Final Exam December 17, 2015
HONORS 259L

Please provide reasoning for your answers. If you need to make additional assumptions to arrive at an answer, be explicit what you are assuming. The best answer typically makes the fewest assumptions.

1. [15 points] A certain university is deciding on a pricing scheme for its (almost professional) college basketball team. It knows that 25% of potential fans are very wealthy and very eager to attend and would be willing to pay $500 for season's tickets. The rest of the fans are eager but poor and will pay at most $100 for season's tickets. The total number of fans is less than the number of seats in the university basketball arena.
   i) Assuming that there is no direct way of distinguishing between the two types of fans, prove that the university raises the most money by only selling to a subset of the fan base. Which subset will it sell to?
   *Let the total number of fans be 100 (for example). If it sold to all the fans, it would raise $10K because it would have to set a price of $100 and since it cannot tell wealthy from poor fans, they all would pay the same price. If it set a price of $500, only wealthy fans would buy and the total revenue would be $500*25=$12.5K.*

   ii) Suppose the university only wants the game to be attended by the poorer fans. Given the information provided so far, is there any way it can achieve this goal through pricing? Prove your answer.
   Given the information so far, for any price charged to the poor fans that they would pay, the wealthy would also pay.
   iii) Suppose that tickets are not transferable and that wealthy fans really dislike camping out overnight on the campus while poorer fans rather enjoy the practice. Supposing the university wishes only to raises as much revenue as possible, suggest a selling scheme that would raise more revenues for the university than in i) above and would allow all types of fans to attend the game.
   Suppose the university offered $100 tickets at (say) 5am in the morning to the first 75 fans who show up. Afterwards, tickets will be available for $500. Since the poor fans like to camp out, they would set up tents the night before and be first in line to get the tickets. The wealthy fans do not like to camp and will wait to buy the more expensive tickets. In this way, the university gets 7.5+12.5 =$20K and all fans attend. To answer this correctly, it was necessary to use the information provided in question.
2. [10 points] Fred and Wilma are negotiating the division of 100 bones. The negotiations take place over four rounds. One person offers a division in the first round, the other accepts or rejects. If the responder accepts, the game ends and the division is realized. If not, some number of bones are lost to BamBam and a new round begins with the roles of offeror and responder reversed. This continues for at most four rounds. If at the end of four rounds, no acceptance occurs, the game ends and each player gets nothing. You may assume that both players want more bones than fewer and that if ever a player is indifferent between accepting and rejecting, the player will accept.
   i) Suppose that the number of bones that BamBam steals after each round is 10. Compute the back to front solution of the game. Is it better to offer first or second?
   Say Wilma offers first. If the game goes to round 4, there will be 70 bones left and Fred will offer (0,70), taking all the bones for himself. Therefore in Round 3, Fred will not accept anything less than 70. There are 80 bones left, so Wilma offers

(10,70). In round 2, Fred offers (10,80) and in round 1 Wilma offers (20,80). In this game, it is best to be the second offeror.

ii) Suppose that the number of bones that are stolen after each round except after round three is 10. After a rejection in round three, *only*, BamBam steals 65 bones. Compute the back to front solution of the game. Is it better to offer first or second?

Again, suppose Wilma offers first. If the game goes to round 4, there will be 15 bones left (100-10-10-65) and Fred will offer (0,15), taking all the bones for himself. Therefore in Round 3, Fred will not accept anything less than 15. There are 80 bones left, so Wilma offers (65,15). In round 2, Fred offers (65,25) and in round 1 Wilma offers (75,25). In this version of the game, it is best to be the first offeror.

3. [20 points] The table below shows the ranking of preferences of three groups of voters in an election. Lower numbers reflect a higher ranking.

	Dems (40%)	Reps(35%)	Wackos(25%)
Tramp	3	3	1
Clifton	1	2	3
Ruby	2	1	2

i) Does this profile of voting preferences exhibit a Condorcet cycle? If so, show it, if not, prove it.
Consider Tramp v. Clifton. Clifton wins 75 to 25.
Consider Clifton v. Ruby. Ruby wins 60 to 40.
Consider Ruby v. Tramp. Ruby wins 75 to 25.
Since Ruby wins all of the pairwise matchings, there is no Condorcet cycle.

ii) Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a Borda count rule.
*Under a sincere Borda count, Tramp would get 40*3+35*3+25=250.*
*Clifton would get 40+35*2+25*3=185*
*Ruby would get 35+40*2+25*2=165*
so Ruby would win.

iii) Under a Borda count rule, determine whether any group would prefer to vote insincerely given that the other groups are voting sincerely. Prove your answer.
Note that if the Clifton voters switched their reported preferences over Tramp and Ruby, Tramp would drop to 210 and Ruby would rise to 205 so Clifton would then win.

iv) Suppose the preferences of the Reps changed so that Tramp was preferred to Clifton while Ruby is still most preferred. The Wackos preferences change so that Clifton is preferred to Ruby but Tramp is still their most preferred. Dems preferences staty the same. Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a Condorcet rule.
Consider Tramp v. Clifton. Tramp wins 60 to 40.
Consider Clifton v. Ruby. Clifton wins 65 to 35.
Consider Ruby v. Tramp. Ruby wins 75 to 25.

Ruby would win because Ruby has the fewest losing votes. I gave 3 points for pointing out the cycle and 2 for correctly using the Condorcet Rule for dealing with the cycle.

4. [20 points] An auctioneer has two identical cases of 2000 vintage Bordeaux wine to sell. There are five bidders, each of whom want a single case. The numbers indicate the maximum amount each bidder would be willing to pay for one case of wine (for example, bidder B3 is willing to pay as much as $90.) The auctioneer sells the two cases via a THIRD price auction. That is, all bidders submit (privately) sealed bids. The two bidders who submit the highest bids will each obtain a case and will pay the third highest price.

Bidder	B1	B2	B3	B4	B5
Value	110	135	90	105	150

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i) Prove that each bidder has a dominant strategy and show what it is.
Let B be the third highest bid. A bidder with value V would like to win whenever B is less than V. If she bids less than V, there is a chance that B is between her bid and V and she could have done better by bidding V. If she bids more than V, there is a chance B could be between V and her higher bid and she would have done better by bidding V. Like in second price auctions with single objects, bidding your value in the third price auction is a dominant strategy. I gave 5 points for the strategy and 5 for the CORRECT reasoning.

ii) Characterize the equilibrium of this game assuming bidders select their dominant strategy you characterize in i). Show who wins the cases and what price is paid.
Using i) above, bidders B2 and B5 will win and pay the bid of B1, 110.

iii) Suppose bidder B2 was not in the auction. Determine the equilibrium outcome in this case and compare it to the outcome you found in ii).
If B2 is not in the auction, then B3 will win the second case and the price paid by both bidders will be 105, B4s bid.

5. [15 points] In the matrix game below, let q be the probability that Column player chooses C1 (and of course, $1-q$ is the probability she chooses C2). Assume that players want to maximize the average (or expected) value of their payoffs. (Row player payoffs are shown first).

i) Compute the minimax/maximin equilibrium of this game.
Consider Column player. When Row chooses R1, her payoff is
$20q+36(1-q)=36-16q$.
When Row chooses R2, her payoff is
$23q+16(1-q)=16+7q$.
These are equal at $23q=20$ *or* $q^*=20/23$.
Consider Row player. When Column chooses C1, his payoff is
$26p+23(1-p)=23+3p$.
When Column chooses C2, his payoff is
$10p+30(1-p)=30-20p$.
These are equal at $23p=7$ *or* $p^*=7/23$.

ii) Suppose the payoffs of player 1 rises by 5 in cell R1,C2 and the payoff of player 2 falls by 5. What is the new minimax/maximin equilibrium of this game?

	C1	C2
R1	(26,20)	(10,36)
R2	(23,23)	(30,16)

Consider Column player. When Row chooses R1, her payoff is
$20q+31(1-q)=31-11q$.
When Row chooses R2, her payoff is
$23q+16(1-q)=16+7q$.
These are equal at $18q=15$ *or* $q^*=15/18$.
Consider Row player. When Column chooses C1, his payoff is
$26p+23(1-p)=23+3p$.
When Column chooses C2, his payoff is
$15p+30(1-p)=30-15p$.
These are equal at $23p=7$ *or* $p^*=7/18$.
Row player chooses R1 more frequently. Column player chooses C1 less frequently.
($15/18<20/23$) *even though the payoff to C2 has fallen.*