

THE THEORY OF CAPITAL UTILIZATION IN LABOR-MANAGED ENTERPRISES*

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I. INTRODUCTION

In the last few years the theory of the labor-managed enterprise and the theory of the capital-utilization decision of the capitalist enterprise have been worked out in some detail.¹ This paper employs these two theories to develop a theory of the shift-work or capital-utilization decision of the labor-managed firm. It is not obvious from the analysis of the capital-utilization decision of the capitalist firm what the results will be for the labor-managed firm; thus, this topic is of scientific interest per se. In addition, it may also provide insights into the explanation of the remarkable growth performance of the Yugoslav economy. For, according to Vanek, one of the main explanations is the ability of a labor-managed economy to use efficiently the resources it withholds from present consumption.² But part of this "efficiency" may simply be a much higher level of shift-work;³ and indeed there is some empirical evidence that Yugoslav firms tend to work more shifts than enterprises in other countries.⁴ Therefore, the

* We would like to thank Robert Dorfman for improving our argument in several places.

1. For examples of the former, see J. Vanek, *The General Theory of Labor-Managed Market Economics* (Ithaca: Cornell University Press, 1970); or B. Ward, "The Firm in Illyria: Market Syndicalism," *American Economic Review*, XLVIII (Sept. 1958), 566-89; for examples of the latter, see R. Betancourt and C. Clague, "An Economic Analysis of Capital Utilization," *Southern Economic Journal*, XLI (July 1975); G. Winston, "Capital Utilization and Optimal Shift-Work," *Bangladesh Economic Review*, II (April 1974), 515-58; and for further references, G. Winston, "The Theory of Capital Utilization and Idleness," *Journal of Economic Literature*, XII (Dec. 1974), 1301-20.

2. J. Vanek, *The Participatory Economy* (Ithaca: Cornell University Press, 1971), pp. 39-50.

3. It has been shown in the context of a Harrod-Domar model that utilization has the same impact on the growth rate as efficiency or saving; see R. Marris, *The Economics of Capital Utilization* (Cambridge: Cambridge University Press, 1964), Ch. 1.

4. In a survey of industrial plants undertaken by UNIDO in various countries, about 90 percent of Yugoslav plants work two or more shifts, whereas about 65 percent of plants in the nearest mixed economy country work two or more shifts; see UNIDO, *Profiles of Manufacturing Establishments*, Vol. I, II (ID/Ser. E/4 and 5).

theoretical framework to be developed here can also be viewed as providing one of the components needed for an explanation of this growth performance. The actual explanation of this phenomenon, however, lies beyond the scope of this work. Instead, we concentrate on analyzing the determinants of shift-work in the labor-managed firm.

We shall assume that the labor-managed firm makes a simultaneous decision regarding the size of the capital stock and its degree of utilization. Thus, our analysis is long run in nature. Capital is assumed to be substitutable for labor *ex ante*, but once the machinery is installed, no substitution is possible. We shall assume that the firm attempts to maximize income per worker. While this is a poor assumption for analyzing short-run behavior, it may be reasonable (at least as a first approximation) for the long run.⁵ The firm is also assumed to be externally financed at a fixed rate of interest.⁶

In the context of the decision to work shifts, the goal of maximizing income per worker must be interpreted in the light of the workers' preferences for day-time or night-time work. We shall assume that the workers agree to the following procedure: the shift premium for night-time work will be set high enough to induce the required number of night workers to come forth voluntarily. Letting α be the premium for second-shift work, the marginal worker will be indifferent between receiving Y on the first shift and $Y(1 + \alpha)$ on the second. In deciding whether to work shifts, the firm compares the incomes of the *day workers* under the single-shift and double-shift systems.⁷

Following Vanek, we shall distinguish two kinds of technology.

5. This theory has been criticized by A. Atkinson, "Worker Management and the Modern Industrial Enterprise," this *Journal*, LXXXVII (Aug. 1973), 375-92; B. Horvat, "Prilog zasnivanju teorije jugoslovenskog proizuđeca" (A Contribution to the Theory of the Yugoslav Firm), *Ekonomska Analiza*, I-II (1967), 7-28 (as cited by Vanek and Miovic); J. Vanek, *The Economics of Workers' Management* (London: George Allen and Irwin Ltd., 1972); and B. Horvat, "Critical Notes on the Theory of the Labor-Managed Firm and Some Macroeconomic Implications," *Ekonomska Analiza*, VI (1972), 288-93. For a partial defense see Jaroslav Vanek and P. Miovic, "Explorations into the 'Realistic' Behavior of the Yugoslav Firm," mimeo, Cornell University, April 1970. The recent literature has been nicely summarized by Laura D. Tyson, "The Yugoslav Firm: Theory and Reality," mimeo, Princeton University, July 1975.

6. On the theory of financing labor-managed firms and its relation to Yugoslavia, see L. Tyson, "Investment Decision-Making in the Yugoslav Self-Managed Enterprise," August 1975; Jaroslav Vanek, "The Yugoslav Economy Viewed through the Theory of Labor Management," *World Development* (Sept. 1973), 39-56; and "The Basic Theory of Financing Participatory Firms" in J. Vanek, ed., *Self-Management: Economic Liberation of Man* (Baltimore: Penguin, 1975), 445-55; and Eirik Furubotn and Svetozar Pejovich, "Property Rights, Economic Decentralization, and the Evolution of the Yugoslav Firm, 1965-1972," *Journal of Law and Economics* (Oct. 1973), 275-302.

7. This rule tilts the balance slightly against shift-work because it ignores the consumers' surplus of the inframarginal night-time workers.

The technology of the first kind is the one that gives rise to the familiar *U*-shaped long-run average cost curve for the capitalist firm; increasing returns to scale give way to decreasing returns to scale as output expands. The technology of the second kind is that of constant returns to scale. In both cases, the CES production function will be utilized to examine the role of the capital intensity, the night-shift wage premium, and the elasticity of substitution in the decision to work shifts. In addition, the role of economies of scale and of the elasticity of demand will be brought out in our discussion of monopoly.

The analysis of the shift-work decision of the labor-managed firm will begin with the assumption of perfect competition (Section II). Section III deals with monopoly. Comparisons with the capitalist firm are made in Section IV, and some concluding observations are contained in Section V.

II. PERFECT COMPETITION

Income per worker under single-shift (Y^1) and income per worker under double-shift (Y^2) operation is defined, respectively, by the following equations:

$$Y^1 = (PX^1 - rK^1)/L^1$$

$$Y^2 = (PX^2 - rK^2)/L^2,$$

where X is the daily rate of output, P is price of X , L is employment, K is capital stock, and r is the price of owning a unit of capital stock for a day. We use subscripts to indicate the particular shift; for example, L_1^2 refers to the number of day workers under the double-shift system (system 2). Since the ex post elasticity of substitution is zero, we have $L_1^2 = (1/2)L^2$. Letting α be the night-shift premium, we see that

$$Y^2 = (1/2)(Y_1^2 + Y_2^2) = (1/2)[Y_1^2(2 + \alpha)].$$

Let us recall that the choice of system 1 or system 2 depends on the comparison of the income of the *day* workers under the two systems, or Y^1 and Y_1^2 . We have

$$(2.1) \quad Y_1^2/Y^1 = [2/(2 + \alpha)]Y^2/Y^1.$$

The production function can be written as

$$(2.2) \quad X^1 = F(S^1, L^1) \text{ and } X^2 = 2F(S_1^2, L_1^2) = 2X_1^2,$$

where S refers to the capital services utilized from the capital stock.

We assume the absence of wear-and-tear depreciation and the presence of perfect information and foresight. Under these conditions, there will be full-capacity utilization within a shift, i.e., $S^1 = u^*K^1$ and $S_1^2 = u^*K^2$, where u^* is the maximum rate of utilization per shift.

As Vanek shows, the first-order conditions for maximizing Y under single-shift operation are the following: the value of marginal product of capital must equal the price of capital; and income per worker must equal the value of marginal product of labor.⁸ More precisely,

$$(2.3) \quad P(\partial X^1/\partial K^1) = r; \quad P(\partial X^1/\partial L^1) = Y^1.$$

The corresponding conditions for the double-shift system are⁹

$$(2.4) \quad 2P(\partial X_1^2/\partial K^2) = r; \quad P(\partial X_1^2/\partial L_1^2) = Y^2.$$

The second equalities in (2.3) and (2.4) imply that

$$(2.5) \quad \frac{Y^2}{Y^1} = \frac{(\partial X_1^2/\partial L_1^2)}{(\partial X^1/\partial L^1)}.$$

These conditions hold whether the firm's technology is of the first or the second kind. The equality of the ratio of income per worker in the two systems to the ratio of the marginal products (2.5) is the basic equation for the analysis of perfect competition. Immediately below, we develop an expression for the ratio of the marginal products under the technology of the second kind. In subsection B we show that the same expression also holds for the technology of the first kind.

A. Technology of the Second Kind

With the technology of the second kind (constant returns to scale), the scale of the firm's operations is indeterminate, since income per worker plotted against the number of workers is a horizontal line. It is still possible, however, to develop an expression for the ratio of the marginal products in the two systems. For this purpose, we assume that the production function is CES, i.e.,

$$X^1 = [\delta(u^*K^1)^{-\rho} + (1 - \delta)(L^1)^{-\rho}]^{-1/\rho}$$

and

$$X_1^2 = [\delta(u^*K^2)^{-\rho} + (1 - \delta)(L_1^2)^{-\rho}]^{-1/\rho},$$

8. Vanek, *The General Theory*, pp. 28-31.

9. $Y^2 = [P2X_1^2 - rK^2]/2L_1^2$. Partially differentiating this expression with respect to K^2 and L_1^2 and setting the results equal to zero yield (2.4).

where $\rho = (1 - \sigma)/\sigma$, and σ is the elasticity of substitution. The marginal products of labor are

$$f_1 = (1 - \delta)(X^1/L^1)^{1+\rho} \text{ and } f_1^2 = (1 - \delta)(X_1^2/L_1^2)^{1+\rho},$$

where we write f_1 for $\partial F/\partial L^1$ and f_1^2 for $\partial F/\partial L_1^2$. Hence,

$$(2.6) \quad f_1^2/f_1 = (L^1/L_1^2)^{1/\sigma}(X_1^2/X^1)^{1/\sigma}.$$

Use of the marginal-product-of-capital equations in (2.3) and (2.4) with the CES production function leads to

$$(2.7) \quad K^2/K^1 = 2^\sigma(X_1^2/X^1).$$

From the production function¹⁰

$$(2.8) \quad \frac{K^2}{K^1} = 2^\sigma \left(\frac{L_1^2}{L^1} \right) \left(\frac{1 - 2^{\sigma-1}\psi}{1 - \psi} \right)^{\sigma/(1-\sigma)},$$

where $\psi = rK^1/PX^1$ is the share of capital costs in value added under system 1 when the optimal capital-labor ratio is employed. Substituting (2.8) into (2.7) and the result into (2.6) yields

$$(2.9) \quad \frac{f_1^2}{f_1} = \left(\frac{1 - 2^{\sigma-1}\psi}{1 - \psi} \right)^{1/(1-\sigma)}.$$

Hence, using (2.1) and (2.5), we see that

$$(2.10) \quad \frac{Y_1^2}{Y^1} = \frac{2}{2 + \alpha} \left(\frac{Y^2}{Y^1} \right) = \frac{2}{2 + \alpha} \left(\frac{1 - 2^{\sigma-1}\psi}{1 - \psi} \right)^{1/(1-\sigma)}.$$

Equation 2.10 can be analyzed to show the effects on the income ratio (Y_1^2/Y^1) of changes in α , ψ , and σ . The treatment of ψ as an exogenous variable calls for some explanation. ψ can be written as a function of the other exogenous variables as follows:

Since

$$P(\partial X^1/\partial K^1) = r,$$

we have

$$P\delta(X^1/u^*K^1)^{1+\rho} = r$$

$$X^1/K^1 = (r/\delta P)^\sigma u^*$$

$$\psi = rK^1/PX^1 = \delta^\sigma(r/u^*P)^{1-\sigma}.$$

An increase in the capital share ψ in (2.10) is the result of an increase

10. For a derivation see the longer version of this paper, which was presented at the Econometric Society Meetings, Dallas, 1975, and which is available upon request from the authors.

in the distribution parameter δ , assuming the price to be constant. A change in σ in (2.10), while ψ is held constant, requires a compensating change in δ . To change σ in this way amounts to changing the degree of curvature of the isoquant at the initial single-shift equilibrium point, while that equilibrium point remains undisturbed.¹¹ This change in σ is the one evaluated in (2.12) below.

Before differentiating (2.10) with respect to the variables, we should note an anomalous case. As $1 - 2^{\sigma-1}\psi$ approaches zero from above, K^2/K^1 approaches infinity. If $1 - 2^{\sigma-1}\psi$ is less than zero, K^2/K^1 becomes negative. Clearly something in our assumptions has to give way; it seems reasonable to say that the elasticity of substitution would not remain constant as the capital-labor ratio becomes very high. In what follows, we shall simply assume $1 - 2^{\sigma-1}\psi$ to be positive. For instance, if ψ has a value of 0.5, then a σ of less than 2 ensures that the expression will be positive.

It is apparent from (2.10) that shift-work will be more advantageous the weaker the workers' preferences against night-time work (the lower the α). While not so apparent, it can be shown by differentiating the right-hand side of (2.10) that shift-work will be more advantageous the more capital intensive the production process under single-shift operation (the higher the ψ).¹² Formally, the impact of ψ on the income ratio is given by

$$(2.11) \quad \frac{\partial(Y_1^2/Y^1)}{\partial\psi} = \frac{1}{(1-\sigma)(1-\psi)} \frac{1-2^{\sigma-1}}{1-2^{\sigma-1}\psi} \frac{Y_1^2}{Y^1} > 0.$$

Finally, differentiating the log of (2.10) with respect to σ gives

$$(2.12) \quad \frac{d \ln \text{R.H.S.}}{d\sigma} = -\frac{2^{\sigma-1}(\ln 2)\psi}{(1-\sigma)B(1-\psi)} + \frac{\ln B}{(1-\sigma)^2},$$

where $B = (1 - 2^{\sigma-1}\psi)/(1 - \psi)$, and we assume that $B > 0$. The expression in (2.12) is shown to be positive in the appendix.

Why is shift-work more advantageous with a higher σ ? The decision to work shifts lowers the price of capital services. In general, the firm benefits more from any factor price reduction when that factor can be more readily substituted for other factors. Application of this general principle to the factor capital leads to the conclusion

11. This procedure for changing the elasticity of substitution has been used in the analysis of effective protection. See W. Max Corden, "The Substitution Problem in the Theory of Effective Protection," *Journal of International Economics*, I (Feb. 1971), 48.

12. Incidentally, the impact of changes in r or P on the income ratio is easily derived through their effect on ψ . That is, with a CES production function, increases in (r/P) will either increase, not affect, or decrease, ψ , depending on whether $\sigma < 1$, $\sigma = 1$, or $\sigma > 1$.

that shift-work is more advantageous when the elasticity of substitution is higher.

B. Technology of the First Kind

Vanek shows that under perfect competition and with a technology of the first kind the firm will operate at the constant-returns-to-scale point in the production function because only at that point can the first-order conditions (2.3) hold simultaneously.¹³ A similar argument leads to the conclusion for the double-shift system that each shift will be operated at the constant-returns-to-scale point in the production function. Otherwise, the two first-order conditions (2.4) cannot hold simultaneously. To see this, note that by definition $Y^2 = (2PX_1^2 - rK^2)/L^2$. If the firm operates each shift at a point other than the constant-returns-to-scale point, $X_1^2 \geq (\partial X_1^2/\partial K^2)K^2 + (\partial X_1^2/\partial L_1^2)L_1^2$. Inserting this expression into the definition of Y^2 and using the first-order condition for the marginal product of capital from (2.4) leads to

$$Y^2 \geq 2P(\partial X_1^2/\partial L_1^2)(L_1^2/L^2) = P(\partial X_1^2/\partial L_1^2).$$

If we now assume that at the constant-returns-to-scale point the production function is CES, the argument of the previous section leading from equation (2.6) to (2.10) goes through in exactly the same manner with the proviso that the two systems must be compared at the optimal level of output for each system. To conclude, the analysis of equation (2.10) in the previous section is equally applicable to a perfectly competitive labor-managed firm using a technology of the first kind.

III. MONOPOLY

The assumption under perfect competition that the firm would be able to market additional output with no loss in average revenue is frequently unrealistic. A reasonable assumption would be that, in order to sell additional output, the firm must either lower its price or incur higher selling costs per unit of output. Either of these possibilities can be represented by a downward-sloping demand curve.

Under monopoly we shall assume that the technology of the first kind is represented by a homothetic production function. Equation (2.2) will be written as $X = G[F(u^*K, L)]$, where F is a CES function with constant returns to scale. The first-order conditions for system

13. *The General Theory*, p. 30.

1 are:¹⁴ The marginal-revenue product of capital must equal the price of capital, and income per worker must equal the marginal-revenue product of labor, i.e.,

$$(3.1) \quad MR^1(\partial X^1/\partial K^1) = r,$$

and

$$MR^1(\partial X^1/\partial L^1) = Y^1.$$

The corresponding conditions for the double-shift system are

$$(3.2) \quad 2MR^2(\partial X_1^2/\partial K^2) = r,$$

and

$$MR^2(\partial X_1^2/\partial L_1^2) = Y^2.$$

The homotheticity of the production function allows us to demonstrate that the optimal level of output will be chosen independently of the optimal capital-labor ratio in both systems of operation.¹⁵ Consider the single-shift system first. From the definition of Y^1 ,

$$L^1 Y^1 + r K^1 = P^1 X^1$$

$$L^1 \cdot MR^1 G'(F^1)(\partial F/\partial L^1) + K^1 MR^1 G'(F^1)(\partial F/\partial K^1) = P^1 X^1,$$

where we used (3.1). Since F is first-degree homogeneous,

$$(3.3) \quad G'(F^1) \cdot F^1/X^1 = P^1/MR^1.$$

Turning now to the double-shift system, we see that the definition of Y^2 , the first-order conditions (3.2), and the production function lead to

$$L_1^2 \cdot MR^2 \cdot G'(F_1^2)(\partial F/\partial L_1^2) + K^2 \cdot MR^2 \cdot G'(F_1^2) \cdot (\partial F/\partial K_1^2) = P^2 X_1^2,$$

or

$$(3.4) \quad G'(F_1^2) \cdot F_1^2/X_1^2 = P^2/MR^2.$$

In conditions (3.3) and (3.4), the right-hand side is related to the elasticity of demand. Specifically, $P/MR = n/(n-1)$, where n is the absolute value of this elasticity. Condition (3.3) therefore says that the optimal level of output under single-shift operation will be chosen to equate the degree of economies of scale, which is what the left-hand

14. *Ibid.*, p. 101.

15. Note that the converse is not true, i.e., the choice of the optimal capital-labor ratio will depend on the optimal level of output.

side of (3.3) measures, to $n/(n - 1)$ at the level of output X^1 . Similarly, condition (3.4) indicates that the optimal level of output $X^2 (= 2X_1^2)$ for the double-shift system will be chosen by equating the degree of economies of scale *per shift* to $n/(n - 1)$ at the level of output X^2 .

As in the case of perfect competition, we are interested in the behavior of the income ratio (Y_1^2/Y^1) and consequently in Y^2/Y^1 . Using the first-order conditions (3.1) and (3.2) for the average product per worker in each system, and recalling that f^1 is defined as $(\partial F^1/\partial L^1)$, we see that

$$(3.5) \quad \frac{Y^2}{Y^1} = \left[\frac{X_1^2/F_1^2 P^2}{X^1/F^1 P^1} \right] \frac{f_1^2}{f^1} = H \frac{f_1^2}{f^1}.$$

The term in brackets, denoted by H , reflects the influences of economies of scale and of demand conditions in ways that will be indicated below. First, an explicit expression for f_1^2/f^1 must be developed.

The derivation contained in equations (3.6) through (3.10) parallels that of equations (2.6) through (2.10):

$$(3.6) \quad \frac{f_1^2}{f^1} = \left(\frac{L^1}{L_1^2} \right)^{1/\sigma} \left(\frac{F_1^2}{F^1} \right)^{1/\sigma}$$

$$(3.7) \quad \frac{K^2}{K^1} = (2H)^\sigma \cdot \frac{F_1^2}{F^1}$$

$$(3.8) \quad \frac{K^2}{K^1} = (2H)^\sigma \left(\frac{L_1^2}{L^1} \right)^{1/\sigma} \left(\frac{1 - (2H)^{\sigma-1}\psi}{1 - \psi} \right)^{\sigma/(1-\sigma)}$$

$$(3.9) \quad \frac{f_1^2}{f^1} = \left(\frac{1 - (2H)^{\sigma-1}\psi}{1 - \psi} \right)^{1/(1-\sigma)}$$

$$(3.10) \quad \frac{Y_1^2}{Y^1} = \left(\frac{2}{2 + \alpha} \right) H \left(\frac{1 - (2H)^{\sigma-1}\psi}{1 - \psi} \right)^{1/(1-\sigma)}$$

Equation (3.10) can be analyzed to show the effects on the income ratio (Y_1^2/Y^1) of changes in α , ψ , σ , and H . Similarly, just as in the case of perfect competition, the expression $1 - (2H)^{\sigma-1}\psi$ will be assumed to be positive in order to rule out nonsensical outcomes. The results of differentiating (3.10) with respect to α , ψ , and σ are qualitatively the same as under perfect competition, provided that $1/2 < H \leq 1$ (see the longer paper for details). The derivative of (3.10) with respect to H , holding α , σ , and ψ constant, is unambiguously positive:

$$\frac{\partial(Y_1^2/Y^1)}{\partial H} = \frac{B^{1/(1-\sigma)}(1 - \psi)}{B} > 0,$$

where $B = [1 - (2H)^{\sigma-1}\psi]/(1 - \psi) > 0$ by assumption. Finally, as H

approaches 1, monopoly turns into perfect competition; and setting $H = 1$ in (3.10) makes the equation identical with (2.10).

In the longer paper we show, by means of some special cases, why it is reasonable to assume that H lies between $\frac{1}{2}$ and 1. We also argue there that H will be increased by a reduction in the magnitude of economies of scale or by an increase in the elasticity of demand. Since the income ratio varies directly with H , either of these changes will favor shift-work.

IV. A COMPARISON WITH THE CAPITALIST FIRM

Comparing the behavior of the labor-managed firm with its capitalist counterpart yields further insights into the determinants of shift-work in both types of firms. In order to facilitate the exposition, we consider separately the cases of perfect competition and monopoly.

A. Perfect Competition

For the capitalist firm the condition for shift-work to be profitable is that the ratio of costs under system 1 to those under system 2 (the cost ratio CR) exceed unity.¹⁶ This cost ratio can be written as

$$(4.1) \quad CR = [2/(2 + \alpha)][\theta(2 + \alpha)^{\sigma-1} + (1 - \theta)]^{1/(\sigma-1)},$$

where θ is the share of capital costs in combined labor and capital costs under system 1, or $rK^1/(rK^1 + W_1L^1)$.

When we speak of the labor-managed firm and its capitalist counterpart as being twins, we mean that they have the same production function, the same price of capital, the same α , and the same demand conditions. The wage rate for the capitalist firm, W_1 , may or may not be the same as Y^1 . If it is, then the capitalist firm would earn no profits under single-shift operation and θ must equal ψ . If W_1 is less than Y^1 , then normally θ will differ from ψ .

Consider first the similarities between the labor-managed and the capitalist firm. The sign of the response of the pay-off function with respect to changes in the parameters σ , α , ψ (or θ), is the same. That is, increases in σ and ψ (or θ) increase the profitability of shift-work in the capitalist firm and the desirability of shift-work in the labor-managed firm. Similarly, decreases in α increase the profitability or the desirability of shift-work in both firms. Moreover, if θ

16. The shift-work decision of the capitalist firm has been analyzed in Betancourt and Clague, *op. cit.*, and all the comparisons in this section are made with respect to the results established there.

= ψ , the labor-managed firm and its capitalist twin will always reach the same shift-work decision. This proposition can be established by showing that the values of the parameters at which the two twins are indifferent between system 1 and system 2 are the same: that is, if Y_1^2/Y^1 equals one, then CR must also equal one, and vice versa:

$$\begin{aligned} \frac{Y_1^2}{Y^1} &= \left(\frac{2}{2+\alpha} \right) \left(\frac{1-2^{\sigma-1}\psi}{1-\psi} \right)^{1/(1-\sigma)} = 1 \\ 2^{1-\sigma} - \psi &= (1-\psi)(2+\alpha)^{1-\sigma} \\ (2+\alpha)^{\sigma} 2^{1-\sigma} &= \psi(2+\alpha)^{\sigma} + (2+\alpha)(1-\psi) \\ [2/(2+\alpha)]^{1-\sigma} &= \psi(2+\alpha)^{\sigma-1} + (1-\psi) \\ (4.2) \quad 1 &= \frac{2+\alpha}{2} [\psi(2+\alpha)^{\sigma-1} + (1-\psi)]^{1/(1-\sigma)} = \frac{1}{CR}. \end{aligned}$$

Another interesting similarity between the two types of firms regarding their capital-utilization behavior is that the necessary (but not sufficient) condition for shift-work to be desirable and at the same time decrease capital productivity is the same: namely, the elasticity of substitution must be greater than unity. The capital productivity for the two systems in the labor-managed firm is easily derived by manipulating equation (2.7) to yield

$$(4.3) \quad \frac{(X^2/K^2)}{(X^1/K^1)} = 2^{1-\sigma}.$$

From this equation it can be seen that capital productivity will be lower under shift-work if σ is greater than one. This is a somewhat surprising conclusion since shift-work is usually thought of as a means of saving capital. However, the explanation of the paradox, as in the capitalist case, lies in the fact that shift-work involves a reduction in the price of capital services, which leads to the choice of a technique with a higher instantaneous capital-labor ratio.

While the similarities are surprising, it would be even more surprising if there were no differences in the behavior of the two types of firms in their capital utilization decision, but there are, of course, several differences. In the capitalist firm α plays a substantial role in determining the relative amounts of capital under the two systems through its impact on the cost ratio;¹⁷ in the labor-managed firm, on the other hand, the relative amounts of capital that would be used under the two systems are independent of α , as (2.7) demonstrates. This difference in behavior stems from the difference in the objective

17. See Betancourt and Clague, *op. cit.*, equation (7), p. 74.

functions of the two firms. In the capitalist firm an increase in α always reduces the profits of the double-shift system and the entrepreneur responds by adjusting the variables under his control; in a labor-managed firm, on the other hand, an increase in α merely changes the distribution of income between day and night workers and does not provide an incentive for the firm to change its behavior, as long as the change in α is too small to tip the balance in favor of single-shift operation.

Another difference in the role of α is that the percentage change in the income ratio (2.10) of the labor-managed firm due to a change in α depends solely on α ; moreover, percentage changes in the income ratio with respect to ψ and σ are independent of α . Neither of these results holds for the capitalist firm.¹⁸

A final difference between the two types of firms arises when the capitalist twin is making profits under system 1, for then Y^1 will in general exceed W_1 and ψ will normally differ from θ . It can be shown that¹⁹

$$(4.4) \quad \frac{\theta/(1-\theta)}{\psi/(1-\psi)} = \left[\frac{Y^1}{W_1} \right]^{1-\sigma}$$

Equation (4.4) shows that θ will exceed ψ if $Y^1 > W_1$ and $\sigma < 1$; however, if $Y^1 > W_1$ and if $\sigma > 1$, ψ must exceed θ . Since capitalist firms normally earn profits (most firms are inframarginal) and σ is widely believed to be less than one, we may argue that $\theta > \psi$ is the normal case.

18. This can be established directly from (4.1) by differentiation.

19. The first-order conditions imply that the ratio of the marginal products of labor and capital equal (W/r) in the case of the capitalist firm, and (Y/r) in the case of a labor-managed firm. Writing out the marginal products for the CES and manipulating, we obtain

$$\left(\frac{K}{L}\right)_C = \left(\frac{W}{r} \frac{\delta}{1-\delta}\right)^\sigma \text{ and } \left(\frac{K}{L}\right)_{LM} = \left(\frac{Y}{r} \frac{\delta}{1-\delta}\right)^\sigma,$$

where the subscripts C and LM refer to capitalist and labor-managed firms. We have suppressed the superscript referring to system 1. Hence,

$$\frac{(K/L)_{LM}}{(K/L)_C} = \left(\frac{Y}{W}\right)^\sigma.$$

Now

$$\frac{\theta}{1-\theta} = \frac{r}{W} \left(\frac{K}{L}\right)_C \text{ and } \frac{\psi}{1-\psi} = \frac{r}{Y} \left(\frac{K}{L}\right)_{LM}$$

Hence,

$$\frac{\theta/(1-\theta)}{\psi/(1-\psi)} = \left(\frac{Y}{W}\right)^{1-\sigma}.$$

B. Monopoly

A comparison of the two types of firms under monopoly is somewhat more difficult because of the increased complexity of the analysis in both cases. To simplify the analysis and to bring out the difference between the shift-work behavior of the two types of firms, we shall assume that for the capitalist firm the degree of homogeneity of the production function (β) is constant, and the elasticity of demand (n) is also constant. For the labor-managed firm it is not very useful to assume that both β and n are constant because in that case equilibrium is not possible unless $\beta = n/(n - 1)$. (See the longer paper for explanation.) Although the labor-managed and capitalist firms will not face identical environments (and therefore we do not speak of them as twins), some interesting comparisons can be made between the two counterpart firms.

Under these assumptions the cost ratio for the capitalist firm becomes²⁰

$$(4.5) \quad CR = [2^{1/\beta}/(2 + \alpha)][\theta(2 + \alpha)^{\sigma-1} + (1 - \theta)]^{1/(\sigma-1)},$$

and the ratio of profits under system 2 to those under system 1 becomes

$$(4.6) \quad \Pi^2/\Pi^1 = CR^{\beta(n-1)/(n+\beta-\beta n)}.$$

The ratio of the optimal outputs under systems 2 and 1 is given by

$$(4.7) \quad X^2/X^1 = CR^{\beta n/(n+\beta-\beta n)}.$$

The second-order condition is that $\beta < n/(n - 1)$, which ensures that the exponents in both (4.6) and (4.7) are positive.

As under perfect competition the sign of the response of the pay-off function with respect to changes in the parameters α , σ , and ψ (or θ) is the same for the capitalist and the labor-managed firms. But the determination of the relative output levels, X^2/X^1 , is quite different for the two types of firms. Under capitalism, anything that increases the cost ratio (while holding β and n constant) will increase X^2/X^1 , as (4.7) shows. Thus, a decrease in α or in σ , or an increase in θ (as a result of either a technological change or a change in the price of capital) will increase CR and thereby increase X^2/X^1 . Under labor management, on the other hand, neither X^1 nor X^2 will be affected by a change in α , σ , or ψ .

20. The analysis of the capitalist firm under these assumptions is contained in C. Clague, "The Theory of Capital Utilization: Some Extensions," mimeo, April 1975, Section II.

The elasticity of demand also plays a different role in the shift-work decisions of the two types of firms. The capitalist firm can determine the more profitable system of operations without knowing the elasticity of demand; shift-work is profitable if and only if the cost ratio in (4.5) exceeds unity.²¹ But for the labor-managed firm an increase in the elasticity of demand can easily change shift-work from being disadvantageous to advantageous.

V. CONCLUDING REMARKS

In this essay we have endeavored to develop the theory of the long-run capital-utilization decision of the labor-managed firm under both perfect competition and monopoly. In both cases shift-work was shown to be more advantageous the more capital intensive the production process (measured by ψ), the higher the elasticity of substitution (σ), and the lower the shift differential (α). In addition, in the case of monopoly, we found that a high elasticity of demand (n) and a low degree of economies of scale (β) increase the desirability of shift-work.

The profitability of shift-work for the capitalist firm is also increased by a higher capital intensity (measured by θ), a higher σ , and a lower α . Moreover, if $\theta = \psi$ under perfect competition, the labor-managed firm and its capitalist twin always reach the same shift-work decision. But there are a number of striking differences in the shift-work behavior of the two types of firms. An increase in α does not affect either the relative amounts of capital (K^2/K^1) or the relative outputs (X^2/X^1) under the two systems for the labor-managed firm; such an increase would affect both for the capitalist firm. Under monopoly the relative outputs (X^2/X^1) for the labor-managed firm would also be unaffected by a change in σ or a change in the price of capital. For the capitalist firm, on the other hand, such changes would normally affect X^2/X^1 through their effect on the cost ratio.

To conclude, we raise several questions that our results suggest should be analyzed empirically and that may help elucidate the issue discussed in the introduction as to the explanation of the remarkable growth performance of the Yugoslav economy. What explains the high level of shift-work in Yugoslavia? Is this explained by the large size of factories? If so, what role does the elasticity of demand play in explaining these large factories? Is shift-work explained by a lower effective α ? If so, would this be explained by governmental subsidies or because the preferences of managers are not given excessive weight?

21. This statement does not hold if β is not constant.

Is shift-work in Yugoslavia explained by a very large capital share? If so, what accounts for this large capital share? The answers to the above questions will provide the information necessary to answer the question of whether or not Yugoslav growth can be explained by higher levels of incentive for shift-work in a labor-managed system. While the above questions involve a substantial research program, this study may provide a framework in which to embed the answers.

APPENDIX: PROOF THAT EXPRESSION (2.12) IS POSITIVE²²

(2.12) can be written as

$$\frac{2^{\sigma-1} \psi \ln 2^{\sigma-1} + (1 - 2^{\sigma-1} \psi) \ln [(1 - 2^{\sigma-1} \psi)/(1 - \psi)]}{(1 - \sigma)^2 (1 - 2^{\sigma-1} \psi)}$$

The denominator is assumed to be positive, as discussed in the text. Let N denote the numerator, and let $A = 2^{\sigma-1} \psi$. Then

$$\begin{aligned} N &= A \ln (A/\psi) + (1 - A) \ln [(1 - A)/(1 - \psi)] \\ &= \ln (A/\psi)^A [(1 - A)/(1 - \psi)]^{1-A}. \end{aligned}$$

Now $N \geq 0$ is equivalent to

$$(A/\psi)^A [(1 - A)/(1 - \psi)]^{1-A} \geq 1$$

or to

$$(1) \quad \psi^A (1 - \psi)^{1-A} \leq A^A (1 - A)^{1-A}.$$

Let

$$f(\psi) = \psi^A (1 - \psi)^{1-A}.$$

Then

$$f(0) = f(1) = 0,$$

and

$$f(A) = A^A (1 - A)^{1-A}.$$

Now let us hold A constant, and let ψ vary. (This implies that σ varies.)

Then

$$f'(\psi) = \psi^{A-1} (1 - \psi)^{-A} (A - \psi).$$

Thus, the following holds for any value of A (where $0 \leq A \leq 1$):

During $0 \leq \psi \leq A$, $f(\psi)$ climbs from 0 to $f(A)$.

During $A \leq \psi \leq 1$, $f(\psi)$ falls from $f(A)$ to 0.

Hence, $f(\psi)$ never exceeds $A^A (1 - A)^{1-A}$, and condition (1) holds.

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22. We owe the idea of this proof to Robert Dorfman.

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