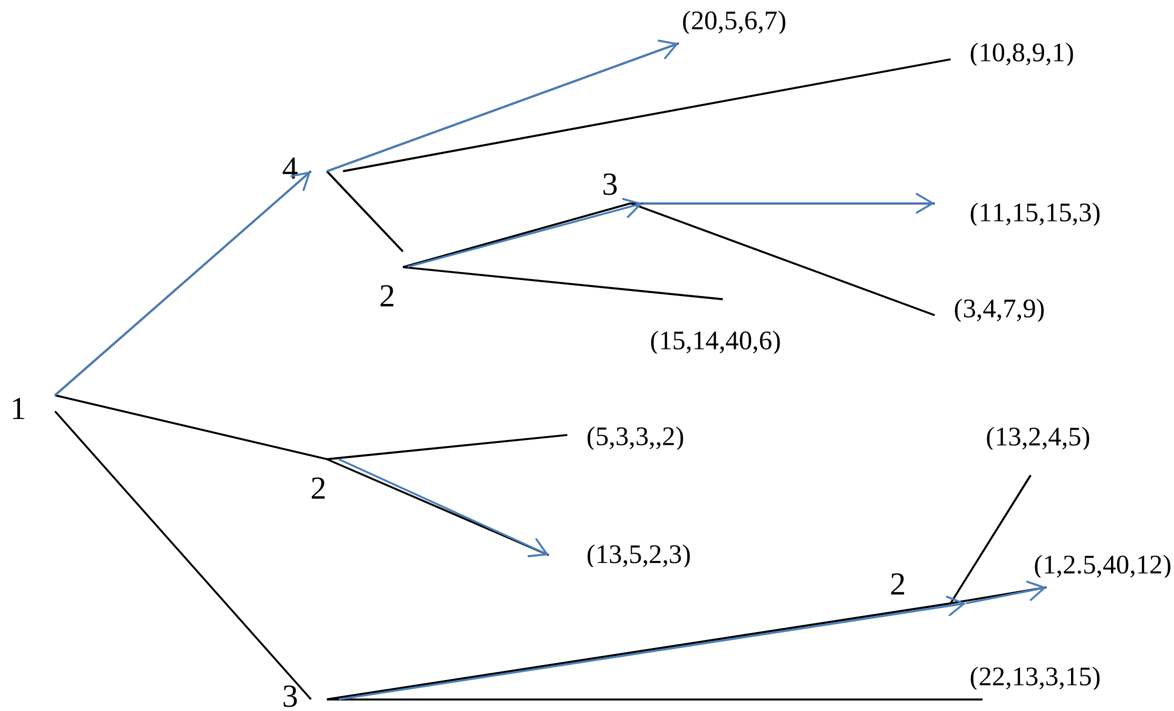
Midterm Exam October, 2013
HONORS 259L
Explain your reasoning clearly.

1. (5 points) Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1,Player 2, Player 3, Player 4). Pay careful attention to who moves when:



2 . (10 points) Find all the (pure strategy) Nash Equlibria of the following game:
The best response for row is in red, for column is in blue. Any cell that is fully colored is a NE.

	C1	C2	C3	C4
R1	(2,3)	(11,5)	(1,10)	(10 ,1)
R2	(10,11)	(7,6)	(0,-1)	(4,0)
R3	(1,6.5)	(5,20)	(2,-4)	(2,7.5)
R4	(4,4)	(1,8)	(3,11)	(4.5,7)

3. (15 points) Emily and James have agreed to divide their collection of 100 chocolate bars they gleaned from Halloween. They will divide their loot according to an offer game with a pre-determined number of offers. At each stage, one of them will propose a division (x,y), where x is the number of chocolate bars for Emily and y is the number of bars for James. The other of the two children either agrees (and the division is as proposed) or rejects. If the stage is not the final stage and there is a rejection, their father will eat one of the chocolate bars. If it is the final stage, their father will force them to split the remaining bars equally (if there is an odd number, the father will eat one and then split them equally.) At any stage of the proceedings, the sum x+y must equal the number of bars that remains in their collection.

i. Provide a back to front solution of the game where Emily offers at most once and then James offers at most once with Emily making the first offer.
In the final stage, no matter what Emily offered in the first stage, let (x,y), x+y=99(=100-1) be the offer of James. If Emily rejects, she will get 98/2=49. Therefore, James will have to offer at least x=49 to Emily and receive 50 for himself. Knowing this, Emily will have to offer (50,50) in the first stage since James will reject any offer that gives him less than 50.

ii. Provide a back to front solution of the same game where Emily makes two offers, the first and last (if the game goes that far) and James makes an offer in between (if it goes that far).
In the final stage, there will be 98 bars remaining so if James rejects, he will get 49. Thus, Emily will have to offer (49,49). Since she knows she can get 49 bars if she rejects James' offer in the second stage, and there are 99 bars remaining in the second stage, James will have to offer (49,50). This means that Emily again will offer (50,50) in the first stage.

4. (10 points) In the simultaneous move, three player game below, Player 1 decides IN or OUT. Player 2 selects Rows and Player 3 selects Columns. The payoffs are listed in each cell with first entry for player 1, then Player 2 and finally, Player 3. Find the Nash equilibrium of the game by successively eliminating dominated strategies.

IN

	C1	C2	C3	C4
R1	(23,2,2)	(-3,9,0)	(33,1,7)	(1,5,8)
R2	(20,3,6)	(-2,7,4)	(34,2,7)	(2,8,10)
R3	(21,4,1)	(0,3,0)	(32,3,2)	(4,4,3)

OUT

	C1	C2	C3	C4
R1	(21,1,3)	(-2,6,-1)	(33,3,3)	(4,2,2)
R2	(20,2,9)	(-3,6,7)	(32,5,9)	(1,6,10)
R3	(23,1,3)	(0,2,-1)	(32,6,3)	(2,7,4)

1) *For Column player, C2 is dominated by C1 (Note that this has to be checked both for IN AND for OUT.) so eliminate that to get.*

2)

IN

	C1	C3	C4
R1	(23,2,2)	(33,1,7)	(1,5,8)
R2	(20,3,6)	(34,2,7)	(2,8,10)
R3	(21,4,1)	(32,3,2)	(4,4,3)

3)

4) OUT

	C1	C3	C4
R1	(21,1,3)	(33,3,3)	(4,2,2)
R2	(20,2,9)	(32,5,9)	(1,6,10)
R3	(23,1,3)	(32,6,3)	(2,7,4)

2) For the Row player, R1 is now dominated by R2 (once again checking for both IN and OUT) so eliminate that.

IN

	C1	C3	C4
R2	(20,3,6)	(34,2,7)	(2,8,10)
R3	(21,4,1)	(32,3,2)	(4,4,3)

OUT

	C1	C3	C4
R2	(20,2,9)	(32,5,9)	(1,6,10)
R3	(23,1,3)	(32,6,3)	(2,7,4)

1) For the Column player, C1 and C2 are dominated by C4.

IN

	C4
R2	(2,8,10)
R3	(4,4,3)

OUT

	C4
R2	(1,6,10)
R3	(2,7,4)

4) For the First Player, Out is Dominated by IN. And if In is played, Row player with choose R2. So the final outcome is (In,R2,C4).

5. (10 points) A child I know enjoys behaving badly and dislikes spending time-out in her room. Each hour of time-out is so bad it destroys the pleasure of one hour of bad behavior. Behaving pleasantly is only OK for her. She has about 20 hours available to spend in time-out, behaving well or behaving badly. Her parents have a short fuse. The first hour of bad behavior costs 5 minutes of time out, the next hour costs an additional 10 minutes, the next hour another 15 minutes, the next hour, 20 minutes and so on. She has just spent 2 hours being pleasant to her relatives.
   i) Use this information to provide examples of marginal costs, sunk costs and opportunity costs. *The marginal cost of an extra hour of bad behavior after misbehaving x hours is x/12. The number of hours spent being nice are sunk. The opportunity cost of one hour of acting nice is an hour (minus x/12) hours of misbehaving.*

ii) How should she allocate the rest of her time to maximize her enjoyment? If you need to make some additional assumptions to answer this, make clear what you are assuming. *I assume given the description, that behaving pleasantly is worth about 0 hours of enjoyment (it neither adds nor subtracts). If she misbehaves x hours, the next hour of misbehavior will cost her x/12 hours of timeout. If x> 12, then the net impact of another hour of misbehavior starts actually taking away her pleasure. Therefore, she should misbehave up to 12 hours and then behave nicely.*