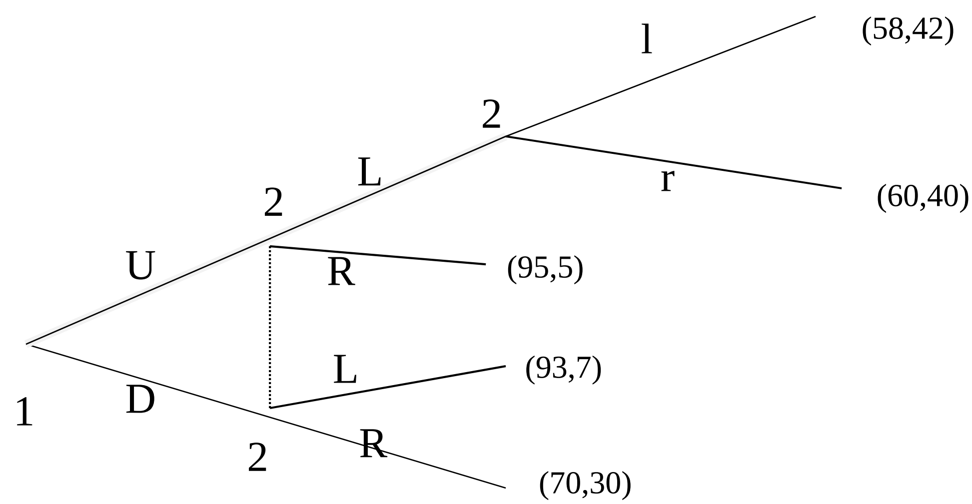
Final Exam December, 2012
HONORS 259L

1. [20 points] Use techniques we developed in class to find the (possibly mixed) strategy equilibrium of this game. This is a two player game. Player 1 and 2 each move simultaneously (the dotted line joining Player 2's first moves indicate 2 makes her choice without seeing what 1 has selected.). After they move, either the game ends or else if (U,L) is played, Player 2 gets to move again and selects either *l* or *r*. Before this second move, Player 2 observes the move of Player 1.The payoffs show first Player one, then Player two. (Hint: use a combination of solving for maximin and back to front reasoning.)

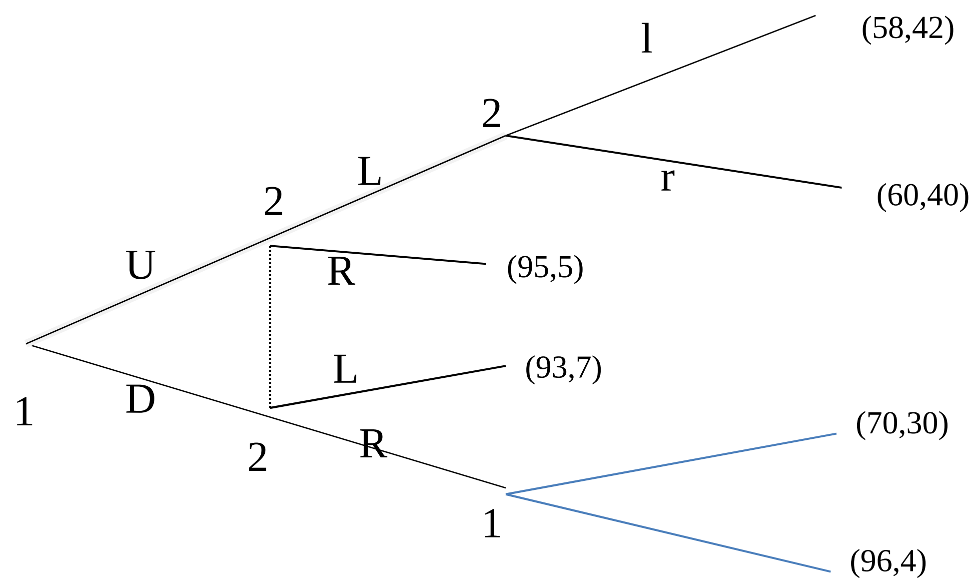


Use back to front reasoning to note that if ever 2 is asked to move after (U,L), she would select l or (58,42). Therefore, this is a simultaneous move game with outcome (58,42) in the upper payoff point. Simultaneous move games can also be described as a bimatrix game, looking like the following:

1 \2	*L*	*R*
U	*(58,42)*	*(95,5)*
D	*(93,7)*	*(70,30)*

Cell by cell examination indicates that there is no deterministic Nash Equilibrium of this game. (If U,L is selected, P1 wants to go to D,L, if D,L is selected, P2 would go to D,R etc.) Therefore, we need to use minmax. Let q be the probability that P2 selects L and p be the probability that P1 selects U. At a minmax, the payoff to each player's strategy has to be the same, therefore for Player 1, we must have 58q+(1-q)95=93q+(1-q)70 or (1-q)25=35q or q=25/60. For Player 2, we must have 42p+(1-p)7=5p+(1-p)30 or 37p=(1-p)23 or p*=23/60. Thus, in the full equilibrium of the game, Player 1 chooses U with probability 23/60 otherwise he chooses D. Player 2 chooses L with probability 25/60 and if (U,L) is ever selected (which happens with probability 23/60*25/60), Player 2 chooses l.*

2. [10 points] The next game is like the game in 1), however, instead of outcome (70 ,30), Player 1 also has an option to accept that outcome (if it occurs out of the first two moves) or else select the option (96 ,4).. Player 1 gets to make that choice after seeing if the play (D,R) has occurred. Draw the game tree that represents this new situation. Determine the equilibrium outcome of this game. Does Player 1 benefit from having this additional choice? Explain. *The new game looks like*



Use back to front reasoning to note that if ever 2 is asked to move after (U,L), she would select l or (58,42). If ever Player 1 is asked to move after D,R, he would choose (96,4). Therefore, this is a simultaneous move game with outcome (58,42) in the upper payoff point and (96,4) in the lower payoff point. Simultaneous move games can also be described as a bimatrix game, looking like the following:

1 \2	*L*	*R*
U	*(58,42)*	*(95,5)*
D	*(93,7)*	*(96,4)*

In this game, both players have a dominant strategy D for Player 1 and L for Player 2. So the outcome of the game is D,L with payoffs (93,7). Note that even though (96,4) is never played, Player 1 gets a higher payoff in the equilibrium.

3. Consider the following simplification of the so-called incentive auctions that the FCC would like to run. There are four TV broadcast stations, A,B,C, and D holding licenses to use frequency in a given area. The values to the four different owners of the stations of continuing to operate (staying on air) are given by *(1.5, 1.3, 1.8, 2.0)* respectively (in

millions of dollars). The FCC will start a clock at \$2.5 million and have it drop by \$0.1 million in each round. Owners each have a button and after each tick down of the clock they can press it. When they press it, they have to leave the auction which means they do not get to sell their station. The clock stops ticking down when there is only one station left. That station sells its license at a price equal to the number on the clock.

i) [10 points] Determine the optimal strategies of each station and find which wins and at what price. *Let p_t be the current clock price at stage t. If a player is the last one in the auction at t, its payoff is $p_t - V$, where V is the value (or opportunity cost) of staying on air. The net payoff is positive as long as $p_t >(=) V$. So each player has a dominant strategy to stay in until the clock reaches V or V+0.1. With all owners following this strategy, the clock will keep ticking down until p_t=1.5, the second lowest station value and the lowest station value will sell its license.*

ii) [10 points] Suppose the FCC wishes to buy two stations and modifies the auction so that the clock stops when only two stations remain. Those stations then sell at the clock price. Repeat your answer for part i) for this case. *Same as above except the clock stops at 1.8 when the third lowest station drops out. The price is now \$1.8M.*

4. Suppose that a committee of 100 people are voting to decide on following one of four alternatives: a,b,c,d. The committee preferences over these can be split into three groups shown in the table below. (The highest row is most preferred, then next preferred in the row below etc.)
   i) [10 points] Describe how a Borda count (with values 4,3,2,1) would operate in this situation and determine the winner assuming all members voted sincerely according to their preferences.
   ii) [10 points] Show that there is at least one group that would actually prefer not to vote sincerely assuming the members of the other group do vote sincerely.

Group	I	II	III
Group Size Rank	35	45	20
1	a	b	d
2	d	c	a
3	c	a	c
4	b	d	b

*i) The Borda count for a is 35*4+45*2+20*3=290. The BC for b is 35+45*4+20=235. For c is 35*2+45*3+20*2=245 and for d is 35*3+45+20*4 = 230 so a would this vote.*

*j) ii) Now consider the interests of Group II. Note that they prefer c to a. Suppose instead of voting (b,c,a,d) they lied and voted (c,b,d,a). (That is, they switched up b and c and a and d.) The new Borda counts would be for a:35*4+45****1****+20*3=245. The BC for b is 35+45****3****+20=190. For c is 35*2+45****4****+20*2=290 and for d is 35*3+45****2****+20*4 = 275 so with this vote they could make sure a loses and c wins which is not their most preferred outcome but is better than what would occur if they voted sincerely.*