

Final Exam December 15, 2018
HONORS 259L
Professor Daniel R. Vincent

Please provide reasoning for your answers. If you need to make additional assumptions to arrive at an answer, be explicit what you are assuming. The best answer typically makes the fewest assumptions. Unless otherwise indicated, all subparts are of equal value. This is a closed book, no calculator exam.

1. [15 points] Consider a Simultaneous Multi-Round Auction (SMA). There are five objects, A,B,C,D,E, for sale. Licenses A through D represent 10 eligibility units. License E represents 20. The activity rule is 100% (if you bid on or are provisionally winning bidder (PWB) on licenses worth X activity units in a given round, your eligibility in the next round is X units.) In any round you may not bid on licenses representing more activity units than your current eligibility and you can only use eligibility that is not already committed to a PWB.
 - i) Suppose you currently have 40 units of eligibility and are not the PWB on any license. Which licenses can you bid on in the next round? Provide the full list of possible bids.
 - ii) Suppose instead, that in the previous round you bid on license A and C only and became the PWB on A. What would have to happen over the next few rounds for you to be able to submit a bid on E?
 - iii) Suppose there were 6 bidders in this auction. Each bidder wants a single license and does not care which one. The value that bidders place on licenses are (by bidder) 200, 150, 175, 140, 135, 180. In each round, prices are raised over the previous round price by 1. The auction ends when no new bids are submitted. All bidders know the value of everyone. Provide a prediction of what the equilibrium will be in this auction and justify your prediction.

2. [15 points] The table below shows the ranking of preferences of three groups of voters in an election. Lower numbers reflect a higher ranking.

	Dems (40%)	Reps(35%)	Wackos(25%)
Tramp	4	1	3
Saunders	1	2	1
Baker	2	3	2
Harpo	3	4	4

- i) Does this profile of voting preferences exhibit a Condorcet cycle? If so, show it, if not, prove it.
- ii) Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a slightly modified Borda count rule where bidders submit a number from 1 through 4 and the lowest count wins.
- iii) Under a Borda count rule, determine whether any group would prefer to vote insincerely given that the other groups are voting sincerely. Prove your answer.

3. [10 points] The table below indicates the private value each bidder places on a painting at sale at an English auction. A bid clock starts at \$70 and rises continuously. Each bidder has a button that turns a light on and off. The light indicates the bidder is in the auction. As soon as the bidder turns the light off the bidder is out of the auction and cannot return. The auction and the bid rise stops when a single bidder remains. That bidder wins the painting and pays the price shown on the bid clock.

Bidder	B1	B2	B3	B4	B5
Value	89	76	94	105	75

- i) [5 points] Characterize the equilibrium strategy of each bidder.
 ii) [3 points] What is the equilibrium outcome?
 iii) [2 points] How does this compare to the equilibrium outcome of a second price sealed bid auction?
4. [15 points] Consider a roommate allocation problem where a college wants to match incoming freshman. The match must pair a student from the west, W, with a student from the east, E. The preferences of all the students are given in the table below (most preferred at the top going down to least preferred.)

	W1	W2	W3	W4	E1	E2	E3	E4
1	E4	E4	E4	E4	W1	W2	W4	W4
2	E3	E3	E3	E3	W3	W3	W3	W3
3	E2	E2	E2	E2	W2	W1	W2	W1
4	E1	E1	E1	E1	W4	W4	W1	W2

- i) Compute the outcome from the use of a deferred acceptance algorithm where W students make proposals.
 ii) Compute the outcome from the use of a deferred acceptance algorithm where E students make proposals.
 iii) Which version do E students prefer? Demonstrate this from the outcomes in i) and ii).
5. [15 points] Applying the reasoning we have used in class, find a Nash Equilibrium of this game. The equilibrium may be in pure or in mixed strategies but it may not use weakly dominated strategies. Player 1 selects Rows and Player 2 selects Columns. The first number in each cell denotes the payoff to Player 1 and the second to Player 2.

Player1\Player 2	C1	C2	C3	C4
R1	(5,4)	(3,3)	(3,1)	(7,2)
R2	(4,4)	(7,2)	(2,-1)	(0,0)
R3	(4,6)	(0,5)	(-2,7)	(2,1)
R4	(3,3)	(0,4)	(4,0)	(1,1)
R5	(6,3)	(1,-1)	(-1,1)	(3,6)

6. [10 points] Two very good game theorists, Bob and Alice are trapped in two separate prison cells. Alice can see 6 trees from her cell. Bob can see 3 trees. It is common knowledge that neither can see the other person's trees and that, between them, they see all the trees and that there are either 9 or 11 trees. A devious magician has devised the following scheme: Every day, Alice is first asked are there 9 or 11 trees? She can choose to answer or pass. If she passes, Bob is asked the same question which he can answer or pass. If they both pass, the same situation is repeated the following day. As soon as either answers correctly, both are freed. If either answers incorrectly, they are imprisoned forever. Use back to front reasoning and game theory to show how they can escape for sure.