Midterm Exam October, 2015
HONORS 259L
Explain your reasoning clearly.

1. (5 points) Solve the four player game below back to front. The number at each node indicates which player gets to move. The numbers at the end show payoffs (Player1,Player 2, Player 3, Player 4). Pay careful attention to who moves when:

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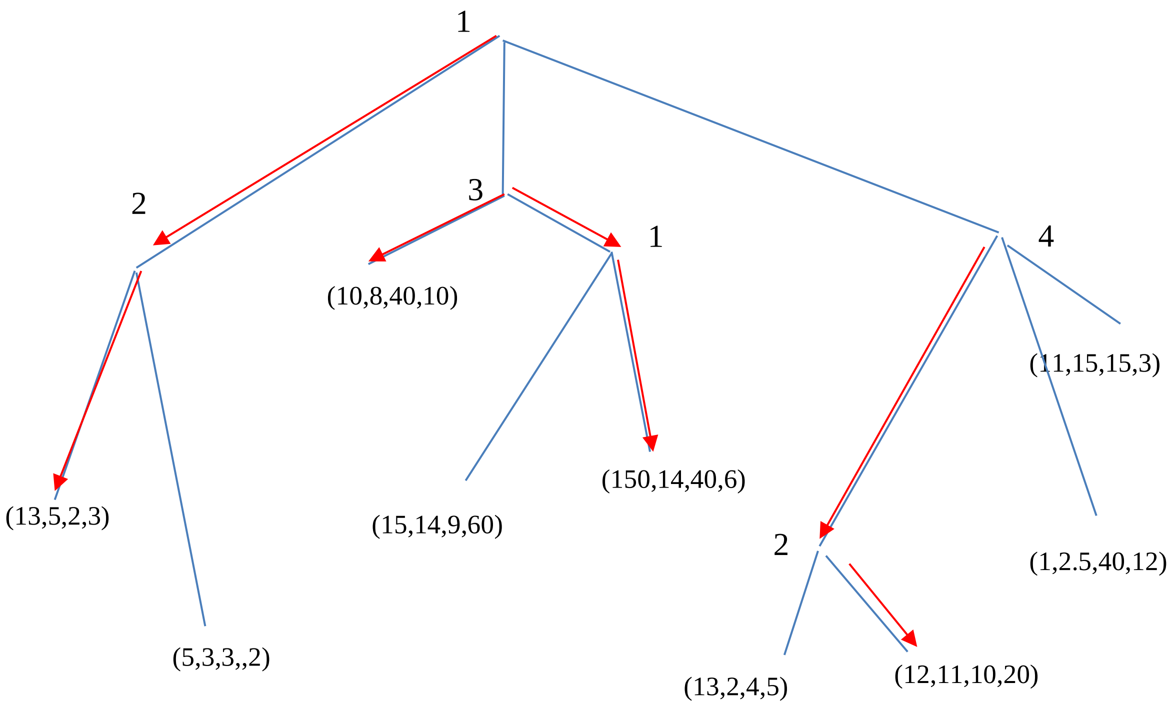

In this case, there are two possible optimal choices for Player 3. This leads to two possible back to front solutions, leading to either (13,5,2,3) or to (150,14,40,6). (It is also possible that player 3 could randomize, leading to a fully randomized outcome as well.) A full credit answer would at least have to recognize this issue even if all the possible solutions were not identified.

2 . (15 points) In the simultaneous move game below, find all the pure strategy Nash Equlibria, if they exist and find any minmax/maxmin equilibria. If necessary, you may use the concepts of dominant or dominated strategies in your approach.

	C1	C2	C3
R1	(2,5)	(11,3)	(6,4)
R2	(10,6)	(7,11)	(2,5)
R3	(1, 5)	(5,20)	(1,21)

Note that R3 is dominated by R2. Therefore, eliminate R3. Once R3 is eliminated, C3 is dominated by C1. Remove C3. This yields the simpler 2 by 2 game,

	C1	C2
R1	(2,5)	(11,3)
R2	(10,6)	(7,11)

This game does not possess a pure strategy Nash Equilibrium. However, because this is not a constant sum game, it will not typically have a maxmin-minmax solution that corresponds to a Nash equilbrium either. Therefore, to compute the maxmin strategy we could proceed two different ways:

Let p be the probability of R1. If C1 is selected, the payoff to Player 1 is

$2p+10(1-p)=10-8p.$

If C2 is selected, the payoff to Player 1 is

$11p+7(1-p)=7+4p.$

Equating the two payoffs and solving gives

$10-8p=7+4p$

$3=12p, p=1/4.$

Similarly, let q be the probability C1 is selected. If R1 is chosen, payoff to Player 2 is

$5q+3(1-q)=3+2q$

If R2 is selected,

$6q+11(1-q)=11-5q.$

Notice that equating the two gives q=8/7 which is impossible since q is a probability. The maxmin strategy for player 2 is actually q=1. However, this is not an equilibrium because if Player 2 selects q=1, (C1 for sure.) Player 1 will not want to randomize. But the pair of probabilities, p=1/4, q=1 do maximize each player's minimum payoff.

An alternative approach is just to find the mixing probabilities so that given the randomized solution for player 1 (p) player 2 is willing to randomize (q) and vice versa.

For Player 1 to randomize, he would have to be indifferent between R1 and R2. This would require that q satisfy

$2q+11(1-q)=10q+7(1-q)$

or

$11-9q=7+3q$, *or* $q= 4/12=1/3$..

Similarly, for Player 2 to randomize, he would have to be indifferent between R1 and R2. This would require that q satisfy

$5p+6(1-p)=3p+11(1-p)$

or

6-p=11-8p, or p=5/7..

I accepted both approaches, since the question asked for max min.

3. (15 points) You and a friend are moving out of an apartment. There are two rooms and in each room there are two boxes. You take turns carrying boxes out of rooms. In each turn, you must choose one room. You can take however many boxes you wish from the room but you must take a box. The person to remove the last box must buy the other person a drink. Your friend moves first.

i. Provide a back to front solution of the game to show who should win this game.
Let (x,y) represent the number of boxes in each room. The first move can only lead to four possible outcomes: (0,2), (1,2), (2,0),(2,1). In all cases, the next player can force the game to either (1,0) or (0,1) and thus win. Therefore, the second mover (you) wins.

ii. Suppose there are five boxes in one room and two in the other. Again, your friend moves first. Who should win now?
The argument in i) shows that if the profile of boxes in rooms is (2,2), the second player to move after that will win. Therefore, if your friend moves three boxes out of the box room, she will confront you with a (2,2) situation and can ensure that she wins.

iii. How would your answer to ii) change if your friend was only allowed to remove one box at a time but you could remove any number greater than zero?
If your friend creates a (4,2) profile, then you can select 2 boxes from the 4 room and ensure a win from the argument in i). If your friend creates a (5,1) profile, you can take 4 boxes from the 5 room and win right away.

4. (10 points) In the simultaneous move, two player game, G, below, Player 1 chooses Rows and Player 2 chooses Columns.

G

	C1	C2	C3
R1	(23,20)	(-3, 0)	(33, 7)
R2	(20,3)	(-2, 4)	(34,2)
R3	(21,4)	(-5,3)	(35,4)

i. (5 points) Characterize all the pure strategy Nash equilibria of the game, G.
The above table is redrawn showing the Best Response of Player 1 in Red and the BR of Player 2 in Blue.

G

	C1	C2	C3
R1	(23,20)	(-3, 0)	(33, 7)
R2	(20,3)	(-2, 4)	(34,2)
R3	(21,4)	(-5,3)	(35,4)

Thus, there are three NE, (23,20), (-2,4) and (35,4).

ii. (5 points) Consider an extended, version of this game where Player 1 first chooses either to exit and receive a sure payoff of 0 or to enter and then play the simultaneous move game, G. Use back to front reasoning to find the Nash equilibria of this

extended game.
Suppose that, when the simultaneous move game is played, both players play (-2,4). In this case, it is better for Player 1 to exit and receive 0. If either of the other NE are played in the simultaneous move game, Player 1 enters.

5. (10 points) Blog is an extra-terrestrial interested in finding a mate. It has spent two sols (a planetary unit of time) developing a relationship with Gloop, a prospective partner from another planet and Gloop looks promising and available. Blog is mildly attracted and believes the current probability of an eternity of happiness with Gloop is 20%. However, Blog would like to know if there are more suitable mates in the universe. Blog has to allocate the next 10 sols of time. Each additional sol Blog spends with Gloop increases the probability of success with this match by 2%. Blog can go out to the Space Bar. The first sol, Blog's probability of meeting a winner is 8%, the second sol it is 7%, the third sol it is 2%. After that, the probability is 1% for each additional sol. (So, for example, if Blog went to the Bar for 5 sols, Blog's probability of success from the Bar is 8+7+2+1+1=19%.)
   i) Use this information to provide examples of marginal benefits, sunk costs and opportunity costs.
   We can think of the amount of time spent as a cost and the gain in probability as a benefit. A marginal benefit, is, for example, the 2% increase in probability of success for each additional sol invested with Gloop. A sunk cost example would be the 2 sols already spent with Gloop. An example of an opportunity cost, is if Blog spends the first available sol going with Gloop, the opportunity cost is the 8% increase in probability that would be gained in the Space Bar.
   ii) Use marginal reasoning to show how Blog should allocate its 10 sols to maximize the total probability of finding a mate? (That is, the sum of the probability of success with Gloop and success at the Bar). If you need to make some additional assumptions to answer this, make clear what you are assuming.
   Order the marginal benefits per sol spent with Gloop versus at the Space Bar as follows

Number of Sols/Action	*Space Bar*	*Gloop*
1	*8*	*2*
2	*7*	*2*
3	*2*	*2*
4	*1*	*2*
5	*1*	*2*
6	*1*	*2*
7	*1*	*2*
8	*1*	*2*
9	*1*	*2*
10	*1*	*2*

After Blog spends 3 sols at the Space Bar, the marginal benefit of the SB is strictly less than the marginal benefit of staying with Gloop. At 3 sols *the marginal benefit is exactly the same. So Blog should choose either 2 or 3 sols at the SB then stay with Gloop. The total increase in probability is 15+8*2=31.*