Optimal Capital-Income Taxation
in a Model with Credit Frictions

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Abstract
The optimality of the long-run capital-income tax rate is revisited in a simple neoclassical growth model with credit frictions. Firms pay their factors of production in advance, which requires borrowing at the beginning of the period. Borrowing, in turn, is constrained by the value of collateral they own at the beginning of the period. This constraint leads to inefficiently low amounts of capital and labor. In this environment, the optimal capital-income tax in the steady state is non-zero. Specifically, with no capital depreciation allowance, the capital-income tax is unambiguously negative so that the distortions stemming from the credit friction are offset by subsidizing capital. When depreciation allowance is introduced, capital-income tax can be either positive or negative depending on the degree of allowance. Quantitative analyses show that for plausible degrees of depreciation allowance the capital-income tax is indeed negative. However, when the government cannot distinguish between capital-income and profits, the capital-income tax is positive as the government levies the same tax rate on both sources of income. These results stand in contrast to the celebrated result of zero capital-income tax of Judd (1985) and Chamley (1986).

Key Words: Optimal capital-income taxation; Credit frictions; Borrowing constraints.

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1 Introduction

A classic result in optimal fiscal policy, due to Judd (1985) and Chamley (1986), is that in an economy with infinitely-lived agents who derive utility from leisure, capital-income should not be taxed in the long run. This paper studies the optimal long-run capital-income tax in a simple neoclassical growth framework in which firms borrow in order to pay factors of production in advance. Borrowing, however, is constrained by their beginning-of-period collateral. The main result of the paper is that the optimal capital-income tax is non-zero in the long run. The analytical analyses suggest that the long-run capital-income tax is unambiguously negative with no capital depreciation allowance, but it may be positive if depreciation allowance is introduced. Numerical analyses show that the optimal capital income tax is negative under plausible rates of the capital depreciation allowance. These results suggest that credit frictions, in and of themselves, lead to departures from the celebrated Judd-Chamley result.

Besides the government, the baseline setup assumes two types of agents in the economy: households and a representative firm. The firm hires labor and rents capital from households at given factor prices. The firm borrows in order to pay for the factors of production at the beginning of the period. Borrowing, in turn, is constrained by the firm’s value of real estate. This corresponds to the usual limited enforcement problem as in Kiyotaki and Moore (1997).

The basic intuition behind the non-zero capital tax result is as follows. Because of the binding credit constraint, firms rent capital so that the marginal product of capital is larger than its rental rate, thus implying inefficiently low demand for capital. By subsiding capital, the supply of capital is boosted towards the first-best level of capital, thus offsetting the effects of credit distortion.

An alternative way of viewing the result is by considering the implications of the collateral constraint. When the marginal product of capital exceeds the capital rental rate, the firm is acting as a monopolistically-competitive firm even though the product market is perfectly competitive (and the difference between the marginal product of capital and the capital rental rate is akin to a "markup"). Judd (2002) shows that with imperfectly competitive product markets, the optimal capital-income tax is negative in order to offset the markup of monopolistically-competitive firms. In this regard, the negative capital-tax result of this paper can also be viewed similarly.
The above results are based on the assumption that the government can distinguish between capital income and profits (which in this paper arise due to the credit friction) and thus it can tax each of them separately. Under this assumption, the government would like to tax profits at a rate of 100 percent. Confiscating all profits has the advantage of generating tax revenues that allow for reducing distortionary taxes without influencing households’ decisions at the margin. The paper shows that when all profits are confiscated, the optimal capital-income tax rate equals the first-best capital-income tax rate.

The government, however, may be unable to tax profits and capital income separately; as discussed in Guo and Lansing (1999), this may happen because the government cannot distinguish between both sources of income or because the firm hides its profits. In this case, the government levies the same tax rate on profits and capital income. Under this restriction on optimal fiscal policy, the paper shows that the optimal long-run capital-income tax is positive (around 32 percent in the benchmark calibration of the model with depreciation allowance of 100 percent).

There is a voluminous literature on the optimal capital-income tax rate when lump-sum taxes are unavailable. Judd (1985) and Chamley (1986) suggest that the optimal capital-income tax is zero in the steady state to eliminate any wedges between the intertemporal rate of substitution and the marginal rate of transformation. Capital taxation discourages savings, due to the fall in the after-tax rate of return on savings, implying taxation of future consumption. Judd (1999) argues that capital-income should not be taxed given its nature as intermediate good. His argument follows the suggestions of Diamond and Mirrless (1971) against taxing intermediate goods. Chari, Christiano, and Kehoe (1994) show that, over the business cycle, optimal Ramsey taxation implies an ex ante capital tax rate that is approximately zero.

Ho and Wang (2007) consider a two-period overlapping generation model in which capital is accumulated through credit-financed investments in a credit market with asymmetric information between borrowers and lenders. They argue for less reliance on capital-income taxation by the government to finance its expenditures (and, instead, more reliance on labor-income taxation). In their model, capital-income taxation worsens the adverse selection problem in the credit market and it leads to a higher deadweight loss due to screening. As a result, capital-income taxation has adverse effects on economic growth.
Other studies suggest mechanisms that justify deviations from the Judd-Chamley result. Judd (1997) and Judd (2002) show the optimality of negative capital-income tax with distortions (in the form of monopolistic power) in the product market. Guo and Lansing (1999) show that the optimal steady state capital-income tax in a neoclassical growth model with imperfectly competitive products markets can be negative, zero or positive depending on various factors (such as the degree of market power, the size of the depreciation allowance, the size of government expenditures and the extent of monopoly profit taxation). Aiyagari (1995) suggests a positive capital-income tax as optimal in an economy with incomplete insurance markets and borrowing constraints. The main intuition behind his result is that, for precautionary reasons, agents tend to over accumulate capital beyond the optimal level. When capital is taxed, households reduce their capital accumulation and, by so doing, the pretax return on capital is equalized to the rate of time preference. Arseneau and Chugh (2009) suggest deviations from a zero-capital-income tax rate in a model with labor market frictions. Analytically, they show that, if only capital and labor incomes are taxed, inefficient labor force participation calls for non-zero optimal capital tax in the long run. Numerically, they demonstrate that the optimal capital-income tax is very sensitive to the size of the deviation of the participation rate from its efficient level.

The economic events of recent years call for studying the effects of various aspects of financial frictions on optimal policies, including optimal fiscal policy. This paper contributes to the existing literature by studying the implications of financial frictions for the optimality of capital-income taxation. In particular, the difficulties of firms in obtaining sufficient credit during the last recession raise questions about the optimal policy that governments should follow in this type of economic episodes. This is essentially addressed in this paper in the context of optimal capital-income taxation.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy with the borrowing constraint and defines the private-sector equilibrium. Section 3 presents the problem of the social planner and section 4 discusses the problem of the Ramsey planner. Section 5 presents some analytical results about the optimal capital-income tax in the steady state. Section 6 describes the calibration and the solution methodology of the model. Section 7 presents the main quantitative results of this paper. Section 8 presents some robustness analyses and section 9 concludes.
2 The Model

The model is a variation of the neoclassical growth model with credit frictions. The economy is populated by households, a representative firm and the government. Households consume, supply labor and rent capital to the firm on spot markets. The firm needs to pay (at least part of) its input costs before production takes place, thus giving rise to borrowing from households. Borrowing is constrained by the value of real estate that the firm owns. This is the source of the credit friction in the baseline model.

2.1 Households

In each period $t$, the representative household purchases consumption $c_t$, supplies labor $l_t$, invests in physical capital $k_t$ and lends $b_t'$ to the firm at the beginning of the period at intra-period gross real interest rate of $R_t'$. The household also has access to a standard one-period real government bond $b_t$ that pays a gross real interest rate of $R_t$.

Households maximize their expected discounted lifetime utility given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \]

where $E_0$ is the expectation operator, $\beta < 1$ is the subjective discount factor and $u(c_t, l_t)$ is the period utility function from consumption and labor. This function satisfies the Inada conditions and the standard properties: $\frac{\partial u(\cdot)}{\partial c} > 0$, $\frac{\partial^2 u(\cdot)}{\partial c^2} < 0$, $\frac{\partial u(\cdot)}{\partial l} < 0$ and $\frac{\partial^2 u(\cdot)}{\partial l^2} < 0$.

Maximization is subject to the sequence of budget constraints of the form:

\[ (1-\tau_c^t)w_tl_t + \left[ 1 - \delta + r_t - \tau_k^t (r_t - \rho \delta) \right] k_t + (1-\tau_c^t)\Pi_t + R_{t-1}b_t + R_t'b_t' = c_t + k_{t+1} + b_{t+1} + b_t' , \]

where $c_t$ denotes consumption, $w_t$ is the real wage, $r_t$ is rental rate of capital, $\delta$ denotes the depreciation rate of capital, $\rho$ is the degree of capital depreciation allowance, $\tau_c^t$ and $\tau_k^t$ stand, respectively, for the labor-income tax and capital-income tax, $\Pi_t$ denotes lump sum profits from the ownership of the firm and $\tau_c^t$ is the tax rate on those profits.
The optimal choices of consumption, bonds, lending to firms, labor supply and capital yield the following optimization conditions (see Appendix A for derivations):

\[ R^f_t = 1, \quad (3) \]

\[- \frac{u_{t,c}}{u_{t,c}} = (1 - \tau^f_t) w_t, \quad (4) \]

\[ u_{t,c} = \beta R_t E_t(u_{t+1,c}) , \quad (5) \]

\[ u_{t,c} = \beta E_t \left[ u_{t+1,c} \left( 1 - \delta + r_{t+1} - (r_{t+1} - \rho \delta) \tau^k_t \right) \right] , \quad (6) \]

where \( u_{t,c} \) is the marginal utility of consumption in period \( t \) and \( u_{t,l} \) is the marginal disutility of supplying labor in period \( t \).

Equation (3) governs the lending of households to firms and, as is in Carlstrom and Fuerst (1998), households are basically passive suppliers of credit to the firm. Equation (4) is the standard labor-supply condition, equation (5) is the standard consumption Euler equation and condition (6) is the intertemporal capital supply condition.

### 2.2 The Firm

The representative firm hires labor and rents capital from households to produce a homogeneous good using the following production function

\[ y_t = z_t f(k_t, l_t) , \quad (7) \]

with \( y_t \) and \( z_t \) being output and total factor productivity, respectively.

I assume that the firm pays its input costs before revenues are realized, which requires the firm to borrow at the beginning of period \( t \). This assumption follows Carlstrom and Fuerst (1998), but with some differences in the specifics of the model. Borrowing, however, is constrained by the value of the firm’s collateral. I assume that the firm’s assets are in the form of real estate and, therefore, its collateral is equal to the beginning-of-period market value of its real estate. Real estate is in fixed supply.

Assuming that firms use real estate as collateral is common in the literature: for example, Kiyotaki and Moore (1997) assume that borrowing is tied to the value of land; Iacoviello (2005) assume that entrepreneurs use real estate (in the form of housing) as collateral. Chaney, Sraer and Thesmar (2011) show that, for U.S. firms over 1993-2007, appreciation in firms’ real estate values led to increases in investment, which is mainly financed through additional debt issuance. This effect is particularly stronger for
credit-constrained firms. I, therefore, follow those studies and use the real estate value as the firm’s collateral.¹

As shown in Appendix B, the firm’s problem with credit frictions can be reduced to the following maximization problem:

\[
\text{Max} \left[ z_t f(k_t, l_t) - w_t l_t - r_t k_t \right],
\]

subject to

\[
\phi(w_t l_t + r_t k_t) \leq \kappa q_t h_t,
\]

where \(h_t\) is the beginning-of-period stock of real estate, \(q_t\) is the price of real estate, \(\kappa\) is the share of assets that can be used as collateral (or the loan-to-value ratio), \(\phi\) is the fraction of factor payments that has to be paid in advance. Clearly, if \(\phi = 0\), then the model collapses to the standard neoclassical growth model with no frictions.

Denoting the Lagrange multiplier on (9) by \(\mu_t\), profit maximization gives the following factor demands:

\[
z_t f_{k_t}(k_t, l_t) = (1 + \phi \mu_t) w_t,
\]

\[
z_t f_{l_t}(k_t, l_t) = (1 + \phi \mu_t) r_t.
\]

The firm thus hires labor and rents capital so that each input’s marginal product is a “markup” over its respective factor price. The net markup is given by \(\phi \mu_t\), and it arises only due to the external financing needs of the firm. This result is similar to the result in the “output model” of Carlstrom and Fuerst (1998). In their model, agency costs, which arise due to the monitoring activity of lenders, induce differences between the marginal products of capital and labor and their respective factor prices. In fact, the use of the term “markup” in this paper is borrowed from their study.

In the second part of the period, the firm chooses the next-period real estate taking into account the role of real estate as collateral and subject to the budget constraint

¹ Differently from Kiyotaki and Moore (1997) and Iacoviello (2005), however, real estate serves here only as collateral. Instead, I could assume that real estate is used also as an factor of production by firms and provides direct utility for households. Assuming that real estate is a factor of production implies that the production function exhibits decreasing returns-to-scale in capital and labor. Decreasing returns to scale, in and of themselves, lead to deviations from the zero capital-income tax result. To keep the focus on the implications of the credit friction for optimal capital taxation, I assume constant returns to scale in capital and labor (and hence, I do not add real estate as another input). Under this assumption, any operating profits will arise only due to the binding borrowing constraint.
\[ z_t f(k_t, l_t) - w_t l_t - r_t k_t + q_t h_t = q_t h_{t+1} + \Pi_t. \]
The left-hand side is the total resources of the firm after production takes places, and they are equal to the sum of operating profits \( z_t f(k_t, l_t) - w_t l_t - r_t k_t \), and the market value of assets, \( q_t h_t \). Those resources are used in two ways: First, to finance the purchase of next-period real estate \( h_{t+1} \). Second, any remaining profits (or resources), denoted by \( \Pi_t \), are remitted to households in a lump-sum fashion. I also assume that in the process of accumulating assets, the firm is more impatient than households (one may think about this firm as being managed by an entrepreneur who is less patient that households). For this reason, the firm’s discount factor is \( \gamma \Xi_{t+1} \), where \( \Xi_{t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}} \) is the stochastic discount factor of households and \( \gamma < 1 \). The parameter \( \gamma \) is introduced to avoid self financing by the firm.

The assumption that the borrower (firm/entrepreneur in this case) is less patient than the lender is standard in this class of models (see, for example, Carlstrom and Fuerst 1997, 1998). In addition, assuming that profits are transferred to households simplifies the optimal policy problem as it reduces the objective function of the Ramsey planner to only the utility function of households. This formulation also allows for better comparisons with the standard neoclassical model.

With this characterization of the firm’s problem, the choice of \( h_{t+1} \) gives the following dynamic equation in the price of real estate:

\[
q_t u_{c,t} = \gamma \beta E_t \left[ u_{c,t+1} (1 + \kappa \mu_{t+1}) q_{t+1} \right],
\]

which makes explicit the roles of the credit friction and the additional discount factor.

Since profits \( \Pi_t \) are transferred to households, one could alternatively assume that the objective function of the firm is to choose labor, capital and real estate in order to maximize \( \Pi_t = z_t f(k_t, l_t) - w_t l_t - r_t k_t + q_t h_t - q_t h_{t+1} \). In this case, the firm’s problem is to maximize \( \sum_{t=0}^{\infty} \gamma^t \Xi_{t+1} \left[ z_t f(k_t, l_t) - w_t l_t - r_t k_t + q_t h_t - q_t h_{t+1} \right] \) subject to the financing constraint (9). It is straightforward to show that the first-order conditions with respect to labor, capital and next-period real estate are exactly the same as (10)-(12). In this respect, both approaches are identical.
2.3 The Government
The government collects capital-income taxes, labor-income taxes, profit taxes and obtains real debt to finance an exogenous stream of real government expenditures $g_t$.

The government budget constraint in period $t$ is thus given by:

$$\tau^k_t w_t l_t + \tau^k_t (r_t - \rho \delta) k_t + \tau^\pi_t \Pi_t + b_{t-1} = g_t + R_{t-1} b_t \tag{13}$$

2.4 Market Clearing
In equilibrium, the resource constraint of the economy reads as follows:

$$z_t f(k_t, l_t) + (1 - \delta) k_t = c_t + k_{t+1} + g_t \tag{14}$$

2.5 The Private Sector Equilibrium

**Definition 1:** Given the exogenous processes \(\{z_t, g_t, \tau^k_t, \tau^\pi_t, \tau^\nu_t\}\), the private-sector equilibrium is a state-contingent sequence of allocations \(\{c_t, l_t, k_t, w_t, q_t, h_t, R^f_t, R_t, b_t, \mu_t\}\) that satisfy the equilibrium conditions (3)-(6) and (9)-(14).

3 Efficient Allocations
It is useful to consider the optimal tax results that emerge as a solution to the social planner’s problem in order to better understand the results of the Ramsey planner later. I refer to the allocations of the social planner as the “efficient allocations” or the “first-best allocations”, interchangeably. Those are the allocations the planner will choose when distortionary taxes are absent.

**Definition 2:** Given the exogenous processes \(\{z_t, g_t\}\), the problem of the social planner is to choose consumption, labor and capital to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{15}$$

subject to the sequence of resource constraints

$$z_t f(k_t, l_t) + (1 - \delta) k_t = c_t + k_{t+1} + g_t \tag{16}$$

As show in Appendix C, the choice of capital yields:

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( 1 - \delta + z_{t+1} f_{k,t+1} \right) \right], \tag{17}$$

which is the usual capital choice condition under the first-best policy.
4 Optimal Capital Taxation - Ramsey Problem

In this section, I present the solution to the second-best capital taxation problem using the standard Ramsey approach (maximizing the utility of households subject to the private-sector equilibrium conditions and the resource constraint). Following Lucas and Stokey (1983) and Chari and Kehoe (1999), I use the primal approach, in which the government only chooses allocations after prices and taxes have been substituted out using the private-sector equilibrium conditions. To do so, I derive the present-value implementability constraint (PVIC) by substituting the equilibrium conditions into the households’ budget constraint (see Appendix D for the derivation of this constraint). Differently from standard Ramsey models, however, the PVIC in this paper does not capture all of the equilibrium conditions of the private sector (in addition to the resource constraint, of course), and therefore, the Ramsey problem will be enlarged beyond just maximizing utility subject to the PVIC and the resource constraint.

**Definition 3:** Given the exogenous processes \(\{z_t, g_t\}\), the Ramsey Planner chooses sequences of allocations \(\{c_t, l_t, k_t, h_t, q_t, \mu_t\}\) to maximize

\[
E_0 \sum_{t=0}^\infty \beta^t u(c_t, l_t),
\]

subject to,

\[
\sum_{t=0}^\infty \beta^t \left\{ c_{t+1}u_{t+1} + l_{t+1}u_{t+1} - u_{t+1} (1 - \tau^*_t) \left[ z_t f(k_t, l_t) - \frac{z_t f_{k,t}}{1 + \phi \mu_t} k_t - \frac{z_t f_{l,t}}{1 + \phi \mu_t} l_t + q_t h_t - q_t h_{t+1} \right] \right\} = A_0,
\]

\[
z_t f(k_t, l_t) + (1 - \delta)k_t = c_t + k_{t+1} + g_t.
\]

\[
q_t u_{t+1} = \gamma \beta E_t \left[ u_{t+1} (1 + \kappa \mu_{t+1}) q_{t+1} \right].
\]

with \(A_0 = u_{c,0} R_0 b_0 + u_{c,0} [1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta)] k_0\).

Under the assumption of a Cobb-Douglas production function \(f(k_t, l_t) = k^\alpha_l^{1-\alpha}\), the solution to Ramsey problem yields the following first-order condition with respect to next-period capital:

\[
\beta \gamma E_t \left[ u_{r_t+1} (1 - \tau^*_t) \frac{\phi \mu_{r_t+1}}{1 + \phi \mu_{r_t+1}} z_{r_t+1} f_{r_t+1} \right] + \lambda_t - \beta E_t \left[ \lambda_{t+1} (1 - \delta + z_{t+1} f_{t+1}) \right] = 0,
\]

\[
(22)
\]
where $\lambda_i$ and $\zeta_i$ denote the Lagrange multipliers on the time-$t$ resource constraint (14) and the PVIC (19), respectively. Comparing (22) with (17), the solution to the Ramsey planner's problem differs from the solution to the social planner's problem due to the first term in (22). The two solutions also coincide if $\zeta_i = 0$ as the problem of the Ramsey planner is essentially reduced to the problem of the social planner (the first-order condition of the Ramsey planner with respect to $\mu_i$ implies that the Lagrange multiplier on condition (21) is zero whenever $\zeta_i$ is zero).

5 Analytical Results in the Steady State

In this section, I present the main analytical results about the optimal long-run capital-income tax.

5.1 First-Best Capital-Income Taxation Policy

In this subsection, I show that, in the (deterministic) steady state, the market solution is not efficient with zero tax on capital. Setting $z$ to 1 in the steady state, the steady state versions of conditions (6) and (17) are, respectively:

$$1 = \beta \left[ 1 - \delta + (1 - r) t + \rho \delta t^k \right].$$

and,

$$1 = \beta \left[ 1 - \delta + f_k \right].$$

Imposing steady state on condition (11) and substituting it into condition (23) yield:

$$1 = \beta \left[ 1 - \delta + (1 - t^k) \frac{f_k}{1 + \phi_t} + \rho \delta t^k \right].$$

By comparing (24) to (25), the market allocation is efficient if the following holds:

$$\tau_{FB} = \frac{-\phi_t f_k}{f_k - \rho \delta (1 + \phi_t)},$$

with $\tau_{FB}$ denoting the capital-income tax rate under the first-best tax policy. Using condition (24), equation (26) can also be written as:

$$\tau_{FB} = \frac{-\phi_t (1 - \beta + \beta \delta)}{(1 - \beta + \beta \delta) - \beta \rho \delta (1 + \phi_t)}.$$
The main results that come out of this equation are summarized in the following propositions.

**Proposition 1:** In an economy with no credit frictions \((\phi=0)\), the first-best capital-income tax is zero in the steady-state.

*Proof:* Substituting \(\phi=0\) in condition (27), we have \(\tau_{FB}^k = 0\). This is the Judd-Chamley result.

**Proposition 2:** In an economy with credit frictions \((\phi>0)\), the first-best capital-income tax is non-zero in the steady-state. Therefore, the market equilibrium achieves efficiency only with non-zero capital-income tax.

*Proof:* when \(\phi>0\), the numerator of condition (27) is non-zero, which suggest that \(\tau_{FB}^k\) is different from zero. The sign of \(\tau_{FB}^k\), however, is ambiguous and it crucially depends on the sign of \((1-\beta+\beta\delta) - \beta\rho\delta(1+\phi\mu)\) as \(-\phi\mu(1-\beta+\beta\delta)\) is unambiguously negative.

**Proposition 3:** In an economy with credit frictions \((\phi>0)\), but with no capital depreciation \((\delta=0)\) or no capital depreciation allowance \((\rho=0)\), the first-best capital-income tax is unambiguously negative.

*Proof:* When \(\phi>0\), \(\delta=0\) or \(\rho=0\), we have \(\tau_{FB}^k = -\phi\mu\), which is unambiguously negative. Therefore, capital should be subsidized. The magnitude of the tax rate is proportional to the “significance” of the credit friction in renting capital \(\phi\mu\). More interestingly, the size of the first-best capital-income subsidy is exactly equal to the “markup” of the marginal product of capital over its rental rate. Therefore, under the first-best tax policy, capital is subsidized by the size of the “markup” that results from the existence of the credit friction. The aim of this policy is straightforward - if capital demand is reduced due to the borrowing constraint, capital supply is boosted so that the economy achieves efficiency.

**Proposition 4:** In an economy with credit frictions \((\phi>0)\) and capital depreciation allowance \((\rho>0)\), the steady state first-best capital-income tax is negative if \(\rho < \frac{1-\beta + \beta\delta}{\beta\delta(1+\phi\mu)}\) and it is positive and does not exceed 100 percent if \(\rho \geq \frac{1-\beta + \beta\delta}{\beta\delta} \).
Proof: Given that the sign of \(-\phi\mu(1 - \beta + \beta\delta)\) is negative, the first-best capital tax rate is negative if \((1 - \beta + \beta\delta) - \beta\rho\delta(1 + \phi\mu) > 0\), which requires \(\rho < \frac{1 - \beta + \beta\delta}{\beta\delta(1 + \phi\mu)}\). Therefore, \(\tau_{FB}^k\) is negative at relatively low rates of capital depreciation allowance.

The first-best capital-income tax is positive if the denominator of (27) is negative, which requires: \(\rho > \frac{1 - \beta + \beta\delta}{\beta\delta(1 + \phi\mu)}\). However, the tax rate on capital should not exceed 100 percent (i.e. \(\tau_{FB}^k \leq 1\)) or households will not rent capital to the firm (instead, they will simply let their capital depreciates and then receive depreciation allowance). Imposing this restriction on (27) implies \(\rho \geq 1 + \frac{1 - \beta}{\beta\delta}\), which is more restrictive on \(\rho\). Therefore, the relevant range for positive capital-income tax rates is given by \(\rho \geq 1 + \frac{1 - \beta}{\beta\delta}\).

This condition suggests that positive capital tax rate is optimal when the rate of capital depreciation allowance sufficiently high (and, in particular, higher than 100 percent). In this case, households’ disincentives to invest in capital, due to capital taxation, are at least offset by the households’ incentives to invest due to the generous depreciation allowance system. Hence, subsidizing capital is not necessary.

Depreciation allowance is important in determining the optimal capital tax rate for the following reason: the higher the rate of capital depreciation allowance, the higher the reduction in tax payments by households (see condition (2)). Put differently, higher rate of capital depreciation allowance implies higher income from capital holdings, which encourages investment in capital. If the depreciation allowance is sufficiently large, households may over invest in capital. In this case, it may be optimal to set a positive tax rate on capital to disincentivize households from over accumulation of capital. Reducing agents’ incentives for over accumulation of capital by taxing capital income is similar to the conclusion of Aiyagari (1995) as presented in the introduction (positive capital-income tax reduces the incentives of credit-constrained agents for a precautionary behavior). Differently from his model, however, capital does not serve in this paper as collateral, but it is actually assumed that capital demand is subject to a borrowing constraint.
5.2 Second-Best Capital-Income Taxation Policy

I turn now to discuss the optimal long-run capital-income tax. Comparing conditions (22) and (6), evaluated at the steady state, gives:

\[
\tau_{SB}^k = \frac{-\phi uf_k \left[ 1 - \frac{\zeta}{\lambda} u_c (1 - \tau^\pi) \right]}{f_k - \rho \delta (1 + \phi \mu)},
\]

(28)

where \(\tau_{SB}^k\) denotes the second-best capital-income tax rate in the steady state. The sign of the optimal capital-income tax rate is ambiguous, and, without specific assumptions, no theoretical conclusions can be made about it.

I summarize the main observations regarding the optimal steady state capital-income tax in the following propositions.

**Proposition 5:** In an economy with no credit frictions (\(\phi = 0\)), the optimal steady state capital-income tax is zero.

*Proof:* Setting \(\phi = 0\) in condition (28), it is clear that the optimal capital-income tax rate is zero. In the absence of credit frictions, the model is a standard neoclassical growth model and, as already has been shown by Judd (1985) and Chamley (1986), the optimal capital-income tax rate is zero in the steady state.

**Proposition 6:** In an economy with credit frictions (\(\phi > 0\)), the optimal steady state capital-income tax is non zero, but the sign of the optimal steady state capital-income tax is ambiguous.

*Proof:* When \(\phi > 0\), condition (28) suggests that \(\tau_{SB}^k\) can be positive, negative or zero. Therefore, without further assumptions, no theoretical conclusions can be made about the sign of the optimal capital-income tax in the steady state.

**Proposition 7:** In an economy with credit frictions (\(\phi > 0\)) and full confiscation of profits (\(\tau^\pi = 1\)), the optimal steady state capital-income tax is non zero and it coincides with the optimal capital-income tax under the first-best tax policy.

*Proof:* When \(\tau^\pi = 1\), the optimal tax rate is given by \(\tau_{SB}^k = \frac{-\phi uf_k}{f_k - \rho \delta (1 + \phi \mu)}\), which exactly equals the \(\tau_{FB}^k\) as shown in expression (26).
When the government confiscates all profits (i.e. \( \tau^x = 1 \)), the government can obtain lump-sum tax revenues that allow for reducing distortionary taxes without affecting the decisions of households. Confiscating all profits is therefore the optimal profit-tax policy as it allows for replicating the first-best tax policy.

The two tax rates also coincide if the government is not constrained by the private-sector equilibrium conditions (i.e. \( \zeta = 0 \)). This result is expected since in this case the problems of the Ramsey planner and social planner are the same.

**Proposition 8:** In an economy with credit frictions (\( \phi > 0 \)), full confiscation of profits (\( \tau^x = 1 \)), capital depreciation (\( \delta > 0 \)) and capital depreciation allowance (\( \rho > 0 \)), the optimal steady state capital-income tax is non zero. The optimal steady state capital-income tax is negative if \( \rho < \frac{1 - \beta + \beta \delta}{\beta \delta (1 + \phi \mu)} \), and it is positive and satisfies \( \tau^x_{SB} \leq 1 \) if \( \rho \geq \frac{1 - \beta + \beta \delta}{\beta \delta} \).

**Proof:** As demonstrated in Proposition 7, the optimal capital-income tax and the first-best capital income tax are the same when \( \tau^x = 1 \). This proves that the capital-income tax is non zero. The other two proofs follow from the proof of Proposition 4.

**Proposition 9:** In an economy with credit frictions (\( \phi > 0 \)), full confiscation of profits (\( \tau^x = 1 \)), no capital depreciation (\( \delta = 0 \)) or no capital depreciation allowance (\( \rho = 0 \)), the optimal steady state capital-income tax is unambiguously negative.

**Proof:** When \( \tau^x = 1 \), \( \delta = 0 \) or \( \rho = 0 \), the optimal capital-income tax is given by \( \tau^x_{SB} = -\phi \mu \), which is unambiguously negative. The optimal capital-income tax rate in this case equals the capital-income subsidy that restores efficiency as presented in Proposition 3.

### 6 Computational Strategy and Calibration

#### 6.1 Parameterization and Functional Forms

I assume a time unit of a year and hence the discount factor \( \beta \) is set to 0.96 implying an annual interest rate of roughly 4 percent. I also assume the following period utility function for households:
\[ u(c_t, l_t) = \ln c_t - \frac{l_t^{1+\theta}}{1+\theta}. \]

The parameter \( \theta \) is set to zero, implying a linear disutility function of labor. This high labor supply elasticity is needed to capture the volatility of total hours in a model with no extensive margin, as is the case in this paper. In each case considered below, the parameter \( \chi \) is calibrated so that the steady state value of \( l \) is 0.3.

Firms produce using the Cobb-Douglas production function:

\[ f(k_t, l_t) = k_t^\alpha l_t^{1-\alpha}, \]

with capital share \( \alpha \) of 0.33, as usually assumed in literature. The steady state value of \( z \) is normalized to 1.

In each case considered below, the deterministic steady state value of government spending \( g \) is set so that the deterministic steady state value of government spending constitutes 20 percent of deterministic steady state output, which is the average government-GDP ratio over 1960-2007. The steady state value of \( b \) is obtained so that \( \frac{b}{y} \) is 0.36. This is the average of the gross federal debt held by the public as percentage of GDP over the period 1960-2007 (see Table B79 of the 2011 Economic Report of the President). I choose 2007 as the final year of the sample because this ratio has increased dramatically in the last three years; including those years in the sample may only bias my results without adding any further insights.

The additional discount factor \( \gamma \) is set to 0.961, implying an annual discount rate of about 0.922 for the firm, in line with Iacoviello (2005). The credit friction parameter \( \phi \) is set to 1 in the benchmark calibration of the model. I set the loan-to-value ratio \( \kappa \) to 0.89, which equals the entrepreneurial loan-to-value ratio as reported in Iacoviello (2005). Giving this value of \( \kappa \) and using the steady state version of condition (12), the implied value of the Lagrange multiplier \( \mu \) is 0.0946. This value of \( \mu \) suggests that the “markup” of the firm is 9.46 percent, very close to the empirically plausible range of markup rates (generally, between 10 percent and 20 percent). Therefore, the choice of \( \kappa \) delivers results that are consistent with NIPA data, making it safer to use the term “markup” to describe the effects of the credit friction on factor demands. In addition, the steady state version of condition (12) suggests that the value of \( \mu \) is inversely related to \( \kappa \). This result is as expected since higher \( \kappa \) implies, for any level of collateral,
more borrowing capabilities by the firm, thus reducing the tightening of the borrowing constraint. In the extreme case, whenever \( \kappa \) approaches infinity, the Lagrange multiplier approaches zero.

The benchmark value of the tax rate on profits \( \tau^\pi \) under the assumption that profits and capital income are taxed separately is set to 35 percent. This is the average effective corporate tax rate for the period 1960-2007 as reported in the NIPA tables of the Bureau of Economic Analysis. During this period, the average effective corporate tax rate declined from roughly 44 percent to about 25 percent (and the top marginal tax rate dropped from 52 percent to 35 percent, respectively). Besides this value, I also consider different values of \( \tau^\pi \) as this variable has significant implications for the optimal tax rate on capital income.

Capital depreciates at a yearly rate of \( \delta = 0.08 \). The capital depreciation allowance \( \rho \) is set to 1 in the benchmark calibration of the model, implying that the depreciation rate for tax purposes \( \rho \delta \) equals the economic depreciation rate \( \delta \). This is the usual assumption of models with depreciation allowance. In what follows I also consider various values of this parameter.

### 6.2 Solution Methodology

The solution to the Ramsey problem is a sequence of allocations \( \{c, l, k, h, q, \mu, \zeta\} \) and a scalar \( \zeta \) that satisfy the PVIC and the first-order conditions of the Ramsey planner with respect to consumption, labor, capital, real estate, the price of real estate and the shadow value of the borrowing constraint. Then, taxes and factor prices that support those allocations are recovered using the private-sector equilibrium conditions.

I impose steady state on those seven conditions to find the (deterministic) steady state values of \( c, l, k, h, q, \mu \) and \( \zeta \). I consider a different value for \( \tau^\pi \) each time and, given the assumed value of \( \tau^\pi \), the corresponding value of \( \tau_{SB}^k \) is pinned down using condition (28). As for the steady state value of \( \tau_{FB}^k \), it is easily found using condition (27), or, as discussed above, it is the value of \( \tau_{SB}^k \) when \( \tau^\pi = 1 \).
7 Optimal Capital- Income Tax- Quantitative Results

This section presents the main findings regarding the optimal capital-income tax in the deterministic steady state.

7.1 First-Best Taxation Policy

The main results about first-best capital taxation are presented in this subsection. Recall that, from Proposition 1, the capital-income tax rate is positive and does not exceed 100 percent if \( \rho \geq 1 + \frac{1 - \beta}{\beta \delta} \). This cut-off value depends only on the depreciation rate and the subjective discount rate and hence it holds for any “markup” value \( \phi \mu \).

Under the baseline calibration of the model, the cut-off value is 1.521. Therefore, regardless of the “size” of credit friction, the first-best capital-income tax rate is likely not to be positive unless the depreciation allowance system is very generous.

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( \tau_{FB}^k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-9.46</td>
</tr>
<tr>
<td>0.25</td>
<td>-11.53</td>
</tr>
<tr>
<td>1.00</td>
<td>-33.74</td>
</tr>
<tr>
<td>1.10</td>
<td>-45.40</td>
</tr>
<tr>
<td>1.75</td>
<td>36.44</td>
</tr>
<tr>
<td>( \rho^* )</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 1: The first-best steady-state capital-income tax rate for various values of \( \rho \).

As shown in Proposition 4, the first-best capital-income tax rate is unambiguously negative if \( \rho < \rho^* \), where \( \rho^* = \frac{1 - \beta + \beta \delta}{\beta \delta (1 + \phi \mu)} \). Therefore, this cut-off depends upon the size of the Lagrange multiplier. I report this cut-off in the bottom line of Table 1, which presents the first-best steady state capital-income tax rates for various values of the depreciation allowance rate \( \rho \).\(^2\) I choose those values for the following reasons: \( \rho = 0 \)

---

\(^2\) Between \( \rho = 1.179 \) and \( \rho = 1.521 \), the first-best capital-income tax rate is positive, but it exceeds 100\%, and it is therefore not in the relevant ranges of capital-income taxation.
and $\rho = 1$ are natural choices; $\rho = 1.1$ is chosen to see the results when the depreciation rate for tax purposes exceeds the economic depreciation rate only slightly; $\rho = 1.75$ is chosen to illustrate the idea that under a relatively high depreciation allowance rate, the capital-income tax rate is positive (and under 100 percent), and $\rho = 0.25$ is chosen to allow for symmetry in the choice of the depreciation allowance rate (i.e. 75 percent lower and 75 percent higher than the economic depreciation rate $\delta$).

The first-best steady state capital tax rate in the benchmark calibration of the model is roughly -34 percent (using condition (27), it is easy to show that the capital subsidy rate increases with the size of the credit friction provided that $\rho < \rho^*$). In addition, as the cut-off values for negative tax rates suggest, the first-best capital-income tax is negative as long as the depreciation allowance rate is not very large; under $\mu = 0.0946$, first-best taxation policy would subsidize capital income for all values of $\rho$ below 1.39. The first-best capital-income tax rate is thus negative provided that the depreciation rate for tax purposes $\rho \delta$ does not exceed the economic depreciation rate $\delta$ by more than roughly 39 percent. Finally, The results for $\rho = 1.75$ confirm that, with sufficiently high values of $\rho$, capital-income subsidy is unnecessary. Moreover, with sufficiently high values of $\rho$, first-best capital taxation will (positively) tax capital to prevent over accumulation of capital beyond the socially optimal level.

### 7.2 Second-Best Taxation Policy

The main results of this paper are presented in Table 2 below. Under the assumption that the government taxes profits at the optimal rate of 100 percent (i.e. $\tau^x = 1$), the optimal steady state capital-income tax is negative and it coincides with the first-best capital-income tax. Subsidizing capital is optimal for almost all empirically plausible values of the depreciation allowance rate.

When the government cannot confiscate all profits, the optimal capital-income tax rate is also negative in the steady state, but it is lower, in absolute value, than the tax rate under $\tau^x = 1$. Table 2 suggests that the optimal capital subsidy is decreasing with the tax rate on profits. For example, moving from $\tau^x = 0.5$ to $\tau^x = 0$, the optimal tax rate is cut by, approximately, two thirds. Intuitively, with a lower profits’ tax rate, the
government has fewer resources for subsidizing capital. The government, therefore, only partially subsidizes capital relative to what the first-best taxation policy suggests.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\tau^\pi = 0$</th>
<th>$\tau^\pi = 0.35$</th>
<th>$\tau^\pi = 0.50$</th>
<th>$\tau^\pi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>-2.62</td>
<td>-5.55</td>
<td>-6.63</td>
<td>-9.46</td>
</tr>
<tr>
<td>0.25</td>
<td>-3.16</td>
<td>-6.71</td>
<td>-8.04</td>
<td>-11.53</td>
</tr>
<tr>
<td>1.00</td>
<td>-8.07</td>
<td>-18.13</td>
<td>-22.17</td>
<td>-33.74</td>
</tr>
<tr>
<td>1.10</td>
<td>-10.18</td>
<td>-23.44</td>
<td>-28.97</td>
<td>-45.40</td>
</tr>
<tr>
<td>1.75</td>
<td>14.50</td>
<td>25.86</td>
<td>29.20</td>
<td>36.44</td>
</tr>
<tr>
<td>$\rho^{**}$</td>
<td>1.22</td>
<td>1.19</td>
<td>1.18</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Table 2: The optimal steady-state capital-income tax rate for various values of $\tau^\pi$ and $\rho$.

The bottom line in Table 2 presents the cut-off values of $\rho$ below which the optimal capital-income tax rate is negative. In all cases reported, the depreciation rate for tax purposes should be at least 18 percent higher than the economic depreciation rate for the optimal capital-income tax to be positive. Moreover, in all of those cases, the capital depreciation allowance rate should be more than 50 percent for the optimal inflation rate to be positive and less than 100 percent (not reported). The optimal long-run capital-income tax rate is thus negative when the government has enough instruments to tax capital income and profits separately. In the next subsection, I show the implications of taxing capital income and profits at the same rate.\(^3\)

### 7.3 Second-Best Taxation Policy when the Government cannot distinguish between capital-income and profits

Up to this point, I have assumed that the government can distinguish between capital income and profits and hence it can levy different tax rates on each of them. In reality, however, the government may not be able to distinguish between both types of incomes and therefore it taxes them at the same rate each period ($\tau_i^\pi = \tau_i^k$ for all $t$). This implies another constraint on the Ramsey planner. I discuss this case in what follows.

\(^3\) The steady state labor-income tax rate under $\tau^\pi = 0.35$ is around 32.91 percent, in line with other estimates of the labor tax rate in models with capital, and slightly above the average of about 30 percent of the federal and social security tax rates combined.
As shown in Table 3, the optimal capital-income tax rate is positive in this case (roughly 32 percent under the benchmark calibration). Therefore, with the inability of the government to distinguish between both types of income and since the government wants to tax profits, capital-income is taxed as well. But, differently from the case when the government can tax profits separately, it is no longer optimal to set a 100 percent tax rate on profits as this policy will discourage households from accumulating capital and renting it to firms. For this reason, the government levies a lower positive tax rate on profits (and capital-income) in order to balance between the willingness to tax profits on one hand, and not taxing capital (or subsidizing capital), on the other.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\tau_{SB}^k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>10.76</td>
</tr>
<tr>
<td>0.25</td>
<td>14.54</td>
</tr>
<tr>
<td>1.00</td>
<td>32.46</td>
</tr>
<tr>
<td>1.10</td>
<td>38.11</td>
</tr>
<tr>
<td>1.75</td>
<td>80.27</td>
</tr>
</tbody>
</table>

Table 3: The optimal steady-state capital-income tax rate for various values of $\rho$ under $\tau^k = \tau^k$.

The rate of capital depreciation allowance is again important in determining the optimal capital-income tax rate. The lower the depreciation allowance rate, the lower the positive capital-income tax rate. Intuitively, for lower depreciation allowance rates, the Ramsey planner tries to avoid excessive taxation on capital income as the exemptions from tax payments are relatively low. In this circumstance, heavy taxation on profits (and hence capital income) will significantly discourage capital accumulation by households. The planner thus sets a relatively low profits’ tax rate that is meant to balance between the willingness to tax profits on one hand, and not discouraging capital accumulation, on the other.

8 Robustness Analyses

The analyses above assumed that $\kappa=0.89$ and hence $\mu=0.0946$, implying a “markup” $\phi \mu$ of 9.46 percent, very close to the lower bound of the empirically relevant markup values (between 10 percent and 20 percent). In this section, I carry on the analyses of
section 7.2 under the assumption that the “markup” is 15 percent, in the middle of the markups’ range. Using condition (12), the implied value of $\kappa$ is about 0.56.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\tau^*=0$</th>
<th>$\tau^*=0.35$</th>
<th>$\tau^*=0.50$</th>
<th>$\tau^*=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>-61.52</td>
</tr>
<tr>
<td>1.10</td>
<td>-14.88</td>
<td>-36.56</td>
<td>-49.15</td>
<td>-89.17</td>
</tr>
<tr>
<td>1.75</td>
<td>19.22</td>
<td>32.96</td>
<td>37.70</td>
<td>46.40</td>
</tr>
<tr>
<td>$\rho^{**}$</td>
<td>1.40</td>
<td>1.31</td>
<td>1.31</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Table 4: The optimal steady-state capital-income tax rate for various values of $\tau^*$ and $\rho$. $\kappa = 0.5611$.

The results for a wide range of profit tax rates and depreciation allowance rates are presented in Table 4. We again learn that, provided that the depreciation allowance rate is not very high, the optimal taxation policy calls for subsidizing capital income. The size of the subsidy in this case, however, is notably higher than in Table 2, suggesting that an increase in the “markup” size requires bigger subsidies of capital income. Also, in most cases, the depreciation allowance rate under which the capital-income tax rate is unambiguously negative is higher than in Table 2. Therefore, with a bigger size of the credit friction, capital income is more likely to be subsidized.

9 Conclusions

The main purpose of this paper is to study optimal capital-income taxation in the presence of credit frictions. The model is a variation of the standard neoclassical growth framework in which firms’ borrowing to finance the hiring of labor and the rental of capital at the beginning of the period is constrained by their collateral.

The main result of the paper is the non optimality of zero tax on capital income in the long run. Analytical analyses suggest that, with no capital deprecation allowance, optimal policy unambiguously sets a negative capital tax rate in the long run. With deprecation allowance, the analytical analyses show that the optimal capital-income tax rate is indeed non zero, but not necessarily negative. However, the quantitative results of the paper do support a negative capital tax rate as optimal for plausible
parameterization of the model. If the government cannot distinguish between profits and capital income, and hence both sources of income are taxed at the same rate, the capital-income tax rate is positive in the benchmark calibration of the model. Credit frictions, therefore, induce departures from the zero long-run capital-income tax rate result of Judd (1985) and Chamley (1986).

The borrowing constraint induces inefficiently low demand for factors of production; the firm rents capital so that the marginal product of capital is higher than its rental rate. Subsidizing capital boosts capital supply towards the efficient level and thus acts towards offsetting the effects of the credit friction on the equilibrium amount of capital. Alternatively, since capital is chosen so that the marginal product is a markup over its rental rate, capital-income subsidy offsets this markup. In this regard, the result is similar to the idea of Judd (1997, 2002) in a model with monopolistic power in the product market.

This study is part of the literature that studies the implications of credit frictions for conducting macroeconomic policies in general, and for optimal fiscal policy, in particular. Once again, credit frictions are proven as important factors in shaping macroeconomic policies well beyond their role in amplifying and propagating exogenous shocks. The events of the last few years make this type of literature very timely.

References


Appendix: Mathematical Derivations

A The Households’ Problem

Households maximize the following objective function:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{A1} \]

subject to the sequence of budget constraints:

\[ (1-\tau_i^t)w_t l_t + \left[ 1 - \delta + r_i \tau_i^k (r_i - \rho \delta) \right] k_i + (1-\tau_i^t) \Pi_i + R_{t+1} b_i^t + \beta b_{i+1}^t = c_i + k_{i+1} + b_{i+1}^t, \tag{A2} \]

Denoting the Lagrange multiplier on the budget constraint by \( \varphi_i \), the first order conditions with respect to \( c_i, b_i, k_{i+1} \), and \( l_t \) are, respectively:

\[ \varphi_i = u_{c_i}, \tag{A3} \]

\[ \varphi_i = \beta R_{i+1} \varphi_i - \beta E_i \left[ \varphi_{i+1} \left( 1 - \delta + r_{i+1} \tau_{i+1}^k (r_{i+1} - \rho \delta) \right) \right] = 0, \tag{A4} \]

and,

\[ u_{l_i} + \varphi_i (1-\tau_i^t) w_t = 0. \tag{A6} \]

Combining (A3) and (A6) gives equation (4) in the text. Combining (A3) and (A4) gives equation (5), and the combination of equations (A3) and (A5) yields equation (6) in the text.

B The Firm’s Problem

At the beginning of the period, the firm obtains a loan \( b_i^t \) from households, which is repaid at the end of the period at a nominal gross interest rate of \( R_i^t \). Borrowing is constrained by the beginning-of-period of the firm’s collateral. Formally, the firm chooses labor, capital and loans to maximize:

\[ z_i f(k_i, l_i) + b_i^t - w_i l_i - r_i k_i - R_i^t b_i^t, \tag{B1} \]

subject to

\[ b_i^t - \phi(w_i l_i + r_i k_i) \geq 0, \tag{B2} \]

and,

\[ \kappa a_i h_i - b_i^t \geq 0, \tag{B3} \]
Letting $\nu_t$ and $\gamma_t$ denote the Lagrange multipliers on the constraints (B2) and (B3), respectively, the optimality condition with respect to $b_t^f$ reads:

$$\gamma_t = R_t^f + \nu_t - 1. \quad (B4)$$

Similarly, the first order conditions with respect to $l_i$ and $k_j$ yield:

$$z_i f_{l,i}(k_i, l_i) = (1 + \phi_l) w_i, \quad (B5)$$

$$z_j f_{k,j}(k_j, l_j) = (1 + \phi_k) r_j, \quad (B6)$$

Recalling that $R_t^f = 1$, equation (B4) becomes:

$$\nu_t = \gamma_t, \quad (B7)$$

and, therefore, the two Lagrange multipliers are equal.

Alternatively, conditions (B2) and (B3) can be combined to get:

$$\kappa q_i h_i - \phi(w_i l_i + r_i k_i) \geq 0, \quad (B8)$$

which is condition (9) in the text.

Substituting $R_t^f = 1$ in (B1), the profit function is given by:

$$z_i f(k_i, l_i) - w_i l_i - r_i k_i, \quad (B9)$$

which is condition (8) in the text. Therefore, the optimization problem of the firm is to maximize (B9) subject to (B8). Letting $\mu_t$ be the Lagrange multiplier on (B8), the demand functions of labor and capital are, respectively, given by:

$$z_i f_{l,i}(k_i, l_i) = (1 + \phi_l) w_i, \quad (B10)$$

$$z_j f_{k,j}(k_j, l_j) = (1 + \phi_k) r_j, \quad (B11)$$

which are, respectively, conditions (10) and (11) in the text. These conditions coincide with (B5)-(B6) respectively with $\nu_t = \gamma_t = \mu_t$.

### C Efficient Allocations

The social planner chooses consumption, labor and capital for the next period to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad (C1)$$

subject to the sequence of resource constraints

$$z_t f(k_t, l_t) + (1 - \delta) k_t = c_t + k_{t+1} + g_t. \quad (C2)$$
Letting $\eta_t$ be the Lagrange multiplier associated with (E2), the first-order conditions with respect to $c_t$, $l_t$, and $k_{t+1}$, respectively, read:

$$u_{c,t} = \eta_t,$$  \hspace{1cm} (C3)

$$u_{l,t} + \eta_t z_t f_{l,t} (k_t, l_t) = 0.$$  \hspace{1cm} (C4)

and

$$- \eta_t + \beta E_t \left[ \eta_{t+1} \left( 1 - \delta + z_{t+1} f_{k,t+1} \right) \right] = 0.$$  \hspace{1cm} (C5)

Combining (E3) and (E4) yields

$$- \frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t} (k_t, l_t),$$  \hspace{1cm} (C6)

and hence efficiency requires the marginal rate of substitution (the left hand side of condition (C6)) to be equal to the marginal product of labor (given by the right-hand side of condition (C6)).

Similarly, combining (C3) and (C4) gives

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( 1 - \delta + z_{t+1} f_{k,t+1} \right) \right],$$  \hspace{1cm} (C7)

which is condition (16) in the text.

**D  The Present-Value Implementability Constraint**

I show here the derivation of the Implementability Constraint (IC) for the Ramsey problem. Recalling that $R_f^t = 1$, the households’ budget constraint becomes:

$$(1 - \tau^t_i) w_t l_t + \left[ 1 - \delta + r_t - \tau^t_i (r_t - \rho \delta) \right] k_t + (1 - \tau^t_i) \Pi_t + R_{t-1} b_t = c_t + k_{t+1} + b_{t+1}. \quad \text{(D1)}$$

By introducing $E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t}$ to (D1) and rearranging, we have:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} \left( 1 - \tau^t_i \right) w_t l_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} \left[ 1 - \delta + r_t - \tau^t_i (r_t - \rho \delta) \right] k_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} (1 - \tau^t_i) \Pi_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} R_{t-1} b_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} k_{t+1} - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} b_{t+1} = 0 \quad \text{(D2)}$$

Recall that, from the solution to the households’ problem, we have:

$$- \frac{u_{l,t}}{u_{c,t}} = (1 - \tau^t_i) w_t,$$  \hspace{1cm} (D3)

$$u_{c,t} = \beta R_t E_t (u_{c,t+1}),$$  \hspace{1cm} (D4)

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( 1 - \delta + r_{t+1} - (r_{t+1} - \rho \delta) \tau^t_i \right) \right],$$  \hspace{1cm} (D5)
Substituting (D3) in the first term of (D2), (D4) in the last term of (D2) and (D5) in the sixth term of (D2) yield:

$$E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left( -\frac{u_{r,0}}{u_{r,0}} \right) l_i - E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left[ 1 - \delta + r_i - \tau^*_i (r_i - \rho \delta) \right] k_i + E_0 \sum_{r=0}^{\infty} \beta^r u_{r,0} \left( 1 - \tau^*_r \right) \Pi_i,$$

$$+ E_0 \sum_{r=0}^{\infty} \beta^r u_{r,0} c_i - E_0 \sum_{r=0}^{\infty} \beta^r \beta u_{r,0} \left[ 1 - \delta + r_{r+1} - (r_{r+1} - \rho \delta) \tau^*_{r+1} \right] k_{r+1} \right) k_{r+1} = \mathbf{0}.$$  

(D6)

Combining the second and sixth terms of (D6) yield:

$$E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left[ 1 - \delta + r_i - \tau^*_i (r_i - \rho \delta) \right] k_i - E_0 \sum_{i=0}^{\infty} \beta^i \beta u_{r,0} \left[ 1 - \delta + r_{r+1} - (r_{r+1} - \rho \delta) \tau^*_{r+1} \right] k_{r+1}$$

$$= u_{c,0} \left[ 1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta) \right] k_0 + E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left[ 1 - \delta + r_i - \tau^*_i (r_i - \rho \delta) \right] k_i,$$

$$- E_0 \sum_{r=0}^{\infty} \beta^r c_i \left[ 1 - \delta + r_{r+1} - (r_{r+1} - \rho \delta) \tau^*_{r+1} \right] k_{r+1} = u_{c,0} \left[ 1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta) \right] k_0,$$

where the last two terms of (D7) were canceled using summation rules.

Similarly, combining the fourth and seventh terms of (D6) gives

$$E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \sum_{r=0}^{\infty} \beta^r \beta R_i R_{r+1} b_{r+1} = u_{c,0} R_{r+1} b_0 + E_0 \sum_{r=0}^{\infty} \beta^r u_{r,0} R_{r+1} b_i - E_0 \sum_{r=0}^{\infty} \beta^r \beta R_i u_{r+1} b_{r+1} = u_{c,0} R_{r+1} b_0.$$  

(D8)

Also, the combination of the first and fifth terms of (D6) gives:

$$E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left( -\frac{u_{r,0}}{u_{r,0}} \right) l_i - E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} c_i = -E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} l_i - E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} c_i.$$  

(D9)

Finally, substituting (D7)-(D9) into (D6) yield:

$$E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} l_i + E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} c_i - E_0 \sum_{i=0}^{\infty} \beta^i u_{r,0} \left( 1 - \tau^*_i \right) \Pi_i - u_{c,0} R_{r+1} b_0 - u_{c,0} \left[ 1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta) \right] k_0 = \mathbf{0}$$

or,

$$E_0 \sum_{i=0}^{\infty} \beta^i \left[ u_{r,0} c_i + u_{r,0} l_i - u_{c,0} \left( 1 - \tau^*_i \right) \Pi_i \right] = u_{c,0} R_{r+1} b_0 + u_{c,0} \left[ 1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta) \right] k_0,$$

(D10)

which is condition (18) in the text with

$$A_0 = u_{c,0} R_{r+1} b_0 + u_{c,0} \left[ 1 - \delta + r_0 - \tau^*_0 (r_0 - \rho \delta) \right] k_0$$

and

$$\Pi_i = \frac{z_i f_{k\ell} k_i}{1 + \phi \mu_i}, \quad h_i = q_i h_i - q_i h_{i+1}.$$