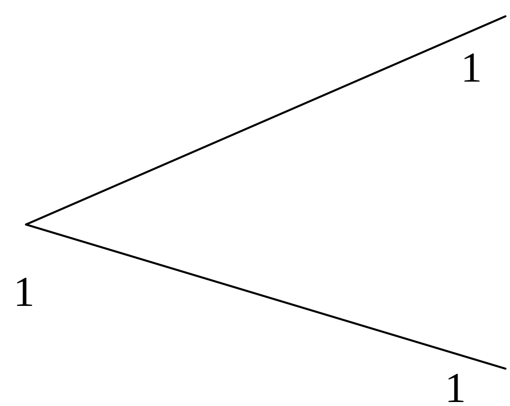
Final Exam December 16, 2013

HONORS 259L

Please provide reasoning for your answers. If you need to make additional assumptions to arrive at an answer, be explicit what you are assuming. The best answer typically makes the fewest assumptions.

1. [10 points] Use techniques we developed in class to find all of the pure strategy Nash equilibria of this game. This is a two player game. Player one selects one of the two boxes and then players one and two select row and column respectively. Both players one and two know which box is selected but they choose row and column simultaneously. The payoffs show first player one, then player two. (Hint: use a combination of solving for Nash Equilibrium and back to front reasoning.)



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	C1	C2
R1	(2,2)	(7,-7)
R2	(-6,20)	(10,10)

2

	C1	C2
R1	(3,1)	(0,-2)
R2	(1,- 3)	(1,0)

We solve this by back to front reasoning and NE analysis. If the upper box is selected by 1, C1 dominates C2 and therefore, row player selects R1. The only equilibrium outcome there is (2,2). In the lower box, there are two pure strategy NE, (3,1) and (1,0). (There is also a mixed strategy equilibrium but I am not asking about that). We do not know which one will be played. If the (3,1) is played, then player 1 should choose the lower box, if the (1,0) equilibrium is played, then Player 1 should choose the upper box. Thus we have two potential NE of this game.

2. [10 points] John and his sister Jane take turns stealing cookies from a jar. There are ten cookies and each can take one, two or three cookies when it is their turn to steal (each must take at least one cookie on their turn). Their father will be very angry at the child who steals the last cookie. Solve this game by back to front reasoning. Is it better to take the first turn or the second turn?

   Solve this by using Back to Front reasoning. The first column shows the number of cookies left, the next column indicates whether the next player to move wins or loses:

Cookies Left	*Next Player to move*
1	*loses*

2 (take one cookie)	*wins*
3(take two cookies)	*wins*
4(take three cookies)	*wins*
5(take any number, move to one of above three rows)	*loses*
6(take one cookie, move to above row	*wins*
7(take 2 cookies, move to row 5	*wins*
8(take 3 cookies, move to row 5	*wins*
9(take any number, move to one of above three rows)	*loses*
10 (take one cookie, move to above row	*wins*

3. [10 points] In a game show, each of two players is given, privately, an envelope with an integer written in it. They are told (that is, it is common knowledge) that the integers that are given out are strictly positive and consecutive. Once they are shown their private number, a clock starts and indicates each time a minute has passed. The players may not speak to each other except to announce once and once only what the other player's number is. They are given the opportunity to announce each time the clock reaches a minute point. If either player announces incorrectly, the game ends and they receive nothing. If one player guesses correctly, they each get $1 million. Show how they can assure themselves the $1 million. (you may assume they can take as long as they need). *In general, players only will know if the other player's number is one higher or one lower. The one exception to this is if a player gets a 1. In that case, that player will announce in the first minute. However, suppose both players realize this. Now consider player with a 2. The other player has either a 1 or a 3. If he has a 1, he will announce in the first minute. So, if he does not announce in the first minute, he must have a 3. The player with a 2 should announce in the second minute. The similar logic holds for the player with a 3. If there is no announcement by the second minute, player with 3 knows the rival has a 4, etc…Many answers just suggested that players wait until the same number of minutes elapse as their number and then announce their number plus one. While this is a winning NE, the logic is not as compelling as the one where players are eliminated iteratively so this answer got 8 out of 10 instead of the full marks.*

4. [20 points] Suppose an auctioneer uses a second price auction to sell a car to one of two dealers. The car may be in superb condition, in which case it can be sold by either dealer at a profit (not including the price paid at the auction) of $15,000 or it may be a lemon, in which case it can be sold for a profit of $5,000. There is a 50-50 chance of either possibility.
    a) Is this an example of private values or common values? *Since both bidders value the car the same, this is common values.*
    b) Supposing the dealers just want to maximize their average profits (taking into account the price they pay at the auction), what should each dealer bid assuming they each use dominant strategies? *The average value of the car is $10,000. Since neither bidder has any inside information, then winning does not convey any more information, so in a sealed bid second price auction, the optimal strategy*

(in fact the dominant strategy) is to bid your own value, \$10,000. Each bidder bids the same, has an equal chance of winning but earns zero profits on average.

c) Suppose that the dealers are given the opportunity to inspect the car and, if they do, can tell whether or not it is good or bad.

   i. How should the informed dealers bid in a second price auction? *The informed bidder should bid the (newly learned) true value of the car. If both bidders are informed, they again get zero profits.)*

   ii. Suppose the dealers had to pay \$50 for the right to inspect the car before the auction (both dealers pay \$25 and they each learn the type.) Would they agree to do this? Why? *No. Since in equilibrium, when they are both informed they get zero profits, there is no advantage to having the information.*

5. [15 points] In the matrix game below, let q be the probability that Column player chooses C1 (and of course, $1-q$ is the probability she chooses C2). Assume that players want to maximize the average (or expected) value of their payoffs. (Row player payoffs are shown first).
   a) Compute the minimax/maximin equilibrium of this game.
   b) Suppose the payoffs of Row player rise by 5 in each cell. What is the new minimax/maximin equilibrium of this game?

	C1	C2
R1	(15,25)	(40,0)
R2	(30,10)	(25,15)

Cell by cell inspection shows there is no pure strategy NE of the game. In a maximin equilibrium, if q is the probability that column player selects C1, the average payoff to row player has to be the same so we must have

$15q+40(1-q)=30q+25(1-q)$ or

$40-25q=25+5q$ or

$15=30q$, $q=1/2$.

If p is the probability that row player selects R1, then we also must have

$25p+10(1-p)=0p+15(1-p)$

$10+15p=15-15p$ or

$30p=5$, $p=1/6$

Observe that if we add 5 points to the row player in every cell, the first equation is now

$20q+45(1-q)=35q+30(1-q)$ or $5+15q+40(1-q)= 5+ 30q+25(1-q)$

which is exactly the same as before, so the equilibrium does not change. (Payoffs for the column player have not changed so that will not change either.