Abstract

Beginning with two Hotelling duopolies where demand for the product in each market is independent of demand for the product in the other, the paper examines the price, profit and welfare consequences that result when first one firm in a market merges with a firm in the other market creating a single two-product firm and then the remaining two firms merge – resulting in a duopoly of two-product firms. The paper demonstrates how to compute the equilibrium in each market structure. Assuming that firms cannot commit not to use all the pricing instruments at their disposal, mixed bundling by two-product firms emerges following each merger. While such behavior is a unilateral best response, the equilibrium consequences of these choices end up lowering total profits and welfare compared to the pre-merger markets suggesting that the opportunity to engage in mixed bundling cannot be the sole motivation for such mergers.

Mixed Bundling and Mergers

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1 Introduction

When a firm adds a new product to its product line, perhaps through innovation or by acquiring another firm, in addition to another revenue stream, it also acquires the opportunity to engage in richer pricing strategies such as offering its products only as individually-priced components, only as a bundle or as both. Adopting the terminology of Adams and Yellen (1976) these three options are known as 'components pricing', 'pure bundling' and 'mixed bundling' respectively. Previous research has shown that the incentive to engage in bundled pricing arises even when the product line of the firm consists solely of independent goods.[1] However, in oligopoly markets, the opportunity to adopt these pricing policies may appear to be a profitable opportunity while ending up as a regrettable temptation.

In this paper, I trace the effects on product offerings, prices, profits and consumer welfare of successive changes in market structure. Starting from two independent differentiated duopolies, a blended market arises when one firm in each of the two markets merge – a so-called conglomerate merger. If a counter-merger by the remaining two firms then occurs, a single duopoly results with two, two-product firms. Following each merger, the newly formed firm adopts mixed bundling as a best response to pricing by the rival firm(s). From a unilateral perspective, a multi-product firm increases its profits by adopting mixed bundling against either two independent single-product firms or another multi-product firm that is engaging in components pricing. However, the equilibrium consequences of this strategy ultimately result in lower profits.

Furthermore, the fall in profits of both firms is even more severe if these remaining firms merge to create a duopoly of multi-product firms. For consumers, the price and welfare consequences of such changes are unevenly distributed. While the price of bundle products falls, when a two-product firm competes against two single product firms, the prices of individual goods rise and the price of the synthetic bundles – those bundles constructed when consumers purchase two goods independently – also rises and, so, these consumers are typically harmed by such mergers. Following a countermerger by the remaining independent firms all prices fall compared to both the market with four single-product firms and the market with a single two-product firm.

These effects have anti-trust implications. Historically, U.S. anti-trust policy has

[1] As demonstrated by various studies of multi-product pricing such as Thanassoulis, (2007) and Armstrong and Vickers (2010).

been relatively uninterested in the effects of conglomerate mergers.[2] This indifference has changed recently. In its review of the ATT-DirecTV merger in 2015, the FCC noted the fact that incumbent cable operators typically provided two products – Multi-channel Video Programming Distribution (MVPD) and residential broadband – and, prior to the proposed merger, the merging parties competed against each of these products singly as well as against bundled offerings by the cable firms. Its consideration of the price effects of the merger included the anticipation that the new formed firm would compete head-to-head on bundled products as well as stand-alone products. Their conclusion was that the resulting competition would lower bundle prices to consumers. The analysis in this paper supports that conclusion but also indicates that another consequence of the merger is a rise in some stand-alone prices, potentially causing harm to some consumers. It also indicates that the opportunity to compete in bundles against their rivals, is a poisoned chalice in that overall profits will fall following the more intense competition.

2 The Model and Related Literature

2.1 Model

There are two *product groups*, 1 and 2. Each product group consists of two differentiated *types*, A and B. Thus, there are four goods, $1A, 1B, 2A$, and $2B$. For example, the two product groups might be broadband and multi-channel video to the home while the types could be coaxial versus fiber for the first and cable versus satellite for the second.

Consumers have unit demand for a single good in each product group (and differentiated preferences over the two types) and, so, will typically buy two goods, a good in group 1 of type A or B and a good in group 2 of type A or B. Consumers are uniformly distributed on the unit square and the two dimensional location, $(x_1, x_2) \in [0,1]^2$ determines a consumer's preference over the four goods. Let $q_j^i \in \{0,1\}$ denote the purchases of a consumer of a good from group j of type i.[3] A consumer located at (x_1, x_2) who makes purchases $q_j^i, i = A, B, j = 1, 2$ and pays a

[2]See, for example, Kolasky (2001), 'After fifteen years of painful experience ....,the U.S. antitrust agencies concluded that antitrust should rarely, if ever, interfere with any conglomerate merger'.

[3]Note that inelastic demand implies that, typically, $q_j^i q_j^{i'} = 0, i \neq i'$ – for most price profiles, we can expect consumers to buy no more than one unit in a product group.

total of m for the purchase, receives utility

$$u(q_1^A, q_2^A, q_1^B, q_2^B, m; x_1, x_2) = \sum_{j=1}^{2} (V^j - t_j x_j)\mathbf{1}_{\{q_j^A > 0\}} + (V^j - t_j(1 - x_j))\mathbf{1}_{\{q_j^B > 0\}} - m. \quad (1)$$

In a Hotelling location interpretation, goods of type A are located at $(0, 0)$ and goods of type B are located at $(1, 1)$ and $x_j, 1 - x_j$ denote the 'distance' the consumer must travel to obtain a good from group j of type A and B respectively. Consider Figure 1. In that case, a consumer with type (x_1, x_2) who purchases product 2 of type A and product 1 of type B obtains utility gross of payments of $V^1 + V^2 - t_1(1 - x_1) - t_2 x_2$. The t_js represent degrees of differentiation within each product group, 1 and 2. These parameters may be different across product groups. For concreteness, assume that good 1 is more differentiated than good 2, so that $t_1 \geq t_2 > 0$. Finally, it is assumed that the gross willingness to pay for a product in each group, V^1, V^2, are high enough that, in equilibrium, all consumers buy one product in each group. That is, the market is covered. This assumption ensures that the equilibria analyzed in this paper have the Hotelling property that the firms compete on the margin for each other's consumers.

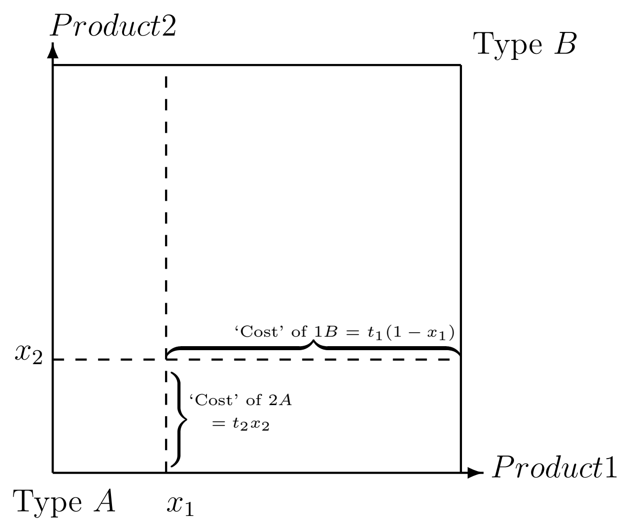

Figure 1: Differentiated A-B Products in Product Markets 1 and 2

Of course, the interpretation of firms' locations is more flexible than the literal model suggests. The 'location' of (say) product $1A$ is not literally the same as the 'location' of product $2A$. All that is required is that a consumer at (x_1, x_2) choosing to purchase product j of type A, incurs a differentiation cost $t_j x_j$ relative to purchasing product j of type B with differentiation cost, $t_j(1 - x_j)$. Indeed, in this model, the only thing that links Firm A's product $1A$ to product $2A$, is the fact that they may be priced together if they are produced by the same firm.

Firms are located either at A or B and the number of firms determines the market structure. If all firms are single-product firms, then there are four firms, each firm producing a different good, $1A, 1B, 2A$, or $2B$.[4] This 'Independent' market is, in fact, two independent Hotelling duopolies. In a 'Blended' market, there are three firms – Firm A produces goods $1A$ and $2A$ and two single-product firms produce $1B$ and $2B$. In a 'Duopoly' market, there are two, two-product firms, one producing $1A, 2A$ and the other, $1B, 2B$. All goods are produced under constant marginal costs. Marginal costs are the same within product group but may differ across product groups. In all equilibria analysed, large enough V_1 and V_2 ensure full market coverage and so, with no loss of generality, costs are normalized to zero and prices can be interpreted as profit margins.

2.2 Related Literature

A number of papers have examined the equilibrium pricing behavior of multi-product firms. Matutes-Regibeau (1988 and 1992) were the first to examine competition in mixed bundle pricing. In the modified Hotelling framework similar to the one employed in this paper, consumers are located on the unit square and purchase two products which are perfect complements – so that one good from each product group is required to enjoy the benefits of consumption. Although the focus of these papers is on design decisions concerning the compatibility of the components, in the event the products are made compatible, firm pricing decisions are in mixed bundling and Matutes-Regibeau characterize equilibrium prices in this scenario.

Chen (1997) also provides an early study of bundling under competition, however, in his model, bundling (by one firm) is motivated by a desire to lessen competition by increasing product differentiation. A consequence is that non-trivial mixed bundling tends to be less profitable for firms as the ability to differentiate each others' product is lessened leading to more vigorous competition. A similar incentive reduces the incentives for both firms to engage in bundling as it increases head to head competition on the bundle. This insight plays a role in the results in this paper as well.

Gans and King (2006) and Thanassoulis (2007) adopt a model of consumer preferences similar to that in this paper, where the product groups are independent in

[4] A firm that produces a single good, say, $1A$ will also be referred to as Firm $1A$, a two-product firm, say, that produces $1A$ and $2A$ will simply be Firm A.

demand (i.e. they replace the assumption of strong complementarity). In Gans and King, there are four firms, two in each product group but bundle pricing can effectively be achieved through pre-competition agreements by pairs of firms. Thanassoulis on the other hand examines a market with two two-product firms directly competing in mixed bundle pricing. An important insight from both of these papers is that, even in the absence of direct complementarity between product groups, when at least one firm engages in mixed bundle pricing, a form of complementarity in demand is generated for rival firm(s) in their need to compete against a bundle price. Armstrong and Vickers (2010) extend the framework of Thanassoulis to allow for elastic demand by consumers and two-part tariffs. In this richer model, per unit prices are set to marginal costs but firms compete in the fixed price component in a fashion very similar to that identified by Thanassoulis for the case of unit demand and linear pricing.

Zhou (2017) examines a model of product differentiation and price competition with much more general preferences, more goods and more firms. However, the focus of that paper is on the consequences of pure bundle pricing.

To the best of my knowledge, this paper is the first to characterize equilibrium behavior in Blended markets where one multi-product firm competes against two single product firms by engaging in mixed bundling. This equilibrium characterization allows us to determine which products are purchased by which consumers and enables us to compare prices, profits and welfare across the three market structures of Independent Goods, Blended and Duopoly.

3 Strategy Spaces, Market Structure and Market Segments

Multi-product firms frequently choose a rich variety of pricing policies. Telecommunications firms offer a la carte pricing as well as bundled pricing for broadband, cell-phone and video services.[5] Insurance firms offer separate pricing for stand-alone

[5]ATT operated its original wireless service well before it completed its acquisition of Cingular Wireless in 2007. Its U-Verse broadband video services was rolled out throughout 2006 and 2007. Nevertheless it was not until August 2009 that it launched its first pricing plan offering cellphone and U-Verse together. A few months after this announcement, Verizon Wireless announced plans to bundle its broadband service FIOS with cellphone plans.'AT&T Plugs Wireless As Part of U-Verse TV Bundle', http://multichannel.com/news/telco-tv/att-plugs-wireless-part-u-verse-tv-bundle/297229, August, 2009. 'Introducing New Service Bundles

policies, as well as discounts for bundling house, car and life insurance policies. Stores with frequent buyer programs offer effective bundled discounts when buyers earn credits based on total dollar purchases.

Firms are assumed to compete by simultaneously choosing prices. Either because consumer types are privately known or because of regulation, it is assumed that prices may not be conditioned on consumer type. For any profile of prices, consumers then select which goods to buy from which firms.[6]

I assume that firms will use all the pricing strategies at their disposal whenever it is unilaterally optimal – that is, they cannot exogenously commit to using fewer pricing strategies than they have available. Furthermore, a firm cannot prevent consumers from constructing their own bundles by buying individual products. This constraint prevents firms from charging consumers who purchase both of their products more than the sum of the component prices. However, firms *can* observe whenever a consumer buys both of its products as a bundle. Such a firm has the capability and possibly the unilateral incentive to offer such consumers a price for the pair of goods that differs (is weakly lower than) the sum of the prices it charges for its individually purchased products. Therefore, in this paper, I assume that the strategy space of two-product firms is a three dimensional real vector, (p_1^i, p_2^i, P^i) – the prices of its two products purchased individually and a separate price for when the products are purchased together by the same consumer, the bundle price. Component pricing is equivalent to price triples where the bundle price equals the sum of the component prices, pure bundling arises when the component prices are so high that no consumer constructs synthetic bundle via individual good purchases, and mixed bundling occurs when some consumers buy bundles and others buy individual goods to form synthetic bundles. Under this assumption, the outcomes of component prices alone or pure bundling alone can occur only if such choices are mutual best-responses for firms with full pricing strategy spaces (outcomes that will be shown in Result 2 and Theorem 1 not to be an equilibrium).[7]

This perspective implies that the strategy space of each firm is governed by the

with Wireless', https://www.verizon.com/about/news/press-releases/verizons-new-quadplay-bundle-offers-customers-wireless-calling-home-phone-tv-and-broadband-moneysaving-combinations, October, 2009.

[6]For a multi-product monopolist, Manelli and Vincent (2006) show that posted price schedules may be dominated by more complex trading mechanisms involving randomization. However, in the case of uniformly distributed types, they show that posted price mechanisms are, in fact, optimal among all mechanisms. In this paper, I only consider posted prices.

[7]Thannassoulis (2007) offers a more detailed argument in support of this position.

market structure. Only in the Independent market, can each firm commit to naming a single price. In the Duopoly market, both firms A and B select price triples and in the Blended market, firm A selects a price triple and each of firms $1B$ and $2B$ select single prices.

With the option of obtaining either a bundle from a single firm or a bundle made up of products from both firms, the market can partition in a variety of ways. The no-monitoring assumption implies that $P^i \leq p_1^i + p_2^i$. If the inequality is strict, sales of the pair of goods of a two-product firm to any single consumer will occur through the bundle price. Even if it is assumed that the market is covered, the market can segment in qualitatively different ways as the next lemma illustrates. Let AB denote the set of consumers who buy good 1 from firm A and 2 from firm B. Sets BA, AA, and BB are defined analogously.

Lemma 1. *Suppose the menu of prices is $\{(p_1^A, p_2^A, P^A), (p_1^B, p_2^B, P^B)\}$ and are such that every consumer buys at least one good from each product group. Define*

$$\begin{aligned} \underline{x}_j &= \frac{t_j - (p_j^A + p_{j'}^B - P^B)}{2t_j} \\ \bar{x}_j &= \frac{t_j + p_{j'}^A + p_j^B - P^A}{2t_j}. \end{aligned}$$

Consumers separate into the intersection of $[0,1]^2$ with the following sets:

(i) $AB = \{(x_1, x_2) | x_2 \geq \bar{x}_2, x_1 \leq \underline{x}_1\}, BA = \{(x_1, x_2) | x_1 \geq \bar{x}_1, x_2 \leq \underline{x}_2\}$.

(ii) $AA = \{(x_1, x_2) | x_2 \leq \bar{x}_2, x_1 \leq \bar{x}_1, x_2 \leq \frac{t_1 + t_2 + P^B - P^A - 2t_1 x_1}{2t_2}\}$.

(iii) $BB = \{(x_1, x_2) | x_2 \geq \underline{x}_2, x_1 \geq \underline{x}_1, x_2 \geq \frac{t_1 + t_2 + P^B - P^A - 2t_1 x_1}{2t_2}\}$.

Proof. The conditions i) through iii) are derived by determining the intersection of half-spaces where each bundle choice dominates. □

Refer to Figure 2. The manifold that separates consumers of (say) the bundle made up of Firm A's product 1 and Firm B's product 2 (the synthetic bundle AB) from those who buy Firm B's bundle (BB) is a vertical line at $\underline{x}_1$ and the manifold that separates consumers of (say) the bundle made up of Firm A's product 1 and Firm B's product 2 (AB) from those who buy Firm A's bundle (AA) is a horizontal line at $\bar{x}_2$. Combining these lines with the lines $x_1 = 0$ and $x_2 = 1$ provide the boundaries of the market segment AB. The market segment BA is defined similarly

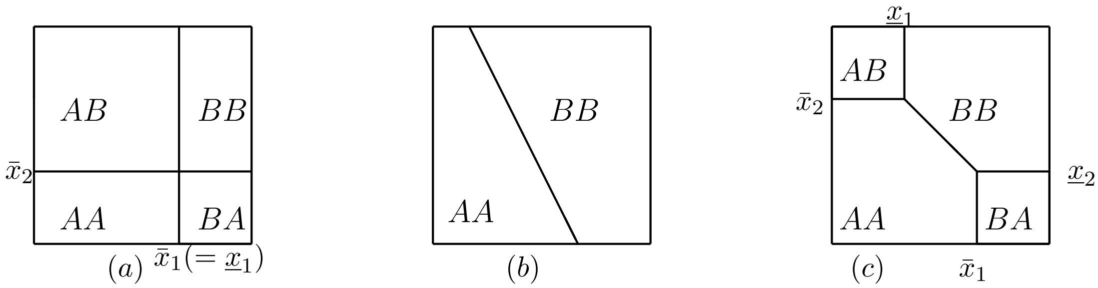

Figure 2: Market Segments: (a)Component Pricing; (b) Pure Bundling; (c) Mixed Bundling.

while the market segments AA and BB partition the remainder and are separated by a line with slope $-t_1/t_2 \geq 1$.

Even when two-product firms have mixed bundling strategy spaces, it is feasible for them to select price triples such that all of component pricing, pure bundling and mixed bundling can emerge. If $P^i = p_1^i + p_2^i$ for both firms, i, then $\bar{x}_j = \underline{x}_j$ and the market partitions into four rectangles corresponding to only independent goods pricing. If, instead, $P^i < p_1^i + p_2^i$ for at least one firm, i, then the set of agents who buy one good from each firm is a rectangle while the set of agents who buy both goods of the same type is a five sided figure. Suppose that $P^A \in [P^B - (t_1 - t_2), P^B + (t_1 - t_2)]$ so the manifold separating the two bundles intersects the top and bottom edge of the support of buyer types. The horizontal line at $\bar{x}_2$ is the lower bound of market segment AB. It approaches one as the component prices, p_1^A and p_2^B increase. When these prices are such that $\bar{x}_2$ is greater than or equal to one, then the set of agents who buy 2 from a B-type firm and 1 from a A-type firm vanishes. If all individual good prices are so high that both of these synthetic bundle sets are empty, the market converges to a pure bundling solution.

Figure 2 illustrates three main different profiles: Figure 2(a) illustrates the case of component (individual goods) pricing (in this case with $p_1^A < p_1^B$ and $p_2^A > p_2^B$); (b) represents the case where only the bundles are purchased, $\{AA, BB\}$; and (c) is the case of Mixed Bundling where at least one firm (say, A), offers a bundle price lower than the sum of the prices, $p_1^A + p_2^A$. [8] Lemma 1 illustrates how the market is partitioned for a given profile of mixed bundle prices. The distribution of consumers along with this partition then generates a demand for each product for each firm corresponding to the measures of each set, $\mu(AB), \mu(AA), \mu(BA), \mu(BB)$. Therefore the profit function for each firm is easily constructed and this function can now be used to determine best responses for the firms in various mixed bundle modes.

[8] It is also conceivable that price profiles are such that all mixed bundles are sold *and* some consumers buy only individual products. This would require (say) p_2^B close to but below V^2, P^A fairly high and p_1^A fairly low. I do not consider equilibria where this might occur.

4 Equilibria in Independent Firm and Duopoly Market Structures

The following results restate previously known conclusions with a few minor extensions.

Result 1. [9] *In the Independent market structure, each market* 1 *and* 2 *is the standard single good Hotelling game with zero costs and uniformly distributed consumers. Equilibrium prices are*

$$p_j = t_j.$$

In this market structure, each market is evenly divided between the two firms and the prices reflect the degree of differentiation of each product. Since, by assumption, each firm can only name a single price, the market segment outcome corresponds to component pricing.

Result 2. [10] *Individual component pricing alone is not an equilibrium in the Duopoly or Blended structures,.*

That independent goods pricing cannot be an equilibrium is a consequence of the insight in McAfee, et. al. (1989), (henceforth MMW) extended to the duopoly case. The MMW model is a multi-product monopoly problem but the same incentives apply. Focus on Firm A. Holding fixed the (individual) pricing of the firm(s) B and starting from any candidate pair of equilibrium individual prices of a multi-product firm A, firm A would like to offer a bundle price equal to the sum of its component prices and raise the price of at least one of the stand-alone prices, say product $1A$. Doing so, loses some consumers at the margin to the rival but since these were selected optimally, this loss is only second-order. The other effect is to shift consumers in AB buying a synthetic bundle into the segment AA, raising firm A's profits (since it earns $p_1^A + p_2^A$ in this segment as opposed to p_1^A alone).

Result 3. [11] *In the Duopoly market,* (V_1, V_2, t_1) *is a symmetric (pure bundling) equilibrium in weakly dominated strategies.*

[9] See, for example Tirole 1988, p. 280. The 'high enough values' assumption ensures that firms are on a competitive margin.

[10] See, for example, Thannassoulis (2006) or Armstrong and Vickers (2010). The argument for the Blended structure is minor modification of Gans and King (2006).

[11] See, for example, Thannassoulis (2006) or Armstrong and Vickers (2010).

Although pure bundling can emerge as an equilibrium in the duopoly model, Armstrong and Vickers (2010) argue that it is implausible since the equilibrium requires the use of weakly dominated strategies. The equilibrium arises via a coordination failure – neither firm is induced to offer stand alone prices because if the other is not, no sales via synthetic bundles can occur. If, instead, (say) Firm A were to offer (t_1, t_1, t_1), there exists an offer by Firm B, for example,$(t_1 - \epsilon, t_1, t_1)$ which would induce more sales via synthetic bundles than via pure bundling and generate the same revenue per sale. Thus, while this profile would be ineffective against the proposed equilibrium prices of the rival, it nevertheless weakly dominates the equilibrium bundling profile. In the next section it is shown that a similar logic will rule out pure bundling even as a weakly dominated equilibrium in the blended market structure though the argument is somewhat more intricate.

In addition to this equilibrium, there typically also exists an equilibrium with fully mixed bundle pricing. Result 4 characterizes necessary conditions for a symmetric equilibrium in non-trivial mixed bundle pricing.[12]

Result 4. [13] *In the Duopoly Market Structure, if* (p_1, p_2, P) *is a symmetric equilibrium of the symmetric mixed bundle pricing game with positive sales of all products, then, defining the bundle discount,* $\Delta = p_1 + p_2 - P$*:*

$$\Delta = (3(t_1 + t_2) - \sqrt{9(t_1 + t_2)^2 - 32t_1t_2})/8. \tag{2}$$

$$P^A = \frac{t_1 + t_2 - 2\Delta}{2t_1t_2 - 2\Delta^2} 2t_1t_2. \tag{3}$$

$$p_i^A = \frac{(t_1 - \Delta)(t_2 - \Delta)}{t_1 + t_2 - 2\Delta} + P^A \frac{t_i - \Delta}{t_1 + t_2 - 2\Delta}, i = 1, 2. \tag{4}$$

Note that equations (2), (3) and (4) can be solved recursively to yield the equilibrium for any values of t_1, t_2.

Result 2 indicates that when duopoly firms have or acquire the ability to market their multiple competing products as a cheaper bundle, then each firm has an incentive to offer its products as a bundle. In simple Hotelling type models where the market is covered, the only determinant of efficiency is through the optimal allocation of differentiated products among consumers with different tastes. If a symmetric equilibrium exists in the independent goods pricing game, then this equilibrium achieves

[12] Armstrong and Vickers(2010) shows that these conditions are sufficient if $t_1 = t_2$.

[13] Armstrong (2008) offers a slightly different representation of this equilibrium. A proof of this result is offered in the Appendix as it provides insights into the results for the Blended market.

the social optimum given the location of firms. Thus, when one or both firms are able to engage in mixed bundling, Results 2,3 and 4 demonstrate that any equilibrium of this new game must result in a loss of efficiency relative to the independent goods pricing game. If the acquisition of a firm producing $1A$ by a firm that produces $2A$ – a merger of horizontally unrelated firms – introduces this new pricing dimension, overall surplus declines.

The results also underline the significance of modeling the strategic mode as either endogenous or exogenous. In Matutes and Regibeau (1992), it is assumed that firms can first commit to a particular strategic mode and then, conditional on that commitment, a pricing game ensues. In Gans and King (2006) it is assumed that cooperating firms first commit to a bundle discount and then compete on individual prices. Since individual pricing typically generates higher prices when the competition is Hotelling-like, both papers conclude that multi-product firms will often unilaterally choose the individual pricing mode. However, while this outcome seems plausible in the case of four independently optimizing firms, it is not obvious how competing multi-product firms could make such a commitment credible. Result 2 illustrates that in the absence of such a strong ability to commit, if multi-product firms are not restricted from mixed bundling, we should expect them to utilize it whenever it is in their unilateral best interest.

5 Mixed Bundling in Blended Market Structures

What results if a multi-product firm faces two separate producers of competing products and engages in mixed bundling? In this structure, the pure bundle price outcome cannot emerge because Firms $1B$ and $2B$ are not offering a bundle price. Their best responses will be to offer prices for their individual goods even if the integrated rival does not offer its goods independently. Theorem 1 demonstrates that, in this structure, no equilibrium exists where the two-product firm chooses pure bundling.

Theorem 1. *In the Blended market, there is no equilibrium in which the two-product firm chooses pure bundling.*

Proof. See Appendix 2. □

The logic of the proof of Theorem 1 is similar to the argument Armstrong and Vickers (2010) employ to show that, in the Duopoly market, pure bundling involves weakly dominated strategies, however, the proof is somewhat more complicated since this structure generally results in asymmetric outcomes. On the other hand, even weakly dominant outcomes are ruled out because, by definition of the market structure, at least two firms always offer individual goods prices.

Result 2 applies in this structure as well, though, also ruling out individual goods pricing as an equilibrium. The integrated firm will wish to offer mixed bundles prices even though the rival does not. Given that the integrated firm is engaging in non-trivial mixed bundling, the independent goods firms are at a disadvantage because of an induced double-marginalization. Even though their products are independent in consumption, when Firm A offers a bundle price for its two goods, a frontier emerges where some consumers are deciding between the synthetic bundle, BB and the actual bundle AA. Firm $1B$, for example, balances the margin it enjoys on the set of consumers $BA \cup BB$ against loss of sales on that competitive frontier. Firm $2B$ does a similar balance but on the set $AB \cup BB$. The failure of the independent firms to internalize the impact of their pricing on the profits of the other induces higher prices than would be offered by an integrated pair.[14]

Equilibria in the Blended market are generally difficult to characterize explicitly. Profit functions are neither quasi-concave nor supermodular, so standard existence results cannot be applied. However, the restriction to uniformly distributed consumer types enables the representation of conditions that must be satisfied in equilibrium. The following Theorem 2 demonstrates how to reduce the solution of the necessary conditions for equilibrium into three simultaneous equations which can then be solved by standard computational software such as Mathematica.

Theorem 2. *Define*

$$\Delta^A = p_1^A + p_2^A - P^A,$$

$$A_i = \left(\frac{t_i - \Delta^A + p_i^A}{2} + \Delta^A \frac{t_j + p_j^A}{4t_j} - \frac{{\Delta^A}^2}{8t_j} \right),$$

for $i = 1, 2, i \neq j$. A necessary condition for an equilibrium of the Blended market mixed bundle pricing game is

$$p_i^B = 4t_i \frac{4t_j A_i - \Delta^A A_j}{16 t_1 t_2 - {\Delta^A}^2}, i = 1, 2, \tag{5}$$

[14]Ironically, this distortion ends up actually helping the independent firms. As noted in the next section, even if the two firms could merge and commit to independent goods pricing and, in this way, eliminate the double-marginalization distortion, the lower BB price induces such a vigorous counter-response by Firm A that overall profits are lower.

and

$$0 = (1-\bar{x}_2)\underline{x}_1 - p_1^A \frac{1-\bar{x}_2}{2t_1} + (P^A - p_1^A)\frac{\underline{x}_1}{2t_2} \tag{6}$$

$$0 = (1-\bar{x}_1)\underline{x}_2 - p_2^A \frac{1-\bar{x}_1}{2t_2} + (P^A - p_2^A)\frac{\underline{x}_2}{2t_1} \tag{7}$$

$$0 = \bar{x}_2\bar{x}_1 - \frac{\Delta^{A^2}}{8t_1t_2} + p_1^A\frac{\underline{x}_1}{2t_2} - p^A\frac{t_1+t_2+p_1^B+p_2^B-P^A}{4t_1t_2} + p_2^A\frac{\underline{x}_2}{2t_1} \tag{8}$$

Proof. See Appendix 2. □

Observe that while the above system, formally, is a system of five non-linear equations in five unknowns, the two equations in (5) yield explicit expressions for p_1^B, p_2^B in terms of P_1^A, p_2^A, P^A. Thus, this system can be solved recursively by directly substituting the expression in (5) into (6),(7) and (8) and solving for P_1^A, p_2^A, P^A. The resulting three-equation, three-variable system is highly non-linear and yields many solutions. However, by focusing on real, subadditive solutions ($p_1^i + p_2^i \leq P^i$) and symmetric solutions for the case $t_1 = t_2$ a single feasible solution arises. (For the asymmetric case, the solution that converges to the symmetric solution as t_2 approaches t_1 is selected.)

These expressions, along with the equilibrium characterizations in Results 1 and 4 are used in the next section to illustrate the effects of changes in market structure from independent firms, to a Blended market and from there to Duopoly.

6 Consequences of Mergers

The figures referred to in this section are in Appendix 1.

6.1 Price Effects

The equilibrium prices for individual products, for synthetic bundles and for marketed bundles all vary with the absolute and relative degrees of differentiation (setting $t_1 = 1$, the latter can be represented by variations in t_2) as well as with the market structure. Recall from Result 1 that in the Independent market structure we have $p_j^i = t_j, j = 1,2$ and the price of the (necessarily) synthetic bundle is $P^i = t_1 + t_2$ for $i = A, B$. The prices in the other two market structures can be computed using the explicit formulae provided in Result 4 and Theorem 2.

Figures 3 and 4 show the behavior of the equilibrium price of the bundled product, across the Independent and Blended market structures under symmetry ($t_1 = t_2$) and asymmetry ($t_1 = 1 \geq t_2$). $Pind$ denotes the sum of the prices of the two-good groups in the Independent market. The variable P^B describes the cost of buying the two B type goods – formally this is also a synthetic bundle but the variables $PSynBlended, PSynBlendedAB, PSynBlendedBA$ refer to the cost of buying one product from A and one from B.[15] Focusing first on the effects of AA and BB prices, in both the symmetric and asymmetric cases, the impact of the merger of the A firms is to lower these prices. The (true) bundle price P^A falls even more than the price of the BB good. In contrast, the prices of the synthetic bundles AB and BA increase following the merger as an MMW-type effect induces Firm A to raise the prices of its stand-alone good. The asymmetric case allows us to distinguish different price effects on the synthetic bundles. The bundle AB involves purchases of the more differentiated product from the two-product firm. This allows Firm A to raise its component price of $1A$ by more than $2A$ and results in a higher price rise in total of AB than of BA, though both prices rise.

Figures 5 and 6 show the effects on bundle prices of a counter-merger by the B firms from the Blended market moving to Duopoly in both symmetric and asymmetric markets. In both cases, the prices of the synthetic bundles AB and BA and the AA and BB bundles fall with the move from the Blended to the Duopoly market structure. The decrease in price of the BB good is particularly strong. This is due to two effects. The merger reduces the double marginalization effect from the Blended market and the new two-product firm cannot resist the temptation to engage in mixed bundling. In doing so, it competes directly with Firm A's bundle. It is well-known that bundled products have a more competitive margin and this more intense competition generates a progression of competitive responses.[16]

Figures 7, 8 and 9 describe the behavior of the equilibrium price of the independently priced products, across market structures first in the case of symmetrically differentiated goods ($t_1 = t_2$) as products become more differentiated and then for the case of asymmetrically differentiated products ($t_1 = 1 \geq t_2$). Since all consumers purchase a unit each of product group 1 and 2 these prices are not especially relevant except insofar as they help to explain the effects on the total price of the pair of products. However, it is informative to note that compared to the Independent market, in the Blended market, the two-good firm raises the stand-alone prices of its products in order to encourage purchases of its bundle while the two single product firms reduce their prices. Consider Firm $2B$. In the Independent market, its price

[15] In the symmetric case, it is irrelevant whether the pair is AB or BA.

[16] See Matutes and Regibeau (1992), for example.

reacts as a strategic complement to the price of $1B$ and is independent of the other two prices. In the Blended market, products AA and BA are on its competitive margin so in the pricing game, the prices of these bundles act as strategic complements while the component prices, p_1^A and p_1^B are now strategic substitutes. The price of AA falls and the price of BA ($p_1^B + p_2^A$) rises. The former falls significantly more than the latter increases since Firm A fully internalizes the market gain from a decrease in that price. Additionally, since p_1^A (a strategic substitute) rises as Firm A attempts to draw market share from AB to AA (the MMW effect), an additional incentive on $1B$ to reduce its price to compete on the $AB - AA$ margin arises. The net effect is to draw down the component prices of $1B$ and $2B$. Interestingly, as can be seen in Figure 8, if the differentiation parameter is significantly different across the two product groups, a counter-merger from a Blended Market to Duopoly results in a *fall* in the component price of the more competitive good (p_2).

6.2 Effects on Profits

Figures 10 and 11 show how firm profits vary across market structures. In the cases of the independent goods market structure and the blended market structure, the sum of the (identical) firms pre-merger profits are shown. In the move from Independent to Blended market structure, the profits of the merging firms $1A$ and $2A$ fall slightly and the unmerged firms' profits fall significantly. In the shift from the Blended market structure to the Duopoly structure, again, both A and B firm profits fall. This impact on the B firms stems from their inability to commit not to compete in mixed bundling. Competition in bundles has been shown to be much more aggressive than individual goods competition and the induced response by its rival results in lower prices and profits. Interestingly, though, even if the B firms were able to commit not to bundle their products post merger, it can be shown that their profits would still fall in equilibrium. The reason is that the merger, in enabling them to eliminate the double-marginalization on the BB goods, in itself reduces the cost of these goods, even if priced stand alone. The subsequent response of Firm A on its bundle price ultimately makes even this merger with commitment unprofitable for the B firms.

The figures demonstrate that total profits fall as the market structure changes from Independent to Blended to Duopoly both for the merging parties and the other parties. Thus, the opportunity to engage in mixed bundling alone does not provide an incentive for firms to merge either unilaterally or as a counter-response. Still, a merger of the independent firms can arise for reasons other than for pricing purposes.[17] Unless firms are able to commit not to engage in mixed bundling, any advantages of a merger should be weighed against the costs that this acquired strategy imposes.

6.3 Total and Consumer Welfare Effects

If the market covered assumption holds in all cases, then total welfare relies solely on the partition of buyer types into the various bundle profiles, AA, AB, BA, BB. The profile associated with the symmetric equilibrium of independent goods pricing, maximizes the welfare available from product variety and is, therefore, optimal among all profiles considered here. Pure bundle prices would be the worst symmetric outcome from the perspective of product variety, however, such an outcome appears unlikely in view of Result 3 and Theorem 1. The mixed bundle pricing that emerges from the Duopoly market is generally intermediate between the two. It offers some additional product variety over pure bundling but not as much as independent goods pricing. The impact on total welfare of a move from a Blended market to a Duopoly is ambiguous. Restricting attention to the symmetric case, $t_1 = t_2$, in a Blended market, the segments AB and BA are larger which generally leads to higher overall welfare because of the increase in availability of differentiated products. However, since bundle prices are different across B firms and the A firm in a Blended market, this discrepancy induces a smaller than optimal BB segment vis-a-vis the AA segment.

Consumer welfare effects are the opposite of total welfare effects when comparing individual goods pricing with the mixed bundling equilibrium in Duopoly. The individual goods prices are strictly lower in the latter in every case except for when product group two is very competitive (in which case, the component price of the less competitive good is slightly higher) but even in that case, the prices of all bundles (AA,BB,AB and BA) are lower under Duopoly. Similarly, consumers are unambiguously better off in the Duopoly market compared to the Blended market as the prices of all pairs of goods fall. The comparison is more ambiguous when comparing the Independent market with the Blended market. Consumers who purchase the AA or BB pair enjoy lower prices in the Blended market but those who consume the AB or BA pair do better in the Independent market.

[17] Or, alternatively, firms may not anticipate the full equilibrium consequences of a merger.

7 Conclusion

Not all multi-product firms have an incentive to engage in mixed bundling. It is unlikely that when General Electric owned both NBC and General Electric Aviation it had any incentive to bundle prices for commercials on Saturday Night Live with jet engines. However, when the same consumers can potentially buy the different products, even when the preferences for the two goods are independent of each other, the scope and incentive for price discrimination arises and thus for mixed bundling.

Multi-product competition with differentiated products generates profit functions that are intrinsically neither supermodular nor quasi-concave. Thus, our standard tools for equilibrium analysis – lattice-based and topological fixed point theorems – are not that usable in these models. The importance and ubiquity of multi-product competition point to the need for additional tools to aid equilibrium analysis.

In the absence of such tools, the environment examined in this paper is necessarily special. Attention is restricted to two firms with two products and to a market where, because of the Hotelling feature, total sales are assumed constant. Furthermore, the ability to compute explicit equilibrium results required assuming uniformly distributed consumers. Nevertheless, I believe the model offers some important insights that are likely to extend to broader environments. Result 2 holds for any pair of independent distributions possessing a density – the MMW intuition that mixed bundling is generally strictly more profitable extends to imperfectly competitive, multi- product firms. If products are differentiated, this behavior imposes potential welfare costs as the resulting pricing induces consumers to select into a suboptimal allocation of variety. Since mixed bundled pricing is, in theory, at least, available to any multi-product firm, this effect should be an additional consequence considered when assessing the implications of mergers, even of horizontally unconnected firms.

Insights from the blended market model are also likely to extend beyond the special case. If two independent firms compete in different markets against the same multi-product firm, then even though their products are not intrinsically complements, they become complements through the mixed bundled pricing of the rival. This puts the independent firms at a disadvantage in the market place because of what might otherwise have been an unexpected source of double- marginalization.

8 Appendix 1: Computational Figures

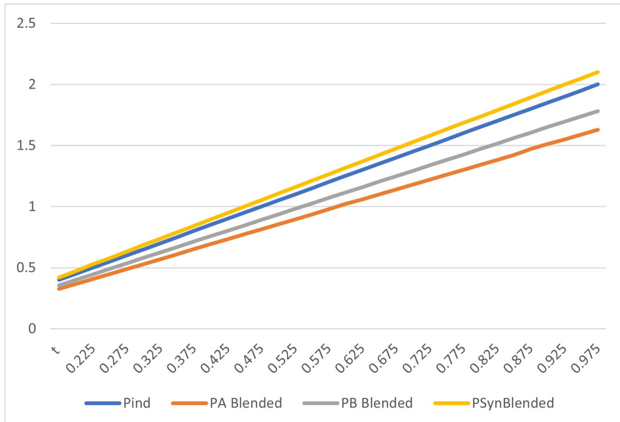

Figure 3: Effect on Bundle Prices in Independent Goods and Blended Markets: $t_1 = t_2 = t$

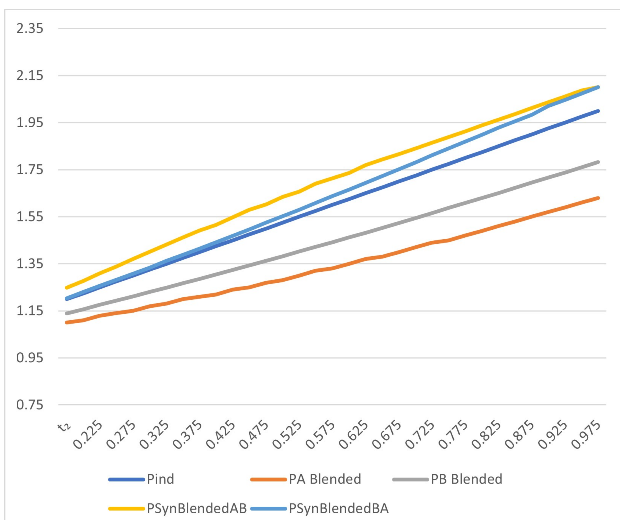

Figure 4: Effect on Bundle Prices in Independent Goods and Blended Markets: $t_1 = 1 \geq t_2$

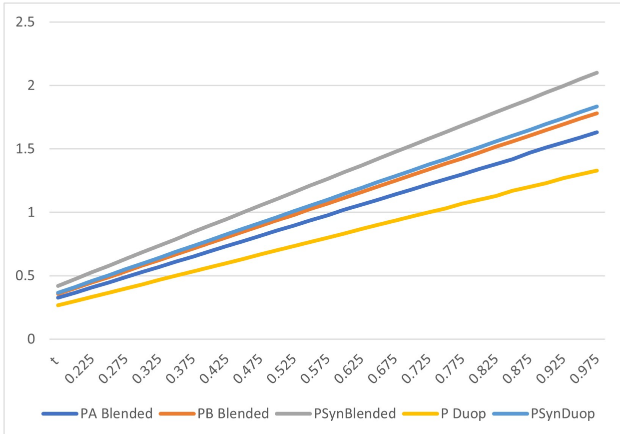

Figure 5: Effect on Bundle Prices in Blended and Duopoly Markets: $t_1 = t_2 = t$

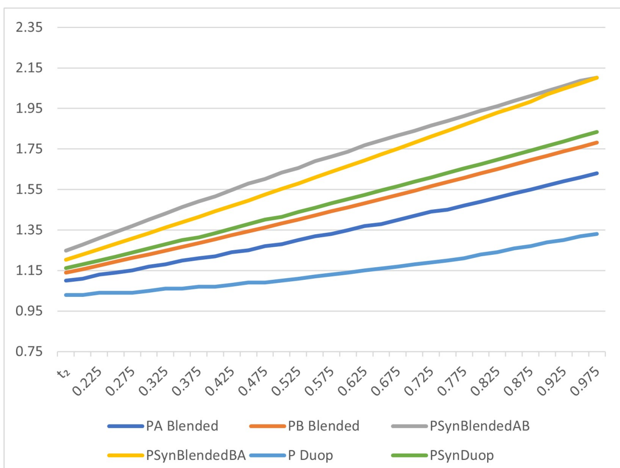

Figure 6: Effect on Bundle Prices in Blended and Duopoly Markets: $t_1 = 1 \geq t_2$

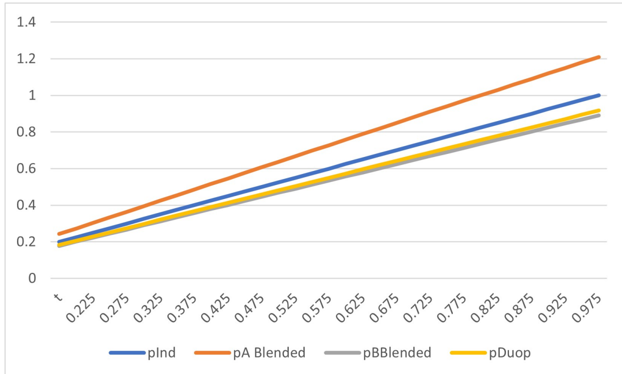

Figure 7: Individual Price Effects of Mergers ($t_1 = t_2 = t$)

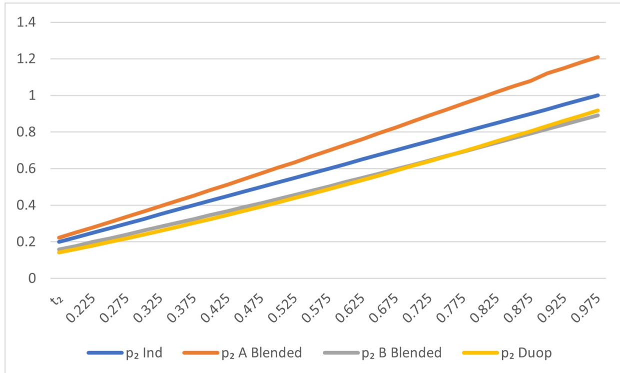

Figure 8: Effects of Mergers on Individual Goods Prices ($t_1 = 1 \geq t_2$): p_2

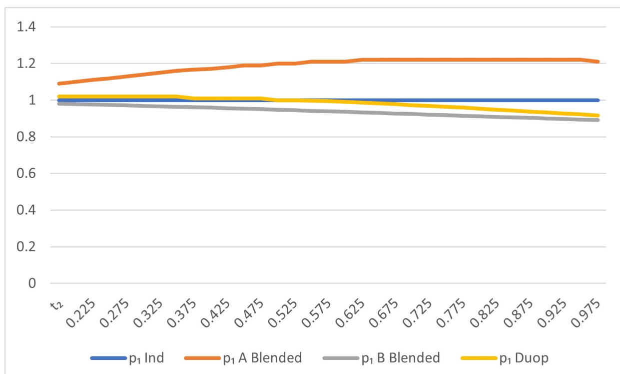

Figure 9: Effects of Mergers on Individual Goods Prices ($t_1 = 1 \geq t_2$): p_1

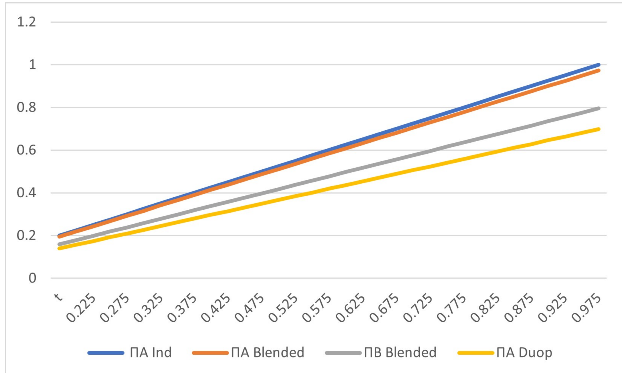

Figure 10: Profit Effects of Mergers ($t_1 = t_2 = t$)

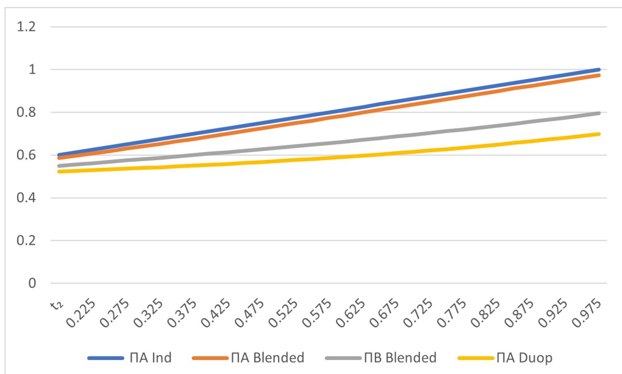

Figure 11: Profit Effects of Mergers ($t_1 = 1 \geq t_2$)

9 Appendix 2: Proofs

9.1 Proof of Result 4

For a fixed vector of prices, $(p_1^A, p_2^A, P^A, p_1^B, p_2^B, P^B)$, assuming all consumers buy all bundles, the market is partitioned as given in Lemma 1. The uniform distribution then gives the measures of the four market segments as the area of the sets AB, AA, BB, BA:

$$\mu(AB) = (1-\bar{x}_2)\underline{x}_1; \mu(BA) = (1-\bar{x}_1)\underline{x}_2; \mu(AA) = \bar{x}_2\bar{x}_1 - (\bar{x}_2 - \underline{x}_2)(\bar{x}_1 - \underline{x}_1)/2.$$

These definitions yield the following:

$$\begin{aligned}
\frac{\partial \mu(AB)}{\partial p_1^A} &= -\frac{(1-\bar{x}_2)}{2t_1} - \frac{\underline{x}_1}{2t_2} \\
\frac{\partial \mu(AB)}{\partial P^A} &= \frac{\underline{x}_1}{2t_2} \\
\frac{\partial \mu(AA)}{\partial p_1^A} &= \frac{\underline{x}_1}{2t_2} \\
\frac{\partial \mu(AA)}{\partial P^A} &= -\frac{\bar{x}_1}{2t_2} - \frac{\bar{x}_2}{2t_1} + \frac{\bar{x}_1 - \underline{x}_1}{4t_2} + \frac{\bar{x}_2 - \underline{x}_2}{4t_1} \\
&= -\frac{t_1(\bar{x}_1 + \underline{x}_1) + t_2(\bar{x}_2 + \underline{x}_2)}{4t_1t_2}.
\end{aligned}$$

Using the fact that

$$t_j(\bar{x}_j + \underline{x}_j) = (2t_j + P^B - P^A + p_j^B + p_i^A - p_j^B - p_i^B)/2,$$

we can also write

$$\frac{\partial \mu(AA)}{\partial P^A} = -\frac{t_1 + t_2 + P^B - P^A}{4t_1t_2}.$$

Firm A's profit function is

$$\Pi(p_1^A, p_2^A, P^A) = p_1^A\mu(AB) + P^A\mu(AA) + p_2^A\mu(BA). \tag{9}$$

Thus the first order condition for p_1^A is

$$0 = \mu(AB) - p_1^A\frac{1-\bar{x}_2}{2t_1} + (P^A - p_1^A)\frac{\underline{x}_1}{2t_2} \tag{10}$$

A symmetric condition holds for p_2^A. The first order condition for P^A is

$$0 = \mu(AA) + p_1^A\frac{\underline{x}_1}{2t_2} - P^A\frac{t_1 + t_2 + P^B - P^A}{4t_1t_2} + p_2^A\frac{\underline{x}_2}{2t_1} \tag{11}$$

Assuming a symmetric solution, $p_j^A = p_j^B, P^A = P^B, \underline{x}_j = 1 - \bar{x}_j$, $\mu(AB) = \mu(BA)$ and $\Delta = p_1^A + p_2^A - P^A$. Summing the first order conditions for p_1^A, p_2^A then eliminates the p_j^As yielding an expression only in terms of Δ:

$$(t_2 - \Delta) + (t_1 - \Delta) = 2(t_2 - \Delta)(t_1 - \Delta).$$

This is a convex quadratic in Δ (and therefore p_j^A) so necessary second order conditions imply selecting the smallest of the roots. Substituting into (10) for p_1^A and (11) for P^A, yields a necessary conditions for a symmetric equilibrium. If $t_1 = t_2 = t$, then the solution for $\Delta = t/2$. This then gives $p_j = 11t/12 + c_j, P = 8t/6 + c_1 + c_2, \underline{x}_j = (1 - \bar{x}_{j'}) = 1/4$ and firm profits are $(2/3 + 1/32)t = .698t$.

9.2 Proof of Theorem 1

Suppose that Firm A offers its product solely as a bundle at price P^A and no synthetic bundling occurs.[18] Define $P^B = p_1^B + p_2^B$ and $\delta = P^B - P^A$. This implies that the market is partitioned by the partition of the unit square determined by intersection of the line

$$x_2 = \frac{t_1 + t_2 + \delta}{2t_2} - \frac{t_1}{t_2} x_1.$$

and the unit square.

The partition could occur in three ways: a) the dividing manifold could intersect the left edge and the bottom of the square ($\delta < t_2 - t_1$); b) the manifold could intersect the top and bottom of the square, ($\delta \in [t_2 - t_1), t_1 - t_2)$), or; c) the top of the square and the right edge of the square ($\delta \geq t_1 - t_2$).

Case a) can be shown never to be an equilibrium since in that case, Firm A's bundle price exceeds the sum of the prices of the independent firms and Firm A always prefers to lower its price to capture more market.

In Case b), suppose $\delta = t_1 - t_2 - \gamma, \gamma > 0$. The firms' first order conditions can be solved directly to yield

$$\begin{aligned} P^A &= \frac{5}{4} t_1 \\ p_i^B &= \frac{3}{4} t_1, \end{aligned}$$

and this case can only arise for $t_2 \in (0, \frac{3}{4} t_1]$.

[18] This could be ensured if there were no free disposal by consumers though as will be seen, this assumption is not necessary to rule out the equilibrium

Suppose instead of offering only the bundle price Firm A offers the equilibrium bundle price and a stand-alone price for product $2A$ of

$$p_2^A = t_2 + p_2^B - 2t_2\epsilon.$$

At this price, consumers in the lower right corner of the unit square will choose to form the synthetic bundle BA since a consumer of type $(\frac{t_1+t_2+\delta}{2t_1}, 0)$ was previously just indifferent between buying the two goods from A and buying the two goods from B. This segment of the market is the rectangle $[x_1^\epsilon, 1] \times [0, \underline{x}_2]$ where

$$\begin{aligned} x_1^\epsilon &= \frac{t_1 + t_2 + \delta - 2t_2\epsilon}{2t_1} \\ &= 1 - \frac{\gamma + 2t_2\epsilon}{2t_1} \end{aligned}$$

and, from Lemma 1,

$$\begin{aligned} \underline{x}_2 &= \frac{t_2 - (p_2^A + p_1^B - P^B)}{2t_2} \\ &= \frac{t_2 - (t_2 + p_2^B - 2t_2\epsilon + p_1^B - P^B)}{2t_2} \\ &= \epsilon \end{aligned}$$

since $P^B = p_1^B + p_2^B$. This rectangle has area $\epsilon(1 - x_1^\epsilon) = \frac{\gamma\epsilon + 2t_2\epsilon^2}{2t_1}$ The intersection of this segment with the segment Firm A was selling under pure bundling is a triangle with vertices

$$\{(x_1^\epsilon, 0), (x_1^\epsilon, \epsilon), (\frac{t_1 + t_2 + \delta}{2t_1}, 0)\}$$

where $\frac{t_1+t_2+\Delta}{2t_1} - x_1^\epsilon = \frac{t_2}{t_1}\epsilon$ so its area is $\frac{t_2}{2t_1}\epsilon^2$. Thus, the deviation generates a gross gain of

$$p_2^A \frac{\gamma\epsilon + 2t_2\epsilon^2}{2t_1}$$

and a gross loss from cannibalized sales of the bundle of

$$P^A \frac{t_2}{2t_1}\epsilon^2.$$

For small ϵ, the gain dominates the loss and this deviation is profitable.

Case c) occurs for $t_2 \in [\frac{3}{4}t_1, t_1]$. In this case, BB is a triangle in the upper right corner of the unit square with vertices

$$\{(\underline{x}_1^c(\delta), 1), (1, \underline{x}_2^c(\delta), (1, 1)\}$$

where

$$\underline{x}_1^c(\delta) = \frac{t_1 - t_2 + \delta}{2t_1},$$

and

$$\underline{x}_2^c(\delta) = \frac{t_2 - t_1 + \delta}{2t_2}.$$

The first order conditions for $1B, 2B, A$ imply

$$P^B = \frac{t_1 + t_2 + P^A}{2}$$

and

$$0 = 8t_1t_2 - (P^B)^2 - 2P^BP^A.$$

Combining the two equations from the FOCs gives

$$0 = 5(P^B)^2 - 2(t_1 + t_2)P^B - 8t_1t_2$$

which implies

$$P^B = (2(t_1 + t_2) + \sqrt{4(t_1 + t_2)^2 + 160t_1t_2})/10.$$

The candidate equilibrium price P^B is increasing in t_2 so it reaches its maximum at $t_2 = t_1$. At that value, $P^B = 1.76t_1$ and assuming symmetry, $p_1^B = .86t_1$. Thus, in this region, any optimal $p_1^B < t_1$.

Suppose Firm A offers instead of the pure bundling solution, the bundle price P^A and a stand-alone price, $p_2^A = P^A$ (or equivalently, suppose a consumer can purchase the bundle, AA and discard $1A$ costlessly.) Applying the definitions in Lemma 1, if $\bar{x}_1 < 1$, the market segment BA will have positive measure.

$$\begin{aligned}
\bar{x}_1 &= \frac{t_1 + p_2^A + p_1^B - P^A}{2t_1} \\
&= \frac{2t_1 + p_1^B - t_1}{2t_1} \\
&= 1 - \frac{t_1 - p_1^B}{2t_1} \\
&< 1.
\end{aligned}$$

Since this implies (using Lemma 1)

$$
\begin{aligned}
\underline{x}_2 &= \frac{t_2 + p_1^B + p_2^B - P^A - p_1^B}{2t_2} \\
&= \frac{t_2 + p_2^B - P^A}{2t_2} \\
&= \frac{t_2 - p_1^B + p_1^B + p_2^B - P^A}{2t_2} \\
&\geq \frac{t_2 - t_1 + \delta}{2t_2} \\
&= \underline{x}_2^c,
\end{aligned}
$$

and $p_2^A = P^A$, Firm A obtains the same revenue per sale in market segment BA and the union of the new segments AA and BA strictly contains the old AA. Thus, this deviation is profitable and breaks the candidate equilibrium.

9.3 Proof of Theorem 2

The market partitions are as outlined in Lemma 1, however, since the price of bundle BB is the sum of the standalone prices, p_j^B, the definitions of the boundaries between BB and AB or BA are simplified somewhat to

$$\underline{x}_j = \frac{t_j + p_j^B - p_j^A}{2t_j}.$$

The upper boundary, $\bar{x}_j$ is the same as before. For convenience, set $\Delta^A = p_1^A + p_2^A - P^A$ and note that

$$
\begin{aligned}
\bar{x}_j - \underline{x}_j &= \frac{(t_j + p_i^A + p_j^B - P^A) - (t_j + p_j^B - p_j^A)}{2t_j} \\
&= \frac{p_i^A + p_j^B - P^A + p_j^A - p_j^B}{2t_j} \\
&= \frac{p_i^A + p_j^A - P^A}{2t_j} \\
&= \frac{\Delta^A}{2t_j}
\end{aligned}
$$

Given the uniform distribution, the measures of the sets are computed in the same way as for the mixed bundling two-firm market. Now, however, the sales of (say) product $2B$, then is the sum $\mu(BA) + \mu(BB)$. This yields

$$\begin{aligned}
\mu(AB) + \mu(BB) &= (1 - \bar{x}_2) + (\bar{x}_2 - \underline{x}_2)(1 - \underline{x}_1) - \frac{(\bar{x}_1 - \underline{x}_1)(\bar{x}_2 - \underline{x})}{2} \\
&= (1 - \bar{x}_2) + \frac{\Delta^A(1 - \underline{x}_1)}{2t_2} - \frac{\Delta^{A2}}{8t_1t_2}
\end{aligned}$$

Only the first term depends on p_2^B so

$$\begin{aligned}
\frac{\partial\mu(AB)}{\partial p_2^B} + \frac{\partial\mu(BB)}{\partial p_2^B} &= \frac{\partial(1 - \bar{x}_2)}{\partial p_2^B} \\
&= -\frac{1}{2t_2}
\end{aligned}$$

Firm $2B$ then selects its best response in price, p_2^B holding fixed the three prices of Firm A and the price of Firm $1B$. The first order condition yields a unique solution

$$\begin{aligned}
0 &= \mu(AB) + \mu(BB) - \frac{p_2^B}{2t_2} \\
&= (1 - \bar{x}_2) + \frac{\Delta^A(1 - \underline{x}_1)}{2t_2} - \frac{\Delta^{A2}}{8t_1t_2} - \frac{p_2^B}{2t_2} \\
&= \frac{t_2 - p_1^A - p_2^B + P^A}{2t_2} + \frac{\Delta^A(1 - \underline{x}_1)}{2t_2} - \frac{\Delta^{A2}}{8t_1t_2} - \frac{p_2^B}{2t_2} \\
&= \frac{t_2 - (p_1^A + p_2^A - P^A) + (p_2^A - p_2^B)}{2t_2} + \frac{\Delta^A(1 - \underline{x}_1)}{2t_2} - \frac{\Delta^{A2}}{8t_1t_2} - \frac{p_2^B}{2t_2} \\
&= \frac{t_2 - \Delta^A + (p_2^A - p_2^B)}{2t_2} + \frac{\Delta^A(1 - \underline{x}_1)}{2t_2} - \frac{\Delta^{A2}}{8t_1t_2} - \frac{p_2^B}{2t_2} \\
&= \frac{t_2 - \Delta^A + p_2^A}{2t_2} + \frac{\Delta^A(t_1 + p_1^A - p_1^B)}{4t_2t_1} - \frac{\Delta^{A2}}{8t_1t_2} - \frac{p_2^B}{t_2} \\
&= \frac{t_2 - \Delta^A + p_2^A}{2} + \frac{\Delta^A(t_1 + p_1^A - p_1^B)}{4t_1} - \frac{\Delta^{A2}}{8t_1} - p_2^B.
\end{aligned}$$

Observe this is decreasing in p_2^B so yields a unique best response. Rewriting, yields

$$p_2^B = \left(\frac{t_2 - \Delta^A + p_2^A}{2} + \Delta^A\frac{t_1 + p_1^A}{4t_1} - \frac{\Delta^{A2}}{8t_1}\right) - \frac{\Delta^A}{4t_1}p_1^B.$$

A similar condition, replacing 1 with 2 and BA for AB, holds for the other independent firm. These equations form a linear system in p_1^B, p_2^B and yield unique solutions as functions of P_1^A, p_2^A, p_1^A.

Writing $p_i^B = A_i - \frac{\Delta^A}{4t_1} p_1^B$ we get

$$\begin{aligned} \frac{\Delta^A p_2^B}{4t_2} &= \frac{\Delta A_2}{4t_2} - \frac{\Delta^A p_1^B}{16t_1t_2} \\ &= A_1 - p_1^B \end{aligned}$$

So,

$$\Delta^A 4t_1 A_2 - \Delta^{A^2} p_1^B = 16t_1t_1A_1 - 16t_1t_2p_1^B.$$

Thus

$$p_i^B = 4t_i \frac{4t_jA_i - \Delta^A A_j}{16t_1t_2 - \Delta^{A^2}} \qquad (12)$$

The derivatives of the profit function for the integrated firm are the same as given in (10) and (11). These solutions are then inserted in (12) to obtain the prices for the single-good firms.

10 References

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