Optimal Long-Run Inflation
with Occasionally-Binding Financial Constraints∗

January 30, 2012

Salem Abo-Zaid †
Ben-Gurion University of the Negev

Abstract
This paper studies the optimal inflation rate in a simple New Keynesian model with occasionally-binding collateral constraints that intermediate-good firms face on hiring labor. For empirically-relevant degrees of price rigidity, the optimal long-run annual inflation rate is around 0.6 percent if the economy is hit by a TFP shock and roughly 0.8 percent if markup risk is the source of uncertainty in the economy. The shadow value on the collateral constraint is akin to an endogenous cost-push shock. Differently from usual cost-push shocks, however, this shock is asymmetric as it takes non-negative values only. Inflation is positive when the collateral constraint is binding and it is zero when it does not. Since the mean of this asymmetric endogenous cost-push shock is positive, inflation is also positive on average. In addition, a binding collateral constraint resembles a time-varying tax on labor, which the monetary authority can smooth by setting a positive inflation rate. More generally, the basic result is related to standard Ramsey theory in that optimal policy smooths distortions over time.

Key Words: Optimal long-run inflation rate; Financial frictions; Occasionally-binding collateral constraints; Endogenous asymmetric cost-push shock; Money demand.
JEL Classification: E31, E32, E44, E52, E58.

∗ An earlier version of this paper appeared under the title “The Optimal Long-run Inflation Rate in Frictional Credit Markets,” and was presented at the “2010 Midwest Macroeconomics Meetings” and at the “2010 Computing in Economics and Finance” conference. I benefited from comments and suggestions by participants in those conferences. I also benefited from comments and suggestions by Sanjay Chugh, John Shea, Enrique Mendoza, Charles Carlstrom, the seminar participants at the University of Maryland, The Board of Governors of the Federal Reserve, Ben-Gurion University, University of Haifa, Bar-Ilan University, The Federal Reserve Bank of Kansas City and The Federal Reserve Bank of Cleveland.

† Email address: salemabo@bgu.ac.il.
1 Introduction

Recent economic events have revived interest in the optimal long-run inflation rate. This paper studies the optimal long-run inflation rate in a simple calibrated New Keynesian (NK) framework with occasionally-binding financial constraints. For empirically-plausible sizes of exogenous shocks, optimal monetary policy entails a strictly positive inflation rate in the long-run. In particular, the optimal annual long-run inflation rate in the benchmark calibration of the model is about 0.6 percent when the economy is only subject to TFP shocks and about 0.8 percent when the economy is hit only by markup shocks. The main result of the paper, namely the optimality of a positive inflation rate, is robust to introducing a motive for holding money.

The baseline setup assumes three types of agents in the economy: households, entrepreneurs (or intermediate-good firms), and sticky-price firms that produce final goods. Financial frictions arise because hiring labor services by an entrepreneur is constrained by the level of her net worth. The collateral constraint is motivated by a type of the hold-up problem. Prior to supplying their labor services, households require the entrepreneur to show collateral that can be seized if needed.\(^1\) The accumulation of net worth is via purchases of shares that are claims on the profits of final-good firms. These shares pay out the profits of final-good firms as dividends to shareholders.

There are two main differences between this paper and typical papers that study optimal monetary policy within a New Keynesian framework featuring financial frictions. First, this paper assumes an occasionally-binding collateral constraint rather than always-binding collateral constraints as usually assumed in this literature. Second, this paper focuses on the optimal long-run inflation rate (i.e. the mean of the inflation rate in the “stochastic steady state” of the model), whereas the focus of most existing literature on monetary policy and financial frictions is on mainly on the short-run dynamics of inflation around the deterministic steady state.

The assumption of an occasionally-binding collateral constraint not only renders the environment more realistic, but it generates asymmetry in the behavior of the economy in response to favorable vs. adverse shocks. The computational approach that I use to deal with occasionally-binding constraints is a penalty-function algorithm within a second-order approximation. This approach has been extensively used recently (e.g. Kim, Kollmann and Kim, 2012).

---

\(^1\) This setup is similar to a model in which the entrepreneur borrows at the beginning of each period to pay wages ahead of production, and borrowing is constrained by collateral.
2010; Den Haan and Ocaktan, 2009; De Wind, 2008 and Preston and Roca, 2007). A detailed description of this methodology can be found in Judd (1998).

When the collateral constraint binds, the shadow value of relaxing the collateral constraint is akin to a cost-push shock that generates inflation. The reason for that is straightforward: other things equal, a binding collateral constraint implies increases in the marginal costs of final-good firms which they accommodate by increasing prices. The inflation rate is positive on average due to the nature of this endogenous cost-push shock; it is asymmetric as it takes only non-negative values. In periods with a binding collateral constraint, inflation is positive. In periods with a non-binding collateral constraint, inflation is zero. Hence, inflation is positive on average since the shadow value on the collateral constraint is positive on average.

The results of this paper also highlight the role of inflation in mitigating the impact of adverse shocks on the economy. A binding collateral constraint distorts the choice of labor by entrepreneurs, and thus it magnifies the wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor (which exists due to the monopolistic competition in the final-good sector). This implies a deviation from the first-best level of output. The wedge (to which I refer as the “labor wedge”) resembles a labor-income tax, and it increases with the shadow value of relaxing the collateral constraint. The analyses show that, under optimal policy, the monetary authority counteracts the effects of a binding collateral constraint, and it thus smoothes the “tax rate” on labor. Since the collateral constraint is more likely to bind during downturns, monetary policy makers aim for, at least, avoiding excessive increase in the “tax rate” during such episodes.

The ability of the monetary authority to smooth the “labor-income tax” (and more generally, the “labor wedge”) is limited due to the monopolistic power of final-good firms and the price rigidity. Put differently, the monetary authority does not have enough instruments to completely and simultaneously close the three distortions in the economy- the nominal distortion due to price rigidity, the monopolistic power of final-good firms, and the financial distortion. Policy makers choose to spread the distortions across margins. Spreading distortions across all margins is well-known in the literature (Dupor, 2002).

Recent work has suggested other factors that justify a positive inflation rate. Related to the current study, Antinolfi, Azariadis and Bullard (2010) point to the role of positive inflation in deepening asset markets and loosening debt contracts. Kim and Ruge-Murcia (2009) show, assuming a neoclassical labor environment, that the optimal long-run inflation rate is positive (around 0.4 percent annually) if nominal wages are downwardly rigid. Abo-Zaid (2012) reports a
significantly higher optimal long-run inflation rate (around 2 percent annually) in a labor search and matching framework in the presence of downward nominal wage rigidity. Fagan and Messina (2009) suggest that the optimal inflation rate for the U.S. ranges between 2 percent and 5 percent when nominal wages are downwardly rigid. This paper contributes to the growing literature that study motives for setting positive long-run inflation rates.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy with the collateral constraint and defines the private-sector equilibrium and the optimal monetary policy problem. Section 3 discusses the labor wedge and the role of inflation in smoothing this wedge. Section 4 describes the calibration and the solution methodology of the model. Section 5 presents the optimal long-run inflation rate suggested by this paper. Impulse responses are also presented in this section. Section 6 presents the results of robustness analyses, section 7 shows the behavior of the labor wedge and section 8 concludes.

2 The Model Economy

The model is a variation of the standard New Keynesian model, with the basic structure by which financial frictions are modeled similar to the recent work of Carlstrom, Fuerst and Paustian (2010, CFP hereafter). The economy is populated by households, entrepreneurs that produce intermediate goods (in what follows, I refer to this sector as entrepreneurs and intermediate-good firms interchangeably), and final-good firms. Households consume differentiated final goods and supply labor on spot markets. Entrepreneurs hire labor services to produce homogenous intermediate goods. Entrepreneurs’ labor demand is constrained by the accumulated value of their net worth. This constraint is the source of the financial friction. Final-good firms are monopolistic competitors that purchase intermediate goods from entrepreneurs and costlessly produce final goods. The pricing of a final-good firm is subject to a direct resource cost, which is the source of price rigidity in this model.

2.1 Households

The representative household purchases the differentiated final goods and enjoy utility from a composite consumption index \( c_t \) and supplies labor \( l_t \) in each period \( t \). Households have access to two financial instruments. The first is a standard one-period bond that pays a riskless nominal gross interest rate of \( R_t \). These bonds are in zero net supply, and, as in CFP, they make explicit
pricing the nominal interest rate. In period $t$, households also purchase $s_t$ shares of final-good firms at a nominal per-share price of $Q_t$. Total shares pay nominal dividends of $D_t$, and their market supply is normalized to unity.

Households maximize their expected discounted lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],$$

(1)

where $\beta < 1$ is the standard subjective discount factor, $E_0$ is the expectation operator, $u(c_t)$ is the period utility function from consumption and $v(l_t)$ is the period disutility function from supplying labor. These functions satisfy: $\frac{\partial u(\cdot)}{\partial c} > 0$, $\frac{\partial^2 u(\cdot)}{\partial c^2} < 0$, $\frac{\partial v(\cdot)}{\partial l} > 0$ and $\frac{\partial^2 v(\cdot)}{\partial l^2} > 0$. As standard in NK models, consumption ($c_t$) is a Dixit-Stiglitz aggregator of final goods ($c_{jt}$) produced by monopolistically-competitive firms:

$$c_t = \left( \int_0^{1} \frac{c_{jt}^{\varepsilon_t^{-1}}}{c_t^{\varepsilon_t}} \; dj \right)^{\varepsilon_t^{-1}},$$

(2)

where $\varepsilon_t > 1$ measures the elasticity of substitution between two varieties of final goods. The elasticity of substitution is allowed to be time-varying in order to allow for shocks to the desired markup, or, put differently, cost-push shocks. Other things equal, an increase in $\varepsilon_t$ leads to a fall in the desired markup (the optimal ratio of price to the marginal cost), and hence to less inflationary pressures in equilibrium. I allow for markup shocks both due to their familiarity in New Keynesian models and because they generate a tradeoff for the monetary authority between stabilizing inflation and stabilizing output. In some of the experiments in section 5, I consider constant elasticity of substitution, and the main results are not, qualitatively, sensitive to whether markups are stochastic or not.

Following standard derivations in Dixit-Stiglitz based NK models, the optimal allocation of expenditures on each variety is given by

$$c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_t} c_t,$$

(3)

where $P_t = \left( \int_0^{1} P_{jt}^{1-\varepsilon_t} \; dj \right)^{1/1-\varepsilon_t}$ is the Dixit-Stiglitz price index that results from cost minimization.

Maximization is subject to the sequence of nominal budget constraints of the form:
with $c_t$ denoting consumption of the final good, $P_t$ is the nominal price of the final good, $w_t$ is the real wage, $\tau$ is a labor market subsidy that is introduced to ensure the efficiency of the deterministic steady state (i.e. to achieve the first-best level of output; see Appendix 2-E for details). Finally, $T_t$ are real lump-sum transfers by the government, and $\Pi_t$ are real profits from the ownership of firms.

The households’ budget constraint may be expressed in real terms as follows:

$$c_t + q_t s_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t,$$

where $q_t = \frac{Q_t}{P_t}$ denotes the real price of shares, $b_t = \frac{B_t}{P_t}$ is real bond holdings at the end of period $t$, and $d_t = \frac{D_t}{P_t}$ stands for real dividends.

The optimal choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions (see Appendix 2-A for derivations):

$$\frac{\nu_{t,j}}{u_{c,t}} = (1 + \tau) w_t,$$

$$u_{c,t} = \beta R_t E_t \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right),$$

$$u_{c,t} = \beta E_t \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right],$$

where $u_{c,t}$ is the marginal utility of consumption in period $t$, $u_{t,j}$ is the marginal disutility of supplying labor in period $t$, and $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross price inflation rate. Equation (6) is the standard labor-supply condition, and equation (7) is the standard consumption Euler equation. Equation (8) prices shares of final-good firms; it equates the period-$t$ marginal utility of consumption to the expected utility of expanding future consumption through the gross one-period return on holding shares, given by $\left( \frac{q_{t+1} + d_{t+1}}{q_t} \right)$. 

$$P_t c_t + Q_t s_t + B_t = R_{t-1} B_{t-1} + P_t (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t,$$
2.2 Entrepreneurs/Intermediate-Good Firms

There is a continuum of long-lived entrepreneurs, each of whom produces intermediate goods. An entrepreneur hires labor services on sport markets in order to produce a homogenous good using the linear production function,

\[ x_t = A_t l_t , \]  

where \( A_t \) denotes total factor productivity, which is identical across all entrepreneurs.

Prior to supplying labor to an intermediate-good firm, households require that a part \( \alpha \) of their wages be backed up by collateral. This is the source of the financial friction in the model, about which more is discussed below. Given that a share of wage payments is collateralized, the intermediate-good firm then hires labor and starts production. Realized operating profits (revenues net of wage costs) and the beginning-of-the-period net worth can then be used to buy shares \( (e_t) \) for the next period. Positive operating profits are possible if the collateral constraint binds (see Appendix 2-D for a proof).

The collateral constraint can be motivated by the hold-up problem, as in Kiyotaki and Moore (1997). Prior to supplying their labor services, households require some “guarantee” from the entrepreneur so that she does not force their wages down ex-post. In other words, the entrepreneur is required to back up the promised wage by some collateral that can be seized if needed. Introducing the financial friction follows CFP and allows me to obtain the main results in a simple way.

Formally, hiring labor is constrained by the-beginning-of-period net worth, as follows

\[ \alpha w_t l_t \leq \kappa e_{t-1} (q_t + d_t) = \kappa n_t , \]  

where, \( e_{t-1} \) stands for the share-holdings by the entrepreneur at the beginning of period \( t \), and \( n_t \) is the real value of net worth. The maximum share of net worth that can be used as collateral is \( \kappa \) (which is equivalent to the loan-to-value ratio in models with borrowing constraints). The parameter \( \alpha \) measures the “significance” of the financial friction: the higher this parameter is, the more “significant” (or “severe”) the financial friction. Clearly, if \( \alpha = 0 \) then the model collapses to a standard new Keynesian model with no financial frictions.

As shown in Appendix 2-J, this setup is isomorphic to a model in which part of wages is required to be paid in advance (“working capital”), the entrepreneur obtains intra-period loans to finance this part of wages, and borrowing is constrained by collateral. The parameters \( \alpha \) and \( \kappa \) come from two different constraints: \( \alpha \) comes from the constraint that requires the
collateralized wage payment to be lower than borrowing, and the parameter $\kappa$ comes from the constraint that limits borrowing. Therefore, I use two separate parameters in condition (10) rather than only one parameter that is equal to their ratio.

The most realistic setup, which is the main focus of this paper, is one in which the collateral constraint may only occasionally bind. For example, the constraint may not bind after a long series of positive shocks (Iacoviello, 2005). Assuming this constraint is always binding, as in CFP and other New Keynesian models with financial frictions do, imposes a restriction on the model’s dynamics. Also, even if the constraint always binds at the deterministic steady state and for *small* (positive) shocks, it does not necessarily bind for *large* shocks. Because large shocks are of course sometimes observed in reality, it is important to understand the model’s dynamics when constraints need not always bind.

To my knowledge, allowing the collateral constraint to only occasionally bind is an innovation compared to studies of monetary policy in the presence of financial frictions. Recent studies assume always-binding collateral constraints (e.g. Iacoviello, 2005; Monacelli, 2009 and Carlstrom, Fuerst and Paustian, 2010). Studying optimal monetary policy with occasionally-binding financial constraints can be viewed as another contribution of the paper. The way I computationally handle the occasionally-binding constraint is discussed in section 4.

I assume that any remaining resources (or “profits”) will be remitted to households in a lump-sum fashion, and that in the process of accumulating shares, entrepreneurs are more impatient than households. For this reason, they discount the future using a discount factor of $\beta \Xi_{t,t+1}$, where

$$\Xi_{t,t+1} = \beta \frac{u_{t+1}}{u_{t,t}}$$

and $\delta < 1$. The parameter $\delta$ is needed to ensure that an entrepreneur will not accumulate enough assets so that the collateral constraint never binds. Finally, as will be discussed in subsection 2.6, the assumption that entrepreneurs remit their “profits” to households simplifies the objective function of the monetary policy maker; the goal is only maximizing the lifetime utility of households. An entrepreneur thus chooses labor demand and shares to maximize expected present discounted value of profit payouts to households,

$$E_0 \sum_{t=0}^\infty \delta^t \Xi_0 \left[ p_t A_t l_t - w_t l_t + e_{t-1}(q_t + d_t) - e_t q_t \right],$$  \hspace{1cm} (11)

subject to the sequence of collateral constraints (10). The variable $p_t$ denotes the relative price of the intermediate good in terms of the final good (and, in equilibrium, equals the marginal cost of
final-good firms). The term in the square brackets is what I refer to as “profits,” and it corresponds, in equilibrium, to part of $\Pi_i$ in the budget constraint of households.

Denoting the Lagrange multiplier on (10) by $\mu_j$, the optimal choices of labor and shares by an entrepreneur are characterized by (see Appendix 2-B for details):

$$A_j p_{j} = w_{j} (1 + \alpha \mu_j), \quad \text{(12)}$$
$$1 = \partial E \left[ \sum_{t, t+1} \left( \frac{q_{t+1}}{q_t} \right) (1 + \kappa \mu_{t+1}) \right], \quad \text{(13)}$$

where, as in CFP, the variable $\alpha \mu_j$ can be interpreted as a “real interest rate” on a loan required for paying the wage bill of $l$ in advance. Equation (12) states that, at the optimum, the marginal product of labor is equated to the real wage adjusted by a “financial markup” (i.e. the effective real wage from the viewpoint of the firm in the beginning of the period). Hence, if $\alpha > 0$, then labor demand will be distorted by the existence of the collateral constraint if it binds. Ex ante, the cost of hiring a unit of labor is higher the tighter the collateral constraint. Moreover, if $\alpha = 0$, this condition reads $p_j = \frac{w}{A_j}$, as is standard in NK models. Finally, equation (13) is a typical asset-pricing condition, but expanded to account for the imposition of the collateral constraint.

### 2.3 Final-Good Firms

Firms in this market are monopolistically competitive. A final-good firm $j$ purchases the homogenous intermediate goods from entrepreneurs at a relative price $p_j$ and transforms each unit of the intermediate good into a final good $y_{j,t}$ using a one-to-one technology. Each firm chooses its own price ($P_{j,t}$) to maximize profits subject to the downward-sloping demand for its product.

---

2 Condition (12) makes clear that profits are positive when the collateral constraint binds: under the optimal choice of the firm, the marginal product of labor exceeds the real wage.

3 Condition (13) is consistent with condition (8) because of the variations of the Lagrange multiplier on the collateral constraint and the additional discount factor ($\delta$). To fix ideas, consider the deterministic steady state versions of the two conditions. In this case, from (13) we get $1 = \delta (1 + \kappa \mu)$, which makes the two conditions consistent.

4 I assume two types of firms in the production sector since the “asset” in this model is shares of final-good firms. To avoid adding an asset (e.g. capital), and hence deviate from the linear-in-labor technology that is typically assumed in NK models, I assume two types of firms and introduce each friction in one sector.
(see Appendix 2-G for more details). The pricing of a final-good firm is subject to a quadratic adjustment cost as in Rotemberg (1982), expressed in units of the final good:

\[
\frac{\varphi}{2} \left( \frac{P_{t,t}}{P_{t,t-1}} - 1 \right)^2 y_t,
\]

where \( \varphi \) is a parameter that governs the degree of rigidity. In a symmetric equilibrium, in which all firms set the same price, Rotemberg pricing leads to the following forward-looking Phillips curve:

\[
1 - \varphi (\pi_t - 1) \pi_t + \beta \varphi E_i \left[ \frac{u_{c,t+1}}{u_{c,t}} \right] (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} = \varepsilon_t (1 - mc_t),
\]

where, \( mc_t \) is the marginal cost of the final-good firm, which equals \( p_j \). As usual, because of the assumptions of one-to-one technology and zero fixed costs, the real marginal cost equals the real average cost. In the case of fully flexible prices (\( \varphi = 0 \)) or fully stable prices (\( \pi_t = 1 \) for all \( t \)), equation (15) collapses to the familiar condition, \( mc_t = \frac{\varepsilon_t - 1}{\varepsilon_t} \). Hence, in the absence of price adjustment costs, the real marginal cost equals the inverse of the optimal price markup.

By combining conditions (6) and (12) and using the fact that \( mc_t = p_j \), the Phillips curve can be written as

\[
(\pi_t - 1) \pi_t = \frac{1 - \varepsilon_t}{\varphi} + \beta E_i \left[ \frac{u_{c,t+1}}{u_{c,t}} \right] (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} + \left[ \frac{\varepsilon_t v_{1,t}}{\varphi (1 + \tau) A_t u_{c,t}} \right] \mu_t + \left[ \frac{\alpha \varepsilon_t v_{1,t}}{\varphi (1 + \tau) A_t u_{c,t}} \right] \mu_t + \left[ \frac{\alpha \varepsilon_t v_{1,t}}{\varphi (1 + \tau) A_t u_{c,t}} \right] \mu_t,
\]

which explicitly shows the relationship between inflation and the financial friction (as measured by the multiplier \( \mu_t \)). This is a key equation since it directly links inflation and the (binding) collateral constraint. The left hand side of condition (16) is increasing in \( \mu_t \), which implies that, other things equal, an increase in \( \mu_t \) leads to an increase in inflation. In this regard, the Lagrange multiplier on the collateral constraint acts as an endogenous cost-push shock that generates inflation. Differently from typical cost push shocks, however, the shock in this model is asymmetric as it may not be negative. This fact has implications for the average inflation rate: since the mean of the endogenous asymmetric cost-push shock is positive (positive when the collateral binds and zero when it does not), the average inflation rate is, accordingly, positive.
It is also worth noting that, other things equal, the impact of $\mu_t$ on inflation is decreasing in the degree of price rigidity and increasing in the “degree” of the financial friction. With very high degrees of price rigidity, the channel introduced through the collateral constraint is expected to be dominated by the cost of deviating from zero inflation. Also, the elasticity of labor supply is another factor that determines the impact of the collateral constraint on inflation. In the limiting case when labor is inelastically supplied, the collateral constraint has no effects on inflation. This can be easily seen by setting $\nu_{t,j}=0$ in condition (16). The intuition behind this result is straightforward: when the equilibrium quantity of labor is independent of the financial friction, there is no inefficiency to correct for. Therefore, zero inflation is optimal for each period $t$ under TFP shocks: when the quantity of labor is efficient, output is efficient as well. Therefore, setting a zero inflation rate does not lead to inefficiencies in production and since zero inflation minimizes the resource cost of adjusting prices, it is the optimal policy.

Finally, due to monopolistic competition, firms in this sector earn positive profits in equilibrium. These profits are paid in the form of dividends to shareholders. Real dividends are thus given by:

$$d_t = y_t - mc_t y_t - \frac{\varphi}{2} (\pi_t - 1)^2 y_t.$$  \hspace{1cm} (17)

### 2.4 Market Clearing

In equilibrium, the resource constraint of the economy reads as follows:

$$y_t = c_t + \frac{\varphi}{2} (\pi_t - 1)^2 y_t.$$  \hspace{1cm} (18)

Finally, market clearing for shares implies:

$$e_t + s_t = 1.$$  \hspace{1cm} (19)

### 2.5 The Private Sector Equilibrium

**Definition 1**: Given the exogenous processes $\{R_t, A_t\}$, the private sector equilibrium is a state-contingent sequence of allocations $\{c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t\}$ that satisfy the equilibrium conditions (6)-(8), (12)-(13), (15) and (17)-(18), and the complementary slackness condition

$$\mu_t (\kappa e_{-t} (q_t + d_t) - \alpha t l_t) = 0.$$
2.6 The Optimal Monetary Policy Problem

I use a Ramsey-type approach to study optimal monetary policy. The monetary authority chooses allocations to maximize the lifetime utility of households subject to the resource constraint and the private-sector equilibrium conditions. The monetary authority is also assumed to solve a commitment problem.

**Definition 2:** Given the exogenous process for technology \( A_t \), the monetary authority chooses a sequence of allocations \( \{c_t, l_t, w_t, mc_t, \pi_t, q_t, d_t, e_t, \mu_t\} \) to maximize (1) subject to the conditions (6), (8), (12)-(13), (15) and (17)-(18).

3 Optimal Monetary Policy and the Labor Wedge

This section presents an alternative way, related to basic Ramsey theory, to view the implications of the collateral constraint for optimal monetary policy. In the basic Ramsey theory, the aim of the planner is to smooth distortions (or “wedges”) over time. In this paper, a binding collateral constraint distorts the choice of labor by entrepreneurs and hence leads to suboptimal choice of labor. To see this, consider first the problem of the social planner who maximizes the expected present discounted utility of households subject to the goods-market resource constraint (see Appendix 2-E for details). The condition characterizing the social planner’s problem is given by

\[
\frac{v_{l,c}}{u_{c,l}} = A_t, \tag{20}
\]

which states that the marginal rate of substitution between labor and consumption should be equal to the marginal product of labor. In the decentralized economy, the equivalent condition is given by:

\[
\frac{v_{l,c}}{u_{c,l}} = A_t \left[ \frac{(1 + \tau)mc_t}{1 + \alpha \mu_t} \right]. \tag{21}
\]

The “labor wedge” is given by the term in the brackets (more precisely, the wedge is the difference between 1 and this term). In this paper, the labor wedge is a function of the Lagrange multiplier on the collateral constraint and the monopoly power of monopolistically-competitive firms.

---

5 The fact that “profits” of entrepreneurs are transferred to households simplifies the problem of the monetary policy maker; instead of maximizing some weighted average of the lifetime objective functions of households and entrepreneurs, the objective function of the monetary policy maker is only the lifetime utility of households.
The role of positive inflation in smoothing the wedge can be seen by substituting for \( mc_i \) using the Phillips curve (condition 15) as follows:

\[
\frac{V_{ld,t}}{u_{c,t}} = A_y \left[ (1 + \tau) \left( \frac{\varepsilon_i - 1}{\varepsilon_i} + \frac{\phi}{\varepsilon_i} (\pi_i - 1)\pi_i - \beta \phi E_i \left( \frac{u_{c,t+1}}{u_{c,t}}(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right) \right) \right] \left( 1 + \alpha \mu_t \right)^{-1}.
\]

To fix ideas, assume that the economy is subject only to a TFP shock (i.e. \( \varepsilon_i \) is constant) and let the difference between 1 and the term in brackets be defined as “labor-income tax”. Under zero-inflation policy, the numerator of (22) is constant, but the denominator varies \( \mu_t \). If the monetary authority implements zero-inflation policy, then a negative shock that leads to an increase in \( \mu_t \) will also result in a higher “tax rate”. This increase in the “tax rate” can be alleviated by appropriate setting of the inflation rate. In this case, by setting a positive inflation rate, the monetary authority can decrease the “tax rate” and smooth its variation.

If \( \varepsilon_i \) is allowed to be exogenously time-varying, setting a positive inflation rate has a similar role. Suppose that \( \varepsilon_i \) falls (which implies a decrease in the degree of competitiveness in the final-good sector). Under zero-inflation policy, the numerator \( \left( \frac{\varepsilon_i - 1}{\varepsilon_i} \right) \) decreases, but the denominator increases. Both effects lead to increases in the “labor-income tax” and thus require greater response by the monetary authority.

More generally, the aim of setting a positive inflation rate is to reduce and smooth the labor wedge, and thus to position the economy as close as possible to the efficient state. In this regard,

---

6 In general, we can write \( \frac{V_{ld,t}}{u_{c,t}} = A_y (1 - \gamma_i) \), with \( \gamma_i \) being the labor-income tax rate. In our case, the “tax rate” is defined as

\[
\gamma_i = 1 - \left( (1 + \tau) \left( \frac{\varepsilon_i - 1}{\varepsilon_i} + \frac{\phi}{\varepsilon_i} (\pi_i - 1)\pi_i - \beta \phi E_i \left( \frac{u_{c,t+1}}{u_{c,t}}(\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right) \right) \right) \left( 1 + \alpha \mu_t \right)^{-1}.
\]

If \( \pi_i = 1 \), then any increase in the shadow value of relaxing the collateral constraint will lead to an increase in \( \gamma_i \).
optimal monetary policy in this paper is in line with basic Ramsey policy of smoothing distortions over the business cycle.

4 Computational Strategy and Calibration

The first subsection presents some discussion about the solution methodology applied in this study. Subsection 4.2 then discusses the parameterization of the model.

4.1 Computational Strategy

Ideally, occasionally-binding constraints should be handled using global computational methods, but this comes at the expense of tractability. Hence, I resort to local methods in order to approximate the solution of the model. However, standard perturbation methods, as they stand, cannot deal with occasionally-binding constraints. Therefore, I modify the problem by using the penalty function approach; this approach allows for any value of $\alpha w_i l_t$ to be possible in principle, but it imposes penalty once the collateral constraint is violated. Since the constraint is imposed on the labor choice of an entrepreneur, her objective function is modified so that it explicitly includes the penalty on violating the collateral constraint. Once the objective function of an entrepreneur is enlarged with the penalty function, the collateral constraint is removed. Thus, the computational problem that I solve, in place of the problem described in subsection (2.2), is

$$\max_{E_0} \sum_{t=0}^{\infty} \delta^t \Xi_{t,0} \left\{ p_t A_t l_t - w_t l_t + e_{t+1} (q_t + d_t) - e_t q_t - \frac{1}{\psi^2} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t,1} (q_t + d_t)) \right] \right\}. \tag{23}$$

The parameter $\psi$ governs the curvature of the penalty function and it will be a key parameter in the analyses below. Also, the penalty approaches zero when the collateral constraint is not violated (see Figure 1).²

Computationally, the optimality conditions (12) and (13) are replaced by

$$A_t p_t = w_t (1 + \alpha \Omega_t), \tag{24}$$

$$1 = \partial E_t \left[ \Xi_{t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) (1 + \kappa \Omega_{t+1}) \right], \tag{25}$$

where, $\Omega_t = \frac{1}{\psi^2} \exp \left[ \psi (\alpha w_t l_t - \kappa e_{t,1} (q_t + d_t)) \right]$.

---

² This function is similar to the one used by Den Haan and Ocaktan (2009).

³ The horizontal axis shows $\alpha w_t l_t - \kappa e_{t,1} (q_t + d_t)$.
Comparing (24) and (25) with (12) and (13), it is apparent that the approximation method replaces the economic variable $\mu_t$ by $\Omega_t$. This variable satisfies the requirement of being nonnegative and it approaches zero when the collateral constraint does not bind.

The decision rules that solve this approximation to the equilibrium are obtained through a second-order approximation to the optimality conditions of the monetary authority. Using a second order approximation, rather than linearization, is necessary in order to capture the asymmetry inherent in the occasionally-binding collateral constraint. A second-order approximation also allows for the long-run mean of a variable to be different from its respective deterministic steady state value. The second-order approximation procedure I apply is the one developed by Schmitt-Grohe and Uribe (2004).

4.2 Parameterization
In what follows, I assume a time unit of a quarter. The period utility function for households is given by:

$$u(c_t) - v(l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\ell_t^{1+\theta}}{1+\theta}. \quad (26)$$

Productivity is governed by the following AR(1) process:

$$\log(A_t) = (1 - \rho_A)\log(A) + \rho_A \log(A_{t-1}) + u_t, \quad (27)$$

with the innovation term $u_t$ being normally distributed with zero mean and a standard deviation of $\sigma_u$. The deterministic steady state value of $A_t$ is normalized to 1.

Similarly, the elasticity of substitution $\varepsilon_t$ evolves according to the following process:

$$\log(\varepsilon_t) = (1 - \rho_\varepsilon)\log(\varepsilon) + \rho_\varepsilon \log(\varepsilon_{t-1}) + v_t, \quad (28)$$

where $v_t$ is normally distributed with zero mean and a standard deviation of $\sigma_v$. The deterministic steady state value of $\varepsilon_t$ is set to 6, implying a deterministic steady state markup of 20 percent, in line with the literature.

To estimate the model parameters, I assume that monetary policy is implemented using an interest rate rule. Alternatively, I could assume that policy makers in the U.S. have been always optimizing, but this could be a strong assumption. The approach I follow here is more common in this literature (e.g. Kim and Ruge-Murcia; 2009, 2011).
I assume an interest rate rule of the form:

\[
\log\left(\frac{R_t}{R}\right) = \rho_\log \log\left(\frac{\pi_t}{\pi}\right) + \rho_\pi \log\left(\frac{\pi_t}{\pi}\right) + \rho_\gamma \log\left(\frac{y_t}{y}\right),
\]

where undated variables denote the deterministic steady state values of the corresponding variables, \( \rho_\log \) is the smoothing coefficient of the interest rate, \( \rho_\pi \) is the coefficient of inflation and \( \rho_\gamma \) is the coefficient of output.

The model parameters are partitioned into two groups. The values of the first group of parameters, \( \Omega_1 \), are set based on US data and frequently used values. I set \( \beta = 0.99 \), implying a yearly interest rate of roughly 4 percent. Following CFP, the parameter \( \alpha \) is set to 0.5 in the benchmark calibration of the model. The maximum loan-to-value ratio \( K \) is set 0.89 in the benchmark calibration in line with Iacoviello (2005).

The parameter governing the adjustment cost of prices \( \varphi \) is set to 18.4729 in my benchmark calibration. This value is based on the recent evidence regarding the duration of price contracts: Bils and Klenow (2004) show that the average duration of prices is between 4.5 and 5.5 months; Ravenna and Walsh (2006) suggest price duration of between 2 and 3 quarters, and Christiano, Eichenbaum and Evans (2005) use price duration of 2.5 quarters. I follow Christiano, Eichenbaum and Evans (2005) and set my benchmark price duration to 2.5 quarters. I map the price duration to the adjustment cost parameter \( \varphi \) using the relationship \( \varphi = \frac{\lambda(\lambda-1)(\varepsilon-1)}{\lambda - \beta(\lambda-1)} \), with \( \lambda \) denoting the price duration. This approach follows Faia and Monacelli (2007). In short, the price rigidity parameter \( \varphi \) is pinned down when the slope of the Philips curve in a linearized model with Calvo (1983)’s parameterization is equalized to the slope of the Philips curve in a linearized model with a Rotemberg (1982)’s parameterization. For more details, refer to Appendix 2-H.

The second set of parameters \( \Omega_2 = (\psi, \sigma, \theta, \rho_\lambda, \rho_\epsilon, \sigma_\epsilon, \sigma_u) \) is estimated using the Simulated Methods of Moments (SMM).\(^9\) The idea of this method is to estimate \( \Omega_2 \) to match observed U.S. data moments. I use quarterly US data sample, available at the FRED database of the Federal Reserve Bank of St. Louis, that covers the period 1964:Q2-2010:Q4. I extract data for

---

\(^9\) See Appendix 2-M for more details about the estimation methodology. Also, setting some parameters at widely accepted values sharpens the estimation of other parameters.
Gross Domestic Product, consumption, the index of aggregate weekly hours of total private industries, the Consumer Price Index (CPI) and the average hourly earnings in total private industries divided by the CPI. All data series have been logged and detrended before the estimation. Finally, in estimating the parameters of the model, I match the standard deviations, autocorrelations and the covariances of the data series.

The benchmark value of $\psi$ is determined under the assumption that the collateral constraint holds with equality in the deterministic steady state of the model. Imposing this condition on the deterministic steady state yields $\Omega = \frac{1}{\psi}$. In addition, combining the steady state versions of (8) and (25) gives $\Omega = \frac{1-\delta}{\delta \kappa}$. The combination of those two relationships gives the value of the additional discount factor $\delta = \frac{\psi}{\kappa + \psi}$. This relationship allows for one additional degree of freedom in determining the model parameters, and therefore it is not part of $\Omega_2$. The parameter $\delta$ is less than 1 provided that $\kappa$ is positive and $\psi$ is finite (as is the case in this model), which is important in order for entrepreneurs not to accumulate enough assets so that the collateral constraint never binds.

<table>
<thead>
<tr>
<th>Shock</th>
<th>$\psi$</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>$\rho_\lambda$</th>
<th>$\sigma_u$</th>
<th>$\rho_e$</th>
<th>$\sigma_v$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>170.3471</td>
<td>1.3569</td>
<td>0.2085</td>
<td>0.9652</td>
<td>0.0116</td>
<td></td>
<td></td>
<td>0.9948</td>
</tr>
<tr>
<td></td>
<td>(54.1057)</td>
<td>(0.3155)</td>
<td>(0.0847)</td>
<td>(0.0168)</td>
<td>(0.0023)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markup</td>
<td>152.4427</td>
<td>1.2164</td>
<td>0.3272</td>
<td>0.9261</td>
<td>0.1433</td>
<td></td>
<td></td>
<td>0.9942</td>
</tr>
<tr>
<td></td>
<td>(47.1572)</td>
<td>(0.2750)</td>
<td>(0.1066)</td>
<td>(0.0183)</td>
<td>(0.0251)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both Shocks</td>
<td>166.1357</td>
<td>1.3864</td>
<td>0.2912</td>
<td>0.9473</td>
<td>0.0108</td>
<td>0.9144</td>
<td>0.1276</td>
<td>0.9947</td>
</tr>
<tr>
<td></td>
<td>(64.8013)</td>
<td>(0.3447)</td>
<td>(0.1625)</td>
<td>(0.0213)</td>
<td>(0.0034)</td>
<td>(0.0207)</td>
<td>(0.0251)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Simulated Method of Moments results. Standard errors are in parentheses.

Table 1 presents the estimated values of the parameters for each type of shock and for the case in which both shocks are introduced simultaneously. Under both markup and TFP shocks, most parameters are significant at the 95 percent confidence level. The estimates of the curvature parameter $\sigma$ are between 1 and 2, in line with literature. The estimates of $\theta$ suggest relatively high labor supply elasticities in the three cases considered. This relatively high labor supply

---

10 To economize in presentation, the coefficients of the interest rate rule are not presented as they are not used in the optimal monetary problem, but the results can be provided upon request.
elasticity is needed to capture the volatility of total hours in a model with no extensive margin, as is the case in this paper. The parameter $\chi$ is calibrated so that the steady state value of $l$ is 0.3 in each case. Finally, the implied values of $\delta$ are reported in the last column of Table 1, and they suggest that the entrepreneurs discount factor $\beta \delta$ is around 0.984 in all cases considered.

5 The Optimal Long-Run Inflation Rate

This section presents the main findings regarding the optimal long-run inflation rate in the presence of financial frictions.

5.1 The Optimal Inflation Rate in the Deterministic Steady State

Before turning to present the optimal long-run inflation rate, a note on the deterministic steady state (i.e. the state with constant technology) is in order. Given the parameter $\psi$, the deterministic steady state of the model is invariant to the degree of price stickiness. The main result is that the optimal deterministic steady state of inflation is exactly zero (see Appendix 2-K for a proof). This result is as expected: in the absence of shocks, inflation is not beneficial, and due to the resource cost of deviations from zero inflation, the monetary authority completely stabilizes prices in the deterministic steady state. This is true regardless of the degree of price rigidity assumed (since there is no benefit from non-zero inflation but there is a cost of non-zero inflation for any positive value of price rigidity) and regardless of whether there is a labor market subsidy or not. Also, given that the deterministic steady state value of inflation is zero regardless of the degree of price rigidity, the deterministic steady state values of other variables will not vary with the degree of price rigidity.

5.2 The Optimal Long-Run Inflation Rate

This subsection presents the main results regarding the optimal long-run inflation rate. Before presenting the results allowing for occasionally-binding constraint, I comment on the optimal inflation rate with in the absence of financial frictions. In this case, the optimal long-run inflation rate is zero regardless of the type of the underlining shock. Furthermore, inflation does not respond to TFP shocks in the short run.

The results with an occasionally-binding collateral constraint are considerably different (Table 2). In this case, optimal monetary policy deviates from full price stability in the long run.
When the economy is subject only to a markup shock, the optimal annual long-run inflation rate is around 0.8 percent. When the economy is subject only to a TFP shock, the optimal annual long-run inflation rate is around 0.6 percent. When both shocks hit the economy simultaneously, the optimal annual long-run inflation rate is around 1.4 percent. Those are important results, since in the real world the economy is subject to ongoing TFP and markup shocks, among others. Finally, the model well accounts for the volatility of output in all cases.

<table>
<thead>
<tr>
<th></th>
<th>TFP Shocks</th>
<th>Markup Shocks</th>
<th>Both Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.5632</td>
<td>0.7814</td>
<td>1.4267</td>
</tr>
<tr>
<td></td>
<td>(0.5211, 0.6272)</td>
<td>(0.7240, 0.8549)</td>
<td>(1.3372, 1.5318)</td>
</tr>
<tr>
<td>Std. Dev. of $y$ (in percents)</td>
<td>1.5364</td>
<td>1.7132</td>
<td>1.8716</td>
</tr>
</tbody>
</table>

Table 2: The optimal long-run inflation rate in annualized percentages terms. The 95% confidence intervals in parentheses. The quarterly standard deviation of U.S. GDP is 1.5570.

5.3 Discussion

When the collateral constraint binds, the Lagrange multiplier is akin to a cost-push shock that leads to positive inflation. When the collateral constraint does not bind, the optimal inflation rate is zero. Since this cost-push shock is asymmetric, as it may not be negative, it has a positive mean. The positive mean of this cost-push shock leads to a positive mean of inflation. This case differs from the standard cost-push shock; the latter has a zero mean and introduced in a long-linearized model, which is symmetric by construction. The endogenous cost-push shock in this paper has implications not only for the short-run dynamics of inflation but also for its long-run mean.

This paper basically suggests a new mechanism that justifies the setting of positive inflation rate over time; if firms face tighter credit markets and their borrowing costs are higher, they will tend to inflate so that households share the burden of the tighter credit markets. In the real world, however, households also face credit constraints that limit their abilities to smooth consumption. This effect in isolation suggests a fall in total demand and, as result, a fall in inflation. The relationship between inflation and credit frictions thus depends on the strength of each channel—the “supply channel” through which inflation increases in tighter credit markets, and the “demand
channel” through which the opposite occurs. The aim of this paper is to study only the implications of credit frictions for optimal inflation policy through the “supply channel”. A unified framework in which both households and firms face credit constraints is left for a future work.

5.4 Impulse Responses

It is useful to observe the behavior of some key variables following TFP and markup shocks in order to gain some insight about the mechanics of the model. Figure 3 displays the responses of some key economic aggregates under the optimal policy following negative and positive markup shocks of the same magnitude. Figure 4 shows the behavior of these variables following TFP shocks. The figures plot the percentage deviation of each variable from its deterministic steady state value. The main observation is the asymmetry in the response of these variables to negative and positive shocks of either type. The asymmetry is more apparent for the case of markup shocks, which, as we have seen above, generate a higher inflation rate.

Following a negative one standard deviation markup shock, the fall in nominal share prices and the increase in good prices lead to a drop in the real price of shares \((q)\) below its steady state value. The asymmetry in the response of net worth is mainly driven by the asymmetry in the real price of shares (notice the similarity of their movements) and, to a lesser extent, the asymmetry in the behavior of dividends. Shares \((e)\) display little asymmetry (and their overall response is relatively small). Output, consumption, labor and the financial friction variable \((\Omega)\) all display clear asymmetry under both types of shocks.

Inflation behaves as expected; a negative TFP shocks leads to an increase in the marginal cost and consequently to an increase in inflation. This is apparent from examining condition (16). In this paper, the existence of the collateral constraint is the reason for inflation to respond to TFP shocks. A negative markup shock (which is modeled here as a fall in the elasticity of substitution between different types of final goods) is akin to a cost-push shock that generates inflation. Clearly, the response of inflation to markup shocks is considerably larger than the response of inflation to TFP shocks.

\footnote{Conceptually, this is similar to the effects of the nominal interest rate on inflation in a NK model with the “cost channel”. On one hand, a higher interest rate reduces demand and thus inflation. On the other hand, a higher interest rate increases the marginal costs of firms and thus inflation. The overall effect of the rise in the interest rate on inflation depends on the strength of each channel.}
6 Robustness Analyses

6.1 Introducing Money Demand

Friedman (1969) suggested that a negative inflation rate is optimal in order to eliminate monetary distortions. In this subsection, I consider the implications of adding a money demand motive for the optimal inflation rate. I assume that households derive utility from holding money (i.e. “Money in the Utility”). Households’ optimization, which is presented in Appendix 2-L, gives the following money demand condition:

\[ m_t = \left[ \phi \frac{R_t}{(R_t - 1)} c_t^\sigma \right]^{1/\zeta} \]  

(30)

Real money holdings is positively related to consumption and negatively related to the nominal interest rate. As the nominal interest rate approaches 1, real money holding approaches infinity (i.e. the economy is satiated with money balances). In addition, the motive to holding money is affected by the parameter \( \phi \); when this parameter is set to zero, the model collapses to the standard cashless New-Keynesian model. The parameter \( \zeta \) measures the interest-elasticity of money demand and it is set here at 13.3333, implying an interest-elasticity of 0.075, in the middle range of previous estimates. This value also implies an interest rate semi-elasticity of money demand of about 7.5, which is close to the estimate of 7 suggested by Lucas (2000).\(^{12}\) Other parameters, including \( \phi \), are estimated using SMM. The results of the SMM estimation are reported in Table 3.

The main result of the paper, that the optimal inflation rate is positive, is robust to the introduction of money demand. Under TFP shocks, the optimal annual long-run inflation rate is 0.4618 percent, with a 95 percent confidence interval of (0.4251, 0.5125). Under markup shocks, the optimal annual long-run inflation rate is 0.6087 percent, with a 95 percent confidence interval of (0.5470, 0.6741). When the economy is subject to both shocks simultaneously, the optimal

\(^{12}\) Taking natural logarithms on both sides of (30) and defining \( i_t = R_t - 1 \) as the net nominal interest rate give \( \log(m_t) = \frac{\log(\phi)}{\zeta} + \frac{\sigma}{\zeta} \log(c_t) - \frac{1}{\zeta} \log\left( \frac{i_t}{1 + i_t} \right) \), which is the familiar condition of the money demand function. The interest-elasticity of money demand is thus given by \( -\frac{1}{\zeta(1 + i_t)} \), and the corresponding semi interest-elasticity is \( -\frac{1}{\zeta(1 + i_t) i_t} \).
The long-run inflation rate is 1.1472, with a 95 percent confidence interval of (1.0964, 1.2182). Interestingly, adding money demand only moderately affects the optimal long-run inflation rate. Therefore, the results of this subsection suggest that the motive for a positive inflation rate introduced in this paper outweighs the motive for a negative inflation rate that arises due to money demand. Finally, the standard deviations of output under TFP shocks, markup shocks and simultaneous shocks are 1.6247 percent, 1.6905 percent and 1.8341 percent, respectively.

<table>
<thead>
<tr>
<th>Shock</th>
<th>$\psi$</th>
<th>$\sigma$</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$\rho_A$</th>
<th>$\sigma_u$</th>
<th>$\rho_c$</th>
<th>$\sigma_v$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>138.3106</td>
<td>1.1745</td>
<td>0.3021</td>
<td>0.0224</td>
<td>0.9520</td>
<td>0.0107</td>
<td></td>
<td></td>
<td>0.9936</td>
</tr>
<tr>
<td></td>
<td>(55.9033)</td>
<td>(0.2431)</td>
<td>(0.1752)</td>
<td>(0.0037)</td>
<td>(0.0136)</td>
<td>(0.0034)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Markup</td>
<td>146.5772</td>
<td>1.2913</td>
<td>0.2276</td>
<td>0.0218</td>
<td></td>
<td></td>
<td>0.9280</td>
<td>0.1274</td>
<td>0.9940</td>
</tr>
<tr>
<td></td>
<td>(47.1456)</td>
<td>(0.2245)</td>
<td>(0.0947)</td>
<td>(0.0026)</td>
<td></td>
<td></td>
<td>(0.0134)</td>
<td>(0.0138)</td>
<td></td>
</tr>
<tr>
<td>Both</td>
<td>153.5406</td>
<td>1.3314</td>
<td>0.2493</td>
<td>0.0204</td>
<td>0.9510</td>
<td>0.0088</td>
<td>0.9067</td>
<td>0.1042</td>
<td>0.9942</td>
</tr>
<tr>
<td>Shocks</td>
<td>(70.6783)</td>
<td>(0.3625)</td>
<td>(0.1154)</td>
<td>(0.0031)</td>
<td>(0.0188)</td>
<td>(0.0042)</td>
<td>(0.0241)</td>
<td>(0.0181)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Simulated Method of Moments results with money demand. Standard errors are in parentheses.

6.2 Changing the Degree of Price Rigidity

Table 4 presents the results for other price durations between 2 and 4 quarters, which are the most relevant durations of price contracts. The optimal inflation rate is positive for those empirically-relevant price durations. Also, for plausible price durations, the optimal inflation target is falling in the degree of price rigidity. This result is due to the higher resource cost associated with a higher inflation rate. The nominal distortion seems to be less dominant for relatively low degrees of price stickiness, but becomes more dominant as the degree of price rigidity increases. The optimal inflation rate reaches zero for very high degrees of price rigidity, but this happens outside the empirically plausible range of $\varphi$ considered in this paper. Therefore, those analyses suggest that, regardless of the exact actual price duration in the U.S., financial frictions, generally speaking, imply, for each type of shock separately, an optimal inflation rate of below 1 percent annually. When the economy is subject to simultaneous shocks, the optimal annual long-run inflation rate is significantly above 1 percent for a wide range of price durations.

13 The results here are, conceptually, in line with the findings of Schmitt-Grohe and Uribe (2007); they show that the optimal inflation rate is highly sensitive to the degree of price rigidity. In their study, there is a tension between the monetary distortion, which calls for a negative inflation rate, and the nominal distortion, which calls for full price stability. In the current study, the tension is between the financial friction, which calls for a positive inflation rate, and the nominal distortion.
7 The Labor Wedge

As discussed in section 3, a binding collateral constraint generates a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor, thus leading to a rise in the “labor-income tax”, as defined above. These effects can be counteracted by setting positive inflation.

Figure 2 shows the standard deviation of the labor wedge for the various optimal inflation rates presented in Table 4. Clearly, the volatility of the wedge at the optimum is decreasing in the optimal inflation rate (put differently, a lower degree of price rigidity is associated with higher optimal inflation and lower volatility of the labor wedge). The Ramsey planner cannot completely close and/or smooth the wedge due to the lack of a sufficient set of policy instruments to completely and simultaneously offset all distortions along the business cycle. Since prices are not fully flexible, the Ramsey planner must tradeoff between stabilizing the wedge and stabilizing inflation.

More generally, the main results of the paper can be related to the basic Ramsey theory of smoothing distortions over time. In this case, smoothing the labor wedge requires smoothing of the “labor-income tax”, which by itself can be achieved through appropriate setting of inflation.

8 Conclusions

The main purpose of this paper is to study the optimal long-run inflation rate in the presence of financial frictions. The model is a variation of the standard New Keynesian framework in which the hiring of labor by entrepreneurs is constrained by their collateral. This study modifies the assumption of always-binding collateral constraints by assuming that the constraint may only occasionally bind. The main result is that optimal monetary policy sets a strictly positive inflation rate in the long-run (i.e. in the stochastic steady state of the model). The optimal annual long-run inflation rate is roughly 0.6 percent when the economy faces a TFP shock and about 0.8 percent when the economy is hit by markup shocks of empirically-relevant magnitudes.
When the collateral constraint binds, the shadow value of relaxing the constraint is equivalent to an endogenous asymmetric cost-push shock that generates inflation. Final-good firms set higher prices when they observe increases in their marginal costs as a result from a binding collateral constraint. Since the constraint binds on average and the shadow value takes non-negative values only, the effects of positive Lagrange multipliers on inflation are not offset in periods of non-binding collateral constraints. The positive average of the endogenous cost-push shock leads to a positive inflation rate on average.

Furthermore, a binding collateral constraint distorts labor demand and thus leads to suboptimal level of output. Basically, a binding collateral constraint is akin to a “tax” on labor which can be both reduced and smoothed by setting positive inflation. More generally, a positive inflation rate helps in smoothing the labor wedge that arise due to the existence of the collateral constraint and the monopolistic power of firms.

The current study also contributes to recent literature that attempts to justify the fact that central banks around the world target positive inflation rates. To my knowledge, the current study is the first to motivate a positive long-run inflation rate in an environment featuring occasionally-binding financial constraints. The recent debate about the optimal inflation rate makes this study particularly timely and significant.

References:


Appendix 1: Tables and Graphs

Figure 1: The Penalty Function ($\psi = 170.3471$)

Figure 2: The standard deviation of the labor wedge for various optimal annual inflation rate (in percents). The driving processes: Markup and TFP Shocks.
Figure 3: Response to negative and positive markup shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Figure 4: Response to negative and positive TFP shocks with financial frictions (percentage deviations from SS levels). Inflation is shown in annualized terms.
Appendix 2: Mathematical Derivations

A The Households’ Problem

Households maximize the following objective function

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - v(l_t) \right], \]  

(A1)

subject to the sequence of budget constraints of the form:

\[ c_t + q_t s_t + b_t = \frac{R_{t-1} b_{t-1}}{\pi_t} + (1 + \tau) w_t l_t + s_{t-1} (q_t + d_t) + \Pi_t + T_t \]  

(A2)

Denoting the Lagrange multiplier on the budget constraint by \( \lambda_t \), the first order conditions with respect to \( c_t, b_t, s_t \) and \( l_t \) are, respectively:

\[ \lambda_t = u_{c,t}, \]

(A3)

\[ \lambda_q = \beta R_t E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} \right), \]

(A4)

\[ -q_t \lambda_t + \beta E_t [\lambda_{t+1} (q_{t+1} + d_{t+1})] = 0, \]

(A5)

and,

\[ -v_{l,t} + \lambda_t (1 + \tau) w_t = 0. \]

(A6)

Combining (A3) and (A6) gives equation (6) in the text. Combining (A3) and (A4) gives equation (7), and the combination of equations (A3) and (A5) yields equation (8) in the text.

B The Entrepreneurs’ Problem

An entrepreneur chooses labor and shares to maximize:

\[ E_0 \sum_{t=0}^{\infty} \delta^t \mathbb{E}_t \left[ p_t A_t l_t - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t \right], \]

(B1)

subject to \( \kappa e_{t-1} (q_t + d_t) - \alpha w_t l_t \geq 0. \)

(B2)

Denoting the Lagrange multiplier on constraint (B2) by \( \mu_t \), the choice of labor yields:

\[ A_t p_t - w_t - \alpha w_t \mu_t = 0, \]

(B3)

or, after collecting terms,

\[ A_t p_t = w_t (1 + \alpha \mu_t), \]

(B4)
which is equation (12) in the text.

Similarly, the first-order condition with respect to \( e \), gives:

\[
\delta^i \Xi_{0,t}(-q_t) + \delta^{i+1}E_t\left[\Xi_{0,t+1}(q_{t+1} + d_{t+1})\right] + \delta^{i+1}E_t\left[\Xi_{0,t+1}(-\kappa(q_{t+1} + d_{t+1}))\mu_{t+1}\right] = 0,
\]

(B5)

By collecting terms and rearranging, condition (B5) can be written as:

\[
1 = \delta E_t \left[ \Xi_{t,t+1}\left(\frac{q_{t+1} + d_{t+1}}{q_t}\right)(1 + \kappa \mu_{t+1}) \right].
\]

(B6)

This is condition (13) in the text.

\section{The Approximated Entrepreneurs’ Problem}

The problem of an entrepreneur in this case is to maximize:

\[
E_0 \sum_{t=0}^{\infty} \delta^i \Xi_{0,t} \left\{ p_t A_t l_t - w_t l_t + e_{t-1}(q_t + d_t) - e_t q_t - \frac{1}{\psi^2} \exp\left[\psi(\alpha w_t l_t - \kappa e_{t-1} (q_t + d_t))\right] \right\},
\]

(C1)

The first order condition with respect to \( l_t \) yields:

\[
A_t p_t - w_t - \frac{\psi \alpha w_t}{\psi^2} \exp\left[\psi(\alpha w_t l_t - \kappa e_{t-1} (q_t + d_t))\right] = 0.
\]

(C2)

Letting \( \Omega_t = \frac{1}{\psi} \exp\left[\psi(\alpha w_t l_t - \kappa e_{t-1} (q_t + d_t))\right] \), condition (C2) can now be written as

\[
 p_t A_t - w_t (1 + \alpha \Omega_t) = 0,
\]

which is equation (21) in the text.

Finally, the first order condition with respect to \( e \), yields:

\[
\delta^i \Xi_{0,t}(-q_t) + \delta^{i+1}E_t\left[\Xi_{0,t+1}(q_{t+1} + d_{t+1})\right] +
\delta^{i+1}E_t\left[\Xi_{0,t+1}\left(-\frac{1}{\psi} \exp\left[\psi(\alpha w_t l_t - \kappa e_{t-1} (q_t + d_t))\right]\right)(-\kappa(q_{t+1} + d_{t+1}))\right] = 0
\]

(C4)

or, by using the definition of \( \Omega_t \),

\[
\Xi_{0,t}(-q_t) + \delta E_t\left[\Xi_{0,t+1}(q_{t+1} + d_{t+1})\right] + \delta E_t\left[\Xi_{0,t+1}(-\Omega_t)(-\kappa(q_{t+1} + d_{t+1}))\right] = 0.
\]

(C5)

Rearranging equation (C5) gives equation (22) in the text.
Operating Profits of Entrepreneurs in the Approximated Model

Let us define the difference between revenues and wage costs by operating profits, as follows:

\[ \Pi^o_t = p_t A_t l_t - w_t l_t, \quad (D1) \]

Recall, from equation (B4) above, that \( p_t A_t = w_t (1 + \alpha \mu_t) \). Hence:

\[ p_t A_t l_t = w_t l_t (1 + \alpha \mu_t), \quad (D2) \]

Using the production function of entrepreneurs \( x_t = A_t l_t \), condition (D2) can be written as:

\[ p_t x_t = w_t l_t (1 + \alpha \mu_t), \quad (D3) \]

or,

\[ w_t l_t = \frac{p_t x_t}{1 + \alpha \mu_t}. \quad (D4) \]

Substituting (D4) in (D1) and using the production function give:

\[ \Pi^o_t = p_t x_t - \frac{p_t x_t}{1 + \alpha \mu_t}, \quad (D5) \]

which, after collecting terms, can be written as:

\[ \Pi^o_t = \frac{\alpha \mu_t}{1 + \alpha \mu_t} p_t x_t. \quad (D6) \]

Condition (D6) states that operating profits are positive in an equilibrium with a binding collateral constraint. Clearly, if \( \alpha \) is zero (i.e. no part of wages is secured by net worth), then operating profits are zero in equilibrium (as one would expect in a perfectly competitive sector). Similarly, if the collateral constraint does not bind, then these profits will be zero as well, since in this case the economy is behaving as if there is no collateral constraint to begin with.

Finally, in the approximated model discussed in section 4, the operating profits will be given by

\[ \Pi^o_t = \frac{\alpha \Omega_t}{1 + \alpha \Omega_t} p_t x_t, \quad \text{with } \Omega_t \text{ as defined in the text.} \]

The derivations are similar to the ones just shown, and therefore they are not presented here.
E Efficient Allocations and the Labor Wedge

The social planner chooses consumption and labor to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],$$

subject to the sequence of resource constraints

$$A_t l_t - c_t = 0.$$  \hspace{1cm} (E2)

Let $\eta_t$ be the Lagrange multiplier associated with (E2), then, the first-order conditions with respect to $c_t$ and $l_t$, respectively, read

$$u_{c,t} = \eta_t, \quad \text{(E3)}$$

and

$$v_{l,t} = \eta_t A_t. \quad \text{(E4)}$$

Combining (E3) and (E4) yields

$$\frac{v_{l,t}}{u_{c,t}} = A_t,$$

and hence efficiency requires the marginal rate of substitution (the left hand side of condition E5) to be equal to the marginal product of labor (given by the right-hand side of condition E5).

Given this result, one can derive the expression for the intratemporal (static) wedge. To do so, combine labor supply condition (6) and the labor demand condition (12) to get

$$\frac{v_{l,t}}{u_{c,t}} = A_t \left( \frac{(1 + \tau) p_t}{1 + \alpha \mu_t} \right).$$

(E6)

Comparison of (E5) and (E6) reveals that the wedge is defined by the term in the parentheses. Clearly, this wedge is directly affected by the existence of the collateral constraint.

F The Labor Market Subsidy

The labor market subsidy $\tau$ is introduced to render the deterministic steady state of the model efficient. In particular, this subsidy is chosen so that, in the deterministic steady state, the marginal rate of substitution ($MRS$) between consumption and labor is equal to marginal product of labor ($MPL$). The derivations for labor market subsidy in the approximated model are similar to what follows, but with $\Omega$ replacing $\mu$ wherever it appears.
In what follows, undated variables denoted the deterministic steady state level of the corresponding variables. From equation (6), we have:

\[
\frac{v_i}{u_c} = (1 + \tau)w, \tag{F1}
\]

The left-hand side of (F1) is the MRS between consumption and labor, hence:

\[
MRS = (1 + \tau)w \tag{F2}
\]

Recalling that (\(MPL=A\)), equation (12) in the text implies:

\[
MPL = \frac{w(1 + \alpha \mu)}{p}. \tag{F3}
\]

Setting \(MRS=MPL\), and canceling \(w\), yields:

\[
(1 + \tau) = \frac{(1 + \alpha \mu)}{p}, \tag{F4}
\]

which, after rearranging terms, gives:

\[
\tau = \frac{(1 + \alpha \mu) - p}{p}. \tag{F5}
\]

Finally, recall that \(p\) equals the marginal cost of final-good firms (\(mc\)), which, in the deterministic steady state, is given by the inverse of the gross markup (i.e. \(mc = \frac{\varepsilon - 1}{\varepsilon}\)). Substituting this result into equation (F5) yields:

\[
\tau = 1 + \frac{\alpha \mu}{\varepsilon - 1}. \tag{F6}
\]

Therefore, the labor market subsidy depends both on the level of the “real interest rate” and the degree of the monopolistic distortion (represented by \(\varepsilon\)). If no wage is required to be secured (i.e. \(\alpha = 0\)), or if the collateral constraint does not bind (i.e. \(\mu = 0\)), then the labor market subsidy should correct only the monopolistic distortion. On the other hand, when \(\varepsilon\) approaches infinity (which corresponds to perfect competition in the final-good sector), \(\tau\) approaches \(\alpha \mu\). This result is as expected since, with perfect competition, the only inefficiency in the allocation of \(l\) comes from existence of the financial friction. Clearly, if the choice of labor is unconstrained and the final-good sector is perfectly competitive, there is no distortion to correct for, and hence the labor market subsidy is zero.
G Deriving the Philips Curve

The problem of a final-good firm $j$ is to choose its price $(P_j)$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\lambda_j}{\lambda_0} \left( \frac{P_j}{P_t} \right)^{y_j} - mc_{jt} y_j - \frac{\varphi}{2} \left( \frac{P_j}{P_{j+1}} - 1 \right)^2 y_j \right]$$

subject to the demand function for its product

$$y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_j} y_t.$$  \hfill (G2)

Rewrite (G1) as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\lambda_j}{\lambda_0} \left( \frac{P_{jt}}{P_t} \right)^{1-\varepsilon_j} y_t - mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_j} y_t - \frac{\varphi}{2} \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right)^2 y_t \right].$$ \hfill (G3)

The first-order condition with respect to the price $P_{jt}$ reads

$$\beta^t \frac{\lambda_j}{\lambda_0} \left[ \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_j} y_t + \epsilon_j mc_{jt} \left( \frac{P_{jt}}{P_t} \right)^{-\varepsilon_j} y_t - \varphi \left( \frac{P_{jt}}{P_{jt-1}} - 1 \right) y_t \right] +$$

$$\beta^{t+1} E_t \left[ \frac{\lambda_{t+1}}{\lambda_0} \left[ \varphi \left( \frac{P_{jt+1}}{P_{t+1}} - 1 \right) \frac{\lambda_{t+1} y_{t+1}}{P_{t+1}} \right] \right] = 0$$ \hfill (G4)

In equilibrium, all firms set the same price (i.e. $P_j = P_t$ for all $j$). Imposing symmetry on condition (G4) and canceling terms give

$$\lambda_t \left[ (1 - \varepsilon_j) \frac{y_t}{P_t} + \epsilon_j mc_{j} - \varphi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{y_t}{P_{t-1}} \right] + \beta E_t \left[ \lambda_{t+1} \left[ \varphi \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{y_{t+1}}{P_{t+1}} \right] \right] = 0.$$ \hfill (G5)

Multiplying by $\frac{P_t}{y_t}$ yields

$$\lambda_t \left[ (1 - \varepsilon_j) \frac{y_t}{P_t} + \epsilon_j mc_{j} - \varphi \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_{t+1}}{P_{t+1}} \right] + \beta E_t \left[ \lambda_{t+1} \left[ \varphi \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{y_{t+1}}{y_t} \right] \right] = 0.$$ \hfill (G6)

Defining $\pi_t = \frac{P_t}{P_{t-1}}$, we get

$$\lambda_t \left[ (1 - \varepsilon_j) + \epsilon_j mc_{j} - \varphi (\pi_t - 1) \pi_{t+1} \right] + \beta E_t \left[ \lambda_{t+1} \left[ \varphi (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \right] = 0,$$ \hfill (G7)

or, after rearranging and using the fact that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{u_{t+1}}{u_{t}}$, yields

$$1 - \varphi (\pi_t - 1) \pi_t + \beta \varphi E_t \left[ \left( \frac{u_{t+1}}{u_t} \right) (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = \epsilon_t (1 - mc_t),$$ \hfill (G8)

which is equation (15) in the text.
H Mapping Between the Price Duration and the Price Rigidity Parameter

I show here the way to map between the price duration and the price rigidity parameter. To do so, let us define the price duration by \( \lambda \) and probability of not resetting the price during a given period by \( \omega \). Hence,

\[
\lambda = \frac{1}{1 - \omega} .
\]  

(H1)

The slope of the Philips curve under the Rotemberg’s approach for price rigidity is given by

\[
\frac{(\varepsilon - 1)}{\varphi} .
\]  

(H2)

Similarly, the slope of the Philips curve when one follows Calvo’s approach for price rigidity is

\[
\frac{(1 - \omega)(1 - \beta \omega)}{\omega} .
\]

Substituting (H1) shows that the slope with the Calvo’s approach can be rewritten as

\[
\frac{\lambda - \beta (\lambda - 1)}{\lambda (\lambda - 1)} .
\]  

(H3)

Setting equation (H2) equals to equation (H3) and rearranging yields

\[
\varphi = \frac{\lambda (\lambda - 1) (\varepsilon - 1)}{\lambda - \beta (\lambda - 1)} ,
\]

which is the equation reported in the text.

I The link between Inflation and the Lagrange Multiplier on the Collateral constraint

Recall that the labor’s demand function is given by \( A_i p_i = w_i (1 + \alpha \mu_i) \) and the real price of an intermediate-good firm equals the real marginal cost of a final-good firm ( \( p_i = mc_i \)). Hence,

\[
mc_i = \frac{w_i (1 + \alpha \mu_i)}{A_i} .
\]  

(I1)

Also, the labor-supply condition implies \( w_i = \frac{v_{i,t}}{(1 + \tau)u_{e,t}} \). Substituting this result in (I1) gives:

\[
mc_i = \frac{v_{i,t}}{u_{e,t} (1 + \tau) A_i} + \alpha \frac{v_{i,t}}{u_{e,t} (1 + \tau) A_i} \mu_i ,
\]  

(I2)

and so, the real marginal cost is positively related to \( \mu_i \). This condition suggests the collateral
constraint (represented by \( \mu \) in condition I2) affects inflation through the marginal cost. To see this more explicitly, rewrite the Philips Curve (equation 15 in the text) as

\[
(\pi_t - 1)\pi_t = \frac{1 - \varepsilon_t}{\varphi} + \beta E_t \left[ \left( \frac{u_{t+1}}{u_{t}} \right) (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \frac{\varepsilon_t}{\varphi} mc_t,
\]

which shows inflation as an implicit function of the expected future inflation and the current marginal cost. Substituting (I2) in (I3) yields

\[
(\pi_t - 1)\pi_t = \frac{1 - \varepsilon_t}{\varphi} + \beta E_t \left[ \left( \frac{u_{t+1}}{u_{t}} \right) (\pi_{t+1} - 1)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \frac{\varepsilon_t}{\varphi} \left[ \frac{v_{t+1}}{u_{t+1}(1+\tau)A_t} \right] + \left[ \frac{\alpha}{\varphi} \frac{v_{t+1}}{u_{t+1}(1+\tau)A_t} \right] \mu_t. \tag{I4}
\]

Basically, \( \mu \) acts as cost-push shock (even when \( \varepsilon \) is constant), so that a rise in \( \mu \) is associated with an increase in inflation at time \( t \). This is similar to the idea in the log-linearized version of CFP, where \( \mu \) manifests itself as an endogenous mark-up shocks.

In the approximated model \( \Omega \), replaces \( \mu \) wherever it appears.

### J The Equivalence to a Model with Intra-Period Loans

I show here that there is equivalence between the main setup of the paper and a model where part of the wage bill needs to be paid ahead of production (the standard “working capital” requirement), entrepreneurs need to borrow in order pay this part of wages, and the borrowing is constrained by their net worth.

The model is modified in the following way. Households are assumed to lend to entrepreneurs (say through a perfectly-competitive intermediation sector). They deposit \( B^h_t \) in the beginning of period \( t \) and earn an interest rate of \( R^h_t \) in the end of the same period. Their problem will now be

\[
\max_{E_0} \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(l_t)],
\]

subject to the sequence of budget constraints of the form:

\[
P_t c_t + Q_t s_t + B_t + B^h_t = R_{t-1}b_{t-1} + R^h_t b^h_t + P_t (1 + \tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t,
\]

where all variables are as in the main test. The households’ budget constraint in real terms reads:

\[
c_t + q_t s_t + b_t + b^h_t = \frac{R_{t-1}b_{t-1}}{\pi_t} + R^h_t b^h_t + (1 + \tau) w_t l_t + s_{t-1} (q_t + d_t) + \Pi_t + T_t,
\]

35
The choices of consumption, bonds, labor supply and shares of final-good firms yield the following optimization conditions:

\[ \frac{v_{i,t}}{u_{c,t}} = (1 + \tau)w_t, \quad \text{(J4)} \]

\[ u_{c,t} = \beta R_e E_t \left( \frac{u_{c,t+1}}{\pi_{t+1}} \right), \quad \text{(J5)} \]

\[ u_{c,t} = \beta E_i \left[ u_{c,t+1} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \quad \text{(J6)} \]

and,

\[ R^b_t = 1. \quad \text{(J7)} \]

As for entrepreneurs, at the beginning of the period each entrepreneur obtains a loan \( B^e_t \) from households, which is to be paid in the end of the period at a nominal gross interest rate of \( R^e_t \). His borrowing, however, is constrained by the beginning-of-the-period net worth. Formally, an entrepreneur chooses labor, loans and shares to maximize:

\[ E_0 \sum_{t=0}^{\infty} \delta^t \Xi_{0,t} \left[ p_t A_t l_t + b^e_t - R^e_t b^e_t - w_t l_t + e_{t-1} (q_t + d_t) - e_t q_t \right], \quad \text{(J8)} \]

subject to

\[ \kappa e_{t-1} (q_t + d_t) - b^e_t \geq 0, \quad \text{(J9)} \]

and

\[ b^e_t - \alpha w_t l_t \geq 0. \quad \text{(J10)} \]

Letting \( \mu_t \) and \( \zeta_t \), denote the Lagrange multiplier on the constraints (J9) and (J10), respectively, the optimality condition with respect to \( b^e_t \) reads:

\[ \zeta_t = R^e_t + \mu_t - 1. \quad \text{(J11)} \]

Similarly, the first order condition with respect to \( l_t \) yields:

\[ A_t p_t - w_t (1 + \alpha \zeta_t) = 0. \quad \text{(J12)} \]

Finally, the first order condition with respect to \( e_t \) yields

\[ \Xi_{0,t} (-q_t) + \delta E_t \left[ \Xi_{0,t+1} (q_{t+1} + d_{t+1}) \right] + \delta E_t \left[ \Xi_{0,t-1} (-\mu_{t-1}) (-\kappa (q_{t+1} + d_{t+1})) \right] = 0. \quad \text{(J13)} \]
In equilibrium, the interest rate that households earn on their deposits will be equal to the interest rate that entrepreneurs pay, and hence $R^e_1 = 1$. Using this fact, equation (J11) becomes:

$$
\zeta_t = \mu_t,
$$

which, by substituting in (J12) gives

$$
A_t p_t - w_t (1 + \alpha \mu_t) = 0.
$$

which is exactly as equation (12) in the text. Rearranging condition (J13) gives condition (13) in the text.

K The Deterministic Steady State

In this appendix, I present some analytical solutions for the deterministic steady state. The starting point is the assumption that households devote 30 percent of their time for work, and hence $l$ is set to 0.3 in the SS. In addition, in the absence of shocks, the optimal inflation rate is zero, and hence $\pi = 1$. This result can be shown by considering the first-order condition of the optimal Ramsey planner with respect to inflation ($\pi_t$) in the deterministic steady state. In this case, this condition reads

$$
\varphi(\lambda_7 + \lambda_8)(\pi - 1) y = 0.
$$

(K1)

$\lambda_7$ and $\lambda_8$ are the Lagrange multipliers on the resource constraint (condition 18) and dividends (equation 19), respectively. Both of these condition holds with equality in the deterministic steady state and hence $\lambda_7$ and $\lambda_8$ are both positive. Hence, the solution is $\pi = 1$.

Imposing deterministic steady state on equation (15) in the text, the deterministic steady state value of $mc$ equals the inverse of the gross markup (i.e. $mc = \frac{\epsilon - 1}{\epsilon}$). The deterministic steady state value of technology ($A$) is set to 1.

Under the assumption that the collateral constraint holds with equality in the deterministic steady state, we have $\Omega = \frac{1}{\psi}$. By setting $mc = p$, equation (8) in the text yields $w = \frac{mc}{1 + \alpha \Omega}$. Substituting for $mc$ and $\Omega$ gives

$$
w = \frac{(\epsilon - 1)\psi}{\epsilon(\alpha + \psi)}.
$$

(K2)
Imposing SS on equation (17) gives the SS value of dividends $d = AI(1 - mc)$, which, after substituting for $A$ and $mc$, can be written as $d = \frac{l}{\varepsilon}$.

Equation (8) in the text yields $q = \frac{\beta}{1 - \beta} d$, and hence

$$q = \frac{\beta}{1 - \beta} \frac{l}{\varepsilon}.$$  \hspace{1cm} (K4)

Since the collateral constraint holds with equality in the SS, shares of entrepreneurs can be written as $e = \frac{\alpha \psi l}{\kappa(q + d)}$.

After substituting for $q, d$ and $w$, we get,

$$e = \frac{\alpha \psi (1 - \beta)(\varepsilon - 1)}{\kappa(\alpha + \psi)},$$  \hspace{1cm} (K5)

which is zero when $\alpha = 0$. Intuitively, if no wage is required to be backed by collateral, then the entrepreneur has no reason to accumulate assets. Also, the SS value of $e$ is increasing in $\psi$, as expected. The higher the curvature parameter in the penalty function the higher the penalty for any violation of the collateral constraint. Hence, in order to avoid occasions where the constraint is violated, the entrepreneur tends to acquire more assets.

Recalling that $\Omega = \frac{1}{\psi}$, equation (K5) can also be written as

$$e = \frac{\alpha(1 - \beta)(\varepsilon - 1)}{\kappa(1 + \alpha \Omega)},$$  \hspace{1cm} (K6)

which implies a negative relationship between $e$ and $\Omega$. Intuitively, the more shares entrepreneurs have the more collateral they will have which reduces the value of the $\Omega$, the equivalent of the Lagrange multiplier.
\textbf{L \ The Households' Problem with Money in the Utility}

Households maximize the following objective function

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ c_t^{1-\sigma} - \frac{1}{1-\sigma} l_t^{1+\theta} - \frac{1}{1+\theta} \left( \frac{M_t}{P_t} \right)^{1-\zeta} \right], \]  

subject to the sequence of budget constraints of the form:

\[ P_t c_t + Q_t s_t + B_t + M_t = R_{t-1} B_{t-1} + M_{t-1} + P_t (1+\tau) w_t l_t + s_{t-1} (Q_t + D_t) + P_t \Pi_t + P_t T_t, \]  

with \( M_{t-1} \) denoting nominal money holdings at the beginning of period \( t \). The optimality conditions read:

\[ \frac{\lambda_t^\theta}{c_t^{-\sigma}} = (1+\tau) w_t, \]  

\[ c_t^{-\sigma} = \beta R_t E_t \left( \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right), \]  

\[ c_t^{-\sigma} = \beta E_t \left[ c_{t+1}^{-\sigma} \left( \frac{q_{t+1} + d_{t+1}}{q_t} \right) \right], \]  

and

\[ m_t = \left[ \phi \frac{R_t}{(R_t-1)} c_t^{-\sigma} \right]^{1-\xi}. \]  

The households’ equilibrium conditions include now the money demand function (condition L6). This condition is obtained by combing the first order condition with respect to \( M_t \) with condition (L4). Clearly, other conditions are not affected by introducing the money demand motive.

\textbf{M \ Simulated Method of Moments}

Let \( \Omega \) be a \( s \times 1 \) vector of parameters of interest, whose values are unknown and \( x_t \) be a \( r \times 1 \) vector of empirical observations obtained from the data. Let also \( x_s(\Omega) \) be the synthetic counterpart of \( x_t \), and it is computed from simulating the model using the parameters \( \Omega \). The length of the data series is assumed to be \( T \), and the length of simulated series is \( N \), with \( N \geq T \). Then, the SMM estimator \( \Omega_{SMM} \) satisfies:
\( \Omega_{SMM} = \arg \min \ D(\Omega)'WD(\Omega), \) \hspace{1cm} (M1)

with \( D(\Omega) \) being the distances between the empirical values of the moments and their estimated values in the simulations. Formally:

\[
D(\Omega) = \frac{1}{T} \sum_{t=1}^{T} x_t - \frac{1}{N} \sum_{n=1}^{N} x_n(\Omega),
\]

\( W \) is a positive semi definite weighting matrix given by the inverse of

\[
V = \lim_{T \to \infty} \text{Var} \left( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} x_t \right).
\]

A necessary, but not sufficient, condition for identification is that \( s \geq r \). When the model is over-identified (i.e. \( s \) is strictly greater than \( r \)), an over-identification test is run using Hansen’s (1982) chi-square test:

\[
T \left( 1 + \frac{T}{N} \right) (D(\Omega_{SMM})'WD(\Omega_{SMM})) \to \chi^2_{s-r},
\]

with \( D(\Omega_{SMM})'WD(\Omega_{SMM}) \) being the minimized value of the objective function at the optimum. The results of this test indicate that, at the 95 significance level, the over-identification hypothesis is not rejected in any of the specifications considered in the paper.