Multilateral Negotiations and Opportunism

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Abstract

Two downstream firms engage in dynamic, non-cooperative bargaining over two-part tariffs with a single upstream firm. Under a sufficient condition, a stationary subgame perfect equilibrium with immediate agreement exists. With forward looking agents and substitute downstream products, fear of opportunism – even with public offers – generates input prices below the profit-maximum but above the static pairwise proof prices. The flow of fixed fee payments reverses as downstream products become closer substitutes. The model's predictions for vertical mergers are contrasted with predictions from various versions of the Nash in Nash bargaining model and shown to yield significantly smaller price effects.

Keywords: Dynamic bargaining; Nash in Nash; Two part tariffs; Vertical mergers.

JEL Codes: C78, D43, L13, L14.

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1 Introduction

The framework of this paper is motivated by questions raised by two important and interesting lines of research – opportunism in vertical contracting and multilateral bargaining using the so-called Nash in Nash approach.

McAfee and Schwartz (1994) (henceforth, MS) examine contracting incentives when a single upstream firm negotiates terms under which it will supply an input to two or more competing downstream firms. In this scenario and where downstream firms compete in prices, total profits would be maximized with input prices above marginal cost of the upstream firm so as to control profit-destroying competition among the downstream firms. MS demonstrate that in the absence of an ability to credibly commit to these jointly beneficial input prices, every separate upstream-downstream pair of firms has an incentive to deviate and set the per unit price of the input to its own advantage and to the detriment of the other downstream firms. This opportunistic behavior prevents the firms as a whole from achieving the maximally profitable outcome. In the MS version, the upstream firm makes take-it-or-leave-it contract offers to downstream firms under various assumptions about observability. With fully observable offers, the full commitment solution can be achieved. With offers observable only to each participating pair (though assuming rivals learn each other's input prices before competing), under one assumption on out-of-equilibrium beliefs ('passive beliefs') any perfect Bayesian equilibrium outcome must be 'pairwise proof', that is, it must involve input prices where there is no further pairwise incentive to alter prices.[1] Under Bertrand competition over substitutes downstream, this behavior typically results in input prices above marginal costs but below the fully optimal prices. Rey and Vergé (2004) extend this logic by examining other assumptions on-out-of equilibrium beliefs and demonstrate that opportunistic behavior remains.

One weakness in the above analyses lies in the assumption that the upstream firm makes take-it-or-leave-it offers.[2] In markets with large and sophisticated firms upstream and downstream, for example, a large content supplier like NBCUniversal

[1]The term 'passive beliefs' refers to the assumption that, if an out-of-equilibrium offer is made to one downstream rival, the receiving firm continues to believe an equilibrium offer has been made to the rival firm. It was first introduced as 'contract equilibrium' in Cremer and Riordan (1987). The concept was then applied in an opportunistic setting by Hart and Tirole (1990) in their examination of contracting between upstream firms and downstream Cournot competitors.

[2]Chemla, 2004, in a model similar to MS, examines a game where, stochastically, either upstream or downstream firms make take-it-or-leave-it offers. While this variation balances surplus shares, the assumption of take-it-or-leave-it offers keeps bargaining *power* one-sided.

negotiating with large Multi-channel Video Program Distributers (MVPDs) such as Comcast or Verizon over terms for its programs, it seems unlikely that either party would be willing to accept a simple take-it-or-leave-it offer from the other side.[3] An alternative branch of literature examines a more balanced form of multi-lateral negotiations known as Nash in Nash bargaining. Originally introduced by Horn and Wolinsky (1988), a single upstream 'firm' (in their application, it may be a labor union) negotiates simultaneously and bilaterally with multiple downstream firms over supply terms in the manner of cooperative Nash bargaining (the second, cooperative, 'Nash'). Given conjectures over the terms agreed to by the other firm pairs, each upstream-downstream pair uses its (single) contracting instrument – the per unit price – to maximize the geometric average of their profits where the exponents are weights representing each side's bargaining power. The full profile of pair-wise solutions then must be mutual best responses (the first, non-cooperative, 'Nash').

These models have become commonly invoked in both academic and policy analyses of multi-lateral bargaining. Crawford and Yurucoglu (2012) employ the approach to model negotiations between content providers and cable companies and provide an estimate of the relative bargaining power of the various parties. The Department of Justice, the FCC, as well as various outside experts, have utilized the approach to determine the impact of vertical mergers between an upstream firm and one of the downstream firms on prices for content in the MVPD industry.[4] Ho and Lee (2017) apply a multi-layer version of this model to the healthcare industry to analyse the formation of premia and hospital reimbursement costs among employers, hospitals and health insurance companies.

While popular, the Nash in Nash model, at least as it has often been used, has some well-known weaknesses. Implicit in determining the solution of the two objective functions in the cooperative layer of the bargaining models is the assumption that any change in contracting terms agreed to by the upstream firm in one bilateral negotiation is not accompanied by a change in the terms with the other bilateral pair. This assumption is maintained even though altering the contract terms with

[3] Chemla (2003), in a model similar to MS, examines a game where, stochastically, either upstream or downstream firms make take-it-or-leave-it offers. While this variation balances surplus shares, the assumption of take-it-or-leave-it offers keeps bargaining *power* one-sided.

[4] The approach was first used in the analysis of the 2005 merger between Newscorp and DirecTV. See Rogerson (2014). It was later employed to a much greater extent in the FCC's analysis of subsequent MVPD mergers such as NBC-Comcast (Federal Communications Commission (2011)). It also played an important role in the Department of Justice case opposing the ATT-TWI merger (*U.S. vs. AT&T, et.al.* (2018)).

the other downstream firm would now be a better response for the upstream firm to such a variation and even though this firm is a party to both negotiations. In effect, this feature is analogous to the assumption of 'passive beliefs' in noncooperative models which, as noted by Rey and Tirole (2007), seems especially problematic with downstream Bertrand competition. The static nature of the cooperative approach does not offer an obvious way to introduce alternative notions of out-of-equilibrium beliefs.

A second weakness (as generally applied) lies in the assumption of the restrictive contracting instruments available to the negotiators. In most applications, the contracting instruments available to the parties have typically been limited to linear pricing.[5] The restriction to linear pricing results in loss of surplus in general.[6] I am not aware of any models that extend the Nash-in-Nash model to nonlinear contracts. In Section 5, I demonstrate that a necessary condition for Nash-in-Nash contracts with two part tariffs is a per unit input price that is the same as the MS 'pairwise proof' prices. Since these are the prices that emerge from a one-shot non-cooperative model (as in MS), it seems important to investigate the outcome from a non-cooperative model with more balanced negotiating powers and dynamic bargaining.

An important paper by Collard-Wexler, et. al. (2019) partially addresses these concerns in a setting with membership externalities. In that paper, firms have the ability to negotiate using only fixed fees, and the pairwise Nash bargaining solution is replaced by an explicit offer-counteroffer bargaining game as in Rubinstein (1982). Thus, in the spirit of the famous 'Nash program', the paper provides a fully non-cooperative foundation for multi-lateral negotiations. In doing so, conditions are provided under which the unique subgame perfect equilibrium outcome approximates the outcome predicted by the Nash in Nash model. However, by limiting attention to only fixed fee contracts, their framework ignores the incentive to use per unit prices to address the sorts of 'contracting externalities' that are often present among competing firms.[7] In many of the applications of the Nash in Nash model such as

[5]This is true of the original Horn and Wolinsky (1988) formulation as well as all the papers cited above.

[6]Consider, for example, the polar case of two symmetric downstream firms, each operating in separate markets and facing demand $Q = 1 - P$, using a linear technology with a single input from an upstream monopolist with zero marginal cost. Joint surplus maximization requires input prices to both firms that equal marginal cost to avoid double-marginalization, however, the Nash in Nash solution when negotiating over input prices alone ranges from 0 to $1/2$ as the bargaining power of the upstream firm ranges from 0 to 1.

[7]This incentive is not present in their framework as the cross-firm externalities are purely the

the MVPD market or health networks, it is entirely likely that input prices of rivals affect the downstream prices and profits of all firms and thus there are both bilateral and multi-lateral incentives to modify these prices.

This paper simplifies the Collard-Wexler, et. al.(2019) model by assuming only a single upstream firm and two downstream Bertrand competing firms but includes two part tariffs. In the environment examined here, MS show that with public offers and the ability for the upstream firm to commit to a one-shot take-it-or-leave-it offer, two part tariffs are sufficiently rich contracts to enable bargaining partners to achieve full efficiency. They serve as proxies for the complex contracts that large and sophisticated negotiators can be expected to employ.[8] I assume public offers and investigate the consequences of the inability to commit not to renegotiate in the event of an initial failure to agree.

The paper provides a single testable condition for the existence of a stationary subgame perfect equilibrium under the assumption of public offers. Importantly, even with public offers, equilibrium input prices reflect the influence of opportunism, as the agreement of any one pair must respect the (contingent) capability of partners to reject current offers and exploit agreements made by other pairs.[9] This concern induces initial input price offers that are below the joint profit-maximum. At the same time, the extent of opportunistic pricing is mitigated because of the dynamic features of bargaining. The payoff to a downstream firm from rejecting depends negatively on the profit flow of the upstream firm from the *accepted* contract with the other rival. In the case of linear downstream demand, an increase in the per unit price in its initial offer improves the (unutilized but nevertheless effective) bargaining position of the upstream firm on this off-equilibrium path and partially countervails the downward

consequence of membership in a network and not due to, for example, pricing decisions by firms.

[8]The complexity of MVPD contracts which contain a variety of non-linear features such as division of advertising slots, bundling of lesser valued networks and channel placement is described in the expert report, *US vs. AT&T et.al.* (2018). Crawford and Yurucoglu (2012) also report that these contracts are typically quite complex. Though they also argue that actual fixed monetary transfers are small, as demonstrated in Section 4.3, since equilibrium transfers can go in either direction, this fact is not, in itself, evidence against their role.

[9]An interesting recent paper by Do and Miklos-Thal (2021) examines opportunism in a dynamic, non-cooperative model with Cournot-like contracting and homogeneous downstream firms. In the version they examine, bilateral recontracting possibilities emerge stochastically over time which creates the incentive to behave opportunistically. They find, in that context, opportunistic behavior is reduced as agents become infinitely patient but only vanishes entirely if the ability to recontract is sufficiently asymmetric across downstream firms. In Section 4.2, I compare further the results of their paper with my results.

push induced by the concern for opportunism.

In a calibrated version of the model with varying degrees of product differentiation downstream, the input prices lie between the joint surplus maximizing prices and the (much lower) pairwise proof prices. Interestingly, as products become less differentiated, the flow of the fixed fee part of the two part tariff is reversed – the upstream firm *pays* each downstream firm a lump sum in order to induce it to agree to terms. This phenomenon arises because each downstream firm must be rewarded for the competitive value-added it brings to the downstream market. This value is significant with close substitutes but flow profits are low in precisely these conditions.

The model can also be used to analyse the impact of a vertical merger between the upstream firm and one of the downstream firms. The prediction of this model is then compared to those of the Nash in Nash model. Primarily because the dynamic and fully non-cooperative model predicts higher pre-merger prices than the Nash in Nash models, it also yields merger-induced price effects that are much lower than would be predicted by these models with either linear pricing or with two part tariffs.

2 Model and Preliminaries

2.1 Market Structure and Technology

Two symmetric downstream firms (D_1 and D_2) compete in prices to sell differentiated products. They each operate a linear production technology using an input supplied by a monopolist upstream firm (U) with linear costs and, therefore, constant marginal cost (henceforth, set to zero). Normalize units so that a unit of input produces a unit of output. Let w_i denote the per unit input price for firm i and q_1, q_2 be demand for inputs of each firm. Firm U's flow profits are thus given by

$$R(w_i, w_j) \equiv w_i q_i + w_j q_j.$$

I will assume throughout that contract offers are publicly observed. This assumption obviously implies that downstream firms compete knowing the marginal cost of their rival – a feature Rey and Vergé (2004) term 'interim observability'.[10] If firm U supplies to both downstream firms, at input prices (w_i, w_j), assume there is a unique symmetric equilibrium yielding downstream output levels $q_i(w_i, w_j)$ and downstream firm profits $\Pi^i(w_i, w_j), i = 1, 2, j \neq i$. Symmetry implies that $q_1(x, y) = q_2(x, y) \equiv q(x, y)$ and $\Pi^1(x, y) = \Pi^2(x, y) \equiv \Pi(x, y)$.

[10]MS use the term 'ex post observability for the same feature.

Define the per period equilibrium joint surplus of all three firms given upstream prices (w_1, w_2) as

$$S(w_i, w_j) = \Pi(w_i, w_j) + \Pi(w_j, w_i) + R(w_i, w_j). \tag{1}$$

and let

$$(\hat{w}_1, \hat{w}_2) = argmax_{w_1,w_2} S(w_i, w_j).$$

be the input prices $(\hat{w}_1, \hat{w}_2)$ that maximize total per period industry profit. Assume there is a unique pair of such prices and that $\hat{w}_1 = \hat{w}_2 \equiv \hat{w}$. In general, with cross-product substitutability, $\hat{w}$ is strictly above the marginal cost of the upstream firm as positive input prices serve as an instrument to control profit-reducing competition among the downstream firms. In the limiting case of independent products, $\hat{w} = 0$, total industry profits are maximized when input prices equal upstream marginal costs.

For any given w_j, define

$$w(w_j) \equiv argmax_{w_i} S(w_i, w_j) - \Pi(w_j, w_i). \tag{2}$$

The function, $w(w_j)$ is the input price that maximizes the pairwise profits of firm U and firm D_i given the input price w_j paid by firm D_j. This function captures the opportunistic behavior of any bilateral pair that was the focus of MS's original study.

I assume the following two properties on equilibrium profits:

A1) The value $w(w_j)$ is unique and yields a differentiable function of w_j;

A2) The function $w(w_j)$ possesses a unique fixed point,

$$\bar{w} = w(\bar{w}). \tag{3}$$

If $S(w_1, w_2) - \Pi(w_j, w_i)$ is twice differentiable and strictly concave in w_i for all w_j, **A1)** is satisfied and $c'(w_j) \in (0, 1)$ is sufficient to satisfy **A2)**. Note that $\bar{w}$ represents the pairwise proof equilibrium in MS.

The Linear Quadratic Model. A parametrized example yielding **A1)** and **A2)** is the linear-quadratic (LQ) model presented in Vives (2001) where demand is generated by a representative consumer with quasi-linear utility from the two downstream products given by

$$U(q_1, q_2, M) = q_1 + q_2 - \frac{1}{2}(\beta q_1^2 + 2\gamma q_1 q_2 + \beta q_2^2) + M, \text{with } \beta, \gamma \geq 0.$$

The parameter $\gamma \in [0, \beta)$ determines the relationship between the goods: $\gamma = 0$ implies independent products, $\gamma > 0$ implies gross substitutes, and as γ approaches β products become homogeneous. This model generates a symmetric demand system

$$Q(p_1, p_2) = \frac{1}{\beta^2 - \gamma^2}(\beta + \gamma + \gamma p_2 - \beta p_1)$$

and yields a unique Bertrand equilibrium in prices for any $(w_1, w_2) \in [0,1]^2$.(See Vives (2001) Chapter 6.) The diversion ratio, $\frac{\partial Q}{\partial p_2} / \frac{\partial Q}{\partial p_1} = \gamma/\beta$, is constant in this model.[11]

By definition, for all w_j, $S(\hat{w}_1, \hat{w}_1) \geq S(w_j, w(w_j))$. In general, with some downstream substitutability across firms, the inequality will be strict. In the LQ model, for $\gamma \in (0, \beta)$,

$$0 < \bar{w} < \hat{w}_1 = \hat{w}_2,$$

and for $\gamma = 0$,

$$0 = \bar{w} = \hat{w}_1 = \hat{w}_2.$$

These relationships capture many of the important pricing incentives in certain vertical relationships. When there is no substitutability across products, industry optimal input prices equal marginal costs and with some substitutability joint profit-maximizing prices are strictly above marginal costs. However, substitutable products also create opportunistic incentives. Given an input price charged to downstream firm D_j, the upstream-downstream pair, (U, D_i) would like to undercut that price in their contract so as to draw some profits away from firm D_j.

If (say) firm D_j and firm U fail to agree and only firm D_i operates at input prices, w_i, the resulting firm profits and outputs are

$$\tilde{\Pi}(c), \tilde{q}(c), \tag{4}$$

where $\tilde{q}(c)$ is the flow profit-maximizing output for a downstream monopolist. If $c > 0$, this outcome typically involves a profit margin resulting in an inefficiency and an overall profit loss due to double marginalization.

In general

$$S(\hat{w}, \hat{w}) \geq \tilde{\Pi}(0).$$

The maximal industry profits that can be achieved with two downstream firms is typically strictly greater than can be achieved with a single firm since downstream differentiation across firms implies a desire for product variety. If products are homogeneous, then the inequality becomes an equality – the optimum on the left side is achieved by pricing the input to both firms at the industry monopoly price and the optimum on the right is achieved by pricing the input at marginal cost and allowing the sole producer to select the industry monopoly price.

[11] In this model, the opportunistic response is $w(w_i) = \left(\frac{\gamma}{\beta}\right)^2 \frac{(2\beta+\gamma)(\beta-\gamma)}{2\beta^2-\gamma^2} + \frac{\gamma\beta}{2\beta^2-\gamma^2} w_i$. Some algebra confirms that $w' \in (0,1)$. The Mathematica code used to find this value as well as for the calculations in Sections 4 and 5 can be accessed at MathematicFilesNINOPP.

2.2 The Bargaining Game and Payoffs

The upstream firm and each downstream firm bargain independently over input prices (w_i) and fixed fees (f_i). In the first period, and in any period t where no agreement has been reached with either firm, firm U names a *publicly observable* pair $(f_1, w_1), (f_2, w_2)$ to firms D_1 and D_2 which make simultaneous decisions whether to accept or not. If accepted, f_i is paid to firm U and w_i is the price paid by D_i for each unit of input demanded by D_i for the remainder of the game. As will be seen, in equilibrium, fixed fees may flow either way. I adopt the convention that $f_i > 0$ implies a payment from the downstream firm i to the upstream firm. For any agreed upon input price, w_i, firm U must supply any quantity of input demanded by D_i. At the end of any period t, downstream firm D_i learns whether its rival D_j also has come to agreement (and, of course, its terms). If (say) D_1 has reached an agreement but D_2 has not, flow profit (net of fixed fee) and output for D_1 are as in (4) and are zero for firm D_2. Firm D_2 and the upstream firm then engage in offer-counteroffers of two-part tariffs beginning the period after an agreement has been reached with D_1 and ending only when an offer is accepted. Following acceptance of contracts by both firms, the firms compete in prices generating flow profits, $\Pi(w_i, w_j)$ and output, $q(w_i, w_j)$, forever.

Firms live for an infinite number of periods and discount future profits according to a common discount factor, $\delta \in [0, 1)$.

An outcome of the bargaining game is a pair of triplets, $(w_i, f_i, t_i), i = 1, 2$, where t_i is the period in which firm D_i agrees to a two part tariff, (w_i, f_i) with firm U. This structure implies the following payoff from a bargaining game. For firm, U a deal with firm i in period t_i at contract (w_i, f_i) and with firm j in period $t_j \geq t_i$ yields profits

$$\delta^{t_i} \left(\sum_{\tau=0}^{t_j-1} \delta^\tau w_i \tilde{q}(w_i) + f_i \right) + \delta^{t_j} \left(\sum_{\tau=0}^{\infty} \delta^\tau R(w_i, w_j) + f_j \right). \tag{5}$$

For firm i, a deal with firm U in period t_i at contract (w_i, f_i) followed by a deal with firm j in period $t_j \geq t_i$ yields it total profits

$$\delta^{t_i} \left(\sum_{\tau=0}^{t_j-t_i} \delta^\tau \tilde{\Pi}(w_i) + \delta^{t_j} \sum_{\tau=0}^{\infty} \delta^\tau \Pi(w_i, w_j) - f_i \right). \tag{6}$$

Profits for firm j that deals in period $t_j \geq t_i$ are

$$\delta^{t_j} \left(\sum_{\tau=0}^{\infty} \delta^\tau \Pi(w_j, w_i) - f_j \right). \tag{7}$$

2.3 Discussion

Some comments about this bargaining game are in order. A simpler structure might have been to have firm U make all the offers either to both firms or to a single remaining uncontracted firm. However, it is well-known that, in full information bargaining games where one side makes all the offers, whether for a finite number of periods or a potentially infinite number of periods, the offering firm obtains all the surplus in all continuation subgames and the equilibrium outcomes collapse to the same outcome as that in a single period offer game. The introduction of the opportunity for a single remaining downstream firm to engage in offer-counter bargaining is a simple way to balance bargaining power in a non-cooperative dynamic game and to parametrize this balance via the familiar tool of a discount factor.

Even so, by assuming only U makes the offers when no deal has been reached, there is an inherent asymmetry in its favor. Is this plausible? In fact, no extensive form bargaining game is plausible in a literal sense. My view is that in designing an extensive form game, the goal is to attempt to capture some intrinsic strategic properties of the environment. In a sense, the structure represents the feature that, with no deal reached, U has an upper hand with respect to multiple downstream firms. Once a deal has been reached with one firm, that deal is sunk and the two remaining firms negotiate on a more equal basis over the incremental gains from trade. A bargaining structure where downstream firms are allowed to make counteroffers whether or not a deal has been reached would probably yield a larger share of surplus to the downstream firms. However, such a game would be significantly more complex and the gain from such an exercise seems small. Even in the current structure, both sides are able to obtain significant shares of surplus and, as observed in Section 3.2, granting the downstream firms more bargaining power is likely only to strengthen the main conclusions of the paper.

The assumption that contract offers are publicly known is admittedly strong. Importantly, though, even with public offers, the equilibrium characterized in the next section exhibits the impact of opportunistic behavior in the presence of contracting externalities. The argument for modeling this way is primarily one of convenience. The structure allows me to avoid the debate over the nature of off-equilibrium path beliefs about offers to the other pair – an issue that has spawned significant discussion in the literature. In a working paper (Vincent, 2020), I also analyse the case of private offers (though still with interim observability). The equilibrium path characterized with public offers generally remains an equilibrium path with private offers and 'symmetry' beliefs – when an out-of-equilibrium offer is observed, the recipient believes the same offer was made to its rival. On the other hand, if the beliefs are 'passive', another commonly used assumption that implies the recipient maintains its original beliefs about the offers to its rivals, then if equilibrium input prices exist, they must be the fully opportunistic prices identified in MS (1994). However, equilibrium existence with passive beliefs even in simpler games is not ensured as products become closer substitutes (Rey and Vergé, 2004).

It is also worth considering how the analysis could be extended to more than two downstream firms. Conceptually, the approach would be similar, though, computationally burdensome. As long as we are willing to maintain the assumption that, with two or more downstream firms remaining to accept contracts, only the upstream firm makes offers, we simply would need to expand the possible off-equilibrium continuation paths. For example, with three downstream firms, when two firms have agreed to contractual terms, the continuation would involve offer-counteroffer bargaining between U and the remaining firm with continuation payoffs determined in part by a Bertrand equilibrium holding the established contractual terms fixed. When only one firm has agreed to terms, the continuation path reverts to a subgame of negotiations as conducted in this paper, though with the pre-negotiated input price impacting the anticipated downstream continuation payoffs.

3 Subgame Perfect Equilibrium

This paper focuses on subgame perfect equilibria with the following three properties:

M: (Markov Strategies) At the beginning of any subgame, continuation strategies depend only on the input price agreed to at that point (if any) or on the contract(s) currently on offer. Specifically, strategies do not vary with previously rejected offers or with previously accepted fixed fees;

I : (Immediate Acceptance) For any history leading to a subgame where contract(s) are to be offered, the equilibrium continuation prescribes offers that lead to acceptance in the subgame that immediately follows.

$PSAB$: (Pure Strategy and Acceptance Bias) For any history leading to a stage where *Accept*, *Accept* is a sequentially rational pair of strategies, these strategies are selected with probability one.

Under Property M, strategies are not dependent on prior histories, h_{t-1} except for any accepted input prices. Property $PSAB$ enables a simpler analysis of player behavior in the event that some (out-of-equilibrium) set of offers is made leading to a simultaneous move game for downstream firms that possesses multiple equilibria, including those using strictly mixed strategies. This property refines out the mixed strategy equilibria when pure strategy equilibria exist.[12]

In this section, a candidate subgame perfect equilibrium path satisfying properties M, I and $PSAB$ is constructed and a simple condition is provided that ensures it is sequentially rational. Section 3.1 characterizes the subgame perfect equilibrium continuation payoffs for all subgames when only one downstream firm has come to agreement and, given this, illustrates two possible continuation paths for generic subgames with all firms negotiating. In Section 3.2, candidate strategies for the full game are presented and Theorem 1 shows when the strategies form a subgame perfect equilibrium.

3.1 Subgame Perfect Continuation Paths

Subgames of this game can be partitioned into two categories, UG and SG. In the upstream offer game (UG), no offers have been accepted and firm U makes a pair of offers. There are two types of subgames within this category. Following a history, h_{t-1}, in the subgame $UG_a(h_{t-1})$ firm U offers a pair of two part tariffs, $(w_1, f_1), (w_2, f_2)$. In the subsequent subgame, $UG_b(h_{t-1}, (w_1, f_1), (w_2, f_2))$, the downstream firms make simultaneous decisions whether to accept or reject. Property M implies that these subgames do not vary with h_{t-1} and so this argument is dropped going forward.

In the single pair category, $SG^k(h_{t-1}, w_j)$, firm D_j and firm U have come to agreement with input price w_j and either firm $k = U$ or firm $k = i$ is making an offer. The history also includes the fixed fee agreed upon between U and D_j, however, as this fixed fee does not affect the future profits of the remaining negotiators, so again, M enables me express this subgame as depending solely on the agreed upon input price, w_j. Note that once a category of this type occurs, subgames of category UG can no longer occur. Again, there are two subgames within this category. In $SG^k_a(w_j)$, firm k makes an offer, (w_i, f_i) and in subgame $SG^k_b(w_j, (w_i, f_i))$ the remaining firm accepts or rejects.

If a subgame perfect equilibrium with properties M, I and $PSAB$ exists, then

[12] The assumption does not rule out mixed strategy equilibria entirely as out-of-equilibrium histories could emerge in which only mixed strategy equilibria exist.

continuation utilities in a subgame (a) depend solely on the category of subgame that the history has led to. The objects $V_{UG}^k, k \in \{1,2,U\}$ denote the (undiscounted) equilibrium continuation payoff of firm k in a subgame (a) of category UG. The objects $V_{SG^s}^k(w_j), k, s \in \{i, U\}, i \neq j$ denote the (undiscounted) equilibrium continuation payoff of firm k in a subgame (a) of category SG when an offer w_j has been agreed to by firm j and firm s is making an offer.

In Lemma 1, necessary properties of such equilibrium payoffs for subgames of type $SG^k(w_j)$ are characterized. Proofs of all results are in the Appendix.

Lemma 1. *Consider any subgame perfect equilibrium continuation of $SG^k(w_j)$ satisfying M and I. For any w_j such that $S(w(w_j), w_j) - \Pi(w_j, w(w_j)) - w_j\tilde{q}(w_j) > 0$:*

i) The equilibrium input price in any subgame $SG^k(w_j)$ is $w(w_j)$.

ii) The equilibrium fixed fees satisfy

$$f^D(w_j) = \frac{\delta}{1+\delta}\frac{\Pi(w(w_j), w_j)}{(1-\delta)} - \frac{1}{1+\delta}\frac{R(w(w_j), w_j) - w_j\tilde{q}(w_j)}{(1-\delta)} \tag{8}$$

when the remaining downstream firm makes offers and

$$f^U(w_j) = \Pi(w(w_j), w_j) + \delta f^D(w_j). \tag{9}$$

when U makes offers.

iii) The equilibrium continuation payoffs are

$$V_{SG^i}^i(w_j) = \frac{1}{1-\delta}\frac{S(w(w_j), w_j) - \Pi(w_j, w(w_j)) - w_j\tilde{q}(w_j)}{1+\delta}$$

and

$$V_{SG^i}^U(w_j) = \frac{1}{1-\delta}\left(w_j\tilde{q}(w_j) + \delta\frac{S(w(w_j), w_j) - \Pi(w_j, w(w_j)) - w_j\tilde{q}(w_j)}{1+\delta}\right).$$

In the continuation equilibrium of the single pair subgame with an agreement (w_j, f_j) established between U and D_j, the remaining pair agree to the opportunistic input price, $w(w_j)$ (Lemma 1i)). The fixed fee is determined so as to assign each firm its Rubinstein share of the incremental profits that are generated by an agreement at that price (Lemma 1ii)). And, agreement occurs instantaneously.[13] Lemma 1iii) illustrates both the opportunistic influence and a countervailing feature due to dynamic

[13] In the event that subgame $SG^k(w_j)$ follows a w_j for which there are no further gains from trade for U, D_i, I assume the continuation path has the remaining firms make unacceptable offers forever yielding zero further profits.

bargaining. The equilibrium utility of the rejecting firm, D_i, depends positively on the joint surplus of U and D_i (the term, $S(w(w_j), w_j) - \Pi(w_j, w(w_j)))$ and negatively on the default profits of firm U should no agreement be made in subgame $SG^i(w_j)$, (the term, $w_j\tilde{q}(w_j)$). In the relevant range of prices, the term $S(w(w_j), w_j) - \Pi(w_j, w(w_j))$ generally rises with w_j. This is the influence of opportunism – a higher input price to the rival increases the temptation for D_i to reject and behave opportunistically in a continuation bargaining game. In the calibrated class of models examined in Section 4, the term $w_j\tilde{q}(w_j)$ typically rises with w_j – this is the countervailing effect of dynamic bargaining along this path of the game, a higher $w_j\tilde{q}(w_j)$ means that in the bilateral bargaining subgame, firm U's outside option rises and D_i's share of the surplus falls. Both of these effects play a role in the equilibrium input price offers of the full game.

In any subgame where firm U makes offers $(w_1, f_1), (w_2, f_2)$ to both downstream firms, the stationarity assumption and Lemma 1 enables us to represent the resulting pruned game tree as in Figure 1 where the payoffs listed at the end of each path present D_1's profit above the profit of D_2. Any such two by two simultaneous move game can be characterized as $G(w_1, f_1, w_2, f_2, \delta V^1_{UG}, \delta V^2_{UG})$ where the parameters of the game are represented by the six-tuple of the pair of two part tariffs and the expected continuation utilities in the event of no agreement. Bimatrix games of this type possess one, two or three Nash Equilibria.[14] Property $PSAB$ ensures that if $(Accept, Accept)$ is an equilibrium, it is selected. In the cases where both $(Reject, Accept)$ and $(Accept, Reject)$ are equilibria (and possibly a mixture), the equilibrium analysis does not depend on which is selected but, for completeness, I assume that $(Reject, Accept)$ is selected. This ensures that there is a unique outcome from any game $G(w_1, f_1, w_2, f_2, \delta V^1_{UG}, \delta V^2_{UG})$, call it $NE^*(w_1, f_1, w_2, f_2, \delta V^1_{UG}, \delta V^2_{UG})$.

To construct the subgame perfect equilibrium of the full game, consider two generic pure strategy paths. In paths of type 1 (P1), both downstream firms are expected to accept immediately in all UG subgames and one or both firms are indifferent between accepting and rejecting, assuming the rival firm accepts. In paths of type 2, (P2) one downstream firm, say firm D_1 accepts immediately and firm D_2 rejects then returns with an acceptable offer in the subsequent SG subgame. The equilibrium path I analyze will be of type 1, but paths of type 2 must be considered when assessing out of equilibrium offers by U.

[14]The case of two equilibria is non-generic for exogenous games, but since these games arise endogenously, they cannot be ruled out, ex ante.

Consider a P1 subgame perfect equilibirium path with price offers $(w_1, f_1), (w_2, f_2)$. By assumption, firm D_j accepts. A necessary condition for firm D_i to accept is that (w_i, f_i) satisfy

$$\frac{\Pi(w_i, w_j)}{1-\delta} - f_i \geq \delta V^i_{SG^i}(w_j).$$

Suppose this holds with equality since firm U's profit is increasing in f_i. This yields firm U profit

$$\frac{w_j q(w_j, w_i) + w_i q(w_i, w_j) + \Pi(w_i, w_j)}{1-\delta} + f_j - \delta V^i_{SG^i}(w_j).$$

Applying a similar logic to (w_j, f_j) yields the following profit function for firm U:

$$V^{P1}(w_1, w_2) = \frac{S(w_1, w_2)}{1-\delta} - \delta V^1_{SG^1}(w_2) - \delta V^2_{SG^2}(w_1)$$

where $V^j_{SG^j}(w_i)$ are derived in Lemma 1. Define

$$(w^*_1, w^*_2) = argmax_{w_1, w_2} V^{P1}(w_1, w_2). \tag{10}$$

In this hypothesized equilibrium, firm U selects prices to maximize total surplus minus the bargaining share that each downstream firm could collect were it to reject the offer and engage in one on one bargaining for its entry. The impact of input prices on $V^i_{SG^i}(w_j)$ is shown in Lemma 1iii) and discussed immediately following the Lemma. A reduction in w_j has a dampening effect on $V^i_{SG^i}(w_j)$ through the opportunistic influence and pulls input prices below the prices that maximize total surplus. At the same time, the dynamic bargaining effect via the terms $w_i\tilde{q}(w_j)$ typically mitigates this impact. These effects are not present in games where U makes take-it-or-leave-it offers nor are they present in the standard single shot offers made in Nash in Nash bargaining games.

Consider a P2 path. Fix δV^1_{UG}, the expected continuation utility of D_1 if it also rejects and triggers a new UG subgame. Under the stationarity assumption, M, this does not vary with any rejected offers (w_1, f_1). By assumption D_2 will reject, thus (w_1, f_1) must satisfy $\tilde{\Pi}(w_1) + \frac{\delta\Pi(w_1, w_2(w_1))}{1-\delta} - \delta V^1_{UG} \geq f_1$. Applying Lemma 1, this implies that the profit of firm U from such a continuation is bounded from above by

$$V^{P2}(w_1, V^1_{UG}) = \tilde{\Pi}(w_1) + w_1\tilde{q}(w_1) + \delta\left(\frac{\Pi(w_1, w(w_1))}{1-\delta} + V^U_{SG^2}(w_1) - V^1_{UG}\right).$$

Define

$$\tilde{w} = argmax_{w_1} V^{P2}(w_1, V^1_{UG}). \tag{11}$$

Note that $\tilde{w}$ is independent of the value, V_{UG}^1 because of M.

As δ approaches 0, the game approximates the take-it-or-leave-it game analyzed by MS and Rey and Vergé (2004). However, in this limiting case, the optimal input prices for type 1 path are the joint profit optimizing prices with both firms and for the type 2 path, they are the prices that maximize industry profits with a single downstream firm. In neither case, does opportunistic behavior play a role. This is a consequence of the assumption of public offers and also mirrors the result of MS with private offers but symmetry beliefs.

More generally, however, with $\delta > 0$, even with public offers, opportunistic incentives come into play. In P1 paths, this is because, even though offers are transparent and are accepted immediately, they are disciplined by the threat by each downstream firm of rejecting and triggering a subgame where opportunism becomes active. The choice of input offers by the upstream firm is affected by its desire to limit the off-equilibrium payoff threat that each downstream firm can impose on it. In P2 paths, an opportunistic input price is agreed to by the downstream firm that rejects the initial offer and joins the upstream firm in ganging up on the other downstream firm.

3.2 Subgame Perfect Equilibrium

In this section, I describe sequentially rational strategies that support the P1 pure strategy path and provide a simple condition under which it forms a subgame perfect equilibrium of the full game.

Table 1 describes a profile of stationary strategies, σ^{P1}. The functions $V_{SG^i}^i(w_j)$, $V_{SG^i}^U(w_j)$, $f^D(\cdot)$ and $f^U(\cdot)$ are defined in Lemma 1 and $w(\cdot)$ is defined in (2). The notation g_a refers to the first stage of subgame type g where an offer is made and g_b refers to the second stage of subgame type g where a response is made. The column 'Subgame' refers, when relevant, to a profile of outstanding offers, or an already accepted input price. The description ignores all other properties of the history of the game as strategies are not dependent on them. Recall that (w_1^*, w_2^*) is a maximizer of firm U's payoff from a Type 1 path, $V^{P1}(w_1, w_2)$ and define its corresponding fixed fees, (f_1^*, f_2^*) as

$$f_i^* = \frac{\Pi(w_i^*, w_j^*)}{1-\delta} - \delta V_{SG^i}^i(w_j^*).$$

The following theorem shows when σ^{P1} forms a subgame perfect equilibrium.

Theorem 1. *If $V^{P1}(w_1^*, w_2^*) \geq V^{P2}(\tilde{w}, \delta V_{SG^i}^i(w_j^*)), i, j = 1, 2,$ the strategy profile σ^{P1} forms a subgame perfect equilibrium of the full game.*

The idea of the proof of the theorem is that, when agents are following the strategy profile σ^{P1}, selecting w_1, w_2 to maximize $V^{P1}(w_1, w_2)$ is optimal for U. Given property M, U would never wish to make offers that induce a rejection by both firms and a discounted repetition of the conjectured equilibrium, so the best alternative it could achieve in a pure strategy response would be to induce one firm to accept and another to reject which generates a $P2$ path. The choice of $\delta V^i_{SG^i}(w^*_j)$ for V^i_{UG} arises because the most attractive fixed fee in a $P2$ path (for U) that is accepted by the one accepting firm is one which makes it just indifferent between accepting and rejecting and inducing a repetition of the UG subgame. By hypothesis, that subgame follows a $P1$ path with input prices (w^*_1, w^*_2) and utility to the accepting firm D_1 given by $\delta V^1_{SG^1}(w^*_2)$. Attention also needs to be paid to the possibility of inducing a mixed strategy response, however, the indifference conditions that are required for such an equilibrium of G can be exploited to show that such equilibria are also suboptimal for U. (See the appendix for the detailed argument.)

It is worth revisiting the question of the extensive form game used here. Some (justified) unease can be felt with a structure that gives the upstream firm continued strategic power in the event of rejection from both firms. (Recall that it is allowed once again to make offers.) However, the role that this assumption plays in the theorem lies solely in the second argument of $V^{P2}(\cdot,\cdot)$, which is the continuation utility of the downstream firms in the event of no agreement. The function V^{P2} is decreasing in this argument, so any variant of the game that *increases* the returns to downstream firms in the event of dual rejections would only increase the likelihood of the condition being satisfied. Additionally, the characteristics of the equilibrium, specifically, the predicted input prices and payoffs are independent of the continuation payoff following a double rejection. In this limited sense, the analysis has some robustness to the specification of the extensive form.

4 Implications of Equilibrium

In this section, I demonstrate that for many relevant environments, the condition for which σ^{P1} forms a subgame perfect equilibrium of the game is satisfied and explore its implications for the predicted prices. In Section 4.1, the polar case of independent products is discussed. Not surprisingly, with richer contracting instruments, firms are able to reach fully optimal solutions. This feature is in contrast to the typical Nash in Nash models with only linear pricing and also the non-cooperative model of

Collard-Wexler, et. al. (2019) where bargaining is only over fixed fees. In Section 4.2, the opposite polar case of homogeneous goods is examined. In this case, the sufficient condition in Theorem 1 is relatively straightforward to characterize and test. Finally, in Section 4.3, the parametrized LQ model is examined for a variety of intermediate degrees of substitution across downstream firms. In all of the cases examined, the condition for Theorem 1 is satisfied, so the strategies in Table 1 form a subgame perfect equilibrium and the resulting equilibrium fixed fees and input prices can be explicitly calculated.

4.1 Unrelated Products

While the focus of this analysis is on opportunistic incentives that arise in the presence of competing downstream products, it is useful to understand behavior in the polar case where the downstream firms produce products that are unrelated in demand. Then, under symmetry, $\Pi(w_i, w_j) = \tilde{\Pi}(w_i)$ for all w_j and $q(w_i, w_j) = \tilde{q}(w_i)$ for all w_j. In this case, the P1 path is identical to the subgame perfect equilibrium of two independent offer-counteroffer games in two-part tariffs. The $P2$ path disadvantages U twice – it delays efficient agreement with one firm by a period and it allows that firm the opportunity to make the final offer so the following corollary is immediate.

Corollary 1. *With downstream products that are unrelated in demand, the P1 path is a subgame perfect equilibrium outcome.*

With (symmetric) independent demand, $\hat{w} = w^* = \bar{w} = 0$ and $f^* = \frac{\tilde{\Pi}(0)}{1-\delta^2}$ – each bilateral pair agrees to a contract with input price at marginal cost and fixed fee at the Rubinstein price splitting the total surplus according to δ as the bargaining parameter. Given the constant returns to scale, this outcome extends a special case of the Collard-Wexler, et. al. (2019) result to an environment where negotiations are over both fixed fees and input prices.[15]

The assumption of constant returns to scale for firm U is important for the existence of a P1 subgame perfect equilibrium even with independent demands. Consider a slight extension of this model where the upstream firm incurs a fixed cost for operating, F^I. In this case, the profit (gross of fixed costs) from a $P1$ path remains

[15] The extension could also accommodate some moderate but strict decreasing marginal contributions by, for example, introducing a fixed cost, F^D, from adding a second firm. In this case, each downstream firm would capture its Rubinstein share of its incremental contribution, $\frac{\delta}{1+\delta}(\frac{\tilde{\Pi}(0)}{1-\delta} - F^D)$. This would capture the spirit of the assumption of Weak Conditional Decreasing Marginal Contribution in Collard-Wexler, et. al. (2019).

$V^{P1}(0,0) = \frac{2}{1-\delta}(1 - \frac{\delta}{1+\delta})\tilde{\Pi}(0) = \frac{2}{1+\delta}\frac{\tilde{\Pi}(0)}{1-\delta}$ since the equilibrium logic establishing the terms of the agreement for each pair treats the incurring of the fixed cost as sunk. As δ approaches 1, the upstream firm total profit from the pair of deals approaches $\frac{\tilde{\Pi}(0)}{1-\delta}$. Thus, even if the startup cost is less than overall joint profits but exceeds the profits generated by a single market ($\frac{\tilde{\Pi}(0)}{1-\delta} < F^I < \frac{2\tilde{\Pi}(0)}{1-\delta}$), the equilibrium path would yield firm U negative profits as δ approaches 1. In this case, the ability of each downstream firm to capture a significant share of its *incremental* contribution to surplus would be so onerous as to prevent the two markets from operating even though it is efficient to do so.[16]

4.2 Homogeneous Products

With homogeneous products, market demand, $Q(P)$, and Bertrand competition downstream, the equilibrium price is $P(w_1, w_2) = max\{w_1, w_2\}$ and equilibrium total demand is $Q(P(w_1, w_2))$. Thus

$$S(w_1, w_2) = P(w_1, w_2)Q(P(w_1, w_2)).$$

Furthermore, assuming w_j is not strictly above the market monopoly price , $w(w_j)$ is any price in the interval $[0, w_j)$ and $\Pi(w_j, w(w_j)) = 0$ as opportunistic pricing allows the lowest cost firm to capture the whole market. If firm D_j agrees to an input price w_j while D_i rejects its offer, I assume a type of closure property from any later agreement, $w_i \leq w_j$ so that $q(w_j, w_i) = 0$ and $q(w_i, w_j) = Q(w_i)$. This assumption allows me to fix $w(w_j) = w_j$ and yield the full market to the firm that agrees last. If $w_i > w_j$ and f_j is such that firm D_j accepts, firm D_i will reject any positive fixed fee f_j.

These properties of homogeneous goods Bertrand markets imply that the profit for the upstream firm from a (symmetric) Type 1 path with $w_1 = w_2 = w$ satisfies[17]

$$(1-\delta)V^{P1}(w, w) = \frac{1-\delta}{1+\delta}wQ(w) + \frac{2\delta}{1+\delta}w\tilde{q}(w). \tag{12}$$

As δ approaches 1, the second term on the right side dominates. Since $\tilde{q}(w)$ represents the profit-maximizing quantity of a monopoly downstream firm with input prices w, the second term corresponds to the flow profits of an upstream firm facing a

[16] This issue is related to the need to rule out increasing marginal contributions of coalition members in Collard-Wexler, et. al.(2019)

[17] Since the focus in this section is on cases where δ goes to 1, the discounted stream of profits are normalized by multiplying by $(1 - \delta)$.

single downstream firm in the presence of double marginalization. Thus, as players become very patient, the upstream firm offers input prices arbitrarily close to its optimal price when it is limited to linear pricing alone (and therefore is subject to double marginalization). The first term represents the profits of the upstream firm as a function of the input price w in the absence of double marginalization (which corresponds to total industry profits under the homogeneous good assumption). For general δ, the right side of (12) is a convex combination of these two terms.[18]

It is interesting to compare the conclusions of this case with that of Do and Miklos-Thal (2021). Recall their model is one of homogeneous downstream goods and contracting over quantities rather than prices. They find that (with symmetric downstream firms) while opportunism falls as agents become more patient, it never completely vanishes. The explanation lies in the fact that, in their model, contracts are short-lived and overlapping so whenever a new contracting opportunity arises, there is a (brief) expected period of time in which the negotiating pair will wish to exploit the other rival.[19] In my model, dynamic bargaining plays a subtly different role. The term $w\tilde{q}(w)$ represents the default payoff of the upstream firm along a path where one downstream firm agrees to input price w and the other refuses and engages in opportunistic negotiations. Even though refusal is an out-of-equilibrium event, the value of this default term affects the anticipated payoffs of a firm if it were to reject an offer and therefore affects the prices it is willing to accept in equilibrium.

The profit for firm U from a Type 2 path satisfies

$$\begin{aligned}
(1-\delta)V^{P2}(w, V^i_{UG}) &= (1-\delta)\tilde{\Pi}(w) + w\tilde{q}(w) + \delta^2 w\frac{Q(w)-\tilde{q}(w)}{1+\delta} - \delta V^i_{UG} \\
&= (1-\delta)(\tilde{\Pi}(w) + w\tilde{q}(w)) + \delta w\tilde{q}(w) + \delta^2 w\frac{Q(c)-\tilde{q}(w)}{1+\delta} - \delta V^i_{UG} \\
&= (1-\delta)(\tilde{\Pi}(w) + w\tilde{q}(w)) + \delta w(\tilde{q}(w) + \delta\frac{Q(w)-\tilde{q}(w)}{1+\delta}) - \delta V^i_{UG} \\
&= (1-\delta)\tilde{P}(w)\tilde{q}(w) + \delta w(\frac{\tilde{q}(w)}{1+\delta} + \delta\frac{Q(w)}{1+\delta}) - \delta V^i_{UG}
\end{aligned}$$

[18] This conclusion assumes both functions on the right side of (12) are quasi-concave. In the case of linear demand downstream, these two optimal input prices are the same so, with linear demand and homogeneous goods, there is no opportunistic behavior. However, this is a highly special case and the result does not hold generally.

[19] Even though this expected period may be vanishingly small, a deviation from jointly optimal prices would bring a positive gain against a smaller loss in the anticipated recontracting. Thus, the persistence in opportunistic behavior in their model appears to be partly due to the ability to recontract quickly whereas in my model, contracts, once signed, are never revised.

where $\tilde{P}(w)$ is the monopoly price when the input price is w. Setting $V_{UG}^{i} = \delta\frac{w^*(Q(w^*)-\tilde{q}(w^*)}{1+\delta}$, Theorem 1 yields that if $V^{P2}(w, V_{UG}^{i})$ is less than $V^{P1}(w^*, w^*)$ for all w, then σ^{P1} is a subgame perfect equilibrium. For low values of δ this will clearly hold as the first term in the expression for $P1$ dominates the first term in the expression for $P2$. For high values of δ, the comparative values are not readily derived. However, for the class of market demand functions,

$$P(Q) = 1 - Q^{\alpha}, \alpha > 0,$$

the components of each expression can be fairly easily computed. If $\alpha \geq 1$, so demand is concave, then $V^{P1}(w^*, w^*)$ dominates the profit from a $P2$ path for any w and any δ. However, for the case where $\alpha < 1$ and δ is (very) close to one, computations show that the hypothesis of Theorem 1 fails and σ^{P1} is not subgame perfect.

The next subsection extends the case of linear demand to differentiated products in the LQ class of preferences and demonstrates that the Type 1 path represents a subgame perfect equilibrium path for this case.

4.3 Linear Demand, Differentiated Products and Opportunism

While the sufficient condition for Theorem 1 is purely algebraic, the inequality relies solely on properties of the underlying competition between firms 1 and 2. Thus, given the functions $q(w_1, w_1), \Pi(w_1, w_2), \tilde{q}(c), \tilde{\Pi}(c)$ and knowledge of the time preferences of agents, both the hypothesis of the theorem can be evaluated, and the predictions of the theorem computed. In this section, I illustrate such an approach using a simple tractable model of imperfect competition. The approach could be used with any underlying assumptions about demand systems and duopoly equilibria.[20]

Consider the predictions of this multilateral bargaining game in the LQ model introduced in Vives (2001). This parametrized environment is helpful as it allows us to trace out the effects of bargaining and opportunism as the degree of substitutability across products varies from independent ($\gamma = 0$) to perfect substitutes ($\gamma = \beta$). In all cases, the profit from a $P1$ path exceed the profit from a $P2$ path so Theorem 1 applies and the strategy profile in Table 1 is subgame perfect. Table 2 reports on w^* and f^* for varying values of γ/β (diversion ratio) and δ (impatience).

[20] The equilibrium argument does not rely on symmetry in any essential way. At the (significant) cost of notational complexity, the approach could be extended straightforwardly to asymmetric downstream firms.

Product Differentiation: Focus first on the predictions for input prices. The equilibrium input prices, w^*, are below the joint profit maximizing prices, $\hat{w}$, reflecting the role opportunism plays in this game. However, they are above the fully opportunistic pairwise perfect prices, $\bar{w}$ (see equation (3)). Concern that a downstream firm could unilaterally reject a current offer leads the upstream firm to deviate from the jointly optimal prices and reduce the attractiveness of rejecting and instigating a one on one bargaining game. This deviation from the optimal tends to be larger for intermediate levels of substitutability. For low levels, opportunistic motives are less important. As γ/β approaches one, demand in the downstream market approaches the case of linear demand with homogeneous goods. As Footnote 17 observes, in this limiting case the jointly optimal price and the optimal price to avoid opportunistic behavior are the same. Thus the difference falls approaching that limit.

Table 2 illustrates an interesting comparative statics on fixed fees when products become homogeneous. As products become more similar, the marginal social contribution of each firm lies in providing competitive discipline rather than product variety. However, flow profits are generally low when products are similar. In order for each downstream firm to reap its bargaining share of the value it adds due to its competitive presence (it reduces the inefficiencies that would otherwise arise because of double marginalization), it must receive a payment *from* the upstream firm. Each firm is therefore compensated via the fixed fee for that function.

Firm Patience: The deviation from jointly optimal pricing is higher, the more patient bargainers become. This is due to the fact that opportunistic behavior enters through the (unrealized) threat of rejection by downstream firms. This threat becomes more potent the more patient firms are and therefore the upstream firm has a greater incentive to mitigate the exercise of opportunism. As δ approaches zero, the game approaches the take-it-or-leave-it game with public offers analyzed in MS (1994) and, as shown there, the input prices converge to the joint profit-maximizing prices.

The effect of higher δ on equilibrium fixed fees is mainly to magnify these payments. When product differentiation is high and equilibrium fixed fee payments flow from the downstream firms to the upstream firm, these payments are higher as δ grows. And, when the flow of payments is reversed, and higher δ means higher payments.

5 Implications for Vertical Merger Analysis

The Nash in Nash model has been used extensively in vertical merger analysis. For example, in several mergers of Multi-channel Video Programming Distribution (MVPD) firms with upstream content providers, a version of the Nash in Nash model with bargaining only over linear prices has been employed to assess the price effects of the deal. It is valuable, therefore, to examine how the predicted price effects of such mergers would differ in a fully non-cooperative model using two part tariffs.

This section demonstrates that using the fully non-cooperative model yields significantly different predicted impacts of certain vertical mergers. In Section 5.1, I adapt the dynamic bargaining model considered above to a market structure where the upstream firm has merged with one of the two downstream firms and characterize the subgame perfect equilibrium in that environment. In Section 5.2.1, I describe what *would have* emerged from using a Nash in Nash approach to predict the price effect of a vertical merger using linear prices, as has been generally the case in the literature. In that approach, one can use the Nash in Nash model to compute prices both before and after a vertical merger while at the same time allowing output to vary with changes in input prices.[21] Section 5.2.2 expands the Nash in Nash analysis to allow for pricing in two part tariffs. Pre-merger input prices in that case correspond to the pairwise proof prices of the MS model with private offers. Post-merger input prices and fixed fees are very similar to the post-merger prices in my model. Section 5.3 compares the predicted price effects of such a merger across the three approaches.

5.1 Vertical Merger in the Non-Cooperative Model

Suppose that firm U and (say) firm D_1 merge and the joint firm $U1$ now engages in negotiations for input supply to firm D_2. The, now bilateral, negotiation takes a more familiar form of Rubinstein offer-counteroffer bargaining, though, in this case, over two-part tariffs. Assume firm $U1$ makes an initial offer, (w_2, f_2). If it is rejected,

[21] The FCC and DOJ typically employ a simpler, partial version of the Nash in Nash model. This approach does not consider the impact of changes in input prices on downstream market prices and output. In the working paper version (Vincent, 2020) I examine predictions from that approach as well. The approach taken here is similar in spirit to the approach taken by Ho and Lee (2017) in analysing the impact of horizontal mergers in the health insurance industry. Recent studies of merger simulations using a full equilibrium Nash in Nash approach under various timing assumptions and downstream market conditions include Das Varma and DeStefano (2018), Domnenko and Sibley (2019) and Rogerson (2020).

firm D_2 makes a counter-offer and so on. Once any offer is accepted, negotiations end and the two firms engage in perpetual downstream competition as before. In this case, however, sequential rationality requires the newly merged firm to operate so as to maximize the flow of profits of firm (or 'division') D_1 plus its total revenues from input sales. Firm $U1$ can achieve this by instructing its new subsidiary, firm D_1, to operate independently and selecting internal transfer prices to maximize this discounted flow. This immediately implies, for any agreed to w_2, the optimal transfer price for the merged firm is $w(w_2)$, that is, the opportunistic response. In the case of vertical merger, the bilateral negotiations no longer depend on observed (or conjectured) agreements with a rival firm. Rather, they are conducted anticipating that the merged firm will operate its downstream subsidiary so as to maximize its profits going forward.[22]

Define

$$w^M = argmax\ S(w(w), w).$$

For convenience, assume this is unique for all w. Recall that $\tilde{\Pi}(0)$ represents the profits of the merged firm when only its newly acquired firm 1 operates. We can now prove the following variation on Lemma 1.

Lemma 2. *Suppose w_j is such that in SG^k, $k = 2, U1$, the maximal net gains from trade between firms $U1$ and 2 are positive: $S(w, w(w)) - \tilde{\Pi}(0) > 0$, for some w.*

i) The equilibrium input price in any subgame SG^k is w^M.

ii) The equilibrium fixed fees satisfy

$$(1-\delta) f^D = \frac{\Pi(w^M, w(w^M))}{1+\delta} - \frac{\delta(R(w(w^M), w^M) + \Pi(w(w^M), w^M) - \tilde{\Pi}(0)}{1+\delta} \quad (13)$$

when the remaining downstream firm makes offers and

$$\Pi(w^M, w(w^M)) + \delta f^D = f^{U1}. \quad (14)$$

when $U1$ makes offers.

iii) The equilibrium continuation payoffs are

$$V^2_{SG^2} = \frac{S(w(w^M), w^M) - \tilde{\Pi}(0)}{1-\delta^2}$$

[22] This corresponds to what Moresi and Schwartz (2017) term the 'centralized' regime model of a vertically integrated firm.

and

$$V^{U1}_{SG^{U1}} = \frac{\tilde{\Pi}(0)}{1-\delta} + \frac{S(w(w^M), w^M) - \tilde{\Pi}(0)}{1-\delta^2}.$$

Bilateral negotiations between a vertically merged firm, $U1$, and its downstream rival have the following properties. Firms are aware that, post-negotiation, sequential rationality forces the merged firm to maximize its future flow profits – that is, for any agreed upon input price w to its rival, $U1$ will select its internal transfer prices opportunistically at $w(w)$. Dynamic bargaining leads to a standard Rubinstein split of the forecast profit increment above operating alone, $S(w(w), w) - \tilde{\Pi}(0)$. The negotiating firms, then have an incentive to select w to maximize this incremental profit.

The non-cooperative model of multilateral bargaining over two part tariffs can be used to derive pre-merger prices. The equilibrium prices characterized in Lemma 2 provide predictions of prices that would emerge following a vertical merger and comparisons of the two equilibria yield the price effects (for both fixed fees and per unit fees) of a vertical merger.

5.2 Equilibrium Nash in Nash Merger Analyis

In this subsection, I compute equilibrium prices using Nash in Nash bargaining over only per unit input prices (as is usually done in these models) as well as over two part tariffs both before and after a vertical merger to derive full equilibrium predicted price effects.

5.2.1 Nash in Nash with Linear Prices

Note that, with only two downstream firms as studied here, multi-lateral negotiations are only relevant for determining pre-merger prices. Post-merger prices are determined by straightforward bilateral Nash bargaining between the merged firm $U - D_1$ and D_2 but, as in the previous section, anticipating that post-agreement, the merged firm will behave opportunistically. For an assumed bargaining power parameter $\mu \in [0, 1]$ of the (merged) upstream firm, this implies that, the post-merger solution involves selecting an input price w for the rival downstream firm to maximize

$$(R(w(w), w) + \Pi(w(w), w) - \tilde{\Pi}(0))^{\mu} \Pi(w, w(w))^{1-\mu}.$$

After a successful agreement at input price w, the merged firm obtains the sum of its input revenues and the profits of its affiliate after pricing opportunistically. In the absence of an agreement, the most it can obtain is the monopoly profits from its downstream firm when inputs are priced at marginal cost.

Pre-merger predictions in the Nash in Nash model are more delicate. For a conjectured agreement on input price, w_j, of the other pair, each bilateral pair (U, i) selects w_i to maximize the Nash product

$$(R(w_i, w_j) - w_j \tilde{q}(w_j))^{\mu} \Pi(w_i, w_j)^{1-\mu} \tag{15}$$

but it is unclear how to form the conjecture about rival input prices. The traditional theoretical approaches have generally assumed involved solving each of these maximization problems separately holding the other prices for the other bilateral pair fixed and searching for mutually optimal solutions. This approach is very similar to the passive belief approach in MS. Analytic solutions to this problem are not available, however, Table 3 reports computational values for the LQ model. As can be seen there, this model has some unattractive properties: as downstream products become independent, the upstream and downstream firms forego significant profits by using only linear prices; and as products become close substitutes, the joint solutions to the two Nash objective functions implies input prices that are almost zero.[23]

5.2.2 Nash in Nash With Two Part Tariffs

Although I am not aware of any Nash in Nash treatments where negotiating firms use two part tariffs, it is possible to extend the analysis to allow for these richer contracts under some simplifying assumptions about beliefs and about threat points in the case of a single failed bilateral agreement.

Assume that, for any candidate profile of equilibrium price offers, $((w_1^N, f_1^N), (w_2^N, f_2^N))$, if a different contract (w_1, f_1) is contemplated by (say) firm U and firm D_1, it is expected that there will still be an agreement between U and D_2 at the equilibrium price (w_2^N, f_2^N). This implies that in the bilateral negotiations between U and D_1, firm U's incremental gain from an agreement at (w_1, f_1) is given by

$$f_1 + f_2^N + \frac{R(w_1, w_2^N)}{1-\delta} - f_2^N - \frac{w_2^N \tilde{q}(w_2^N)}{1-\delta} = f_1 + \frac{R(w_1, w_2^N) - w_2^N \tilde{q}(w_2^N)}{1-\delta}.$$

[23] An alternative is to assume the analogue of symmetry beliefs which would result in a joint Nash in Nash solution selecting w to maximize

$$(R(w, w) - w\tilde{q}(w))^{\mu} \Pi(w, w)^{1-\mu}.$$

Results from this approach are reported in Vincent (2020). The computed prices are higher than with the passive belief approach but still much lower than the non-cooperative equilibrium prices.

Since the threat point for firm D_1 is 0, this yields the cooperative Nash objective function for the bilateral negotiation between firms U and D_1 is

$$(f_1 + \frac{R(w_1, w_2^N) - w_2^N \tilde{q}(w_2^N)}{1-\delta})^{\mu}(\frac{\Pi(w_1, w_2^N)}{1-\delta} - f_1)^{1-\mu}. \tag{16}$$

A symmetric argument holds for the bargaining between firms U and D_2.

Post-merger, sequential rationality implies that the effective input price for the merged entity will be its opportunistic price $w(w_2)$. Thus the cooperative Nash objective function when negotiating with D_2 is

$$(f_2 + \frac{R(w(w_2), w_2) + \Pi(w(w_2), w_2) - \tilde{\Pi}(0)}{1-\delta})^{\mu}(\frac{\Pi(w_2, w(w_2))}{1-\delta} - f_2)^{1-\mu}. \tag{17}$$

Equations (16) and (17) imply the following lemma yielding necessary conditions for a solution to Nash in Nash bargaining in two part tariffs.

Lemma 3. *A solution to Nash in Nash bargaining with two part tariffs and with Nash bargaining power parameter μ for firm U must satisfy:*

i) The pre-merger in prices are $(w_1^N, w_2^N) = (\bar{w}, \bar{w})$;

ii) The pre-merger equilibrium fixed fees are $(f_1, f_2) = (f^N, f^N)$ where

$$(1-\delta)f^N = \mu\Pi(\bar{w}, \bar{w}) - (1-\mu)(R(\bar{w}, \bar{w}) - \bar{w}\tilde{q}(\bar{w}));$$

iii) The post-merger equilibrium input price to the unmerged firm is w^M, that is, the maximizer of $S(w(w), w)$.

iv) The post-merger equilibrium fixed fee to the unmerged firm satisfies

$$(1-\delta)f^M = \mu\Pi(w^M, w(w^M)) - (1-\mu)(R(w(w^M), w^M) + \Pi(w(w^M), w^M) - \tilde{\Pi}(0)).$$

Lemma 3 indicates that Nash in Nash bargaining over two part tariffs results in the standard Nash split of incremental surplus via the fixed fees both pre- and post-merger. The pre-merger prices are the pairwise proof prices in MS while the post-merger input prices are the prices that maximize total surplus under the expectation that the merged firm will ultimately act opportunistically. Observe that the post-merger fixed fees emerging from Lemma 2ii) also correspond to the post-merger fixed fees in Lemma 3iv) when we set the bargaining power parameter μ equal to $\frac{1}{1+\delta}$.

5.3 Comparisons

Table 3 compares the predicted price effects of a merger across the three approaches over various parameters in the LQ model. It assumes $\delta = .99$ for the dynamic non-cooperative model and sets $\mu = \frac{1}{1.99} = .53$ for both versions of the Nash in Nash models with the effective assumption of passive beliefs. Various levels of substitutability of products are considered from independent goods to virtually homogeneous products.

The non-cooperative model predicts pre-merger input prices that are generally closer to joint surplus maximizing prices than does either of the Nash in Nash models. For the linear Nash in Nash model, this reflects the parties' inability to utilize richer contracts to reach an efficient agreement. The Nash in Nash model with two part tariffs yields input prices that are closer to the jointly profit maximizing prices but not as close as the non-cooperative dynamic model. As noted earlier, this is due to the dynamic incentives of the latter model. The desire to reduce the attractiveness of further bargaining upon rejection by either of the downstream firms induces the upstream firm to push input prices higher than the pairwise proof prices that arise in the Nash in Nash model with two part tariffs.

Table 3 presents substantially different predictions of the effects of the merger on input prices depending on which negotiating framework is used. In the non-cooperative model, predicted marginal price effects are much lower than in either of the Nash in Nash models and this difference in prediction grows as downstream products become closer substitutes. These effects are important since, of course, changes in marginal input costs can be expected to directly alter consumer prices. This suggests that a merger policy using a consumer welfare standard along with Nash in Nash modeling may well overstate the impact of a proposed vertical merger.[24]

Both models predict significant negative effects of the merger on the non-merged party. In Table 4, the term ΔV_{UG}^2 represents the percentage change in profits for the non-merged downstream firm under either framework. For the dynamic bargaining model, this impact is felt mostly through large changes in negotiated fixed fees while the Nash in Nash linear model, by construction, can only have the impact through increases in input prices. The Nash in Nash model with two part tariffs predicts a somewhat larger overall impact on the non-merged firm profits than the non-cooperative

[24]The differences between pre- and post- merger prices are based on comparisons of equilibria *predicted* by underlying parameters of the model. Anti-trust agencies often have access to *actual* pre-merger prices. This ability could conceivably mitigate the differences in the table. I am grateful to Andrew Sweeting for this observation.

model though the amounts are roughly in the same order of magnitude under either scenario.

6 Conclusions

A usable model of multi-lateral negotiations among large savvy agents should enable all participants to claim a share of the available surplus, should be flexible enough for agents to achieve efficient solutions when preferences or strategic circumstances allow and should incorporate opportunistic behavior when they do not. One-shot take-it-or-leave-it models such as MS or Rey and Vergé (2004) posit circumstances where one side retains all the bargaining power and therefore cannot deliver on the first requirement. Nash in Nash bargaining models allot bargaining power to both sides of an agreement but the models with linear pricing predict that negotiating parties fail to utilize all the pricing tools at their disposal to achieve full efficiency even in the absence of contracting externalities. Nash in Nash bargaining in two part tariffs address this problem, however, its intrinsically static nature ignores potential dynamic bargaining effect on equilibrium prices. Models of bargaining over only fixed fees such as Collard-Wexler, et. al. do not extend to situations where firms wish to use input prices to control profit-destroying competition in the presence of contracting externalities.

The model presented in this paper avoids these problems by considering dynamic negotiations over richer contracts. Its dynamic property introduces opportunistic behavior implicitly as powerful agents cannot credibly keep from exerting a threat to break off current negotiations and resume them later against the interests of another party who is negotiating in parallel. The model is modular in the sense that a variety of downstream equilibrium profit functions and derived demands can be used to test the conditions for equilibrium and derive predictions on prices and profits. While previous models typically involve strong and implausible assumptions about the conjectures about the agreements of other agents who are not parties to a given bilateral bargain, my model avoids this by making another strong and probably implausible assumption about the information available to bargaining parties. Nevertheless, it presents an internally consistent benchmark for non-cooperative behavior which can be further built upon to include features such as private offers and alternative extensive forms.

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8 Appendix: Proofs

8.1 Proof of Lemma 1

Proof. Assume equilibrium prescribes immediate acceptance in both the subgame when D_i offers and when U offers. Suppose Firm D_i makes the first offer. By assumption, for Firm U to accept an offer immediately, then the offer must satisfy

$$\frac{1}{1-\delta}\left(w_j q(w_j, w_i) + w_i q(w_i, w_j)\right) + f_i \geq w_j \tilde{q}(w_j) + \delta V^U_{SG^U}(w_j).$$

Firm D_i can ensure acceptance of any (w_i, f_i) satisfying this inequality. Since its payoffs are decreasing in f_i, any optimal pair must satisfy it with equality.

The profit for Firm D_i from an immediately accepted offer then is

$$\frac{1}{1-\delta}\left(S(w_i, w_j) - \Pi(w_j, w_i)\right) - w_j \tilde{q}_j(w_j) - \delta V^U_{SG^U}(w_j).$$

Only the first two terms depend on w_i. Thus, any equilibrium offer resulting in immediate acceptance must involve $w_i = w(w_j)$. Similarly, if Firm U is making an offer, to be acceptable, the offer must satisfy

$$\frac{1}{1-\delta}\Pi(w_i, w_j) - f_i \geq \delta V^i_{SG^i}(w_j).$$

Firm U would wish to raise f_i until this held with equality yielding its payoff as a function of w_i as

$$\frac{\Pi(w_i, w_j) + w_j q(w_j, w_i) + w_i q(w_i, w_j)}{1-\delta} - \delta V^i_{SG^i}(w_j) = \frac{S(w_i, w_j) - \Pi(w_j, w_i)}{1-\delta} - \delta V^i_{SG^i}(w_j).$$

Thus, any immediately acceptable equilibrium input price offered by Firm U must also satisfy, $w_i = w(w_j)$.

Immediate acceptance in both subgames implies that, in equilibrium, $f^U(w_j)$ and $f^D(w_j)$ must satisfy

$$\begin{aligned} \frac{R(w(w_j), w_j)}{1-\delta} + f^D(w_j) &= R(w(w_j), w_j)(1 + \frac{\delta}{1-\delta}) + f^D(w_j) \\ &= w_j \tilde{q}(w_j) + \delta f^U(w_j) + \frac{\delta R(w(w_j), w_j)}{1-\delta}. \end{aligned}$$

The first equality expands the fraction $1/(1-\delta)$ the second equality uses the fact that the same input price is agreed to in the following period if a rejection occurs in the first period. Combining the last two equalities yields

$$R(w(w_j), w_j) - w_j \tilde{q}(w_j) + f^D(w_j) = \delta f^U(w_j) \tag{18}$$

A similar argument applied to the maximal fixed fee acceptable by Firm D_i in subgame $SG^U(w_j)$, implies $f^D(w_j), f^U(w_j)$ must satisfy

$$\Pi(w(w_j), w_j) + \delta f^D(w_j) = f^U(w_j)$$

which is (9). Together, (18) and (9) yield (8) for $SG^i(c_)$.

The equilibrium equations for (w_i, f_i) imply that the (undiscounted) continuation equilibrium payoff for Firm D_i beginning with subgame $SG^i(w_j)$ is

$$\begin{aligned} V^i_{SG^i}(w_j) &= \frac{\Pi(w(w_j), w_j)}{1-\delta} - f^D(w_j) \\ &= \frac{\Pi(w_i(w_j), w_j)}{1-\delta} - \frac{\delta \Pi(w(w_j), w_j) + w_j \tilde{q}(w_j) - R(w(w_j), w_j)}{1-\delta^2} \\ &= \frac{(1+\delta)\Pi(w(w_j), w_j)}{1-\delta^2} + \frac{R(w(w_j), w_j) - w_j \tilde{q}(w_j) - \delta \Pi(w(w_j), w_j)}{1-\delta^2} \\ &= \frac{S(w(w_j), w_j) - \Pi(w_j, w(w_j)) - w_j \tilde{q}(w_j)}{1-\delta^2} \end{aligned} \tag{19}$$

Since $\frac{1}{1-\delta^2} = \frac{1}{1-\delta}\frac{1}{1+\delta}$, this is the familiar result that Firm D_i obtains a slightly favorable share ($\frac{1}{1+\delta}$ versus $\frac{\delta}{1+\delta}$) of the maximal net gains from trade starting from SG(w_j, i).

Similarly, the gross payoff for Firm U when it makes offers in the two firm subgame is,

$$\begin{aligned} V^U_{SG^U}(w_j) &= \frac{R(w(w_j), w_j)}{1-\delta} + f^U(w_j) \\ &= \frac{R(w(w_j), w_j)}{1-\delta} + \Pi(w(w_j), w_j) + \frac{\delta^2 \Pi(w(w_j), w_j) + \delta w_j \tilde{q}(w_j) - \delta R(w(w_j), w_j)}{1-\delta^2} \\ &= \frac{(1+\delta)R(w(w_j), w_j) + (1-\delta^2)\Pi(w(w_j), w_j)}{1-\delta^2} + \frac{\delta^2 \Pi(w(w_j), w_j) + \delta w_j \tilde{q}(w_j) - \delta R(w(w_j),}{1-\delta^2} \\ &= \frac{R(w(w_j), w_j) + \Pi(w(w_j), w_j) + \delta w_j \tilde{q}(w_j)}{1-\delta^2} \\ &= \frac{w_j \tilde{q}(w_j)}{1-\delta} + \frac{S(w(w_j), w_j) - \Pi(w_j, w(w_j)) - w_j \tilde{q}(w_j)}{1-\delta^2} \end{aligned}$$

The first equality is by definition, the second use (8) and (9). The following three equalities are rewriting where the last follows because $R(w(w_j), w_j) + \Pi(w(w_j), w_j) = S(w(w_j), w_j) - \Pi(w_j, w(w_j))$ and $\frac{\delta}{1+\delta} w_j \tilde{q}(w_j) = (1 - \frac{1}{1+\delta}) w_j \tilde{q}(w_j)$.

The indifference condition then implies that

$$V^U_{SG^i}(w_j) = w_j \tilde{q}(w_j) + \delta V^U_{SG^U}(w_j) \tag{21}$$

which is the second expression in iii). □

8.2 Proof of Theorem 1

Proof. That the strategies following any subgame $SG^j_a(w_i), j = 1, 2, U$ are sequentially rational follows from Lemma 1. Therefore focus on the strategies following any UG subgame. The conjectured equilibrium is a Type 1 path and thus yields Firm U an expected payoff of $V^{P1}(w_1^*, w_2^*) = V_{UG}^U$ and downstream firm D_i an expected payoff of $\delta V_{SG^i}^i(w_j^*)$. Consider any two part tariff offer $(w_1, f_1), (w_2, f_2)$. If $NE^*(w_1, f_1, w_2, f_2, \delta V_{SG^1}^1(w_2^*), \delta V_{SG^2}^2(w_1^*)) = (Reject, Reject)$ then offering $(w_1^*, f_1^*), (w_2^*, f_2^*)$ dominates since $\delta < 1$.

If $NE^*(w_1, f_1, w_2, f_2, \delta V_{SG^1}^1(w_2^*), \delta V_{SG^2}^2(w_1^*)) = (Accept, Reject)$ (say) then the prescribed continuation path is a Type 2 path yielding Firm U an expected payoff no greater than $V^{P2}(\tilde{c}, \delta V_{SG^2}^2(w_1^*)) \leq V_{UG}^U$ by hypothesis. If $NE^*(w_1, f_1, w_2, f_2, \delta V_{SG^1}^1(w_2^*), \delta V_{SG^2}^2(w_1^*)) = (Reject, Accept)$ the parallel argument applies.

The path σ^{P1} selects a mixed strategy equilibrium of the game

$$G(w_1, f_1, w_2, f_2, \delta V_{SG^1}^1(w_2^*), \delta V_{SG^2}^2(w_1^*))$$

only if there are no pure strategy equilibria. The equilibrium is unique and in mixed strategies if and only if

$$\begin{aligned}
\frac{\Pi(w_1, w_2)}{1-\delta} - f_1 &< \delta V_{SG^1}^1(w_2) \\
\frac{\Pi(w_2, w_1)}{1-\delta} - f_2 &> \delta V_{SG^2}^2(w_1) \\
\delta^2 V_{SG^2}^2(w_1^*) &> \tilde{\Pi}(w_2) + \frac{\delta \Pi(w_2, w(w_2))}{1-\delta} - f_2 \\
\delta^2 V_{SG^1}^1(w_2^*) &< \tilde{\Pi}(w_1) + \frac{\delta \Pi(w_1, w(w_1))}{1-\delta} - f_1
\end{aligned}$$

or the reverse inequalities.

Let $(p, q) \in (0,1)^2$ be the equilibrium mixed strategies where $p(resp.q)$ represents the probability of acceptance by Firm 1 (2). For $p \in (0,1)$ to be a best response to q, it must be the case that

$$q\frac{\Pi(w_1, w_2)}{1-\delta} + (1-q)(\tilde{\Pi}(w_1) + \frac{\delta \Pi(w_1, w(w_1))}{1-\delta}) - f_1 = q\delta V_{SG^1}^1(w_2) + (1-q)\delta V_{UG}^1$$

or

$$pf_1 = pq\left(\frac{\Pi(w_1, w_2)}{1-\delta} - \delta V_{SG^1}^1(w_2)\right) + p(1-q)\left(\tilde{\Pi}(w_1) + \frac{\delta \Pi(w_1, w(w_1))}{1-\delta} - \delta V_{UG}^1\right) \tag{22}$$

and similarly,

$$qf_2 = qp\left(\frac{\Pi(w_2, w_1)}{1-\delta} - \delta V_{SG^2}^2(w_1)\right) + q(1-p)\left(\tilde{\Pi}(w_2) + \frac{\delta \Pi(w_2, w(w_2))}{1-\delta} - \delta V_{UG}^2\right). \tag{23}$$

The expected return to Firm U from this profile of strategies is

$$pq\frac{R(w_1,w_2)}{1-\delta}+p(1-q)(w_1\tilde{q}(w_1)+\delta V^U_{SG^2}(w_1))+q(1-p)(w_2\tilde{q}(w_2)+\delta V^U_{SG^1}(w_2))+pf_1+qf_2+(1-p)(1-q)\delta V^U_{UG}$$

Substituting, using (22) and (23), yields the expected return is

$$\begin{aligned}
&pq\left(\frac{R(w_1,w_2)}{1-\delta}+\frac{\Pi(w_1,w_2)}{1-\delta}-\delta V^1_{SG^1}(w_2)\frac{\Pi(w_2,w_1)}{1-\delta}-\delta V^2_{SG^2}(w_1)\right) &+\\
&p(1-q)\left(w_1\tilde{q}(w_1)+\delta V^U_{SG^2}(w_1)\tilde{\Pi}(w_1)+\frac{\delta\Pi(w_1,w(w_1))}{1-\delta}-\delta V^1_{UG}\right) &+\\
&q(1-p)\left(w_2\tilde{q}(w_2)+\delta V^U_{SG^1}(w_2)\tilde{\Pi}(w_2)+\frac{\delta\Pi(w_2,w(w_2))}{1-\delta}-\delta V^2_{UG}\right) &+\\
&\qquad\qquad\qquad\qquad\qquad\qquad(1-p)(1-q)\delta V^U_{UG} &.
\end{aligned}$$

By the hypothesized equilibrium continuation, $V^i_{UG}=\delta V^i_{SG^i}(w^*_j)$ and so the bracketed terms in the first three lines are, respectively, $V^{P1}(w_1,w_2)$, $V^{P2}(w_1,\delta V^1_{SG^1}(w^*_2))$, and $V^{P2}(w_2\delta V^2_{SG^2}(w^*_1))$. Therefore, the continuation profit for U from inducing any strictly mixed strategy equilibrium is

$$pqV^{P1}(w_1,w_2)+p(1-q)V^{P2}(w_1)+q(1-p)V^{P2}(w_2)+(1-p)(1-q)\delta V^U_{UG}.$$

By definition, $V^{P1}(w^*_1,w^*_2)\geq V^{P1}(w_1,w_2)$ for all (w_1,w_2) and by hypothesis, $V^{P1}(w^*_1,w^*_2)\geq V^{P2}(\tilde{c},\delta V^i_{SG^i}(w^*_j))\geq V^{P2}(c,\delta V^i_{SG^i}(w^*_j))$ for all c and $V^{P1}(w^*_1,w^*_2)>\delta V^U_{UG}$ for $\delta<1$. The proposed equilibrium returns $V^{P1}(w^*_1,w^*_2)$ with probability one while any mixed strategy equilibrium returns a probabilistic mixture of values that are dominated by this return and so is a better response for Firm U.

□

8.3 Proof of Corollary 1

Proof. In this environment,

$$(1-\delta)V^{P1}(w_1,w_2)=\frac{\Pi(w_1)+w_1\tilde{q}(w_1)+\Pi(w_2)+w_2\tilde{q}(w_2)}{1+\delta}$$

and

$$(1-\delta)V^{P2}(w_i)=\Pi(w_i)+w_i\tilde{q}(w_i)+\frac{\delta\Pi(w_j)+w_j\tilde{q}(w_j)}{1+\delta}-\delta V^i_{UG}$$

It is immediate that $(\hat{w}_1,\hat{w}_2)=(w^*_1,w^*_2)=(\tilde{c}_1,\tilde{c}_2)=(0,0)$ (or marginal cost, more generally) Assuming symmetric markets this implies

$$(1-\delta)V^{P1}(w^*_1,w^*_2)=\Pi(0)\frac{2}{1+\delta}$$

and, substituting $\delta V^i_{UG} = \Pi(0)\frac{\delta^2}{1+\delta}$

$$(1-\delta)V^{P2}(w_1, \delta V^i_{SG^i}(w^*_j)) = \Pi(0)\left(1 + \frac{\delta - \delta^2}{1+\delta}\right).$$

Since

$$\begin{aligned}\frac{2}{1+\delta} - \left(1 + \frac{\delta-\delta^2}{1+\delta}\right) &= \frac{2 - ((1+\delta) + \delta - \delta^2)}{1+\delta} \\ &= \frac{(1-\delta)^2}{1+\delta} \\ &> 0\end{aligned}$$

the payoff from the best Type 1 path dominates the best Type 2 path and by Theorem 1, it forms a subgame perfect equilibrium. □

8.4 Proof of Lemma 2

Proof. Assume equilibrium prescribes immediate acceptance in both the subgame when D_i offers and when $U1$ offers. Suppose Firm D_2 makes the offer. By assumption, for Firm $U1$ to accept an offer immediately, then the offer must satisfy

$$\frac{R(w(w_2), w_2) + \Pi(w(w_2), w_2)}{1-\delta} + f_2 \geq \tilde{\Pi}(0) + \delta V^{U1}_{SG^{U1}}.$$

Firm D_2 can ensure acceptance of any (w_2, f_2) satisfying this inequality. Since its payoffs are decreasing in f_2, any optimal pair must satisfy it with equality.

The profit for Firm D_2 from an immediately accepted offer then is

$$\frac{S(w_2, w(w_2))}{1-\delta} - \tilde{\Pi}(0) - \delta V^{U1}_{SG^{U1}}.$$

Only the first term depends on w_2. Thus, any equilibrium offer resulting in immediate acceptance must be w^M.

Similarly, if Firm $U1$ is making an offer, to be acceptable, the offer must satisfy

$$\frac{\Pi(w_2, w(w_2))}{1-\delta} - f_i \geq \delta V^2_{SG^2}.$$

Firm $U1$ would wish to raise f_i until this held with equality yielding its payoff as a function of w_2 as

$$\frac{S(w(c), c)}{1-\delta} - \delta V^2_{SG^2}.$$

Thus, any immediately acceptable equilibrium input price offered by Firm $U1$ must also equal w^M.

Immediate acceptance in both subgames implies that, in equilibrium, f^{U1} and f^D must satisfy

$$\begin{aligned}\frac{R(w^M, w(w^M)) + \Pi(w(w^M), w^M)}{1-\delta} + f^D &= (R(w^M, w(w^M)) + \Pi(w(w^M), w^M))(1 + \frac{\delta}{1-\delta}) + f^D \\ &= \tilde{\Pi}(0) + \delta f^{U1} + \frac{\delta}{1-\delta}(R(w^M, w(w^M)) + \Pi(w(w^M), w^M)).\end{aligned}$$

Combining the last two equalities yields

$$R(w^M, w(w^M)) + \Pi(w(w^M), w^M) - \tilde{\Pi}(0) + f^D = \delta f^{U1} \tag{24}$$

Similarly the maximal fixed fee acceptable by Firm 2 in subgame SG^U, implies f^D, f^U must satisfy

$$\Pi(w^M, w(w^M)) + \delta f^D = f^{U1}$$

which is (14). Together, (24) and (14) yield (13) for SG^2.

The equilibrium equations for (w_2, f_2) imply that the (undiscounted) continuation equilibrium payoff for Firm 2 beginning with subgame SG^2 is

$$\begin{aligned}V^2_{SG^2} &= \frac{\Pi(w^M, w(w^M))}{1-\delta} - f^D \\ &= \frac{\Pi(w^M, w(w^M))}{1-\delta} - \frac{\delta\Pi(w^M, w(w^M)) + \tilde{\Pi}(0) - \Pi(w(w^M), w^M) - R(w^M, w(w^M))}{1-\delta^2} \\ &= \frac{(1+\delta)\Pi(w^M, w(w^M))}{1-\delta^2} - \frac{\delta\Pi(w^M, w(w^M)) + \tilde{\Pi}(0) - \Pi(w(w^M), w^M) - R(w^M, w(w^M))}{1-\delta^2} \\ &= \frac{S(w^M, w(w^M)) - \tilde{\Pi}(0)}{1-\delta^2}\end{aligned} \tag{25}$$

Since $\frac{1}{1-\delta^2} = \frac{1}{1-\delta}\frac{1}{1+\delta}$, this is the familiar result that Firm D_i obtains a slightly favorable share ($\frac{1}{1+\delta}$ versus $\frac{\delta}{1+\delta}$) of the maximal net gains from trade starting from the default point at which Firm $U1$ is a monopoly provider downstream.

Similarly, the gross payoff for Firm U when it makes offers in the two firm subgame is,

$$V^U_{SG^U} = \frac{\tilde{\Pi}(0)}{1-\delta} + \frac{S(w^M, w(w^M)) - \tilde{\Pi}(0)}{1-\delta^2}.$$

The newly merged firm, $U1$ can guarantee itself the discounted stream of profits as a downstream monopolist plus its bargaining share of the incremental profits from adding a second downstream firm. □

8.5 Proof of Lemma 3

Proof. (ii) Consider firms U and D_1. Fix an expected input price w_2 for D_2. Take logs of (16) and differentiate first with respect to f_2 to get

$$\frac{\Pi(w_1, w_2^N) - (1-\delta)f_1}{(1-\delta)f_1 + R(w_1, w_2^N) - w_2^N \tilde{q}(w_2^N)} = \frac{1-\mu}{\mu}. \tag{26}$$

Rearranging, yields (ii).

(i) Similarly, the necessary condition for optimizing with respect to w_1 requires

$$\frac{\mu \frac{\partial R(w_1, w_2^N)}{\partial w_1}}{(1-\delta)f_1 + R(w_1, w_2^N) - w_2^N \tilde{q}(w_2^N)} = -\frac{(1-\mu)\frac{\partial \Pi(w_1, w_2^N)}{\partial w_1}}{\Pi(w_1, w_2^N) - (1-\delta)f_1}. \tag{27}$$

Combining (26) and (27) yields that the optimal choice of w_1 must be the opportunistic price $w(w_2^N)$. Symmetrically applying this logic to the negotiations between U and D_2 imply that pairwise proof input prices are necessary for Nash in Nash negotiations in two part tariffs.

iv) Similarly, post-merger, the optimal choice of f^M satisfies

$$\frac{\mu(\Pi(w_2, w(w_2) - (1-\delta)f_2)}{(1-\mu)((1-\delta)f_2 + R(w(w_2), w_2) + \Pi(w(w_2), w_2) - \tilde{\Pi}(0))} = 1. \tag{28}$$

iii) Differentiate with respect to w_2 and apply (28) to get

$$\frac{\partial}{\partial w_2}\left(\Pi(w_2, w(w_2)) + R(w(w_2), w_2) + \Pi(w(w_2), w_2)\right) = 0. \tag{29}$$

That is, the optimal post-merger input price maximizes joint profits subject to the knowledge that post-deal the merged firm will select an internal transfer price that corresponds to the opportunistic price.

□

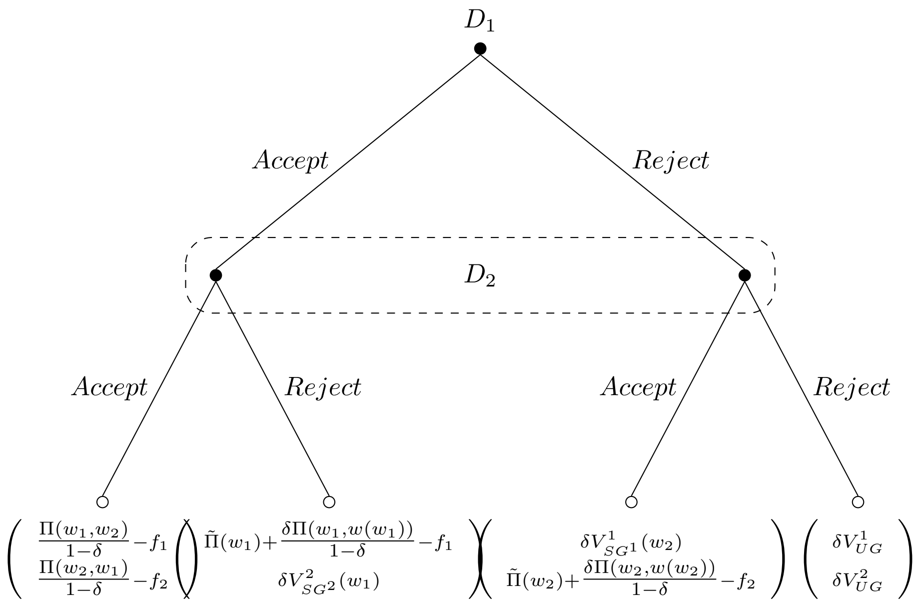

Figure 1: The bimatrix game, $G(w_1, f_1, w_2, f_2, \delta V^1_{UG}, \delta V^2_{UG})$

Table 1: Strategy Profile σ^{P1}, for any UG Subgame

Subgame Type	**'Subgame'**	**Response**
UG_a :		Firm U offers $(w_1^*, f_1^*), (w_2^*, f_2^*)$.
UG_b :	$(w_1, f_1), (w_2, f_2)$:	Firms 1 and 2 play the unique equilibrium strategies $NE^*(w_1, f_1, w_2, f_2, V_{UG}^1, V_{UG}^2)$.
$SG_a^j(w_i)$:		Firm j offers $(w(w_i), f^D(w_i))$.
$SG_b^j(w_i)$:	(w_j, f_j) :	Firm U accepts any offer (w_j, f_j) such that $\frac{R(w_i,w_j)}{1-\delta} + f_j \geq V_{SG^i}^U(w_i)$.
$SG_a^U(w_i)$:		Firm U offers $(w(w_i), f^U(w_i))$.
$SG_b^U(w_i)$:	(w_j, f_j) :	Firm j accepts any offer such that $\frac{\Pi_j(w_j,w_i)}{1-\delta} - f_j \geq \delta V_{SG^j}^j(w_i)$.

Table 2: Equilibrium in the LQ Model					
δ	γ/β	$\hat{w}$ (jointly optimal)	$\bar{w}$ (pairwise proof)	w^* (equilibrium input)	f^* (equilibrium fixed fee)
.99	0	0	0	0	18.8
	1/2	.25	.0625	.216	1.65
	3/4	.375	.141	.330	−4.87
	.90	.45	.20	.418	−7.44
	.98	.488	.238	.477	−8.93
.5	0	0	0	0	0.5
	1/2	.25	.0625	.23	0.10
	3/4	.375	.141	.35	−0.02
	.90	.45	.20	.43	−0.09
	.98	.488	.238	.482	−0.12
.1	0	0	0	0	0.38
	1/2	.25	.0625	.24	0.12
	3/4	.375	.141	.37	0.04
	.90	.45	.20	.45	0.00
	.98	.488	.238	.486	−0.01

Table 3: Comparison of Merger Effects on Input Prices										
		Non-cooperative Model			Nash in Nash ($\mu = .53$)					
		($\delta = .99$)			Linear			Two Part Tariffs		
γ/β	$\hat{w}$	w^*	w^M	Δw	w^*_{Nash}	w^M	Δw	w^*_{Nash}	w^M	Δw
0	0	0	0	0	0.25	0.25	0%	0	0	0%
1/2	0.25	0.2	0.28	30%	0.23	0.37	60%	0.0625	0.28	344%
3/4	0.38	0.33	0.41	24%	0.17	0.43	153%	0.141	0.41	191%
.90	0.45	0.42	0.47	13%	0.1	0.45	350%	0.20	0.47	135%
.98	0.49	0.48	0.49	4%	0.03	0.49	1500%	0.238	0.497	109%

Table 4: Comparison of Merger Effects on Fixed Fees and Profits									
($\delta = .99$)	Non-cooperative Model				NiN Linear	NiN Two Part Tariffs			
γ/β	f^*	f^M	Δf	ΔV^2	ΔV^2	f^*	f^M	Δf	ΔV^2
0	18.8	18.8	0	0	0%	0	0	0	0%
1/2	1.7	3.7	126%	−51%	−51%	4.2	4.1	−3.8%	−78%
3/4	−4.1	0.5	112%	−78%	−75%	1.3	1.2	−87%	−74%
.90	−7.4	−0.1	99%	−92%	−84%	−0.5	−0.1	−90%	−92%
.98	−8.9	−0.1	99%	−98%	−92%	−1.6	−0	−98%	−99%