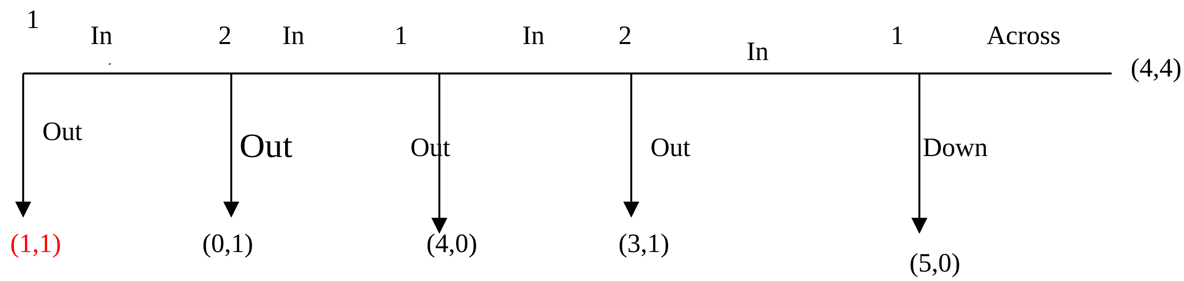
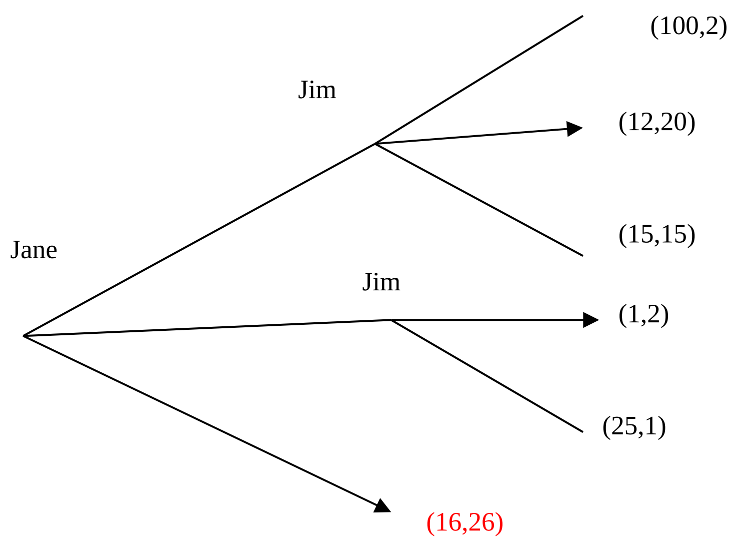
Problem Set 2
HONORS 259L
Due October 1 in class.
(Two of these questions will be graded.)

1) Solve the following games back to front: *The arrows show how the game will be played back to front, the red font color illustrates the final choice.*

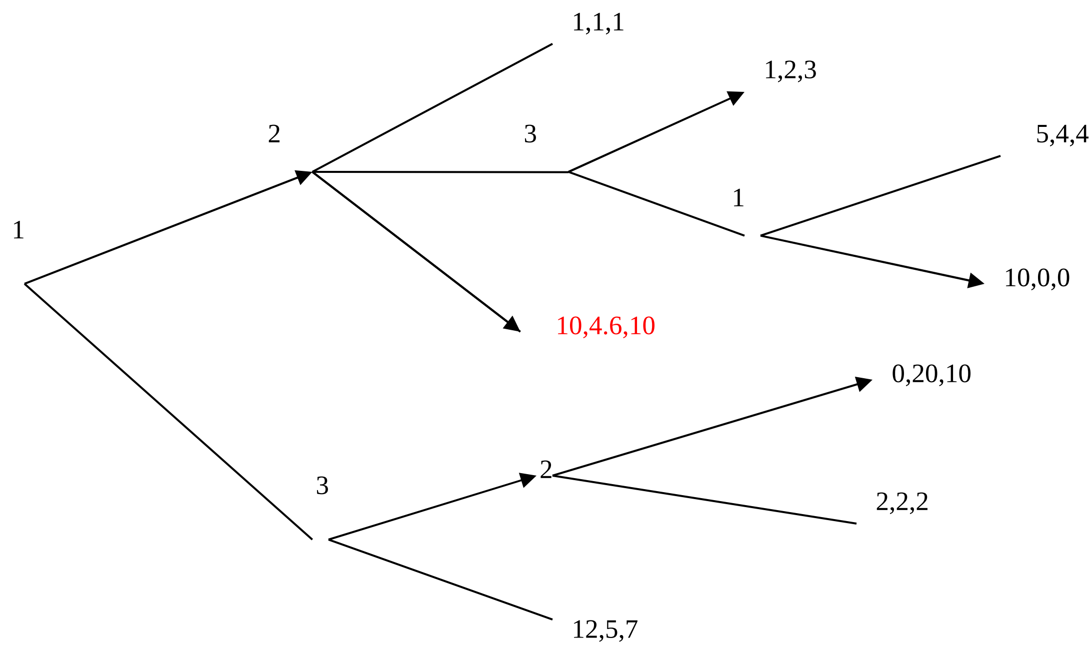
i) First number is payoff to 1, second is payoff to 2.



ii) Jane's payoffs are shown first.



iii) Player 1 payoff, then Player 2 payoff then Player 3 payoffs are shown.



2) Suppose two players are bargaining over how to divide one quart of ice cream. The game proceeds as follows: Player A proposes a split. If Player B accepts, the split is as suggested. If Player B rejects, ¼ of the quart of ice cream melts and Player B proposes a split. Solve the game back to front for the following end of game scenarios:

a) If Player A accepts, the remaining icecream is split as suggested. If Player A rejects, the ice cream melts and no one gets anything.

Consider the final period. For any split (x,3/4-x), x >0, A will have to accept so B should be able to expect virtually ¾ in the final period. This means that B would accept any split (1-y,y), y>3/4 in the first period so A will drive this down to as close as (1/4,3/4) as possible.

b) If Player A accepts, the remaining ice cream is split as suggested. If Player A rejects, another quarter of a quart of ice melts and Player B makes a final proposal. If A accepts, the ice cream that remains is split as suggested, else all the ice cream melts.

Consider the final (third) period. For any split (x1/2-x), x >0, A will have to accept so A should expect virtually 0 in the final period. This means that in the second period, A is in exactly the same position as A was in in part a). and the reasoning for 2a) now applies. B can expect to get ¾ in the second period. This means that B would accept any split (1-y,y), y>3/4 in the first period so A will drive this down to as close as (1/4,3/4) as possible. This is a common feature of these games. If one player gets to make all the offers for the rest of the game, that player gets to drive the other player's payoff to what they would get by refusing (n this case, 0).

3) Consider the following addition game between two players. When it is his or her move, each player can add any number of apples to a bucket between 1 and 9. Each player must add at least one apple when it is their turn. Bob goes first, then Brenda moves. The first player to get 100 apples in the bucket wins. Solve this game from back to front and determine who should win.

Suppose there are between 91 and 99 apples left. Call this stage [01,99]. Then the first mover when the bucket is in this stage wins.

Suppose there are 90 apples. The first mover cannot help but put the bucket into stage [91,99]. in which case the next mover wins so in stage [90], the second mover from this stage forward wins.

When the stage is [81,89], the first mover (A) can choose the number between 1 and 9 that puts the bucket into stage [90] where we have shown the second mover in the [81,89] stage (B) loses so the first mover should win.

Thus, whenever the number of apples is a multiple of 10, the second mover wins. Whenever it is not, the first mover wins. Starting out with 0 apples (a multiple of 10) implies that the second mover wins.