Final Exam December 15, 2018
HONORS 259L
Professor Daniel R. Vincent

Please provide reasoning for your answers. If you need to make additional assumptions to arrive at an answer, be explicit what you are assuming. The best answer typically makes the fewest assumptions. Unless otherwise indicated, all subparts are of equal value.

1. [15 points] Consider a Simultaneous Multi-Round Auction (SMA). There are five objects, A,B,C,D,E, for sale. Licenses A through D represent 10 eligibility units. License E represents 20. The activity rule is 100% (if you bid on or are provisionally winning bidder (PWB) on licenses worth X activity units in a given round, your eligibility in the next round is X units.) In any round you may not bid on licenses representing more activity units than your current eligibility.

   i) Suppose you currently have 40 units of eligibility and are not the PWB on any license. Which licenses can you bid on in the next round? Provide the full list of possible bids. *You can bid on any individual licenses and any pair or triple of licenses. You can bid on A,B,C,D but you cannot bid on all five.*

   ii) Suppose instead, that in the previous round you bid on license A and C only and became the PWB on A. What would have to happen over the next few rounds for you to be able to submit a bid on E? *If you bid only on A and C, your eligibility must drop to 20. The only way you can bid on E, is if you continue to bid on another license to keep your eligibility (and two licenses if you are bid off of A) AND if, at some point, you are no longer the PWB on any license so that you have 20 free points to bid on E.*

   iii) Suppose there were 6 bidders in this auction. Each bidder wants a single license and does not care which one. The value that bidders place on licenses are (by bidder) 200, 150, 175, 140, 135, 180. In each round, prices are raised over the previous round price by 1. The auction ends when no new bids are submitted. All bidders know the value of everyone. Provide a prediction of what the equilibrium will be in this auction and justify your prediction. *This will work like a 5th price auction. The lowest value bidder (135) should not win. That bidder should bid whenever the minimum acceptable bid on the lowest price license is higher than 135. Once that bidder drops out, there should be no new bids on licenses. Some bidders might accidentally bid on the same license and this could raise the price somewhat for a round or two but since bidders do not care which license they get the five high value bidders should ultimately get a license each at a price close to 135.*

2. [15 points] The table below shows the ranking of preferences of three groups of voters in an election. Lower numbers reflect a higher ranking.

	Dems (40%)	Reps(35%)	Wackos(25%)
Tramp	4	1	3
Saunders	1	2	1
Baker	2	3	2

Harpo	3	4	4

i) Does this profile of voting preferences exhibit a Condorcet cycle? If so, show it, if not, prove it.
T vs S: 35 –65
T vs B: 35 –65
T vs H: 60-40
S vs B: 100 – 0
S vs H: 100 – 0
B vs H: 100 – 0
Since Saunders wins in all pairings, there is no cycle.

ii) Assume all voters vote sincerely according to their preferences. Determine who would be the winner under a slightly modified Borda count rule where bidders submit a number from 1 through 4 and the lowest count wins.
*Tramp earns 4*40+35+3*25=270*
*Saunders earns 40+2*35+25=135.*
*Baker earns 2*40+3*35+2*25=235*
*Harpo earns 3*40+4*35+4*25=360.*

iii) Under a Borda count rule, determine whether any group would prefer to vote insincerely given that the other groups are voting sincerely. Prove your answer.
The only possible switch could be to get more votes for Baker and fewer for Saunders but all groups prefer S to B and would not wish to do this.

3. [10 points] The table below indicates the private value each bidder places on a painting at sale at an English auction. A bid clock starts at $70 and rises continuously. Each bidder has a button that turns a light on and off. The light indicates the bidder is in the auction. As soon as the bidder turns the light off the bidder is out of the auction and cannot return. The auction and the bid rise stops when a single bidder remains. That bidder wins the painting and pays the price shown on the bid clock.

Bidder	B1	B2	B3	B4	B5
Value	89	76	94	105	75

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i) [5 points] Characterize the equilibrium strategy of each bidder.
Let B be the price on the clock. Each bidder keeps the light on until B reaches its value and then it turns it off.

ii) [3 points]What is the equilibrium outcome?
The outcome is for B5 to drop out at 75, B2 to drop out at 76, B1 to drop out at 89, B3 to drop out at 94 and the clock stops. B4 wins at price 94.

iii) [2 points]How does this compare to the equilibrium outcome of a second price sealed bid auction?
In a SPSB auction, all bidders submit bids equal to their value. The highest bidder (B5) wins at the second highest price (94) so the outcome is the same.

4. [15 points] Consider a roommate allocation problem where a college wants to match incoming freshman. The match must pair a student from the west, W, with a student from

the east, E. The preferences of all the students are given in the table below (most preferred at the top going down to least preferred.)

	W1	W2	W3	W4	E1	E2	E3	E4
1	E4	E4	E4	E1	W1	W2	W4	W4
2	E3	E3	E3	E3	W3	W3	W3	W3
3	E2	E2	E2	E2	W2	W1	W2	W1
4	E1	E1	E1	E4	W4	W4	W1	W2

i) Compute the outcome from the use of a deferred acceptance algorithm where W students make proposals.

Round 1. W1,W2,W3, pick E4, W4 picks E1. E4 accepts W3.
Round 2. W1,W2, E3. E3 accepts W2.
Round 3. W1 picks E2. End.
Outcome: (E1,W4), (E4,W3),(E3,W2),(E2,W1).

ii) Compute the outcome from the use of a deferred acceptance algorithm where E students make proposals.

Round 1. E1 picks W1,E2 picks W2,E3 and E4 pick W4. W4 accepts E3
Round 2. E4 picks W3. W3 accepts. End.
Outcome: (E1,W1), (E2,W2),(E4,W3),(E3,W4).

iii) Which version do E students prefer? Can you demonstrate this from the outcome?
There is a general result that the Proposer achieves the best of all stable outcomes. DAA always gives a stable outcome so E prefers the second version. We can see this in the outcomes. E1, E2 and E3 get pairings they prefer while E4 gets the same pairing so some are strictly better off and none are worse off.

5. [15 points] Applying the reasoning we have used in class, find a Nash Equilibrium of this game. The equilibrium may be in pure or in mixed strategies but it may not use weakly dominated strategies. Player 1 selects rows and Player 2 selects Columns, the first number denotes the payoff to Player 1 and the second to Player 2.

Player1\Player 2	C1	C2	C3	C4
R1	(5,4)	(3,3)	(3,1)	(7,2)
R2	(4,4)	(7,2)	(2,-1)	(0,0)
R3	(4,6)	(0,5)	(-2,7)	(2,1)
R4	(3,3)	(0,4)	(4,0)	(1,1)
R5	(6,3)	(1,-1)	(-1,1)	(3,6)

The red entries highlight player 1's best responses, the blue the best responses of player 2. Note there are no cells that are fully colored so there is no pure strategy NE. Note as well that for player 1, R3 is never a best response so we can check that to see if it is dominated. In fact, it is dominated by R5. So eliminate R3. Once R3 is out, C3 is never a BR. Check that and see that it is now dominated by C1. Eliminate C3. Now R4 is never a BR. Check it to see it is dominated by R5. Now C2 is dominated by C1 and eliminate it. This leaves us with R1,R5, *C1, C4 as the only strategies. Observe that these do not have a pure strategy NE. However, they form a constant sum game and we can apply maximin/minimax to the subgame below:*

	C1	*C4*
R1	*(5,4)*	*(7,2)*
R5	*(6,3)*	*(3,6)*

Let p be the probability Player 1 selects R1.
When C1 is selected by Player 2, 1's payoff is
$5p+6(1-p)=6-p$.
When C4 is selected by Player 2, 1's payoff is
$7p+3(1-p)=3+4p$.
These two equations are equal at $p^*=3/5$.

Let q be the probability Player 2 selects C1.
When R1 is selected by Player 1, 2's payoff is
$4q+2(1-q)=2+2q$.
When R5 is selected by Player 1, 2's payoff is
$3q+6(1-q)=6-3q$.
These two equations are equal at $q^*=4/5$.

Thus player 1 selecting R1 with probability 3/5 and player 2 selecting C1 with probability 4/5 is a NE of the game.

6. [10 points] Two very good game theorists, Bob and Alice, are trapped in two separate cells. Alice can see 6 trees from her cell. Bob can see 3 trees. It is common knowledge that neither can see the other person's trees and that, between them, they see all the trees and that there are either 9 or 11 trees. A devious magician has devised the following scheme: Every day, Alice is first asked are there 9 or 11 trees? She can choose to answer or pass. If she passes, Bob is asked the same question which he can answer or pass. If they both pass, the same situation is repeated the following day. As soon as either answers correctly, both are freed. If either answers incorrectly, they are imprisoned forever. Use back to front reasoning and game theory to show how they can escape for sure.
Note that it is common knowledge that no player sees more than 11 trees.
If Alice actually saw 10 or 11 trees, she would know the answer. On the first day, then, she passes. Bob knows she has passed (since otherwise he would not have been asked) it thus becomes common knowledge that Alice sees 9 or fewer trees. If Bob saw 0 or 1 trees, he would know there could not be 11 trees so in that case he would know so then he passes. Thus, it becomes common knowledge that Bob sees 2 or more trees.
If Alice actually saw 8 or 9 trees, since she knows Bob sees 2 or more, she would know the answer would have to be 11. On the second day, then, she passes and when Bob is asked again it becomes CK that she has seen 7 or fewer trees. If Bob saw 2 or 3 trees, he would know there could not be 11 trees so in that case he would know and say 9. Therefore on the second day he would say 9 and they would be freed.