The Optimal Timing of Procurement Decisions and Patent Allocations

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ABSTRACT

We illustrate by means of a dynamic research and development race that while at some points in the race social incentives and private incentives may coincide at other points they may diverge -- too many researchers remain in the race. If the social planner cannot determine what stage the researchers have achieved, this informational constraint poses difficulties in ensuring a socially optimal outcome. We show that there is a mechanism which allows the planner to exploit the researchers' private information to determine when and to whom to allocate the exclusive rights to pursue the final prize. This mechanism does not require any transfer of resources and, therefore, will not distort earlier incentives to invest. Furthermore, it is solvable by the iterative elimination of dominated strategies.

JEL Classification: C72, D44, O31.

1. Introduction

In research and development races, social incentives and private incentives may often diverge -- the private pursuit of profit can induce too many researchers to remain in the race. Of course, this possibility depends on the environment. If the costs of independent research are not too high and the social rewards great, it may be desirable to allow multiple paths of research. We illustrate that both phenomena can occur within the same dynamic research and development race and that, as a result, a non-trivial social planning problem arises. The decision to grant the sole rights to pursue a prize may depend not only on where the leader is but also on his position relative to his rival.

This decision may be complicated even further if the social planner cannot observe what stage the researchers have achieved. We show that, if the researchers are aware of their relative positions, there is a mechanism which allows the planner to exploit their private information in order to determine when and to whom to allocate the exclusive rights to pursue the final prize. This mechanism does not require any transfer of resources and, therefore, will not distort earlier incentives to invest as would the use of a price mechanism. Furthermore, it is simply implemented by only a few rounds of elimination of dominated strategies.

2. A Motivating Example

When the US government decided in the spring of 1942 to commit substantial resources to develop the atomic bomb, it remained unresolved which would be the best source of fissionable material. The expectation was that either separation of the U-235 uranium isotope or the manufacture of the element plutonium would yield fuel for the bomb but there were numerous avenues that could be explored, none of which guaranteed certainty of success.[1] In the end, the commander of the Manhattan Project, Leslie Groves, authorized four separate pilot projects to be pursued: in Chicago, at Berkeley, Columbia, and the Naval Research Laboratories.[2] A review committee was created to assess the progress of each venture and decide which would be the most promising method. This decision was evidently not an easy one. Even after Fermi demonstrated the feasibility of using a controlled chain reaction to produce plutonium, the Chicago team was not confident that the decision would go their way. Arthur Compton, the team's leader wrote a letter to Conant complaining that the committee (more specifically, the representatives of Dupont who were to put the process into production) tended to be too pessimistic about the possibilities of their approach and hinted very strongly that perhaps General Electric or Westinghouse should instead be asked to develop plutonium. This implicit threat may or may not have had an effect. In any case, it was decided to implement Fermi's approach to produce fissionable material as well as an additional method.[3]

This episode illustrates the main features of the dynamic incentive problem we analyze. A number of researchers are available to pursue a commonly valued, stochastic prize. Even though effective participation requires the investment of some sunk costs, the urgency of the task or the great uncertainty of success may make it worthwhile from the standpoint of a social welfare criterion to invite more than one agent to participate in the race at the outset. However, at some later point, social efficiency may require that only one agent remain to complete the race. Unfortunately, the private value to the researchers of winning the prize will often imply that we can not rely on private incentives to induce the optimal set of investigators to drop out of the race even when one of them has made substantially more progress than the rest. An outside authority or principal may be required to force some agents to leave. For example, if the race is a patent race, an early granting of a patent right could be the appropriate instrument. If the race is for a procurement contract, making the contract award before the final product is actually developed could achieve this goal.

Even if such an instrument is available, though, in many cases, informational problems remain. It can be very costly or even impossible for the principal to determine when laggard agents should be forced out or even which of them should be the ones to leave the race. Although the review committee did succeed in establishing to its satisfaction that the Fermi approach was superior, Compton's letter suggests that the issue was not an easy one and perhaps even that it was one open to manipulation. Of course, even if the choice of approaches was the correct one, the timing of the decision could easily have been different. Throughout 1942, scarce economic and intellectual resources were devoted to a variety of pursuits that might have been more productively focussed on a single path.

The difficulty inherent in making the determination of when and to whom to allocate the exclusive rights to pursue a prize is the focus of this paper. We construct an idealized model of a stochastic race for a prize and show that there exist situations where it is socially efficient to invite many participants at the initial phase but to restrict participation at later phases. The need for a later restriction creates an implementation problem if private incentives alone do not induce the appropriate participants to drop out. In this environment, we show that even if the principal cannot tell when a researcher is far enough ahead to grant him the sole rights to continue the race, as long as the competitors themselves are aware of each other's position, there exists a simple mechanism which can ensure that the optimal decision is made at the desired time without distorting the initial incentives to invest.

For concreteness, we focus on a stylized example with private agents pursuing the development of an innovation in an economy where a benevolent social planner has the authority to limit their participation in the race by the early grant of a patent. However, the model can also be interpreted as a solution to a procurement problem. Peck and Scherer (1962) describe the importance of timing in the awarding of contracts for military aircraft in the U.S. The costs of development of these weapons have risen dramatically since the beginning of air warfare. As a result, the U.S. government has often found it preferable to limit competitive development races and to award contracts well before a prototype is ever constructed.[4] The desire to encourage and then restrict multiple research paths can arise for private firms as well. In its race with BASF to develop a synthetic indigo dye in Germany at the end of the nineteenth century, Hoechst sponsored four separate pilot projects to test differing approaches.[5]

Consider an invention which can only be acquired after researchers pass through two or more stages. Firms that reach stages sooner have an advantage over laggard firms. Once a single firm moves sufficiently ahead of the rest, it may be socially desirable to grant that firm the exclusive rights to continue the project. Nevertheless, the attractiveness of the final objective may continue to induce all firms to stay in the race. One goal of an efficient patent or procurement system should be to discourage such socially inefficient but privately profitable pursuits. The social planner may then find herself in the position of having to decide whether there is a sufficient asymmetry across researchers to justify the endowment of a patent and to decide to which firm the patent is to be awarded. Very often this decision will have to be made in the absence of any concrete evidence in support of the ruling and or in the absence of an affordable measure for ranking the firms in question.[6] Even when such investigations are technologically feasible, society often will need to rely on an individual such as a patent officer to make the determination for it. The complexity and subjectivity of these decisions make it very difficult for society to monitor its own principal (who is, in fact, acting as society's agent) and this creates a situation ripe with the potential for corruption. A system that does not yield the principal either too much responsibility or too much autonomy to make the decision is needed.

3. The Social Problem

Consider the following dynamic allocation problem. Two agents (who we call firms) seek to develop a product that has a known monetary value V. The development of the product requires passage through separate stages. Let X denote the set of such stages. The movement from stage to stage occurs consecutively and stochastically, and is controlled by the investment of research resources period by period. Time has value in that earlier discovery is preferred to later discovery. Players attempt to maximize their discounted expected profits and, in the absence of outside intervention, select strategies which are best responses to each other in every period -- that is, they select strategies which are subgame perfect Nash equilibria.

The stage that each player has attained is assumed to be common knowledge between the players but not observable by an outside principal. Although the common knowledge assumption is strong, it is made in many analyses of R&D races.[7] While firms may not enjoy the precise knowledge that their rivals have acquired, they may well be aware of the stage of progress achieved by their rivals. That the relative position is unobservable to outsiders can be interpreted as a consequence of the degree of technical sophistication required to assess firms' true positions. Even if all the relevant data is made public, only specialists who are well versed in the research area may be able to evaluate quite readily the achievements of both themselves and their rivals. A less well-informed outsider may not be able to conduct this evaluation at reasonable cost even though she has access to the same observable data.

The closer the researcher is to the final stage, the higher are his expected profits from the race. This monotonicity implies that any stage in the R&D race can be represented by the expected payoff of being at that location in the absence of any competition. For convenience, we choose this representation. Thus, the set X can be represented by a discrete set of real numbers. A generic element of this set, x_j, denotes both the stage attained by firm j and the expected profits j would enjoy if it were the only firm in the race. Higher values correspond to stages which are closer to the ultimate goal. In typical R&D races, social costs are incurred by the wasteful duplication of research activity. On the other hand, the more firms engaged in research, the sooner the expected time of discovery. If the wasteful duplication is too great then it may be preferable to limit entry into the race right at the outset. More generally, though, we argue that it can be preferable to wait until one or both of the firms have made preliminary progress. Furthermore, whether or not restricting the number of racers is desirable will depend not only on the absolute position of a firm but also on its position relative to its rival -- that is, how far ahead it is. A "social choice" function, $f{:}X \rightarrow \mathbb{R}$, is introduced to capture this dependence. It can be interpreted as follows. Suppose firm j is at stage x_j. Only if it is at least $f(x_j)$ ahead of firm i, is it socially desirable to force i to drop out of the race.

The function $f(\bullet)$ allows for the possibility that the distance a firm needs to be leading its rival can vary depending how close it is to the final prize.[8] In the next section we show how a function of this type can arise from a standard R&D race however, for the purposes of the mechanism, this social choice function should be thought of as exogenous to the implementation problem. Note, though, that this function is very general and simply captures the dependence of the allocation decision on both the absolute and relative position of the leader.

Although it is known that privately induced patent races can result in greater than efficient investment (see, for example, Loury, (1979) or Reinganum, (1982)), we are not aware of any models showing that a divergence of social and private incentives can occur at different stages of the race if privately acquired knowledge is not transferable.[9] In the next section, we describe a class of multi-stage R&D games which have this feature and thus provide a motivation for a social choice function of the form, $f(\bullet)$. Using the sum of the private welfare of the two firms as the welfare criterion, there exists a subset of this class of games where total welfare is maximized by allowing multiple firms at the outset and then restricting competition after one of the firms makes a preliminary advance.

4. A Research and Development Race

In this section a simple model is described which shows that rational behavior on the part of firms as well as the social planner can create the type of dilemma illustrated in the introduction. We characterize a class of environments with the following features: there are two firms, each of which maximizes its expected utility (profits) by entering the race and staying in until the end even if it falls behind; and the social optimum calls for both firms to stay in the race *only* as long as they are symmetric, and for one firm to quit the race once the other is (far enough) ahead.

Two firms begin a race in an initial stage 1.[10] Time is measured in periods. In any period, a firm may move out of one stage to another stochastically as long as it invests in an R&D cost in that period.[11] The passage from one stage to another is independent of the actions or position of their rivals. There are three stages. Once one or both firms move out of stage 2 to stage 3 and acquire the prize, V, the game ends but the number of periods that the firms may potentially invest in R&D is infinite. Firms discount the future at a common discount factor, δ, per period. For simplicity, we assume that the costs of R&D, c, is the same whether the firm is in stage 1 or 2. Similarly, the probability of exiting either stage is the same, denoted by p.[12] If a firm is in stage 2 and successfully develops the innovation alone it earns the full value of the product immediately and the laggard firm earns nothing. If both firms pass out of stage 2 by simultaneously making the discovery, then they split the prize equally. The ex post payoff of a firm is the discounted value of this discovery (if it makes the discovery) minus the discounted R&D expenditures. Firms maximize expected payoffs given the strategy of their rival.

We use as our social welfare function, the discounted expected value of the discovery minus the discounted expected costs of both firms. This criterion can arise in a variety of ways. If the example described a pure patent race and the patent winner was subsequently able to capture all the consumer surplus, then the welfare function describes a standard social welfare problem. Alternatively, the social welfare function could represent the preferences of a large research firm wishing to create overall profit-maximizing incentives for research among its independent research subdivisions (as in the example of Hoechst). Or, the problem may simply describe the incentives for two independent research firms to coordinate their activities optimally so as to maximize their joint profits.

Private Incentives:

We begin by calculating backward, from the end of the game, the expected payoffs of the firms in different stages of the race. Denote by U_{st} the expected payoff for a firm in stage s when its rival is in stage t. Thus the continuation value for a firm when both firms in stage 2 and both choose to invest is

$$U_{22} = p(1-p)V + p^2\left(\frac{V}{2}\right) + (1-p)^2\delta U_{22} - c = \frac{\frac{V}{2}(2p-p^2)-c}{1-\delta(1-p)^2}.$$

Similarly, the expected payoff for a firm in the second stage when its rival is still in the first stage is

$$U_{21} = pV + (1-p)\delta[pU_{22} + (1-p)U_{21}] - c = \frac{pV + p(1-p)\delta U_{22} - c}{1-\delta(1-p)^2}.$$

$U_{21} > U_{22}$ -- it is better to be alone in the final stage than with a rival. Continuing along the same line,

$$U_{12} = \delta[(1-p)pU_{22} + (1-p)^2U_{12}] - c = \frac{U_{22}\delta(1-p)p - c}{1-\delta(1-p)^2}.$$

Since

$$\frac{\delta(1-p)p}{1-\delta(1-p)^2} < 1,$$

it follows that $U_{12} < U_{22} < U_{21}$.

We are interested in races in which a firm which finds itself behind still wishes to remain in the race, that is, where $U_{12} > 0$. This implies that the cost of research must be relatively small so that $c \leq \delta p(1-p)U_{22}$ or

$$c \leq \frac{\delta p^2(1-p)V(1-p/2)}{(1+\delta(1-p))(1-\delta(1-p)^2)}. \tag{1}$$

Inequality (1) (which also implies $U_{22} > 0$) is the binding constraint in the analysis which follows. Note that for any $\delta > 0$, there exist c, V and p all strictly positive for which (1) holds.

We can now evaluate the expected payoff for a firm from entering the race when its rival is in a symmetric position.

$$\begin{aligned} U_{11} &= \delta[p(1-p)U_{21} + (1-p)pU_{12} + p^2U_{22} + (1-p)^2U_{11}] - c \\ &= \frac{\delta[p(1-p)U_{21} + (1-p)pU_{12} + p^2U_{22}] - c}{1-\delta(1-p)^2}. \end{aligned} \tag{2}$$

Since $U_{21} > U_{22} > 0$, (2) implies that $U_{11} > U_{12} > 0$ if (1) holds. Therefore, as long as inequality (1) holds, it is always a dominant strategy for each firm to remain in the race.

Welfare Analysis:

Let W_{st} denote the gross welfare generated by having two firms compete, one at stage s and the other at stage t. W_s is the welfare generated from having only one firm in the race at stage s. Note that our measure of social surplus implies that W_s is the same as the private value of being the only firm in the race. Also, since we represent each stage, s, by W_s, X is $\{W_1,W_2\}$. Employing the same approach used to calculate private payoffs,

$$W_{22} = [p^2 + 2p(1-p)]V + \delta(1-p)^2 W_{22} - 2c = \frac{Vp(2-p)-2c}{1-\delta(1-p)^2} = 2U_{22},$$

$$W_2 = pV + \delta(1-p)W_2 - c = \frac{pV-c}{1-\delta(1-p)},$$

$$W_{12} = pV + \delta(1-p)[pW_{22} + (1-p)W_{12}] - 2c = \frac{pV-c+2\delta(1-p)pU_{22}-c}{1-\delta(1-p)^2}.$$

and

$$W_1 = p\delta W_2 + (1-p)\delta W_1 - c = \frac{p\delta W_2-c}{1-\delta(1-p)}.$$

Finally we derive W_{11}. For future use, the calculation is performed under the assumption that if only one firm advances to the second stage then the other firm is forced to quit. This yields

$$W_{11} = \delta[p^2 W_{22} + 2p(1-p)W_2 + (1-p)^2 W_{11}] - 2c = \frac{\delta[p^2 W_{22} + 2p(1-p)W_2]-2c}{1-\delta(1-p)^2}.$$

The following lemma illustrates the potential conflict between the private motives of the firms and the objective of maximizing the joint welfare of the firms ex ante. It establishes that it may be optimal from a social standpoint to have both firms start the race, and once one is ahead, to force the one that is behind to quit.

Lemma 1: *If δ is small enough, then there exist values c, p, and V for which $0<U_{12}$ (inequality (1) is satisfied), yet $W_{22} > W_2 > W_{12}$, and $W_{11} > W_1$.*

Proof: See appendix. ■

Thus, at least for small discount factors, there are values of *c, p,* and *V* for which a conflict is generated between private and social incentives in the race for the prize. In this example, social preferences can be represented by the function $f(\bullet)$ such that $f(W_1) = \infty$ and $f(W_2) = \Delta$, where $\Delta = W_2 - W_1$. No firm in stage 1 should ever be awarded the patent right and a firm in stage 2 should only be awarded the right if its rival is in stage 1. The proof of Lemma 1 suggests either that the costs of R&D be small or the urgency of achieving the prize be great (or equivalently, the length of time between periods be large).[13] The intuition for this is that the social benefit from having more than one firm stems from the effect that more R&D speeds the expected discovery. The cost arises from the possibility that (ex post) duplicative R&D occurs. As a result, for values of δ close to one, there is generally never any social value to having more than one firm in the race.

Note that in this class of R&D games, $W_{12}>W_2$ implies that $U_{12}>0$ so, whenever the social planner wishes both firms to remain, the laggard firm will find it profitable to remain on its own. Green and Scotchmer (1990) show in a sequential innovation model where interim innovations can be profitably marketed that, depending on the patent rule in effect, laggard firms may remain in the race inefficiently or may drop out inefficiently. They show that first to file and first to invent rules only incompletely resolve the inefficiencies. Since in our model, whenever it is socially efficient for both firms to remain in the race, it is privately profitable as well, we look at a narrower class of problems. However, the mechanism we propose fully resolves the problem of insufficient exit.

The example is easily modified to illustrate a more complex implementation problem. With more stages, a social planner must be concerned not only with the question of to whom to allocate the sole right to remain in the race but must also determine the correct time, if ever, to make the allocation. For example, suppose that stage 1 was preceded by an earlier stage, call it stage 0. Let the probability of moving from stage 0 to stage 1 also be p but suppose the investment cost for this period are very low or zero. Lemma 1 can be extended to show that for p,δ,c and V such that $U_{12} > 0$, $W_{22} > W_2 > W_{12}$, and $W_{11} > W_1$ then we also have $W_2 > W_{02}$, $W_{01} > W_1$ and $W_{00} > W_0$. Because early research costs are low, it is best to allow both firms to remain in the race even if one firm has moved ahead to stage 1. However, in this case it is desirable to restrict the race once a firm is ahead and in stage 2. From the perspective of the social planner, it must be determined not only if one firm is ahead but also whether the leading firm has achieved stage 2 before forcing the other firm out of the race.

The timing problem illustrates why a simple auction to allocate the rights to be the sole firm remaining in the race typically fails to achieve the goals of the mechanism. One problem with the use of a standard price mechanism is that, in forcing a transfer of wealth at some later stage, it reduces the initial value of the enterprise and therefore may discourage entry at the outset. However, if the auction is modified so the winner does not have to pay for the right, in equilibrium, there must be some way to discourage frivolous attempts to force the exit of a rival. The need to achieve this goal and maintain a strong solution concept poses a significant mechanism design problem. In the next section we present an institution that implements the socially efficient outcome using the elimination of dominated strategies.

5. An Optimal Mechanism

The R&D game described in the previous section indicates the need for a mechanism which can extract enough information from participants in a race so as to enable an ill-informed principal to grant to one agent the sole right to remain in the race if and only if it moves sufficiently ahead of its rival. In order to be generally applicable, the mechanism should allow for a relatively flexible definition of what "sufficiently ahead" means. That is, it should work for a large class of social choice functions, $f(\bullet)$. In addition, it should not distort the firms' incentives to invest -- the mechanism, in equilibrium, should impose no cost to any participant. It is also important that the optimal strategies in the mechanism not be too difficult for participants to compute. In the proposition in this section, we show that a mechanism exists that satisfies all of these criteria.

We wish, then, to find an allocation mechanism with the following properties:

i) for any firm, j, in any period, if $x_j \geq x_i + f(x_j)$, the mechanism allocates to j the exclusive rights to continue to pursue the prize, otherwise, both firms are allowed to remain;

ii) in equilibrium, no monetary transfers are required from either firm to make the allocation;

iii) the mechanism relies only on the information reported by the two firms in the race;

iv) the mechanism can be implemented by a small number of rounds of iterative elimination of dominated strategies.

Condition iv) is required to ensure the practicality of the institution. It constrains the designer to select a mechanism that is simple for the firms to understand and play appropriately. The more transparent the mechanism (and the fewer the orders of elimination required), the more plausible it will be that the equilibrium will indeed be played.

Condition iii) can be understood as an explicit consequence of the assumption that the principal who is implementing the mechanism cannot directly observe the respective positions of the firms. There are many situations where outsiders cannot observe the actual positions of the researchers and must rely only on their (self-interested) reports. For example, many of the initial stages of a research program are either unpatentable because of the nature of patent law (US patent law does not allow the granting of patents for the discovery of abstract principles, algorithms or laws of nature) or because the cost of patenting exceeds the potential benefits to the innovator.[14] However, this informational disadvantage is not the only reason it may be desirable to limit the implementation of the mechanism to publicly observable reports by the firms. If the principal herself cannot be trusted, an institution that operates openly and publicly may be the best way to prevent corruption on the part of the official operating the mechanism.

The restriction on monetary transfers (condition ii)) stems from the assumption that agents must sink research expenditures initially. If they anticipate having to pay additional costs when their research venture is successful in order to convince an authority to force their slower rival out of the race, the initial incentives to invest may be weakened.[15] Condition i) simply reflects the potential inconsistency between social goals and private incentives that is illustrated in the previous section. Researchers who are maximizing private expected utility may not voluntarily leave the race when a social planner (whose preferences are represented by $f(\bullet)$) would like them to.

The implementation of the mechanism we present requires that x-$f(x)$ be strictly increasing as was the case for the example. This appears to be a weak restriction. It implies that for any stage, x, attained by the leader, there is a unique $y=x-f(x)$ for the follower such that if the follower is at a stage below y then the patent should be awarded and if the follower is at a stage above y, it should not be awarded. Recall that X is the set of possible locations in the R&D race. The proposition is for the case where X is finite. If X has the cardinality of the continuum, a similar proof can be employed, as long as the R&D race is such that in the cross-product space, (v_1,v_2), the set of points, $v_j-f(v_j)=v_i$ occur with zero probability at any stage. This would be the case, for example, if the realization of any given stage is determined by a non-atomistic probability function.

To define the mechanism, we to extend $f(\bullet)$ so that it is defined over the real interval $[W_1,W_n]$. We also modify it to cope with the slightly problematic case, $v_j-f(v_j)=v_i$. For any s,s' such that $W_{s'}=W_s-f(W_s)$, let $f^*(W_s)=f(W_s)-\delta(W_s)$ so that $W_{s'}<W_s-f^*(W_s)<min\{W_{s'+1},W_{s+1}-f(W_{s+1})\}$. Otherwise, set $f^*(W_s)=f(W_s)$. For $z\in[W_s,W_{s+1}]$, $z=\alpha(z)W_s+(1-\alpha(z))W_{s+1}$, let $f^*(z)=\alpha(z)f^*(W_s)+(1-\alpha(z))f^*(W_{s+1})$. Note that $f^*(\bullet)$ is defined over $[W_1,W_n]$, is continuous, and, since x-$f(x)$ is strictly increasing, so is z-$f^*(z)$. The construction ensures that there are no $W_{s'},W_s$ such that $W_{s'}=W_s-f^*(W_s)$, The mechanism below implements $f(\bullet)$ in three rounds of elimination of weakly dominated strategies:

(i) In each time period, first firm 1 and then firm 2, chooses whether to announce "I am ahead" or not. If neither of them announces, then nothing happens in that period. If only one announces, then the mechanism proceeds to (ii). If both firm announce, then only 1's announcement counts, and the mechanism proceeds to (ii).

(ii) Let firm *j* be the announcer. Firm *i* then either "agrees" or "challenges" by submitting a sealed bid $\tilde{r} \in \mathbb{R}$. If firm *i* chooses to "agree", then firm *j* is awarded the patent, and the mechanism ends. If firm *i* challenges, then *j* pays a fine $P>0$ and the mechanism proceeds to (iii).

(iii) Firm *j* submits a counterbid, *x*. If $x \geq \tilde{r}$, *j* wins the patent for a price of $\tilde{r}$ (to be paid to the planner). Firm *i* then pays a fine $Q > 0$. If $x<\tilde{r}$ then firm *i* buys at price $x - f^*(x)$. **End**.

Proposition 2: *Suppose X is finite and x-f(x) is strictly increasing. This mechanism implements f(x) in three rounds of elimination of weakly dominated strategies.*

Proof of Proposition: Without loss of generality, we assume that firm *j* is the proclaimed leader, while firm *i* is the challenger.

Round 1: *For firm j, given the firm's (true) expected value is* v_j, *and a challenge has been made, eliminate every strategy other than bidding* $x=v_j$.

That any other strategy is dominated by $x=v_j$ is standard from arguments from second price auctions, noting that the fine *P* is now sunk.

Round 2: a) *Eliminate all strategies for firm i in which he challenges when* v_j-$f(v_j) \geq v_i$.

This strategy is dominated by not challenging. If i challenges with $\tilde{r} > v_j \geq f(v_j) + v_i$ then, given the elimination from Round 1, firm j will lose the auction and firm i will be forced to buy at price v_j - $f^*(v_j)$. Since $f^*(v_j) \leq f(v_j)$ and $v_j = f(v_j) + v_i$ implies $f^*(v_j) < f(v_j)$, this yields utility $v_i - v_j + f^*(v_j) \leq v_i - v_j + f(v_j) \leq 0$, where at least one of the inequalities is strict. If i challenges with $\tilde{r} \leq v_j$, j will win and i will have to pay the fine and gain negative utility, $-Q$.

b) *Also eliminate all strategies in which he does not challenge, when* $v_j\text{-}f(v_j) < v_i$.

Not challenging yields a payoff of zero. This is dominated by any strategy of the form, Challenge and bid $\tilde{r} > v_{j.}$ This strategy yields $v_i - v_j + f^*(v_j)$. Let i^* be the lowest W_i such that $v_j\text{-}f(v_j) < W_i$. (This exists, since X is finite). By definition, $W_{i^*-1} \leq v_j\text{-}f(v_j) < v_j\text{-}\ f^*(v_j) < W_{i^*}$. . Therefore, for any $v_i \geq W_{i^*}$, challenging with a bid $\tilde{r} > v_j$ yields a strictly positive payoff.

Round 3: *Eliminate all remaining strategies for j except those in which he claims to be ahead when his expected value is* v_j, *his opponent's value is* v_i *and* $v_j\text{-}f(v_j) \geq v_i$.

When $v_j\text{-}f(v_j) \geq v_i$, by Round 2, i does not challenge and j earns the patent for free. When $v_j\text{-}f(v_j) < v_i$, i challenges and wins and j pays a $P>0$ fine. ■

Intuitively, the mechanism operates as follows. When a firm is ahead enough to satisfy the social choice function, the firm announces this fact and posts a bond. The rival firm is then invited to invoke an asymmetric second price auction. If the auction is called for, the announcer’s bond is forfeited. The payments in the auction are constructed so that following, the elimination of weakly dominated strategies, the challenger wins if and only if she is close enough to the announcer that the social choice function would call for no award. Frivolous calls for an auction (which are revealed when the challenger loses the auction) are punished by fining the challenger when she loses. Thus a firm is awarded the patent for free only if it is enough ahead that the social planner would wish to award it. If it calls for an award any other time, it creates an incentive for its rival to call an auction which imposes a penalty on the announcer and which the announcer loses.

Why would a more standard auction mechanism (with asymmetric payments and bidding costs) not equally well implement the social choice rule?[16] For example, what if we just had a second price auction with similar payment rules in the presence of bidding costs? First, observe that in order to satisfy the requirement that no payments arise with an appropriate allocation, it is necessary to have the auction only along a "punishment" path, that is, out of equilibrium. Second, to deter firms from invoking the mechanism at an inappropriate stage, it is necessary that a firm that calls for an auction and loses, incur a strict loss. Third, in order to impose such a cost and at the same time maintain a solution in weakly dominated strategies, a carefully applied system of fines must be established. The presence of bidding costs alone are not sufficient because for at least one bidder, a decision not to participate in an auction allows the bidder to avoid the bid costs. A consequence is that the solution of the auction game in weakly dominant strategies is lost. A similar but slightly more involved argument applies as well when bidding costs are reimbursed contingent on winning the auction. A driving feature of our mechanism is that the "bond" posted by the announcer creates a sunk cost which does not affect his optimal bidding strategy.

The mechanism above does not prescribe a unique bid, r, for the challenger to name when an inappropriate claim is lodged. Any $r>v_j$, is undominated and achieves the goal. If a mechanism with a unique equilibrium strategies is sought, the mechanism could be modified so that the challenger's payment at auction is $r\text{-}f^*(r)$, rather than $x\text{-}f(x)$, however, the space of allowable challenges would have to be discretized since, otherwise, the optimal challenge is the lowest $\tilde{r}<v_j$ which yields a familiar openness problem.

6. Conclusions

An R&D race is often modeled as one with a well defined finishing line where the first to reach that line gets the exclusive rights to pursue the final prize. Where exactly to put the finishing line, can also be an important policy decision. In discussing this policy variable, it is usually implicitly assumed in the literature, that references can be made only to the leader's position in the R&D race. (Such is a policy stating that exclusive rights should be awarded to the first to reach a certain stage.) The optimal rule, however, depends not only on the leader's stage of research, but also on the location of its rivals. If the follower is very close to the leader then his investment might not be socially wasteful even in a relatively late stage of the R&D. At the same time, a big jump ahead by one of the firms in an early stage of the race, could make further investments of lagging firms inefficient. One of the reasons for ignoring the location of the slower firm (by the literature as well as policy-makers) is of course an observability problem. The pursuit of profits may induce the leader to file for a patent (say), but it is much more difficult to get the follower to reveal his position. Our mechanism provides incentives for the follower to report exactly when the leading firm is not far enough ahead.

Two informational assumptions make our model somewhat special: i) the assumption that participants in the race enjoy common knowledge about each other's position; and ii), despite this knowledge, the assumption that outsiders cannot observe the same information. We argue that the second assumption is not implausible. It can be interpreted literally on the grounds that even in the presence of a great deal of information, the specialist skills required to interpret that information may be too great for an outside observer to evaluate. Alternatively, it can be interpreted as a reflection of society's mistrust of the integrity of the official who is required to make the important allocation decision.

Green and Scotchmer (1990) address a related problem, asking in a similar environment of sequential innovation, whether minor or major innovations should be patentable. They show that, in some circumstances, major innovations may be preferable to minor innovations even though they may encourage laggard firms to stay in too long. One can view our model as addressing the case where the only patent option is a major innovation, corresponding to the grant of the sole right to pursue the single, final prize. This limited option may arise because interim innovations are not profitably marketable or because a firm is reluctant to provide its rival with the informational value that patent disclosure would require. Given this limitation, the question remains as to *when* the patent should be allocated, or equivalently, when the laggard should be forced to exit the race.

The assumption that relative positions are common knowledge, while typical of many models of R&D races, is more significant. Its role, though, is primarily to allow us to utilize the very powerful solution concepts of subgame perfection and iterative elimination of dominated strategies. We conjecture that similar results can be obtained in a model with incompletely informed players, however, it is evident that a much more sophisticated solution concept such as sequential equilibrium would need to be employed in solving both the underlying R&D game and any implementation mechanism. While solving such a game would be a formidable task, it is not clear how much additional insight would be gained from this extension.

Our approach may be thought of as an application of the research on the optimal design of patents to the issue of the timing of patent allocations. Recently, a U.S. research company, Human Genome Sciences, generated controversy by attempting to acquire patents for partial gene sequences, the functions of which were not well known at the time. (The Washington Post, April 17, 1996). The R&D model we described illustrates a situation where such an award would be socially desirable. In addition, we present a very simple mechanism which would enable a policymaker to implement the optimal allocation rule if and only if the leading firm has made enough progress relative to the location of its rivals. An attractive feature of the mechanism is that while the leading firm reveals its location truthfully, it does so without the need to reveal details of the innovation. Furthermore, because the mechanism does not require the decision-maker to have full information, it eliminates the need for this agent to bear the costs of evaluation. As a result, it also reduces the opportunities for corruption by the officers who must implement the social decision.

Appendix

Proof of Lemma 1: We start by showing the following claim.

Claim: For every $\delta > 0$, there exist values c, and V for which $U_{12} > 0$ (i.e. inequality (1) is satisfied), yet $W_2 > W_{12}$.

$$W_{12} = \frac{pV - c + 2\delta(1-p)pU_{22} - c}{1-\delta(1-p)^2}$$
$$= \frac{pV-c}{1-\delta(1-p)}\frac{1-\delta(1-p)}{1-\delta(1-p)^2} + \frac{\delta(1-p)pU_{22} - c}{1}-\delta(1-p)^2 + \frac{\delta(1-p)pU_{22}}{1-\delta(1-p)^2}$$
$$= W_2\left[1 - \frac{\delta(1-p)p}{1-\delta(1-p)^2}\right] + \frac{\delta(1-p)pU_{22} - c}{1-\delta(1-p)^2} + \frac{\delta(1-p)pU_{22}}{1-\delta(1-p)^2}$$

Recall that we assumed in (1) that $\delta(1-p)p\ U_{22} \geq c$. So let (1) hold with equality. The third term in the last line above becomes zero and we can write

$$W_{12} = W_2 + \frac{\delta(1-p)p}{1-\delta(1-p)^2}[U_{22} - W_2].$$

Note that W_2 is also the private profit of a firm which finds itself alone in stage 2. Therefore,

$U_{22} < W_2$ and we have $W_{12} < W_2$ for V, p, c such that (1) holds close enough to equality.

We now continue with the proof of Lemma 1.

$$W_{11} = \frac{\delta[p^2W_{22} + 2p(1-p)W_2] - 2c}{1-\delta(1-p)^2}$$
$$= \frac{\delta(p^2W_{22} + p(1-2p)W_2 - c}{1-\delta(1-p)^2} + \frac{\delta pW_2 - c}{(1-\delta(1-p))}\frac{(1-\delta(1-p))}{(1-\delta(1-p)^2)}$$
$$= \frac{\delta p(p(W_{22}-W_2) + (1-p)W_2) - c}{1-\delta(1-p)^2} + W_1(1 - \frac{\delta(1-p)p}{1-\delta(1-p)^2})$$
$$= W_1 + \frac{\delta p(p(W_{22} - W_2) + (1-p)(W_2 - W_1)) - c}{1-\delta(1-p)^2}.$$

So $W_{11} > W_1$ iff

$$\delta p(p(W_{22} - W_2) + (1-p)(W_2 - W_1)) \geq c. \qquad (*)$$

Recall that $W_{22} = 2U_{22}$. Setting (1) again as an equality, we get

$$c = \frac{W_{22}(1-p)p\delta}{2}.$$

Can (*) now hold? Substituting and dividing by δ and p yields

$$p(W_{22} - W_2) + (1-p)(W_2 - W_1 - \frac{W_{22}}{2}) \geq 0. \tag{**}$$

Now express W_{22} and W_1 in terms of W_2:

$$\begin{aligned} W_{22} &= \frac{Vp-c}{1-\delta(1-p)}\frac{1-\delta(1-p)}{1-\delta(1-p)^2} + \frac{Vp(1-p)-c}{1-\delta(1-p)^2} \\ &= W_2 + \frac{p(1-p)(V-\delta W_2)-c}{1-\delta(1-p)^2} \end{aligned}$$

and

$$W_1 = \frac{\delta p W_2 - c}{1-\delta(1-p)}.$$

Since

$$V - \delta W_2 = V - \frac{\delta(pV-c)}{1-\delta(1-p)} = \frac{(1-\delta)V + \delta c}{1-\delta(1-p)}$$

it follows from the way we rewrote W_{22} that $W_{22} > W_2$ iff

$$\frac{p(1-p)((1-\delta)V+\delta c)}{1-\delta(1-p)} > c$$

or

$$(1-\delta)p(1-p)V > c[1-\delta(1-p) - \delta p(1-p)] = c[1-\delta(1-p^2)].$$

As $\delta \Rightarrow 1$, the above inequality can not hold. However, as $\delta \Rightarrow 0$, it holds iff $p(1-p)V > c$. Thus, if δ is small enough and $p(1-p)V > c$, then $W_{22} > W_2$. Inequality (**) now becomes

$$\frac{p^2(1-p)V-c}{1-\delta(1-p)^2} - \frac{\delta p^2(1-p)W_2}{1-\delta(1-p)^2} + (1-p)\frac{(1-\delta)W_2+c}{1-\delta(1-p)} - \frac{1-p}{2}\left(W_2+\frac{p(1-p)(V-\delta W_2)-c}{1-\delta(1-p)^2}\right) \geq 0.$$

As $\delta \Rightarrow 0$, the left hand side approaches

$$p^2(1-p)V - c + (1-p)(W_2 + c) - \frac{1-p}{2}(W_2 + p(1-p)V - c)$$

$$= V[p^2(1-p)-p\frac{(1-p)^2}{2}] + \frac{1-p}{2}W_2 + c[(1-p) + \frac{1-p}{2}-1]$$

$$= V[p(1-p)(p-\frac{(1-p)}{2})] + \frac{1-p}{2}W_2 + c[\frac{1-p}{2}-p].$$

$$= \frac{1-p}{2}W_2 + \frac{1}{2}(3p-1)[p(1-p)V-c].$$

Therefore, a sufficient condition for the lemma to hold is for (1) to hold with (almost) equality, for δ small, for $p \geq 1/3$ and $p(1-p)V \geq c$. Note that $p(1-p)V > c$ is exactly the condition we needed for $W_{22} > W_2$. ■

ENDNOTES

[1]"In a May 14, 1942 memo to [Vannevar] Bush, James Conant stated that there were five separation or production methods which were about equally likely to succeed." Smyth (1945). Bush and Conant were members of the Military Policy Committee which oversaw the Manhattan Project.

[2]The costs of the individual laboratories were large. For example, the Berkeley facility alone required $30 million in set-up costs (Groves (1962)) while Conant estimated that the total cost to the pilot plant stage would be more than $500 million (Smyth (1945)).

[3]See Groves (1962) and Jones (1985).

[4]According to Peck and Scherer (1964) early procurement decisions were not made until prototypes were actually built. Later, decisions were made at the design phase. Now, with even design projects demanding substantial upfront expenditures, decisions are often made on the basis of the business plans of various applicants.

[5]Freeman, 1982.

[6]For example, the costs of assessing the relative merits of designs for US military aircraft are often enormous. Peck and Scherer (1963) estimated that in the early 1960's, the fulltime services of more than 200 engineers over several months were required by the US Airforce to assess proposed aircraft designs.

[7]See, for example, Fudenberg, et al. (1983) or Reinganum (1982). Green and Scotchmer (1990) similarly assume that the stage of innovation is common knowledge among firms but not the information embodied in that stage itself.

[8]If the planner simply wishes to award the prize whenever one firm moves ahead, the

King Solomon mechanism suggested by Glazer and Ma (1989) would achieve many of the desired goals. However, typically the optimal time to award an exclusive prize may be a much more complicated function of the firms' positions. The function $f(\bullet)$ here is intended to capture this complexity.

[9]Reinganum (1982) shows that the levels of socially efficient and privately induced R&D may cross over time if jointly conducted research enables the transmission of knowledge.

[10] Green and Scotchmer (1990) analyze equilibria in a similar two period sequential innovation race but allow for the possibility that interim stages of development may be marketable. In these versions, there may be both excessive incentives for the laggard to remain in the race (as in ours) and excessive incentives for the laggard to drop out depending on the patent rule in force.

[11]The notion that innovation requires the progression through various discrete stages is common in R and D races. Fudenberg, et al. (1983), characterize the distinction as that between the conceptual definition of research program and the actual implementation of it. Riordan and Sappington point out that the procurement of military equipment in the US very often occurs in stages. In the early, design, stages, a number of firms compete for the (often) sole rights to provide the products to the government. At some point in this process, government procurement officials must decide which firm wins this valuable right.

[12]It is more reasonable to allow probabilities and costs to vary with the stage achieved. We allow for this in a later discussion of the implications of extending the number of stages but for the purposes of the formal analysis, this is not required and the assumption of similar

costs and probabilities eases the notation significantly.

[13]In the memo referred to above, Conant writes "All five methods will be entering very expensive pilot-plant development within the next six months; furthermore, if time is to be saved, the production plants should be under design and construction before the pilot plant is finished. To embark in the Napoleonic approach to the problem would require the commitment of perhaps $500 million and quite a mass of machinery. Anything less than this will mean either the abandonment or the slowing down of one of the methods. While all five methods now appear to be about equally promising, clearly the time to production by the five routes will certainly not be the same but might vary by six months or a year. Therefore, if one discards one or two or three of the methods now, one may be betting on the slower horse unconsciously." Smyth (1977).

[14]Scotchmer (1991) argues "Patent law requires disclosure for the same reason that innovators dislike it: it is the vehicle by which technical knowledge is passed from the patenting firm to it competitors.[...] As a consequence the innovator could hold the product off the market until it develops the second more valuable generation product."

[15]A distortion similar in spirit arises in Green and Scotchmer (1995). They point out that the inability to capture all of the social value of a first stage discovery reduces the initial incentive to invest. On the other hand, if another researcher who makes an incremental improvement on this discovery is forced to pay all of the incremental social surplus to the first stage inventor, the incentives of the later researcher to incur further sunk costs in research are diminished and the second stage may not be developed.

[16] We thank an anonymous referee for raising this question.

REFERENCES

Freeman, Christopher, 1982, *The Economics of Industrial Innovation*, Second Edition, Cambridge, MA: MIT Press.

Fudenberg, D., Gilbert, R., Stiglitz, J. and Tirole, J., 1983, "Preemption, Leapfrogging, and Competition in Patent Races," *European Economic Review*, 22, 3-31.

Glazer, J. and Ma, C.-T. (1989), "Efficient Allocation of a "Prize" -- King Solomon's Dilemma," *Games and Economic Behavior*, 1, 222-233.

Green, J. and Scotchmer, S. (1995), "On the Division of Profit in Sequential Innovation," *RAND Journal of Economics*, 26(1), 20-33.

Green, J. and Scotchmer, S. (1990), "Novelty and Disclosure in Patent Laws," *RAND Journal of Economics*, 21(1), 131-146.

Groves, Leslie (1962), *Now it can be told: The Story of the Manhattan Project*, New York, NY: Harper Brothers.

Jones, Vincent (1985), *Manhattan: The Army and the Atomic Bomb*, Washington, DC: Center of Military History, United States Army.

Loury G. (1979), "Market Structure and Innovation," *Quarterly Journal of Economics*, 93, 395-410.

Peck, M. and Scherer, F. (1962), *The Weapons Acquisition Process: An Economic Analysis*, Cambridge, MA: Harvard University Press.

Reinganum, J. (1982), "Dynamic Games of Innovation," *Journal of Economic Theory*, 25, 21-41.

Riordan, M. and Sappington, D.E.M. (1989), "Second Sourcing," *RAND Journal of Economics* 20(1), 41-58.

Scotchmer, Suzanne (1991), Standing on the Shoulders of Giants," *Journal of Economic Perspectives*, 5, 29-42.

Smyth, H.D. (1977), "A General Account of the Development of Methods of Using Atomic Energy for Military Purposes Under the Auspices of the United States Government. (*The Smyth Report*, 1945). Reprinted in *The Secret History of the Atomic Bomb*. Anthony Caves Brown and Charles MacDonald (eds.), Vol. 20, No. 1, New York, NY: Delta Books.

The Washington Post (1996), April 17.