose C2, at (R1,C2), Row wants to choose R2, at (R2,C2), Column wants to choose C1, and at (R2,C1), row wants to choose R1. Thus, no cell is a best response of each player to the other and there is no pure strategy NE.

Let $p = \text{Prob}[\text{Row chooses R1}]$, $q = \text{Prob}[\text{Column chooses C1}]$. When Column chooses C1, Row's probability of success is $60p + (1-p)50 = 50 + 10p$, when column chooses C2, it is $55p + (1-p)70 = 70 - 15p$. Drawing these lines on the graph show the maximin point is at $70 - 15p = 50 + 10p$ or $20 = 25p$, $p^* = 4/5$. Similarly, the minimax choice of q for column player can be computed in the same fashion to be $q^* = 3/5$.



5. Apply elimination of dominated strategies to solve the following game. Be sure to indicate which rows or columns are dominated and by what other rows or columns in each round of elimination.

	C1	C2	C3	C4
R1	(2,4)	(3,4)	(3,5)	(5,4)
R2	(4,10)	(3,20)	(4,5)	(1,10)
R3	(3,10)	(3,15)	(2,20)	(4,2)
R4	(2,2)	(5,3)	(4,4)	(3,2)

For column, C1 is dominated by C2. For Row, R2 is dominated by R4. For column, C2 is dominated by C3. For row, R3 is dominated by R1. C4 is dominated by C3. R4 dominates R1. So the outcome that survives elimination of dominated strategies is R4, C3.

6. A certain college basketball player is attempting to decide how to practice for an upcoming game. The player had 8 hours in which to practice. She has already spent 3 of those hours trying to improve her left handed layup. It is still not reliable but she thinks that if she spends two more hours, it will increase her average points per game by 4 points. Any less, and she does not trust it, any more hours will have no effect. She can also work on her jump shot. One hour of jump shot practice will raise her average ppg by 6, two hours will raise it by 9, three hours will raise it by 10, four hours will raise it by 11, and five hours will raises the ppg by 12 points. She can always work on her free throw. Each hour of practice on free throws raises her average points per game by 1.5.

a. From this information, characterize an example of each of the following:

i) Opportunity cost—*A variety of answers are possible, but notice in the final answer, two hours of layup practice forces the opportunity cost of the 1.5 ppg gained from free throw.*

ii) Sunk cost – *The three hours spent working on the layup are sunk.*

iii) Fixed cost – *In order to get any benefit from working further on the layup, you need to put in two hours, which is a form of fixed cost.*

b. Assuming the player only wants to maximize her points per game, how much should she practice?

Order the marginal benefits measured in ppg as

Hours/activity	layup	jump shot	free throw
1	0	6	1.5
2	4(average MB=2)	3	1.5
3	0	1	1.5
4	0	1	1.5
5	0	1	1.5

The player never wants to practice the jump shot more than two hours since after that the marginal benefit of free throw practice is higher. She never wants to do two hours of layup practice before two hours of jumpshot since the benefit of two hours of jumpshot gives 9 ppg versus 4. Finally, two hours of layup yields a better payoff of

two hours of free shots, so combining gives the solution: 2 hours jumpshot, then two hours layup, then 1 hour free shot.

If she only had six hours in total, then she has only 3 hours to allocate. For sure she wants to spend one hour on jump shot. The last two hours can be spent either on layup, yielding 4 ppg, or jump shot and free throw gives 4.5 for this last is the better solution, ie. 2 hours jumpshot and one hour free throw.

- c. Assuming the player only wants to maximize her points per game, how would she practice if she only had six hours in which to practice?