Problem Set 5
HONORS 259L
Due December 8 electronically or in the ECON Department Office (Tydings Hall, Third Floor).

1. Suppose 5 sawmill owners are bidding to acquire a plot of timber land from the U.S. Forestry Service. The profits (without counting their bid payments) they expect to gain from the plot are given in the following table:

	SM1	SM2	SM3	SM4	SM5
Profit	$1M	$1.25M	$3.35M	$3M	$2.5M

You should assume that all bidders know everyone's expected profits.

i) Suppose that bidder SM1, for whatever reason, acquired the plot. What incentives for the other owners arise assuming SM1 could resell? *Since all the other bidders can make higher profits than bidder SM1, they are likely going to try to buy it from her. A good guess is that bidder SM3 will succeed. The price will likely have to be $3M or more. Otherwise, SM4 would try to buy it.*

ii) What would be the equilibrium bids and the outcome of the auction in a second-price auction? Would there be an incentive to resell? *All bidders bid their valuation (determined by their profits. So the outcome will be that SM3 wins at a price of $3M. Since the highest valuation bidder wins the object, there will be no incentive to resell afterwards.*

iii) Suppose the plot was sold by first price auction. For bidder, SM3, let $B be the highest of the other bids. Assuming bids are allowed in increments of $100,000 what would SM3's best response bid be to that? *As long as B<$3.25M, SM3 will bid B+$100,000 to obtain the plot. Observe that this best response is different from the equilibrium we calculated in class because the question assumes you KNOW the highest bid of the other bidders.*

iv) Suppose the plot was sold by first price auction. What bid amounts represent dominated strategies for SM4? *Any bid above $2.9M is dominated by a bid of $2.9M (bids below that are not dominated since if other bidders bid $100K below that bid, then SM4 would be happy with that bid.*

v) Suppose the plot was sold by first price auction. What would be an equilibrium bid and outcome? *We only really need to focus on SM4 and SM3. One natural equilibrium is SM4 bidding $2.9M and SM3 bidding $3M and obtaining the plot. Again, because you know the other bidders' values, we do not need to conduct the complicated analysis we did in class. Bidder SM4 knows it will likely be outbid by SM3. It can bid $2.9M in the (faint) hope that SM3 will make a mistake. SM3, if it knew that SM4 was bidding $2.9M as computed in iii) above, will want to bid $3M. These are best response bids to each other and therefore are equilibrium bids. The bids of the others are somewhat irrelevant.*

vi) Suppose that bidders SM1, SM2 and SM5 were not present. How would the equilibrium outcomes change in either a first price or a second price auction? *Even if these bidders were not present, the same outcome would be an equilibrium. The point here is that it is the highest and second highest bidder (in IPV auctions) that determine the outcome.*

vii) Suppose that bidder SM4 only was not present. How would the equilibrium outcomes change in either a first price or a second price auction? *Without SM4, now the second highest bidder is SM5, and the equilibrium price would fall to $2.5M.*

2. For each of the following auctions, explain whether you think they are more appropriately described as private value or common value. Give explanations for your reasoning.

   i) Bidding by book publishers for Sarah Palin's new memoir. *Probably CV since all book publishers care about the same basic thing, how many sales the book will make. They might differ in their predictions though.*

   ii) Bidding to supply a local school board with milk for school lunches. *Most likely private values since the suppliers will have their own costs. To the extent they all buy milk from the same dairy farms, there could be a CV element though.*

   iii) Bidding by major league baseball teams in Japan and the U.S. to sign a promising new pitching prospect. *Likely CV since teams generally all value higher talent, however, if each team has an idiosyncratic need for a particular skill you could argue that there is a PV element*

   iv) Bidding at an art auction for a new painting that will hang in someone's living room. *PV since all the bidders care about is the suitability for their particular room*

   v) Bidding to supply one hundred thousand new uniforms to the U.S. Army. *PV I think since each supplier likely has their own costs of supply.*

3. A committee to select a commencement speaker consists of five people. The following table indicates the preferences of five people for the four different choices they are considering:

	John	Alice	Robert	Beth	Carol
Donald Trump	1	4	4	4	2
Malala Yousafzai	4	3	1	3	1
Chris Rock	3	2	2	1	4
Hilary Clinton	2	1	3	2	3

Lower numbers indicate higher preferences.

i) Does a Condorcet cycle exist in this profile of preferences?

To test for a Condorcet cycle, check to see how each voter does against each of the other voters:
Trump v. Malala: 1 vote to 4
Trump v. Rock: 2 to 3
Trump v. Clinton: 2 to 3
Malala v. Rock: 2 to 3
Malala v. Clinton: 2 to 3
Rock v. Clinton: 2 to 3.

Since Clinton beats all three candidates in head to head competition, there is no Condorcet cycle.

ii) Construct a Condorcet rule voting system and describe who would be the winner assuming voters voted sincerely according to their preferences.

In the absence of a cycle, Clinton would win in a Condorcet rule. She does not lose to any candidate, and the rule says that in head to head competition, the one who loses by the fewest votes will win.

iii) Construct a Borda scheme voting system. (Voters give 4 votes for their favorite candidate, 3 for their next favorite, etc. and the candidate with the most votes wins.) Determine who would win if voters voted sincerely. If voters voted sincerely, is there at least one voter who would prefer to vote insincerely?

In a (sincere) Borda rule vote, Trump would gain10 (=4+1+1+1+3) points. Malala would gain 13 (=1+2+4+2+4) votes. Rock would gain 13(=2+3+3+4+1) votes. Clinton would gain 14(=3+4+2+3+2) votes and would win. To see if anyone might want to lie, note first that one of the next highest vote getters is Rock. Consider Beth. If she lied and said that Clinton was last on her list instead of second, she could take two points away from Clinton and then Rock (her favorite) would win.

4. Consider an alternating offer bargaining game where two agents bargain over the division of a basket of six chocolate bars. In each round, one player makes an offer of how to divide, the other players either accepts and the game ends with that division, or the player rejects and either the game ends or it goes to another round where the roles are reversed. There are two possible games. In game 1, there are at most two rounds. If no agreement is reached in the first round, the number of chocolate bars is reduced by 3. If no agreement is reached in the second round, the number is reduced by 3 again so the game ends. In game 2, there are 6 rounds, in each round, if no agreement is reached, the number of bars is reduced by one.
   i) Determine the subgame perfect (back to front) equilibrium of each game.
   Let F go first, L go second. In the equilibrium of the two round game, in the last period, there are 3 left, so L gets all 3. Therefore in the first round, the most F can get is 3.
   In the six round game, going back to front, last period, L gets 1, so 5^{th} period, F gets 1, so 4^{th} period (there are 3 bars left), L gets 2. Thus in 3^{rd} round, F can get 2, so in 2^{nd} round (5 bars left), L can get 3. Thus, the best F can get in the first round, is 3.
   ii) In which game is it best to be the first offeror?