## Final Exam December 17, 2015 <br> HONORS 259L

Please provide reasoning for your answers. If you need to make additional assumptions to arrive at an answer, be explicit what you are assuming. The best answer typically makes the fewest assumptions.

1. [15 points] A certain university is deciding on a pricing scheme for its (almost professional) college basketball team. It knows that $25 \%$ of potential fans are very wealthy and very eager to attend and would be willing to pay $\$ 500$ for season’s tickets. The rest of the fans are eager but poor and will pay at most $\$ 100$ for season's tickets. The total number of fans is less than the number of seats in the university basketball arena.
i) Assuming that there is no direct way of distinguishing between the two types of fans, prove that the university raises the most money by only selling to a subset of the fan base. Which subset will it sell to?
ii) Suppose the university only wants the game to be attended by the poorer fans. Given the information provided so far, is there any way it can achieve this goal through pricing? Prove your answer.
iii) Suppose that tickets are not transferable and that wealthy fans really dislike camping out overnight on the campus while poorer fans rather enjoy the practice. Supposing the university wishes only to raises as much revenue as possible, suggest a selling scheme that would raise more revenues for the university than in i) above and would allow all types of fans to attend the game.
2. [10 points] Fred and Wilma are negotiating the division of 100 bones. The negotiations take place over four rounds. One person offers a division in the first round, the other accepts or rejects. If the responder accepts, the game ends and the division is realized. If not, some number of bones are lost to BamBam and a new round begins with the roles of offeror and responder reversed. This continues for at most four rounds. If at the end of four rounds, no acceptance occurs, the game ends and each player gets nothing. You may assume that both players want more bones than fewer and that if ever a player is indifferent between accepting and rejecting, the player will accept.
i) Suppose that the number of bones that BamBam steals after each round is 10 . Compute the back to front solution of the game. Is it better to offer first or second?
ii) Suppose that the number of bones that are stolen after each round except after round three is 10. After a rejection in round three, only, BamBam steals 65 bones. Compute the back to front solution of the game. Is it better to offer first or second?
3. [20 points] The table below shows the ranking of preferences of three groups of voters in an election. Lower numbers reflect a higher ranking.

|  | Dems (40\%) | Reps(35\%) | Wackos(25\%) |
| :--- | :--- | :--- | :--- |
| Tramp | 3 | 3 | 1 |
| Clifton | 1 | 2 | 3 |
| Ruby | 2 | 1 | 2 |

i) Does this profile of voting preferences exhibit a Condorcet cycle? If so, show it, if not, prove it.
ii) Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a Borda count rule.
iii) Under a Borda count rule, determine whether any group would prefer to vote insincerely given that the other groups are voting sincerely. Prove your answer.
iv) Suppose the preferences of the Reps changed so that Tramp was preferred to Clifton while Ruby is still most preferred. The Wackos preferences change so that Clifton is preferred to Ruby but Tramp is still their most preferred. Dems preferences stay the same. Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a Condorcet rule.
4. [20 points] An auctioneer has two identical cases of 2000 vintage Bordeaux wine to sell. There are five bidders, each of whom want a single case. The numbers indicate the maximum amount each bidder would be willing to pay for one case of wine (for example, bidder B 3 is willing to pay as much as $\$ 90$.) The auctioneer sells the two cases via a THIRD price auction. That is, all bidders submit (privately) sealed bids. The two bidders who submit the highest bids will each obtain a case and will pay the third highest price.

| Bidder | B1 | B2 | B3 | B4 | B5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Value | 110 | 135 | 90 | 105 | 150 |

i) Prove that each bidder has a dominant strategy and show what it is.
ii) Characterize the equilibrium of this game assuming bidders select their dominant strategy you characterize in i). Show who wins the cases and what price is paid.
iii) Suppose bidder B2 was not in the auction. Determine the equilibrium outcome in this case and compare it to the outcome you found in ii).
5. [15 points] In the matrix game below, let $q$ be the probability that Column player chooses C1 (and of course, 1-q is the probability she chooses C2). Assume that players want to maximize the average (or expected) value of their payoffs. (Row player payoffs are shown first).
i) Compute the minimax/maximin equilibrium of this game.
ii) Suppose the payoffs of player 1 rises by 5 in cell R1,C2 and the payoff of player 2 falls by 5 . What is the new minimax/maximin equilibrium of this game?

|  | C1 | C2 |
| :--- | :--- | :--- |
| R1 | $(26,20)$ | $(10,36)$ |
| R2 | $(23,23)$ | $(30,16)$ |

