

Final Exam December, 2009
HONORS 259L

1. [10 points] Use techniques we developed in class to find all of the pure strategy Nash equilibria of this game. Row player payoffs are shown first in each cell, then column player payoffs.

| | C1 | C2 | C3 | C4 | C5 |
|----|--------|--------|---------|--------|--------|
| R1 | (1,4) | (2,7) | (-5,10) | (0,1) | (5,5) |
| R2 | (1,0) | (4,0) | (-1,2) | (4,5) | (1,1) |
| R3 | (2,-3) | (3,-2) | (0,4) | (0,2) | (3,2) |
| R4 | (1,5) | (2,2) | (-2,10) | (-1,4) | (1,20) |

R4 is dominated by R3. Eliminate R4. C5 is dominated by C3, eliminate C5. R1 is dominated by R3, eliminate R1. C1 and C2 are dominated by C3. Eliminate C1 and C2. This now leaves

| | C3 | C4 |
|----|--------|-------|
| R2 | (-1,2) | (4,5) |
| R3 | (0,4) | (0,2) |

This matrix leaves two Pure Strategy NE. R3, C3 and R2, C4.

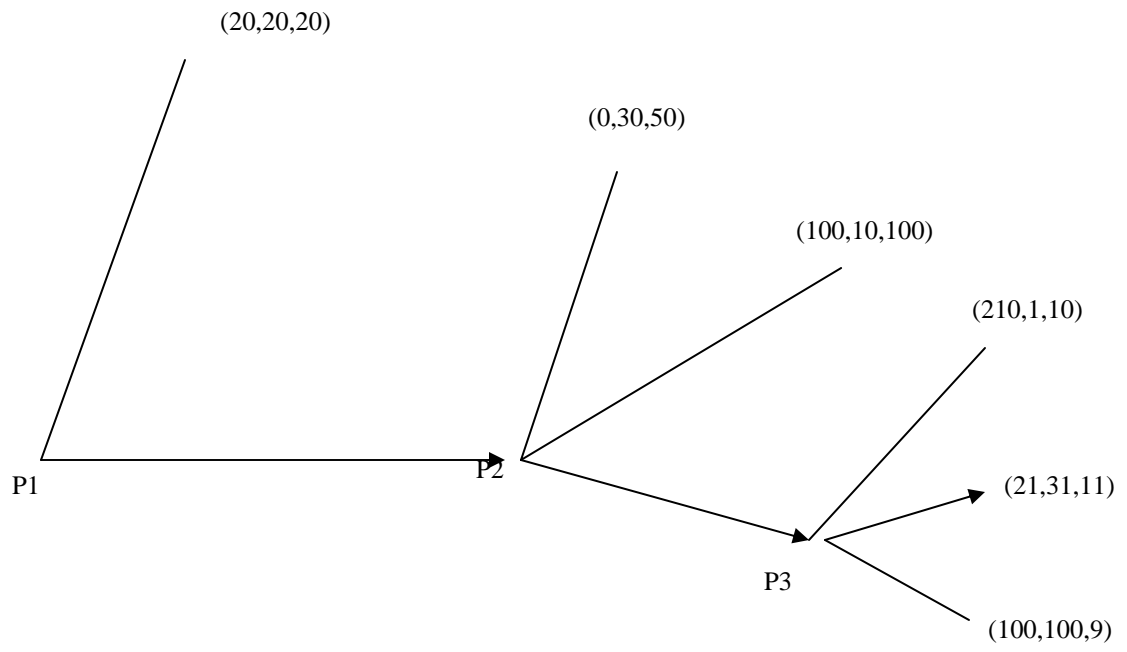
2. [10 points] There are 12 students in an MBA class. Four of the students will get an A, four will get a B and four will get a C. All students know what grade they will get. Employers can only learn the student's grades if they take a final exam. Students who get an A will get a starting salary of \$150,000, those with a B will get a starting salary of \$100,000 and those with a C get a starting salary of \$50,000. The professor offers to pay each student \$10,000 if they choose to take the course Pass/Fail to save himself the trouble of grading. (You can assume they will all pass). If a student obtains a Pass/Fail, employers will pay the average salary of all the student types who choose this option. Use back to front reasoning and adverse selection to predict what will happen in this class.

Suppose all students choose Pass/Fail. The average salary is \$100,000. But A students can get \$150,000 if they choose not to participate so they will opt out of Pass/Fail. If only B and C students do P/F, the average salary is \$75,000 and B students will opt out. Therefore, only C students take the course P/F and they get \$50,000 in salary (plus \$10,000 from the prof).

3. [10 points] Identify two to four concepts that we developed in class that were useful in producing your class project. Explain what the concept was and how you used it.
4. [20 points] Consider the following procurement or "reverse" auction. Four banks want to sell a mortgage-backed security to the federal government under the TARP. Each security has a face value of \$1M but the banks know the true value is much less. Bank 1 knows the value of its asset is really \$0, bank 2 knows the value of its asset is really \$200,000, and banks 3 and 4 know the value of their asset is each really \$300,000. The government will buy one security through a reverse second price auction. It asks the banks to submit a sealed bid and offers to buy the asset from the bank with the lowest bid by paying it the

bid submitted by the second lowest bidder. Banks want to maximize their profit which is the difference between the price they sell the asset for to the government and the value that they know the asset really is worth.

- a) For any bank, characterize its dominant strategy and argue why that is dominant. Consider any bank who knows the true value of its security is v . Let B be the lowest bid of all the other banks. If $B < v$, and B is more than its bid, the bank would lose money by selling the security. If $B > v$, and B is more than its bid, the bank earns $v - B > 0$. Thus, like in any second price auction, submitting a bid equal to the true value of v is a dominant strategy.
 - b) Which bank will sell its asset and what price will it get paid? How much money does the government lose or win in this auction? Bank 1 will offer its security for \$0 and be paid the second lowest bid of \$200,000. The government loses \$200,000 on this auction because it has to pay \$200,000 and gets an asset that is worth nothing.
 - c) Suppose the government offered just to pay a fixed price $\$P$ to buy the asset from the winner of an equal chance lottery. Every bank that agrees to the lottery gets an equal chance of being selected and if it is selected, it will sell the asset for $\$P$. Only one bank will be selected.
 - i) What would be the smallest $\$P$ that ensures all banks enter the lottery? Banks 3 and 4 will only enter if $\$P > 300,000$ so \$301,000 is the lowest price that can ensure all banks enter.
 - ii) Assuming all banks enter the lottery, what is the average value of the asset? The average value of the security over all banks is $\$800,000/4 = \$200,000$.
 - iii) What is the average profit or loss of the government in this scheme? How does it compare to the second price auction? Explain. Here the government pays \$300,000 but gets an asset that is worth, on average, \$200,000 so it loses only \$100,000 which is better than the auction. In the auction, there is a form of adverse selection as the low bid auction ensures the lowest value asset wins.
5. [5 points] Use back to front reasoning to solve for the equilibrium outcome of this game. The payoffs show first player 1's payoff, then player 2 and then player 3.



Outcome is (21,31,11)

6. [15 points] In the matrix game below, let q be the probability that Column player chooses C1 (and of course, $1-q$ is the probability she chooses C2). Assuming that players want to maximize the average (or expected) value of their payoffs, for what values of q is R1 a best response for Row player and for what values of q is R2 a best response? When is Row player just indifferent? (Row player payoffs are shown first).

| | C1 | C2 |
|----|--------|--------|
| R1 | (0,30) | (30,0) |
| R2 | (20,0) | (0,20) |

R1 pays $30(1-q)$

R2 pays $20q$.

when $30(1-q) > 20q$ or $q < 3/5$, R1 is better. If $q > 3/5$, R2 is better, if they are equal, then Row player is just indifferent.