APPENDIX

Let the representative individual have preferences characterized by a modified Stone-Geary utility function:

$$u = \left[c(t) - \overline{Z}(t,x)\right]^{\gamma} \tag{1}$$

where c denotes consumption, and $\overline{Z}$ is some capabilities or basic need. Capabilities vary through time and can be affected by some policy variables, represented by x. For simplicity, we assume that Z is proportional to income:

$$\overline{Z} = \beta(x)y(t)\ . \tag{2}$$

The individual is assumed to maximize lifetime (dynastic) utility, i.e.,

$$\begin{aligned} U &= \int_0^{\infty}\left[c(t) - \overline{Z}(t,x)\right]^{\gamma} e^{-\delta t}dt \\ &= \int_0^{\infty}\left[c(t) - \beta(x)y(t)\right]^{\gamma} e^{-\delta t}dt\ . \end{aligned} \tag{3}$$

In order to obtain the steady state capital labor ratio and the canonical equations for the economy's path for consumption and capital accumulation, we maximize the Hamiltonian:

$$\begin{aligned} H &= e^{-\delta t}\left\{(c-\overline{Z})^{\gamma} + \lambda\left[y - nk - c\right]\right\} \\ &= e^{-\delta t}\left\{(c-\beta k^{\alpha})^{\gamma} + \lambda\left[k^{\alpha} - nk - c\right]\right\} \end{aligned} \tag{4}$$

subject to

$$\dot{k} = i - nk\ , \tag{5}$$

$$k(0) = k_0\ , \tag{6}$$

$$y = c + i\ , \tag{7}$$

$$y = k^{\alpha}\ , \tag{8}$$

and the transversality condition $\lim_{t\to\infty} e^{\delta t}\lambda k = 0$.

The first order conditions for the above maximization problem are:

$$\frac{\partial H}{\partial c} = \gamma\left(c - \beta k^{\alpha}\right)^{\gamma-1} - \lambda = 0, \tag{9}$$

$$\frac{d(\lambda e^{-\delta t})}{dt} = -\frac{\partial H}{\partial k}, \text{ or} \tag{10}$$

$$\dot{\lambda} = \delta\lambda - \left\{\gamma\left(c - \beta k^{\alpha}\right)^{\gamma-1}\left(-\beta\alpha k^{\alpha-1}\right) + \lambda\left(\alpha k^{\alpha-1} - n\right)\right\}. \tag{11}$$

Since along the optimal path equation (9) holds, we can differentiate it with respect to time yielding

$$\dot{\lambda} = \gamma(\gamma-1)\left(c - \beta k^{\alpha}\right)^{\gamma-2}\left(\dot{c} - \alpha\beta k^{\alpha-1}\dot{k}\right). \tag{12}$$

Substituting (9) and (12) into equation (11) and rearranging gives that

$$\dot{c} = \alpha\beta k^{\alpha-1}\dot{k} + \left(\frac{c - \beta k^{\alpha}}{\gamma - 1}\right)\left(\delta + \alpha(\beta-1)k^{\alpha-1} + n\right). \tag{13}$$

Substituting the state equation (5) for the capital stock yields the optimal path for c.

$$\dot{c} = \alpha\beta k^{\alpha-1}\left(k^{\alpha} - c - nk\right) + \left(\frac{c - \beta k^{\alpha}}{\gamma - 1}\right)\left(\delta + \alpha(\beta-1)k^{\alpha-1} + n\right). \tag{14}$$

In the stationary state, $\dot{k} = \dot{c} = 0$, which implies that

$$k^{*} = \left[\frac{n+\delta}{\alpha(1-\beta)}\right]^{\frac{1}{\alpha-1}}. \tag{15}$$

Suppose that there is an exogenous path for policy x(t) such that β is changing through time. Then it is clear from both (14) and (15) that k* is at a supremum when β = 0. Therefore, at a given moment in time, if β is declining then k* is increasing and so is income and consumption. On the other hand, a country's growth rate will decline as β increases, e.g., if the citizenry wants a larger portion of output allocated for basic needs. This may explain the decline in the growth rate in Europe and the US over the last couple of decades.