Problem Set 3
HONORS 259L
Due October 30 electronically or in class.
(Two of these questions will be graded.)

1. Apply elimination of dominated strategies to solve the following game. Be sure to indicate which rows or columns are dominated and by what other rows or columns in each round of elimination.

This question illustrates that when strategies are weakly dominated (for some strategies of the rival player, two of your strategies give equivalent payoffs) then elimination of dominated strategies does not yield a unique outcome.

	C1	C2	C3	C4	C5
R1	(1,4)	(2,7)	(3,10)	(4,1)	(5,5)
R2	(0,0)	(4,0)	(3,2)	(4,5)	(1,1)
R3	(.5,2)	(3,3)	(3,1)	(0,20)	(1,2)

Column C1 is weakly dominated by C2 and C5. Eliminate that.

	C2	C3	C4	C5
R1	(2,7)	(3,10)	(4,1)	(5,5)
R2	(4,0)	(3,2)	(4,5)	(1,1)
R3	(3,3)	(3,1)	(0,20)	(1,2)

R3 is weakly dominated by R2. Eliminate that.

	C2	C3	C4	C5
R1	(2,7)	(3,10)	(4,1)	(5,5)
R2	(4,0)	(3,2)	(4,5)	(1,1)

C5 is strictly dominated by C3.

	C2	C3	C4
R1	(2,7)	(3,10)	(4,1)
R2	(4,0)	(3,2)	(4,5)

From now on, the outcome will depend on which strategy is eliminated.
R2 weakly dominates R1. If we eliminate R1, the outcome must be (R2,C4)
However, it is also the case that C2 is strictly dominated by C3. If we eliminate C2, then no more strategies can be eliminated and we end up with

	C3	C4
R1	(3,10)	(4,1)
R2	(3,2)	(4,5)

and we could have (R1,C3) and (R2,C4) as Nash equilibria.

2. Determine whether the following game possesses a dominant strategy for one or both players. Solve the game, explain your answer.

	C1	C2	C3	C4
R1	(1,1)	(11,0)	(1,-2)	(15,1)
R2	(2,20)	(15,11)	(61,18)	(19,21)
R3	(6,-3)	(12,1)	(14,0)	(17,1)
R4	(4,14)	(21,14)	(12,10)	(-1,15)

Row player does not have a dominant strategy. However, C4 is dominant for Column player. Since we can predict C4 will be played, the Best Response for Row Player is R2. Thus the outcome in this game should be (R2,C4)

3. A tennis coach of mine once claimed that 30-15 or 15-30 is most important point. Use back to front reasoning to explain why this claim is not logically sound.

   Here is my reasoning: Suppose it was correct that whoever was ahead 30-15 or 15-30 had an overwhelming chance to win the game. Now suppose the point before at 15,15. In that case, whoever wins that point, should have an overwhelming chance to win, therefore, 15,15 is a critical point in the game. The point before that is 0-15 or 15-0, if the leader wins, he moves the game to 0-30 or 30-0 which is clearly better than 15-30 or 30-15, therefore, this must be the critical point. But if this is the critical point, then whoever wins at 0-0 is in a dominant position and this should be the critical point.

   Suppose, instead, the previous point was 30 Love or Love 30. This is clearly better than 15,30 or 30,15 for the leader, so this must grant the leader an even higher chance of wining. But the point before this must be 15=0 or 0-15 and we can revert to the reasoning above to dispute this.

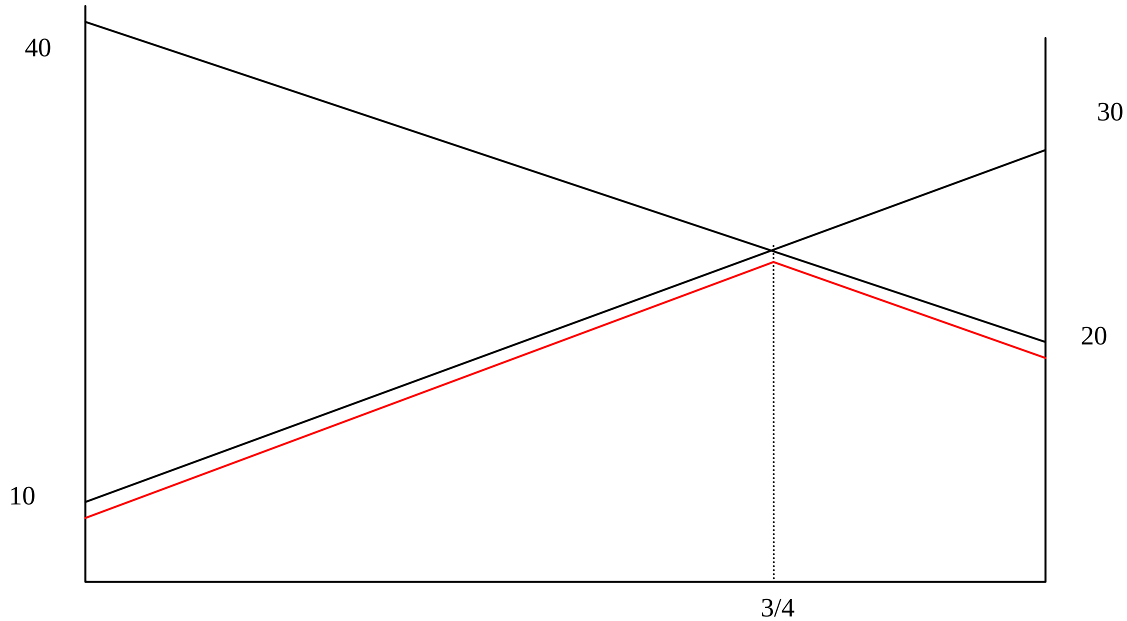
4. Find all the (pure strategy) Nash Equlibria of the following game: Marked in Bold

	C1	C2	C3
R1	**(2,6)**	(1,5)	(1,0)
R2	(0,9)	(0,12)	(0,12)
R3	**(2,60)**	(-5,6)	(-20,-40)

5. Following a touchdown, the Ravens prepare to kickoff to the Patriots. The Ravens are down by 5 points with 3 minutes to play and no timeouts remaining. They must decide to try a short (onside) kick or kick it deep. The Patriots must decide whether to keep their kick return specialists on the field or bring on their sticky handed receivers. With the regular return team in. a deep kick off gives the Ravens a 20% chance of winning the game, an onside kick gives them a 40% chance. With the receivers in, a deep kick gives them a 30% chance of winning and an onside kick gives them a 10% chance of winning.
   a) Describe this situation as a two by two matrix game.
   b) Find a mixing strategy for the Ravens that gives them the same chance of winning whether the Patriots use the regular return or the receivers.
   c) Find the mixing strategies for each team that yield the minimax and maximin probabilities of winning.

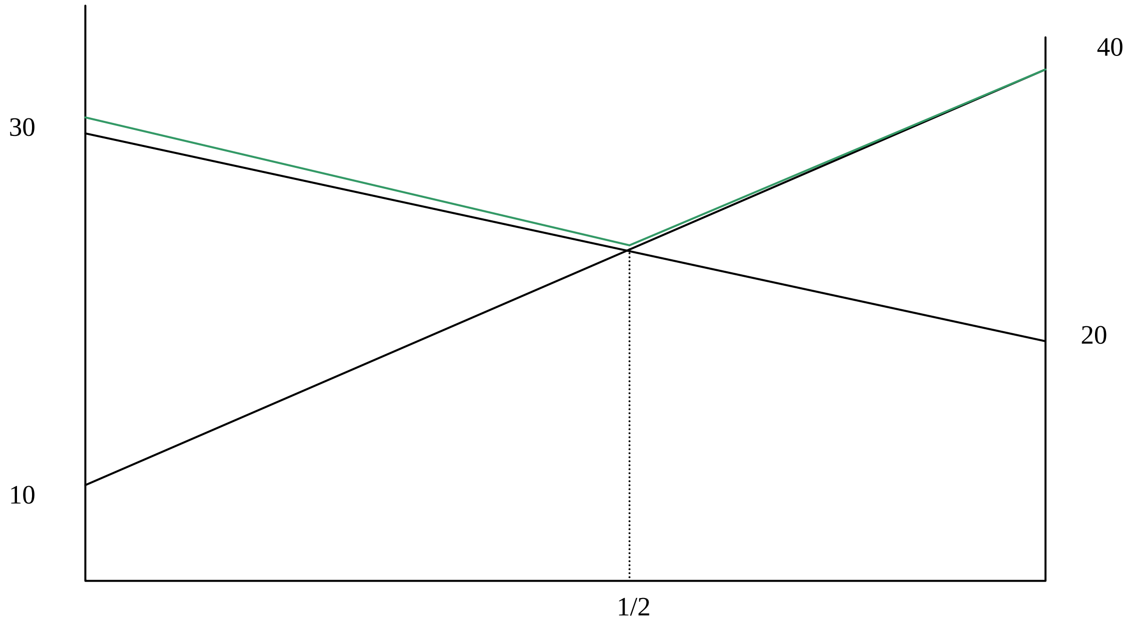
Ravens\Patriots	Regular	Receivers
Deep	(20,80)	(30,70)
Onside	(40,60)	(10,90)

Cell by cell observations shows no Pure Strategy Equilibirum.
Let p be the probability that Ravens kick Deep so 1-p is probability of onside kick.
When the Regular team is in, the probability of winning is: 20p+40(1-p)=40-20p
When the Receiver team is in, the probability of winning is: 30p+10(1-p)=10+20p
This gives the following picture:



Setting 40-20p=10+20p gives p=3/4 for the intersection point/

Let q be the probability that Patriots choose Regular so 1-q is probability of Receivers.
When the Ravens kick Deep, the probability of Ravens winning is: 20q+30(1-q)=30-10q
When the Ravens try an Onside Kick, the probability of Ravens winning is: 40q+10(1-q)=10+30q
This gives the following picture (the green line shows the Worst case scenario from the point of view of the Patriots):



Setting 30-10q=10+30q gives q=1/2 for the intersection point/

Thus the minimax/maximin solution is (3/4,1/2), that is, Probability[Deep]=3/4, Probability[Regular]=1/2.

This gives a probability of Ravens winning of 25% and probability of a Patriots win of 75%.