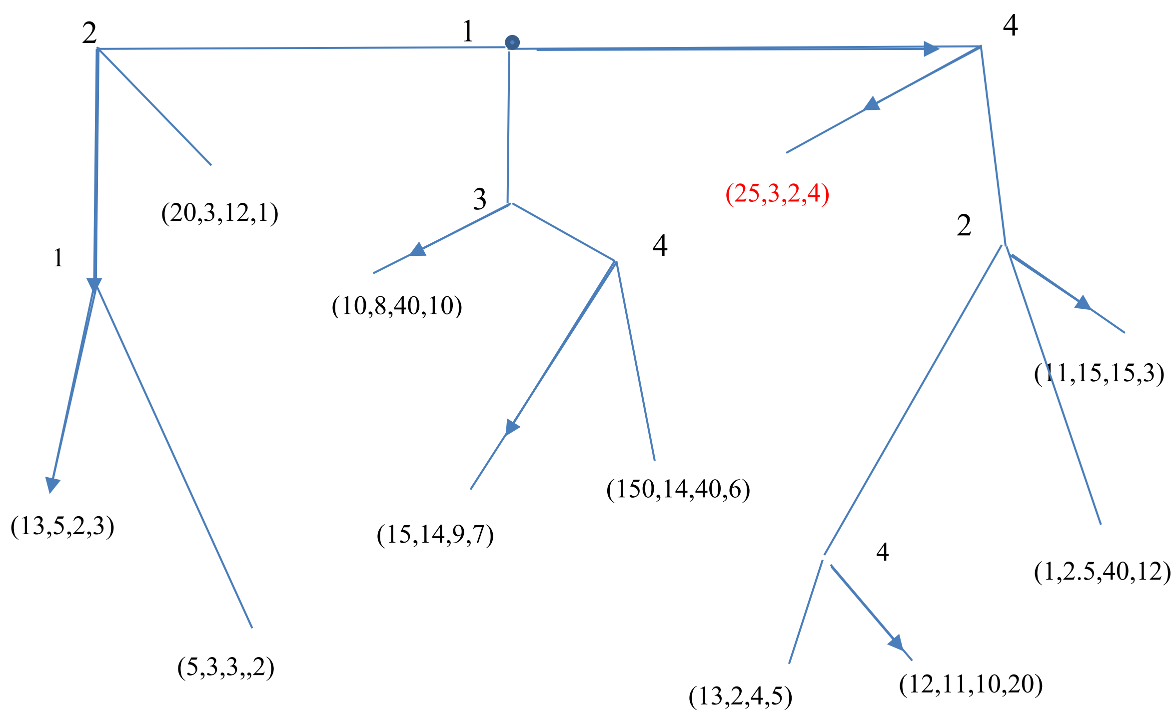
Midterm Exam October, 2018
HONORS 259L
Explain your reasoning clearly.

1. (5 points) Solve the four player game below back to front. The number at each node indicates which player gets to move. Player 1 moves first from the top middle node. The numbers at the end show payoffs (Player1,Player 2, Player 3, Player 4). Pay careful attention to who moves when:



2 . (15 points) Consider the simultaneous move game below. The first entry indicates the payoff to the Row Player, the second is Column Player payoff. The symbol, *A,* represents a non-negative integer less than or equal to 8.

	C1	C2
R1	(3,5)	(4,4)
R2	(A,8-A)	(2,6)

i) Find a value of A such that there is a dominated strategy for at least one player and find the Nash Equilibrium. *If A=0,1,2,3 then R1 dominates R2 for row player.*

ii) For what values of *A* does a pure strategy Nash Equilibrium *NOT* exist? Please give the full range. *If A=4,5,6,7,8 then there is no pure strategy NE. Notice that if A=3, R1,C1 is a NE.*

iii) Suppose $A=5$. Find the maximin/minimax equilibrium of the game.

Observe this is a constant sum game as all payoffs add to 8.
Let p = Prob[Row chooses R1].
If Column player selects C1, Row player's payoff is $3p+5(1-p)=5-2p$.
If Column player selects C2, Row player's payoff is $4p+2(1-p)=2+2p$.
These two lines intersect at $p^*=3/4$ and Row player's maximin payoff is 3.5.

Let q = Prob[Column chooses C1].
If Row player selects R1, Row player's payoff is $3q+4(1-q)=4-q$.
If Row player selects R2, Row player's payoff is $5q+2(1-q)=2+3q$.
These two lines intersect at $q^*=1/2$ and the minimax payoff (to Row player) is 3.5. (Note that I could have computed this using Column player's payoff but the notion of minimax applies to minimizing the maximum of the rival payoff which in this case is Row player.

2. (10 points) You and a co-worker are taking turns placing eggs in a carton that holds 12 eggs. At each turn, you can choose to place 1,2 or 3 eggs in. The worker who places the last egg in the carton wins. The carton is empty and you go first. Provide a back to front solution of the game to show who should win this game.
   The left column describes how many eggs are in the carton, the middle the optimal choice of the 1st player to move next and the right column shows who will win.

#Eggs	*Optimal Choice*	*Winner*
9.10,11	*Choose to 12*	*1*
8	*Any number*	*2*
5,6,7	*Choose to 8*	*1*
4	*Any number*	*2*
1,2,3	*Choose to 4*	*1*
0	*Any number*	*2*

   Therefore, you lose.

3. (10 points) In the simultaneous move, two player game, G, below, Player 1 chooses Rows and Player 2 chooses Columns. The entry, *A,* in (R2,C3) will vary.

G

	C1	C2	C3	C4
R1	(30,30)	(30, 40)	(31, 7)	(31,30)
R2	(20,40)	(46, 30)	(34,A)	(20,20)
R3	(10,28)	(40,30)	(30,40)	(15,50)
R4	(35,15)	(31,10)	(31,12)	(32,10)
R5	(12,8)	(35,9)	(12,10)	(35,9)

i. (5 points) Characterize all the pure strategy Nash equilibria of the game, G when *A =31*. Explain your reasoning.
   R1 is dominated by R4
   G

	C1	C2	C3	C4
R2	(20,40)	(46, 30)	(34,31)	(20,20)
R3	(10,28)	(40,30)	(30,40)	(15,50)
R4	(35,15)	(31,10)	(31,12)	(32,10)
R5	(12,8)	(35,9)	(12,10)	(35,9)

C2 is dominated by C3

	C1	C2	C3	C4
R2	(20,40)		(34,31)	(20,20)
R3	(10,28)		(30,40)	(15,50)
R4	(35,15)		(31,12)	(32,10)
R5	(12,8)		(12,10)	(35,9)

R3 is dominated by R2

	C1	C2	C3	C4
R2	(20,40)		(34,31)	(20,20)
R3				
R4	(35,15)		(31,12)	(32,10)
R5	(12,8)		(12,10)	(35,9)

	C1	C2	C3	C4
R2	(20,40)		(34,31)	
R3				
R4	(35,15)		(31,12)	
R5	(12,8)		(12,10)	

C4 is dominated by C3
R5 is dominated by R4

	C1	C2	C3	C4
R2	(20,40)		(34,31)	
R3				
R4	(35,15)		(31,12)	
R5				

C3 is dominated by C1 and R2 is dominated by R4

	C1	C2	C3	C4
R2	(20,40)			
R3				
R4	(35,15)			
R5				

4. (12 points) Albert is attempting to learn to speak Japanese. He has already taken two online courses of 10 hours each and has a rudimentary capability. He is now trying to improve it by reading newspapers and studying vocabulary. He can devote anywhere from zero to five hours a day. It takes a full hour for him just to get into the mindset of thinking in Japanese and then, for every 15 minutes he can learn 5 new words. Every minute he spends on improving Japanese is a minute less that he can work as a tutor for which he is paid $20 an hour. With this information, provide an example of each of the following and brief justification for your answer.
    i) Sunk fixed cost or benefit. *The hours spent already learning Japanese are sunk fixed costs because they cannot be recovered.*
    ii) Non-sunk fixed cost or benefit. *In order just to get in the mindset, Albert has to spend an hour thinking in Japanese. At this stage, this is not sunk but it is a fixed cost that he has to bear no matter how many words he learns.*
    iii) Opportunity cost or benefit. *Each minute spent on Japanese costs him his next best use of time which is tutoring at $20 per hour.*
    iv) Marginal cost or benefit. *The marginal cost of learning 5 new words is 15 minutes or, in terms of OC, $5.*