Midterm Exam October, 2012
HONORS 259L
Explain your reasoning clearly.

1. Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1,Player 2, Player 3, Player 4). Pay careful attention to who moves when:


2 . Find all the (pure strategy) Nash Equlibria of the following game:

|  | C1 | C2 | C3 | C4 |
| :--- | :--- | :--- | :--- | :--- |
| R1 | $(20,3)$ | $(-11,5)$ | $(1,10)$ | $(1,4)$ |
| R2 | $(10,1)$ | $(-7,0)$ | $(0,-1)$ | $(5,3)$ |
| R3 | $(10,6)$ | $(-5,8)$ | $(-2,-4)$ | $(2,7)$ |
| R4 | $(8,5)$ | $(-1,8)$ | $(3,1)$ | $(4,7)$ |
| R5 | $(9,6)$ | $(-9,1)$ | $(-2,1)$ | $(2,3)$ |

3. Jack and Jill bargain over the division of 10 pails of water. They both want as many pails as they can get. Generally, one player will offer a division of the pails and the other player can either accept the split or refuse. Then, depending on the game, either that player makes a counteroffer or the game ends. Whenever Jack refuses to accept an offer, if Jack gets to make another offer, one pail is lost before his offer is made. Whenever Jill refuses to accept an offer, if she gets to make another offer, no pails are lost. If the game ends with no player accepting an offer, they both get zero. You may assume that when a player is indifferent between two options, the player always chooses the option preferred by the rival.
i. Provide a back to front solution of the game where Jill offers at most once and then Jack offers at most once with Jill making the first offer.
ii. Provide a back to front solution of the same game except Jack makes the first offer.
iii. Provide a back to front solution of the game where Jill offers at most three times and Jack offers at most two times with Jill making the first offer.
4. In the game below, Row player is serving to Column player. Row wants to maximize probabilities and Column player wants to minimize probabilities.
i. Argue that there is no pure strategy Nash Equilibrium of the game and find the minimax/maximin equilibrium in mixed strategies:
ii. Show how probabilities of serving to the backhand change when the Row player (server) improves her chances of winning on forehand serves by 5 percentage points no matter what the returner guesses. Find the new minimax equilibrium.

|  | Guess Backhand | Guess Forehand |
| :--- | :--- | :--- |
| Serve to Backhand | $30 \%$ | $55 \%$, |
| Serve to Forehand | $70 \%$ | $40 \%$ |

5. An enterprising college student who earns $\$ 15$ an hour creating websites also makes money over the summer by operating a lawn service firm. She earns $\$ 30$ per lawn serviced per week for the first 50 lawns, $\$ 20$ per lawn for the next 50 lawns, $\$ 15$ per lawn for the next 50 and $\$ 10$ per lawn for the last 50. Each lawn requires $\$ 1$ of fuel and 1 hour of labor. Workers can work at most 40 hours per week (they can also work less). For every worker hired in any week, she must pay $\$ 10$ for insurance. She can hire workers at a constant wage of $\$ 10$ per hour but each requires a lawn mower. She has five new ones already. They cannot be resold. She can also service lawns herself.
a. From this information, characterize an example of each of the following:
i) Opportunity cost
ii) Sunk cost
iii) Fixed cost
b. Assuming she wants to maximize her profits per week, how many lawns should she service and how many workers should she hire and how much should she work on the lawn service?
c. Suppose she only has one new lawn mower and lawn mowers cost $\$ 100$. Suppose that a summer lasts 20 weeks. How many lawns and workers should she hire?
