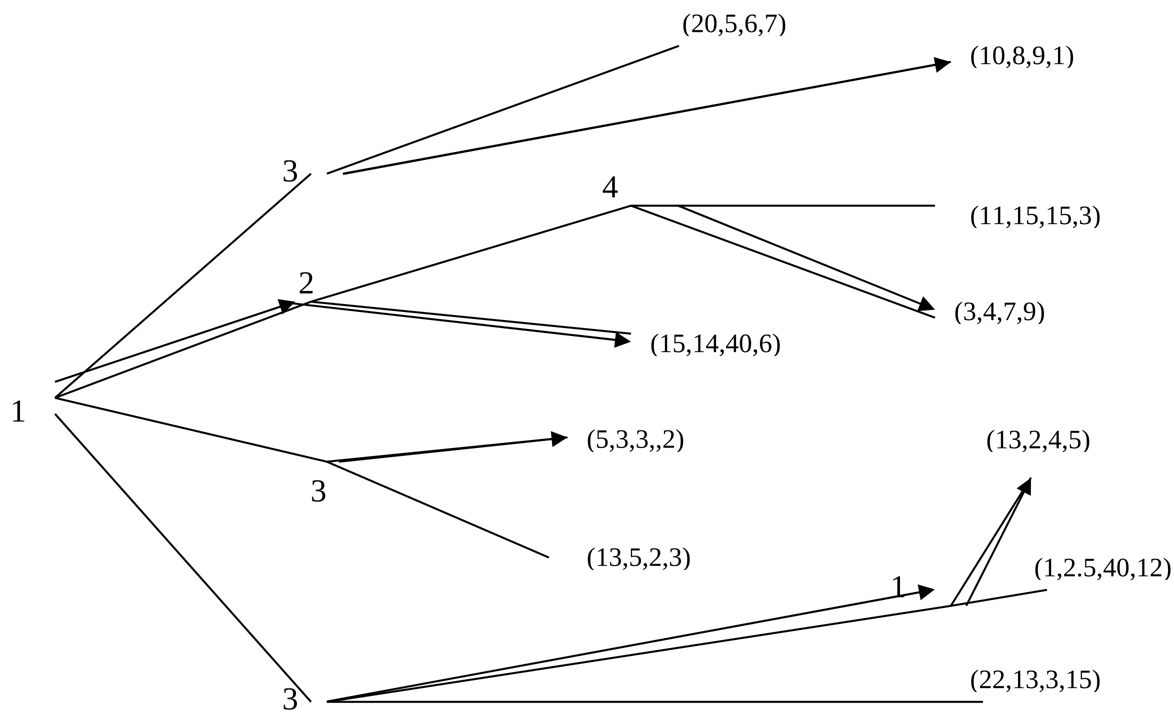
Midterm Exam October, 2012
HONORS 259L
Explain your reasoning clearly.

1. Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1,Player 2, Player 3, Player 4). Pay careful attention to who moves when:



2 . Find all the (pure strategy) Nash Equlibria of the following game:

	C1	C2	C3	C4
R1	(20,3)	(-11,5)	(1,10)	(1 ,4)
R2	(10,1)	(-7,0)	(0,-1)	(5,3)
R3	(10,6)	(-5,8)	(-2,-4)	(2,7)
R4	(8,5)	(-1,8)	(3,1)	(4,7)
R5	(9,6)	(-9,1)	(-2,1)	(2,3)

R2 dominates R5 so eliminate R5.

	C1	C2	C3	C4
R1	(20,3)	(-11,5)	(1,10)	(1 ,4)
R2	(10,1)	(-7,0)	(0,-1)	(5,3)
R3	(10,6)	(-5,8)	(-2,-4)	(2,7)
R4	(8,5)	(-1,8)	(3,1)	(4,7)

C4 dominates C1 so eliminate C1.

	C2	C3	C4
R1	(-11,5)	(1,10)	(1 ,4)
R2	(-7,0)	(0,-1)	(5,3)
R3	(-5,8)	(-2,-4)	(2,7)
R4	(-1,8)	(3,1)	(4,7)

R4 dominates R3 and R1 so eliminate those rows.

	C2	C3	C4
R2	(-7,0)	(0,-1)	(5,3)
R4	(-1,8)	(3,1)	(4,7)

C3 is dominated by either column so this leaves

	C2	C4
R2	(-7,0)	(5,3)
R4	(-1,8)	(4,7)

There are two pure strategy NE in the remaining game: (R4,C2) and (R2,C4).

3. Jack and Jill bargain over the division of 10 pails of water. They both want as many pails as they can get. Generally, one player will offer a division of the pails and the other player can either accept the split or refuse. Then, depending on the game, either that player makes a counter-offer or the game ends. Whenever Jack refuses to accept an offer, if Jack gets to make another offer, one pail is lost before his offer is made. Whenever Jill refuses to accept an offer, if she gets to make another offer, no pails are lost. If the game ends with no player accepting an offer, they both get zero. You may assume that when a player is indifferent between two options, the player always chooses the option preferred by the rival.

i. Provide a back to front solution of the game where Jill offers at most once and then Jack offers at most once with Jill making the first offer.

Suppose that Jack rejects the first offer, the rules imply there are now 9 buckets to divide. Suppose Jack offers (9-x,x) where x is his share. Since Jill will accept any split that gives her 0 or more (by the assumption that if she is indifferent she will accept), Jack should offer the split (0,9) and Jill would accept. Now move to the first stage. Suppose Jill has offered (10-y,y) as a split. Jack now knows if he rejects, he can get 9 buckets when it comes to his turn to offer. Therefore, he will accept any offer y better than or equal to 9. Jill will then offer (1,9) and Jack will accept. (Since Jill prefers this outcome to a rejection and then the split (0,9).)

ii. Provide a back to front solution of the same game except Jack makes the first offer.\

Suppose that Jill rejects the first offer, the rules imply there are still 10 buckets to divide. Suppose Jill offers (10-x,x) where x is Jack's share. Since Jack will accept any split that gives him 0 or more (by the assumption that if he is indifferentshe will accept), Jill should offer the split (10,0) and Jack would accept. Now move to the first stage. Suppose Jack has offered (10-y,y) as a split. Jill now knows if she rejects, she can get 10 buckets when it comes to her turn to offer. Therefore, she will accept any offer y less than or equal to 0. Jack will then offer (10,0) and Jill will accept. (Jack could also offer any other worse offer, Jill would reject and then return with the offer (10,0))..

iii. Provide a back to front solution of the game where Jill offers at most three times and Jack offers at most two times with Jill making the first offer.

The game in terms of maximum number of possible stages (and lost buckets) would go (1) Jill (Reject)(2) Jack -9 buckets (reject)(3) Jill (reject) (4) Jack 8 buckets (reject) (5) Jill. From stage (4) on, there are 8 buckets and the reasoning from (ii) above indicates that the split would be (8,0). Therefore, after an offer at stage 3 of (9-z,z), Jack would accept any z greater than or equal to zero. Which means at the 3 offer stage, Jill does best to offer (9,0). The same reasoning would lead her to reject any offer that gives her less than 9 in Stage 2, so Jack offers (9,0) then. This means that in Stage 1, if the offer is (10-w,w), Jack would accept any offer with w greater than 0. Thus, Jill offers (10,0) in the first stage and Jack accepts.

4. In the game below, Row player is serving to Column player. Row wants to maximize probabilities and Column player wants to minimize probabilities.

   i. Argue that there is no pure strategy Nash Equilibrium of the game and find the minimax/maximin equilibrium in mixed strategies:
   If the NE is Serve To Backhand/Guess Backhand, Server wants to Serve to Forehand, if the NE is Serve ToForehand/Guess Backhand, Returner wants to Guess Forehand, and so on. In the minimax/maximin, let p be the probability Server serves to Backhand (Top Row) and q be the probability that Returner Guesses Backhand (Left Column). The Maximin choice of p for the server must satisfy the property that the probability of winning when the Returner Guesses Bakchand equals the probability when the Returner guesses Forehand:

$p30+(1-p)70=p55 +(1-p)40$ or $70-40p=40+15p$
Solving for p gives $p=30/55=6/11$
Similarly the minimax choice of q must equate the probabilities the returner wins for each choice of the server:
$q70+(1-q)45=q30+(1-q)60$ or $45+25q=60-30q$
Solving for q yields $q=15/55=3/11$.

ii. Show how probabilities of serving to the backhand change when the Row player (server) improves her chances of winning on forehand serves by 5 percentage points no matter what the returner guesses. Find the new minimax equilibrium.

The probabilities in the bottom row now rise to 75 and 45. Just following the same method as above gives:

$p30+(1-p)\mathbf{75}=p55 +(1-p)\mathbf{45}$ or $75-45p=45+10p$
Solving for p gives $p=30/55=6/11$ ***which is the same as above.***
Similarly the minimax choice of q must equate the probabilities the returner wins for each choice of the server:
$q70+(1-q)45=q\mathbf{25}+(1-q)\mathbf{55}$ or $45+25q=55-30q$
Solving for q yields $q=10/55=2/11$. ***The change is that the Returner guesses backhand now less frequently.***

5. An enterprising college student who earns $15 an hour creating websites also makes money over the summer by operating a lawn service firm. She earns $30 per lawn serviced per week for the first 50 lawns, $20 per lawn for the next 50 lawns, $15 per lawn for the next 50 and $10 per lawn for the last 50.. Each lawn requires $1 of fuel and 1 hour of labor. Workers can work at most 40 hours per week (they can also work less). For every worker hired in any week, she must pay $10 for insurance. She can hire workers at a constant wage of $10 per hour but each requires a lawn mower. She has five new ones already. They cannot be resold. She can also service lawns herself.

a. From this information, characterize an example of each of the following:

i) Opportunity cost *The student's foregone earnings of $15 should she decide to mow the lawns herself would be an opportunity cost (she would not choose to do this of course)*

ii) Sunk cost *The cost of the lawn mowers is a Sunk Cost*

iii) Fixed cost *We could include lawn mowers here, also, the $10 per week insurance cost is a form of a fixed cost if it is measure in per lawn mowed.*

b. Assuming she wants to maximize her profits per week, how many lawns should she service and how many workers should she hire and how much should she work on the lawn service?

The student never wants to mow more than 150 lawns per week since she would earn at most $10 for each lawn then and the variable cost for each lawn is $11 (not even including the insurance cost). Also, since she can always hire a student for $10 while her opportunity cost for mowing the lawn herself is $15, she will never choose to mow the grass herself. We can guess that she will want to do the first 100 lawns and then just see if and how many lawns she wants to mow at the $15 per lawn rate.

100 lawns require at least 3 workers, and the third worker having already paid the $10 per week cost, should certainly mow 20 of the next 50 lawns at $15 per lawn since the per lawn income $15 > the marginal cost ($10+1=11). That leaves 30 more lawns that can be mowed at $15 per lawn. The marginal cost per lawn is still $11, the revenue is $15 generating a total gross"profit" of $120 which is much more than the extra $10 of insurance, so the student should hire 4 workers and mow 150 lawns.

c. Suppose she only has one new lawn mower and lawn mowers cost $100. Suppose that a summer lasts 20 weeks. How many lawns and workers should she hire?

The only question here is should she hire fewer workers since now she needs to see if the profits for each worker is worth sinking the cost in the mower. Lets just look at that last worker since that one generates the least profit. That worker generates a net profit of $110 per week which is already more than the cost of a mower for that worker. So that worker should still be hired. The first two are even more profitable so the answer above remains the same here.

	Guess Backhand	Guess Forehand
Serve to Backhand	30%	55%,
Serve to Forehand	70%	40%